MAKING THE MOST OF ERRORS IN FIRST-GRADE MATHEMATICS CLASSROOMS

BY

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THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Educational Psychology
in the Graduate College of the
University of Illinois at Urbana-Champaign, 2011

Urbana, Illinois

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Abstract

This research examines teacher-student discourse surrounding errors in 10 first-grade mathematics lessons. The qualitative analysis begins with a teacher's initial response to an error and continues until the error is resolved. The goal of this exploration is to gain a deeper understanding of how teacher and student contributions shape the discourse following an error until its resolution. The study identifies three common teaching strategies that may present challenges for the exploration of errors. If teachers can be shown the value as well as the possible obstacles involved in discourse surrounding errors, they may be encouraged to welcome student errors into mathematical discourse.

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Chapter 1

Introduction

"Mistakes are the portals of discovery." --James Joyce

Errors are opportunities for discovery, transformation, and learning. Intuitively, this view of errors seems correct. Everyone makes mistakes, and seeing one's errors as opportunities as opposed to obstacles is an encouraging view for anyone to adopt. However, this approach to learning does not always carry into classrooms. Sometimes, errors are treated as undesirable behaviors; signals that a student has not adequately grasped material. So, discouraging errors, mitigating them, and searching for correct answers may be favored over an open discourse about their underlying misunderstandings (Santagata, 2004; Schleppenbach, Flevares, Sims, & Perry, 2007). Research suggests that welcoming errors into mathematical discourse may be a valuable approach to errors as opposed to a model that suggests they should be avoided (National Council of Teachers of Mathematics [NCTM], 1991, 2000; Santagata, 2004; Schleppenbach et al., 2007; Siegler, 1996).

Early on in students' mathematics education is an ideal time to explore the treatment of student errors, for both pragmatic and theoretical reasons. Pragmatically, if students' errors can be dispelled early, then students will have a stronger basis for building mathematical understanding (e.g., Ma, 1999). Also, in terms of conducting research, errors tend to be more black and white in mathematics than in other domains. Theoretically, exploring mathematics in elementary education allows for an exploration of how students are taught to deal with errors in the classroom early on in their education.

Theoretical Framework

Piaget's theory of cognitive disequilibrium (e.g., 1960) suggested that the learning process has moments of discomfort when the learner encounters something new that does not fit into his current body of knowledge. Errors in the classroom may signal these moments. When an error is made, the learner has encountered information that does not fit into his current knowledge base. Understanding a student's current knowledge, his gaps in knowledge, and what new information is necessary to close those gaps is critical for the construction of new knowledge (Piaget, 1960; Vygotsky, 1978). Whether a student acquires the knowledge necessary to relieve cognitive discomfort caused my erroneous understandings and to learn depends on what happens after the error is made (Graessar, Lu, Olde, Cooper-Pye, &Whitten, 2005; Schleppenbach et al., 2007).

Hiebert and Grouws (2007) suggested that once a mathematical error is made, grappling with the error is critical. They said that, "some form of struggle is a key ingredient in students' conceptual learning" (p. 388). Welcoming errors into classroom discourse may be one way to encourage this struggle in a controlled and encouraging environment. Allowing students the room to grapple with their errors as opposed to, for example, immediately correcting them, affords students the "cognitive discomfort" that may be critical to students making the desired cognitive leaps (e.g., Hiebert & Grouws, 2007; Piaget, 1960).

The value of unpacking errors is further supported by Siegler's (1996) theory of competition and selection. Siegler (1996) proposed that learners must focus on their errors as well as successes. By doing so, learners will be more capable of correcting their misunderstandings and acquiring new knowledge (Siegler, 1996). Because of the strong theoretical support for unpacking student errors, research has started to take a closer look at how errors are treated in the classroom (Santagata, 2004; Schleppenbach et al., 2007; Siegler, 1996).

Discourse Surrounding Errors

Research has emphasized the importance of creating a discourse in the classroom that unpacks student errors (Santagata, 2004; Schleppenbach et al., 2007; Siegler, 1996). Borasi (1994) proposed that errors be used as "springboards for inquiry," suggesting that errors were more than missteps on the road to learning, but rather chances for students to more deeply question and understand mathematics. It may appear that a focus on the student removes responsibility from teachers, but this is not the case. Borasi (1994) pointed out that "far from diminishing the role of the teacher, an inquiry approach imposes new demands on mathematics teachers as they are expected to plan and orchestrate classroom activities providing stimuli and structure to support the students' own inquiries" (p. 168). In this model, the teacher is not the "keeper of the mathematics," but the stimulus and support for student questioning following an error. This inquiry approach to student errors is the inspiration for the current study.

The student's role in the inquiry approach is not a singular effort. Creating a public discourse surrounding errors provides an opportunity for multiple students to participate in the construction of mathematical knowledge (Cobb & Bauersfeld, 1995; Inagaki, Hatano, & Morita, 1998). Inagaki et al. (1998) emphasized that when students participate in mathematical discussions with their classmates and not just the teacher, they are able to construct knowledge through that discussion. For example, students use social cues, like what student is offering an idea, to inform their judgment of the plausibility of that idea. There are also many ideas and strategies brought into the discussion because of the sheer variability in perspectives and interests represented by the students (Inagaki et al., 1998). When errors are part of the whole-class discussion, multiple students may participate in the discourse, contributing to the class's understanding of errors and their resolutions through multiple perspectives (Ball, 1993). This strategy also allows for questioning from student-to-student, pushing each other to defend their mathematical positions (Ball, 1993; Lampert, 1990).

Perry, McConney, Flevares, Mingle, and Hamm (2010) also discuss the importance of encouraging first-grade students to recognize that they have a role in their own learning and that they are important contributors to the mathematical discourse that occurs in the classroom. The current study focuses on the first few weeks of first-grade for similar reasons. If teachers can recognize errors as an opportunity to establish students' roles in their own mathematical learning early on, the may confidently expose and tackle their errors in future learning experiences.

The inquiry approach to errors, creating an open discourse after errors are made, positions both teachers and students as active participants in the conversation. Both teacher and student have important responsibilities and both determine the direction of the discourse. Sherin (2002) identified the challenge presented by these responsibilities in what she terms "a balancing act." She identified two tensions in creating a discourse community in mathematics. On one hand, the teacher must encourage student contributions and to use those contributions to shape the discourse. On the other hand, teachers have to be sure that the discourse accomplishes something mathematically (Sherin, 2002). Since the teacher has such a critical role in creating and maintaining this balance, it is important to examine what may affect that ability.

Influences on Teachers' Treatment of Errors

Research has begun to examine what may be influencing a teacher's ability or willingness to stimulate mathematical discourse surrounding an error (Santagata, 2004; Schleppenbach et al., 2007; Stipek et al., 2001). It has been proposed that teachers' core mathematical beliefs may be one factor that affects the treatment of errors. Stipek et al. (2001) showed that teachers' beliefs about mathematics contribute to students' comfort in tackling their errors. Teachers who believe the goal of learning mathematics is to arrive at the correct answer through the use of the correct procedures tend also to believe that the teacher should exercise tight control over the mathematical activities (Stipek et al., 2001). These teachers and their sets of beliefs often create "high-risk" learning environments where errors are viewed as bad and, therefore, things to be avoided (Stipek et al., 2001).

At the other end of the spectrum, teachers who subscribe to "inquiry-related" beliefs create an environment where there is more student autonomy and a feeling that errors are a natural part of the learning process (Stipek et al., 2001). As a result, they place great value on allowing students to grapple with errors and to participate in extended mathematical discourse, structured by the teacher (Lampert, 1990). In doing so, teachers with "inquiry-related" beliefs set up a classroom structure for students to openly discuss errors and the underlying mathematical gaps in knowledge (Lampert, 1990; Schleppenbach et al., 2007).

Research has also argued that there are important cultural differences in teachers' beliefs about mathematics and that this may influence how mathematical errors are treated (Santagata, 2004, Schleppenbach et al., 2007). Stigler and Hiebert (1999) noted:

> Cultural activities, such as teaching, are not invented full-blown but rather evolve over long periods of time in ways that are consistent with the stable web of beliefs and assumptions that are part of the culture. The scripts for teaching in each country appear to rest on a relatively small and tacit set of core beliefs about the nature of the subject, about how students learn, and about the role a teacher should play in the classroom. (p. 87)

Because every culture has its own unique teaching scripts, activities in the classroom become reflections of the teacher, students, and the culture itself. Correa et al. (2008) noted that cultural influences on the classroom are often unrecognizable, but critical for researchers to consider. Correa and his colleagues also pointed to culture as a possible obstacle. These beliefs and the behaviors they create are difficult to change, because they have been developed over long periods of time and are extremely complex. It is important to understand the possible cultural influences on teachers' treatment of errors, because it has been suggested that U.S. teachers may be culturally biased toward a mitigation of errors.

In a study of "mistake-handling strategies" in the U.S. and Italy, Santagata (2004) found that 38% of U.S. teachers' responses to student mistakes per lesson were mitigated whereas, in Italy, 6% were mitigated. Santagata (2004) also noted two themes that often came up in teacher focus group interviews. Italian teachers emphasized that students should take ownership of the mathematics and that they were responsible for putting effort in to learn it (Santagata, 2004). In contrast, U.S. teachers emphasized the self-esteem of the student and that students rarely took ownership of their mistakes (Santagata, 2004).

Similar results were found in a comparison of the treatment of errors in Chinese and U.S. classrooms. Schleppenbach et al. (2007) found that U.S. teachers did not place as much emphasis as Chinese teachers on the freedom to make errors in the classroom. Schleppenbach et al. found that U.S. teachers were more likely than Chinese teachers to say things that seemed to mitigate errors, like "you're so smart, you don't even make mistakes." Given Santagata's findings, these types of statements are likely an attempt to protect the self-concept of the child. At the same time, the statements may create a classroom where absence of errors—including avoiding errors altogether—is equated with intelligence, and making an error means you're not smart. In addition to these cultural beliefs, it is critical to explore other factors that may influence how errors are treated throughout the discourse that surrounds them.

The Current Study

A powerful research foundation has been established that supports a deeper look at the value of unpacking errors in teacher-student discourse (Ball, 1993; Lampert, 1990; Santagata, 2004; Schleppenbach et al., 2007; Siegler, 1996). In their study of errors in U.S. and Chinese classrooms, Schleppenbach and her colleagues (2007) also identified multiple types of teacher responses to errors. For example, some teachers followed up student errors with questions, while others corrected a student's error and moved on with the lesson. This research suggested that some teacher responses to errors, like questioning, might allow students more room to tackle their errors than other teacher responses, like immediate correction, that restrict the subsequent discourse. The current study builds on this work.

Although research has identified common teacher responses to errors across cultures and suggested the value of some of these responses (Santagata, 2004; Schleppenbach et al., 2007), what occurs *after* a teacher's *initial* response to an error has not been closely examined.

The inquiry approach to errors does not suggest that teachers should follow-up an error and then step back and let the students handle the rest (Borasi, 1994). Rather, the teacher is the mediator, and responsible for creating a balance between the mathematical content, and student contributions (Ball, 1993; Borasi, 1994; Sherin, 2002). Since teachers are expected to take on that responsibility, their role is important from the beginning to the end of discourse surrounding an error. To identify strategies that encourage student contributions, and that requires students to participate in the mathematical work, it is critical to explore how teacher and student contributions shape the discourse. The contributions made throughout the discourse surrounding an error may be just as important or perhaps more so than the teacher's initial response to an error.

This study is an exploratory analysis of errors during the first weeks of first-grade mathematics lessons. It aims to gain a richer understanding of the teacher's role in managing the "balancing act" between student contributions and mathematical productivity in discussions of mathematical errors (Sherin, 2002). It investigates the value of certain strategies, and the challenges presented by various discourse paths.

Chapter 2

Method

Data Source

This study includes 10 first-grade lessons from 5 teachers. Each of the teachers was videotaped on two consecutive days in the first month of the school year. The length of a lesson ranged from 31 min, 28 s to 1 hr. The average length of a lesson was 48 min, 36 s. Because of the significance this study places on the discussion of errors and the student participation in this discussion, analyses were limited to whole-class time. Whole-class time is defined as class time when a teacher is not working with individual students, but is addressing the whole class. Whole-class time ranged from 12 min to 38 min, 30 s. Average whole-class time across the 10 lessons was 26 min, 20 s.

Participants

Teachers. District administrators from a diverse small urban community in the Midwest were solicited and they identified two schools for this project. The five first-grade teachers from these two schools volunteered to participate. All five teachers were female and had taught for at least 10 years prior to the study. Two of the teachers taught for more than 20 years.

Students. The sample reflected the general socioeconomic and ethnic makeup of the community; 69% of the students were European American, 23% African-American, 7% Asian-American, and 1% Latino. Both schools had more than 30% low-income students (32% in one and 42% in the other school). In one of the schools, there were two non-native English-speaking students in each of three classrooms. These six students were all from Korea, and one was judged to be fluent in English by her teacher and the others were not. One of the classrooms had 17 girls and 4 boys. The other four classrooms were more balanced. The number of girls in the four classrooms ranged from 7 to 11, and the number of boys ranged from 7 to 9. The four classrooms had an average of 10 girls and 8 boys.

Coding

The coding system required several levels of analysis to capture the complexities of teacher-student discourse. The analysis included the following levels of coding: error episodes, types of teacher and student contributions, and engagement of the mathematics. Each is explained below.

Error episodes. Identifying error episodes began by identifying each student error. In addition to blatant errors, such as incorrect calculations, I also kept track of student contributions that, although not incorrect, the teacher treated as incorrect. For example, if there were multiple correct answers for a question, but the teacher was searching for a particular response, the teacher often treated correct answers as incorrect. It was important to keep track of these instances, because such responses contributed to how the students may have come to understand errors.

After I identified each error, I identified the error episode. Error episodes were defined as (1) beginning with the question or statement that provoked the initial student error (or perceived student error), (2) the error itself, (3) the teacher's response to the error, and (4) any subsequent discourse that followed, and the episode ended when the error was resolved. There were often several errors embedded within one error episode. I kept track of those errors as well. Resolution of an error, which signaled the end of an error episode, was when the subject of the discourse was no longer motivated by the original error. This often included "teacher wrap-ups." These wrap-ups usually involved the teacher repeating the correct answer or the reason that the given answer was correct, and then explicitly moving to the next problem or topic.

For reliability of error episodes, two raters identified error episodes by watching 1 selected 20-min segment of the videos from each teacher for a total of 1 hr, 40 min. This accounted for 37% of the whole-class data. Each rater noted the start and end times of each episode. Reliability for identifying episodes ranged from k=.94 to k=1.00. All disagreements for all teachers were discussed until the two raters came to an agreement.

Teacher and student contributions. After errors and error episodes were identified, teacher and student contributions were coded. Borrowing from Schleppenbach et al. (2007), I identified the following types of teacher responses: teacher ignores error, teacher breaks problem into steps, teacher probes for reasoning, teacher searches for an answer, teacher re-asks original question, and teacher corrects student.

In addition to teacher behaviors, I also coded student contributions. I identified the following types of student contributions: student makes error, student responds to another student, and student responds to the teacher.

For reliability of types of teacher and student contributions, the same two raters who identified episodes independently coded 20% of the whole-class lesson time for each teacher. Reliability ranged from k=.766 to k=1.00. All disagreements for all teachers were discussed until the two raters came to an agreement.

Active engagement. Judging the level of active engagement in mathematical discourse in first-grade is a challenge. Borrowing from previous research on errors (Borasi, 1994; Santagata, 2004, Schleppenbach et al., 2007), I examined three aspects of the discourse in error episodes that indicate high student engagement of the mathematics in a first-grade classroom: (1) high student participation (2) low teacher control of the mathematics, but high stimulus of discussion, and (3) discussion about what the correct answer is and why the initial error was incorrect. I defined low teacher control of the mathematics and high stimulus of the discussion as instances when a teacher encouraged students to ask questions and/or to further explain their answers or disagreements rather than providing the correct mathematics for the student.

Using these factors as a guide, I evaluated the engagement level of the discourse as either: (1) low, (2) medium, or (3) high engagement. Table 1 shows the behaviors associated with each level of engagement. The behaviors listed are possible behaviors that can be observed, but an episode did not need to contain all the behaviors to be placed into that category.

Table 1
Levels of engagement categories

Level of Engagement	Behaviors
Low	• teacher ignored an error • there was little attempt to explain why an error was incorrect • a resolution of an error that further confused the discourse (Ex: final answer was mathematically incomplete or incorrect)
Medium	• moderate teacher control of mathematical work • low student participation in the mathematical work • an error that did not call for much discussion (Ex: student simply needed clarification on what the question asked)
High	• low teacher control of mathematical work • a follow-up to an error that probed for the student to explain their reasoning and/or required the student to do the mathematical work • welcoming of other student contributions

For reliability, a Ph.D. candidate in mathematics served as a rater and coded error episodes as high, medium, or low engagement. The researcher and rater discussed three examples, one from each category, to define the characteristics of each category. The rater then coded the 42 remaining episodes. Reliability was k=.796 between the two coders. All disagreements for all teachers were discussed until an agreement was reached.

Chapter 3

Results

The first part of the results examines the basic features of error episodes. The goal of this initial analysis is to gain a basic understanding of what the episodes in these observations look like. I report the frequencies of the types of teacher responses, the number of students participating, and the number of student and teacher discursive turns. The second part of the results is a more qualitative analysis of the teacher and student contributions in the episode. I give examples of high, medium, and low engagement episodes, and the teacher and students' roles in these episodes. Finally, I identify three teaching strategies that were used differentially in the high versus the medium and low engagement episodes, and report the frequencies of those episodes across the 5 teachers.

Features of Error Episodes

There were 77 errors that students made in 45 error episodes across the 10 lessons. There were more errors than episodes because there were often multiple errors within one episode. The mean number of errors per episode was 1.7 (*SD*=1.01). There was a mean of 9 error episodes (*SD*=3.74) per teacher, across the teacher's 2 lessons. The length of an error episode ranged from 10 s to 2 min, 40 s. The average length of an error episode was 35 s.

Teacher and student contributions

Table 2 shows the frequency of each teacher's initial response, for each error episode, in each of the 5 classrooms. It is important to note that these numbers reflect the teacher's first response to the error, and not her subsequent responses throughout the resolution of that error. The subsequent discourse is analyzed qualitatively. The most common teacher response to the initial error was to correct the student, followed closely by searching for the answer. As can be seen in an examination of Table 2, teachers in this sample displayed all of the teacher responses reported by Schleppenbach et al. (2007). The most common initial responses to student errors were for teachers to correct students (attempted by all teachers, except Teacher A), search for a particular answer (attempted by all teachers, except Teacher A), or break the problem into steps (attempted by all teachers, except Teacher E). The least frequent responses were for teachers to ignore the error or to probe for reasoning.

Table 2
Frequency of types of teacher responses to initial error in each episode

Type of Response	Teacher A	Teacher B	Teacher C	Teacher D	Teacher E	Totals
Teacher ignored error	0	1	2	1	0	4
Teacher corrects student	0	2	3	4	3	12
Teacher re-asks original question	0	2	2	0	3	7
Teacher searches for answer	0	1	5	2	2	10
Teacher breaks problem into steps	3	3	2	1	0	9
Teacher probes for reasoning	1	2	0	0	0	3
Totals	4	11	14	8	8	45

Next, I examined what happened in the error episode after the teacher's initial response to the error. In Table 3, I present the mean number of students contributing to the discourse after each type of teacher initial response. Table 3 also shows the mean number of discursive turns taken by the teacher and students throughout the error episode. The teacher responses were collapsed into "Teacher follow-ups" and "No teacher follow-up" to examine whether a teacher's initial response to an error was related to how many students participated in the subsequent discourse. Based on previous research (e.g., Schleppenbach et al., 2007), it would be expected that following up an error leaves room for more students to participate in the discourse. "Teacher ignored error" and "Teacher corrects student" were placed into the "No follow-up" category. The others were follow-ups. The chi-square analysis ($X^2=3.09, p=.08$) revealed that the teacher's initial response does not paint the whole picture. Later, in the qualitative analysis section, I present an analysis of how episodes proceed and are resolved.

Table 3

Student and teacher contributions after each type of initial teacher response to errors

Discursive Contributions	Teacher ignored error (n=4)	Teacher corrects student (n=12)	Teacher re-asks original question (n=7)	Teacher searches for answer (n=10)	Teacher breaks problem into steps (n=9)	Teacher probes for reasoning (n=3)
Number of students contributing	1.75 (*SD*=.96)	1.25 (*SD*=.45)	1.3 (*SD*=.49)	3.1 (*SD*=1.79)	1.89 (*SD*=.78)	2.3 (*SD*=2.31)[a]
Student discursive turns	.5 (*SD*=.58)	.67 (*SD*=.78)	1.3 (*SD*=.49)	3.2 (*SD*=2.49)	3.5 (*SD*=3.57)[b]	7.3 (*SD*=7.5)[c]
Teacher discursive turns	1.25 (*SD*=.5)	1.25 (*SD*=.62)	1.3 (*SD*=.49)	3.1 (*SD*=2.23)	3.6 (*SD*=2.96)	4.6 (*SD*=2.89)

[a]One of the episodes had 5 students contributing [b]One of the episodes had 12 student turns [c]The episode that had 5 students contributing also had 16 student turns

Qualitative analyses of active engagement

It is important to conduct a qualitative analysis of the entirety of the error episodes to gain a better understanding of how both teacher and student contribute to the resolution of an error, and the possible challenges they face. The next steps of the analyses focus on what happens throughout the discourse surrounding an error until the resolution of the error. Students and teachers were given pseudonyms to maintain their anonymity.

The categories used to judge the active engagement of the mathematics were: low, medium, and high. There were 19 episodes in the low, 19 in the medium, and 7 in the high categories. Table 4 shows the frequencies for each category across the 5 teachers. I first present an example of an episode in the "high" category to identify strategies teachers use that seem to stimulate student inquiry while also maintaining control of the classroom and resolution of the initial error. I then present examples of the medium and low episodes.

Table 4
Frequency of episodes with different levels of engagement across teachers

Level of Engagement	Teacher A	Teacher B	Teacher C	Teacher D	Teacher E
Low	0	2	7	5	5
Medium	2	6	6	2	3
High	2	3	1	1	0
Totals	4	11	14	8	8

High engagement of mathematics. In this episode, the students are sitting in a circle, and the teacher asks them about patterns. There is a box of shapes available for the students to use as well as shapes laid out in an A-B-C pattern in the middle of the floor. Early on in the episode, a student makes an error:

Ms. Smith: Can anyone else explain (what a pattern is) to—what about you, Michael?

Michael: Umm A-B pattern? [Error-Incomplete answer]

Ms. Smith: Well, what do you mean by A-B pattern?

Ray (callout): A, B, A, B, A, B

Michael: Like, A,B,A,B,A,B,A (S1 points to A-B-C pattern) [Error]

Anne (callout): No, it would be A,B,C,A,B,C (points to A-B-C pattern like Michael did)

Ms. Smith: Michael, do you agree with Anne?

Michael: No!

Anne (callout): (gesturing to A-B-C pattern again) Because there's three things, there's three things, it's three things

Michael (callout): That's not supposed to go the (that) way.

Anne (callout): There's three things and if there's two—(continues to explain her reasoning)

Ms. Smith: Well, now, Anne—

Anne (callout): and it's like this (starts making A-B pattern) and it could be A, B, A, B

Ms. Smith: Then, what would you still have to ta- oh I see what you're doing. Michael, do you know what c- if I put the shapes out (gets the box of shapes and puts them in front of Michael) leave this one the way it is (gesturing to pattern of shapes already laid out) alright you show me what you think an A-B pattern is. (Anne reaches for the shapes) Anne let Michael have his chance. What do you mean by A-B? Can you come up here and make an A-B pattern?

Michael: (Starts to make an A-B pattern with shapes)

Ray: Move these (gesturing to pattern already laid out) move these and then A, B, A, B, A, B

Ms. Smith: Well, let's see what Michael makes as an A-B and Ray I'll let you have a chance to show me an A-B. What do you think is an A-B (talking to Michael who is laying out shapes)? If we're running out of hexagons (reaches in bin) no, there's a couple more left.

Michael: (continues to lay out correct A-B pattern)

Ray: Ms. Smith, I have a cold. [off-track]

Ms. Smith: Let's see, okay. (referring to what Michael has laid out)

Anne: That's what I was saying (referring to what Michael has laid out)

Ms. Smith: (Pointing to pattern Michael laid out) A, B, A, B…

Ms. Smith&Ss: A

Ms. Smith: And what comes next?

Ss: B

Ms. Smith: So, I think you just told me a lotta things you already know about patterns.

In this episode, Ms. Smith utilizes several strategies. First, her initial response to the error, "Well, what do you mean by A-B pattern?" requires the student to provide his reasoning. This is a strategy that the teacher maintains throughout the episode. Ms. Smith does not simply give the correct answer nor does she take over the rest of the explanation of a pattern. She requires the students to provide their reasoning and places the responsibility for resolving the error on their shoulders.

Ms. Smith also allows other student contributions, even though Anne's correction of Michael is unsolicited. Instead of disregarding Anne's contribution, the teacher brings that contribution into the discourse surrounding the original error. Here is where, I argue, this episode takes off. Because of a teacher's willingness to embrace an unexpected student response to an error, there is a debate between two first-graders about patterns. Ms. Smith still includes Michael's contributions, and this keeps the focus of the discourse on the resolution of the original error. So, although Anne's contribution is included, Michael is still able to explain and defend his original response.

In this example, the teacher's role in the discourse following the original error is relatively hands-off. When Ms Smith does contribute, it is in a way that guides the student contributions, but does not take over the mathematical work. For example, she allows Anne's contribution but mediates the debate and says, "*Then, what would you still have to ta- oh I see what you're doing. Michael, do you know what c- if I put the shapes out…Anne let Michael have Michael's chance. What do you mean by A,B? Can you come up here and make an A-B pattern?*" again allowing Michael to defend and explain his answer, placing the responsibility for the mathematical work on the students. The strategies implemented by the teacher in the high engagement episode demonstrate an effort throughout the entire episode to maintain focus on the mathematical content while also leaving room for student contributions. She also allows these contributions to shape the discourse surrounding the original error.

Medium engagement of mathematics. In the episode below, the same teacher discusses patterns earlier on in the class-time.

Ms. Smith: Abby, explain it (what a pattern is) to me.

Abby: About like di--like about umm [Error]

Ms. Smith: It's like shapes?

Abby: Like hexagon

Ms. Smith: Like a hexagon. Do you see a hexagon out there? (referring to shapes on floor)

William (callout): No…yea [Error]

Ms. Smith: Can you come point to it, Abby?

(William starts to point to the hexagon)

Ms. Smith: Uh, Abby's name, Abby's.

Abby: (Points to hexagon)

Ms. Smith: So, what do you think a pattern is, Abby, when hexagons come over (gesturing in circles with hands) and over again? Any old place or in a certain place? What do you think? Can I just go ahead and put this down here? (Puts wrong shape next to the others)

Ss: No!

Ms. Smith: So, what would have to go there, then, Abby?

Lisa (callout): hex—(interrupted)

Abby: hexagon

Ms. Smith: A hexagon (replaces incorrect shape with hexagon). So, is that what you mean when I have to put a hexagon in a certain place, that's what makes a pattern?

In the above example, Ms. Smith keeps her focus on Abby, who makes the first error. Ms. Smith sticks with Abby despite callouts from two other students, and one contribution from the class. In an effort to get Abby to the correct answer, and perhaps to avoid other students taking over the discourse, Ms. Smith actually seems to do the opposite: students continue to chime in throughout this episode, and there is no way to know whether Abby arrived at the correct answer because of the teacher's guidance or because of the other student contributions.

Ms. Smith also explains most of the answer. In her attempt to assist Abby, she tries to breakdown the problem. Ms. Smith says, for example, "Can you come point to it [a hexagon], Abby?" But, again, there is another student trying to participate. So, Ms. Smith then explains Abby's reasoning for her. There are more callouts, and Abby does little work to resolve the error. Finally, Ms. Smith provides Abby's answer for her, which is actually a confusing explanation of a pattern. Ms. Smith says, "So, is that what you mean when I have to put a hexagon in a certain place, that's what makes a pattern?" In this case, Ms. Smith's attempt to take responsibility for explaining the answer and to stick with one student did not seem to clarify the erroneous answer.

This episode was placed in the medium engagement category, because of the control the teacher ultimately took over the mathematical work. While the teacher initially attempted to get Abby to explain herself, and to do the mathematical reasoning, other students kept interrupting. So, the teacher eventually took over the explanation of the mathematics, leaving little room for Abby or the other students to participate in the math.

Low engagement of mathematics. In this episode, there are a group of students standing in the front of the room. The teacher asks the other students in the class to sort them.

> *Ms. Davis: Now, there's another way they can sort themselves (Ss are standing in front of room). How else could they sort-(interrupted)*
>
> *Eric: Boy, girl, boy, girl [Error]*

Ms. Davis: Sshhh nope. No, Eric, stop (Eric is sorting Ss into pattern). You're making a pattern. I want you to sort yourselves into two groups. Now, look at them, one group has one thing and the other group has something else.

Eric: I was gonna make a girl, boy— [Error]

Ms. Davis: Nope. Anybody got an ide-you guys stand still (referring to group standing). Turn around Molly and stand still Kelly. Eric, go stand still so we can look at you guys. We gotta figure out how we're gonna sort 'em. Ms. Davis had something in her head you gotta think about it. What do you think, Isaac?

Isaac: Uh, girl or a boy [Error-Not answer Ms. Davis wanted]

Ms. Davis: Well, uh, this time the girls and boys might mix up, because that's not the group I'm lookin' for. Kelly, if you can't stand, I'll have to put someone else in my group. Look at those six people. Three of 'em have this and then the other three have something else. What do you think, Victor?

Victor: (Inaudible) [Error]

Ms. Davis: No, that's not quite it, but that's getting' close. That's not quite it. I'm gonna give you a hint. Look at the top part of them

Heather: I know, Ms. Davis.

Ms. Davis: I know I know. I want these people to think. What do you think, Anita? No, just tell us.

Anita: One boy should be on the other side and some girls should be on this side [Error-Not answer T wanted]

Ms. Davis: No, that's not it. Eric, you gotta go stand up there or I gotta put somebody else in your place 'cause I got somebody else I could add to that group. Look at the top of them and tell me how you think I sorted 'em. Cheryl?

Cheryl: Umm long hair and short hair [Error-Not answer T wanted]

Ms. Davis: I didn't do long hair and short hair, but I could've. I did it a different way. It's their hair, but tell me what I did. I looked around and I found people with (points to student to answer)

Anita: Blonde hair and black hair

Ms. Davis: Okay blonde haired people go to one side and black haired people go to the other side. Molly, you're a blonde hair, go with your blonde people. Okay. Thank you. You six may sit down.

In this episode, Ms. Davis tries to teach the students about grouping things based on common characteristics. Originally, Ms. Davis frames the question in a way that suggests there are multiple correct answers. She says, "Now, there's another way they can sort themselves. How else could they sort…" However, she then changes the question slightly and says, "Ms. Davis had something in her head you gotta think about it." Ms. Davis engages the students in a sort of guessing game until the specific response she wants is given.

Because Ms. Davis has a specific answer in mind, other student contributions that are different, but plausible, are not explored. When a student makes an error or gives an answer that Ms. Davis treats as incorrect, she says, "nope" and moves on, or gives a short explanation so that she can move to the next student. The students are not given the chance to engage very deeply with the mathematics, aside from their guesses. So, while there are many students contributing, there is little feedback on why their answer was not accepted.

Teaching strategies illustrated by the low and medium engagement episodes

The qualitative analysis of the levels of engagement revealed three common responses that teachers had to errors, which seemed to affect how the errors were discussed in these first-grade mathematics classrooms. I have labeled the three common responses the guessing game, the search for answers, and sticking with one student.

The guessing game occurs when the teacher has an answer in mind that she wants the students to guess, even though there are several plausible answers to the question. The second strategy, the search for answers, often overlaps with the guessing game. That is, the guessing game often requires the teacher to also search for answers, but searching for answers may be a separate strategy. For example, there are times when there is only one correct answer to a question, and the teacher calls on many students to get to that answer. In contrast, there may be many plausible answers in the guessing game strategy, but the teacher requires students to guess what's in the teacher's head. Because there are only 3 teacher responses in which the search for answers does not overlap with the guessing game, the guessing game and search for answers strategies will be collapsed here, and explained more in-depth in the qualitative analysis. The third strategy, sticking with one student, is when a teacher attempts to stick with one student after an error is made. So, if a student makes an error, the teacher focuses on that one student until that student resolves his or her own error.

Table 5 presents the frequency of these response strategies for each of the 5 teachers. The teachers used one of these three strategies in all but 15 error episodes. In these 15 episodes, teachers most commonly addressed the whole class, and the whole class responded, and they typically responded correctly. Also, in 2 of the 15 episodes, the teacher allowed for other student contributions without searching for answers or playing the guessing game. An example of this can be seen in the high engagement episode.

Table 5
Mean proportion of student and teacher contributions after types of initial teacher responses to errors

Strategies	Teacher A	Teacher B	Teacher C	Teacher D	Teacher E	Totals
Search for answers/ guessing game	0	3[a]	4	0	2	9
Sticking with one Student	3	7	7	2	2	21

[a]These 3 responses were the only search for answers that were not guessing game

All of the teachers utilized at least one of these strategies in more than one error episode. All of the teachers stuck with one student, and each did this more than once. Teachers C and E were the only teachers to use the guessing game strategy, and Teacher B was the only teacher who searched for answers.

Chapter 4

Discussion

Sherin (2002) described discourse in mathematics as a "balancing act," incorporating students' ideas while also maintaining a focus on the mathematical content. This struggle for balance is also applicable to the treatment of errors. The inquiry approach to errors suggests that student contributions should be encouraged and that students should take responsibility for doing the work to resolve an error (Borasi, 1994). Monitoring these contributions, and maintaining balance falls on the teacher. How can teachers address errors, allow students to do the mathematical work necessary to resolve errors, respect other student contributions, while also keeping a focus on the main lesson and not losing control of the classroom?

Overall, these observations showed teachers managing the discourse surrounding errors, and trying to maintain a balance; deciding which student contributions would be included, which errors would be addressed, and how in depth each error was treated. I found that teachers' responses to errors fell into the categories identified by previous research (Schleppenbach et al., 2007). In general, teachers ignored errors, corrected them, or followed them up with various types of questions. All of the teachers responded to initial errors in more than one of these ways.

Research also suggested that following up an error versus ignoring or immediately correcting it would allow for students to ask more questions, and have more opportunities to do the work to resolve an error (Santagata, 2004; Schleppenbach et al., 2007). If teachers ignore an error or correct it and move on, of course there would be fewer opportunities for students to respond. This study sought to look past a teacher's initial response to an error, and more closely at what happens *throughout* an error episode. The data reported here supported the idea that initial follow ups, like probing a student for reasoning or breaking the problem into steps, provide opportunities for students to do the mathematical work and explain their answers, characteristics of the inquiry approach to errors. However, the teachers' roles in mediating and encouraging those contributions are much more complex and require a closer examination of what happened in every error episode.

The qualitative analysis showed that the various error episodes differed in how they unfolded. In the example of the low engagement episode, the teacher at first tried to break down and re-explain the original question, but the rest of the episode unfolded as a guessing game. In the example of the medium engagement episode, the teacher probed the student for an explanation, but the rest of the episode became disrupted by other student callouts. This suggested that there are more challenges that occur as an episode unfolds than an initial response to an error can predict.

The Struggle for Balance

The most interesting finding that came out of a close examination of the episodes in the low, medium, and high engagement categories, was that teachers wanted to talk about errors. Past research suggested that U.S. teachers may have culturally embedded beliefs about how mathematical errors should be dealt with, often emphasizing a protection of the self-concept of the child over an acknowledgement of errors (Santagata, 2004; Schleppenbach et al., 2007).

While I agree that teaching is a cultural activity, much like any other activity, I did not see a resistance in these teachers when it came to discussing errors. Instead, I found that there were three strategies commonly used by all the teachers that presented challenges for the type of student participation that has been advocated in the teaching of mathematics and the treatment of mathematical errors (Borasi, 1994; NCTM, 1991, 2000; Sherin, 2002). Although these strategies may have been intended to get students participating, sharing the reasoning behind their errors, and resolve their errors, certain characteristics of these strategies made it difficult to maintain the balance between that student participation and the mathematical content.

Guessing Game/Search for Answers. When a teacher plays a guessing game or searches for answers, there leaves little room for the explanation and resolution of erroneous responses. In the low engagement example, the teacher had an answer in her mind that she searched for, requiring the students to guess. The draw of using a guessing game approach or searching for answers especially in first-grade, is understandable. It often grabs the attention of students and allows many students to give an answer and participate in the activity. However, it is important to examine what happens to the mathematics and the errors that occur in the discourse.

The mathematics often gets lost in these episodes. Math becomes represented as having a solution, not arrived at by going through steps or reasoning, but by guessing until the correct answer is found. This is particularly difficult for children at the preoperational stage of cognitive development, who are unable to grasp concrete logic, and hold multiple ideas in their head (e.g., Piaget, 1960). So, one could imagine the difficulty a guessing game may pose for them, where the answer to questions change, depending on what the teacher is thinking. This misrepresentation is not a foundation on which a student's mathematical understanding should be built. Rather, they should begin to understand, early on, that mathematics is a subject with procedures, logic, and reasoning that are critical to the arriving at the correct solution. If asked to provide the mathematical reasoning, they will be able to hone these skills early, and may be in a better position when confronted with more challenging material.

The search for answers and guessing game also limit how deeply each error is treated. There is little room for an in-depth exploration of the misunderstandings behind the errors, because the teacher moves quickly from student to student. Without this exploration, errors are left "up in the air." So, there is rarely an opportunity to bring the reasons behind an error into the open. Without this elaboration, it is hard to know whether students understand why a particular answer was incorrect.

Sticking With One Student. The other strategy that may have both benefits and trade-offs for the teacher's role in managing the discourse surrounding errors is when a teacher sticks with one student. This was the most common strategy used across the teachers, and may be an effective way to treat an error in many cases. If a student makes an error, and the teacher can get that student to work through the problem, then that is certainly a valuable treatment of an error. However, sticking with one student may also mean ignoring other student contributions.

In my observations, teachers often disregarded other student errors and contributions in an effort to make the initial student understand their error. The first-graders were often eager to give their answers and participate in the discussion, a quality that is valuable in all academic pursuits, and that should be encouraged in first-grade. However, when a teacher became too focused on one student, as in the medium engagement episode, other student contributions could be ignored or discouraged. Again, sticking with one student can certainly be a valuable strategy. However, when other student contributions become overwhelming, it may be beneficial to incorporate those into the discourse surrounding the error rather than treating them as obstacles to getting the original student to resolve his error.

Implications for teaching

This study has its limitations. The sample size is small, and no claim can be made that certain teacher responses improves student learning. However, there is strong theory and research to support the idea that an open discussion of students' errors will allow students to entertain multiple perspectives, discuss their own understandings of the material, and build on their current mathematical knowledge (e.g., Ma, 1999). Despite these limitations, this qualitative exploration has important implications for teaching.

The strategies identified, their frequencies, and the challenges they present for the inquiry approach to errors are important for teachers and for future research. Allowing for more opportunities to unpack student errors may mean a simple change to one or more strategies that teachers already use. If small changes are made, as small as how one frames a question, students may have more opportunities to explore their erroneous understandings.

For example, if teachers that like to stick with one student, but other students constantly seem to chime in, perhaps they can be given strategies that incorporate those callouts, while also helping the original student to resolve his own error. By making that small change, the original student may have more room to do the mathematical work, because the callouts are more controlled. So, the callouts become a conversation with the original student rather than a distraction that can drown out the original student. And, these other students will also be able to explain their own ideas and possibly resolve their own erroneous understandings. If a teacher wants to utilize a guessing game approach, it may be more beneficial to say something like, "There are many different ways we can sort this group of people. Can you name some?" as opposed to "I thought of one way to sort this group of people, what is it?" so that it is clearer that there are multiple answers to the question.

A teacher's tendency to stick with one student may also be changed slightly when several students are calling out and trying to participate. I understand that the teacher must maintain control of the classroom. However, if the students' callouts are mathematically related, and interfere with the original student's ability to examine his error, it may be valuable to encourage the inquiries from other students. This may actually result in an even better controlled discussion, because the students' callouts will be acknowledged as important contributions rather than interruptions. If these callouts are errors, it may allow the teacher the opportunity to deal with several similar errors at one time. The high engagement episode showed that this is possible without losing control of the discourse, and that it doesn't carry huge time demands with it.

The high engagement episode represents another important implication for teaching. That is, creating a balance between student contributions and mathematical content regarding errors is possible, even in first grade. In the high engagement episode, the teacher required the student to explain his answer and to defend his answer. Requiring the student to provide his reasoning and to defend his answer places responsibility on the student to do the mathematics as opposed to waiting for the teacher's evaluation of his response as either correct or incorrect. This is an important strategy for several reasons. The student must give a reason for his answer, demonstrating his individual understanding of the mathematical task at hand. The teacher's response to the error also opens up the error for further discussion. This is important because it represents errors not as things that should be hidden, but as opportunities for discussion. So, the student is allowed to unpack his error in front of the class rather than shutting that conversation down. This mathematical discussion allows students an opportunity to correct erroneous understandings, acquire new knowledge, and build on their current foundation of knowledge (Inagaki, Hatano, & Morita, 1998).

Another important part of this episode is that the teacher maintains an inquiry approach *throughout* the episode. She allows an opportunity for another student to weigh in on the error, and also requires that this student explain her reasoning. The type of student-to-student debate that follows does not occur in any other episode, but the fact that this is possible in a first-grade classroom is exciting. Because the teacher did not discourage disagreement from another student, she allowed an opportunity for the discussion of the error between two students in front of the whole class. Research has emphasized the importance of students questioning, discussing, and sharing ideas in mathematics (Ball, 1993; Lampert, 1990). Multiple perspectives are respected and encouraged in this episode, and I argue that a balance between teacher control of the mathematics and student inquiry is achieved.

Finally, this episode showed that asking students to explain their answers, and allowing for other student contributions doesn't take up valuable class time. This is an important practical point for teachers. Recall that the average length of an error episode was 35 seconds and this exceptional episode lasts 1 minute, 40 seconds. Although longer than the average error episode by more than a minute, the total time spent on this error was still less than 2 minutes. By investing a small amount of time, the teacher may accomplish a lot. This episode shows that it is possible, even in first grade, to allow for student participation and inquiries without losing control of the classroom or time.

Conclusions

The creation of an open discourse that allows students to unpack their ideas and resolve erroneous understandings is increasingly popular in mathematics education reform and research (Ball, 1993; NCTM, 1991, 2000; Santagata, 2004; Schleppenbach et al., 2007). Many concepts like "discourse community in mathematics," and "student contributions as the foundation for mathematical discourse" have emerged as goals for mathematics teachers (Borasi, 1994; Inagaki, Hatano, & Morita, 1998; Sherin, 2002). However, research has only just started to explore how these concepts look in the classroom (Santagata, 2004; Schleppenbach et. al, 2007; Sherin, 2002).

This study continued this exploration, and extended past a teacher's initial response to a mathematical into a close examination of what teachers do throughout conversations with students about mathematical errors. I identified three teaching strategies that may create challenges for that type of discourse. While these strategies most likely have good intentions behind them, and demonstrate the teacher's desire to unpack student errors, they may also be problematic for the inclusion of student contributions and the resolution of errors. It may be valuable for future research to look more closely at how frequently these strategies are used, why they are used, and how they may be easily changed to create a better balance between the student-centered discourse and resolution of mathematical errors.

A close examination of more real-world examples in which teachers allow students to unpack their errors, without losing control of the classroom may further anchor the more abstract theoretical ideas. By doing so, teachers will have a more concrete understanding of how these conversations can be facilitated. Also, by identifying the challenges that various strategies pose, teachers can be given practical solutions and suggestions for small changes so that their classroom discourse reflects the type of discourse that has been advocated in mathematics education (e.g., NCTM, 1991, 2000). Ultimately, with a deeper look at how errors are treated, both teachers and students will have a better window into the value of errors in the learning process and how students can tackle errors in the classroom with confidence.

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