THREE DIMENSIONAL SLOPE STABILITY ANALYSES FOR
NATURAL AND MANMADE SLOPES

BY

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DISSERTATION

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in Civil Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 2011

Urbana, Illinois

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Slope stability analyses in practice mostly rely on limit equilibrium (LE) procedures rather than finite element (FE) or finite difference (FD) procedures. Presently, most slope stability methods are two-dimensional (2D) which assumes the failure surface is infinitely wide and therefore three dimensional (3D) shear resistance/forces are negligible when compared to the overall driving and resisting forces. Most, if not all, slopes are not infinitely wide and have a 3D geometry. Therefore, application of 2D analyses to a 3D problem is not theoretically correct but believed to be conservative/sufficient for engineering practice. 2D analyses are conservative because the shear resistance along the two sides of the slide mass (end effects) are neglected in the analysis. This conservatism may be acceptable for slope designs, but in the case of back-analyses of landslides, 2D analyses may result in unconservative values of back-calculated shear strength by as much as 30% (Stark and Eid 1998). In addition, 3D analyses are important in slope failure causation analyses, especially in litigation, to accurately assess the relative effects of slope changes, precipitation, shear resistance and remedial measures.

This study presents a LE methodology for calculating the 3D factor of safety (FS) for natural and manmade slopes and an accompanying user friendly software package. A comparison of different 2D and 3D slope stability methods e.g., LE and continuum mechanics methods, is also presented to verify the new LE methodology. Using known slope stability examples from published literature and field case histories, 2D and 3D slope stability analysis were performed by LE and continuum methods to investigate the applicability and/or limitations of each method to different slope stability problems and geometries.

An inherent advantage of continuum analyses is the failure surface geometry, i.e., rotational or translational, does not have to be specified and it is located as part of the solution for the lowest FS. However in a back-analysis, the
field failure surface and slide mass geometry must be used instead of searching for the failure surface that yields the lowest back-calculated strength. Because there is no provision for specifying a failure surface, current continuum mechanics procedures can be used for design of slopes and probably not for back-analysis. In addition, LE procedures are more user friendly than FD and FE procedures, consume less computational time, and are preferred for routine analysis for design. For important projects, results of LE analysis can be checked using FD or FE procedures.

Based on a review of existing 3D literature, and LE, FE, and FD analyses performed in the present study, it may be concluded that the minimum 3D FS is greater than the minimum 2D FS for all conditions considered herein. If the actual shear strength is used in the design of a slope, the assumption of an infinitely wide 2D failure is conservative. However, the same assumption may lead to an overestimate of the back-calculated shear strength from a 2D analysis. The findings that 2D analyses yield lower FS values than 3D analyses is significant for design of slopes. For example, municipal solid waste (MSW) landfill design is regulated by state and federal codes that require a minimum static FS of 1.5. These codes do not specify whether this is a 2D or 3D FS. However, it is implicitly understood that state or federal regulations require a minimum 2D FS greater than 1.5. With the acceptance of 3D stability analyses in practice, some designers have used a 3D FS of greater than 1.5 to satisfy the state or federal code. However, a 3D FS of 1.5 is less stable than a 2D FS of 1.5 which results in a less stable landfill slope but more airspace for the facility. Therefore, it is recommended that regulatory codes specify “minimum 2D FS of 1.5” to achieve the current level of stability for man-made slopes.

Stark and Eid (1998) show that 3D LE software does not consider the effects of shear resistance offered by the vertical sides that parallel the direction of movement of a translational landslide mass. Based on results of a parametric study conducted herein using FE and FD analysis, it was found that the use of an earth pressure coefficient \( K_{τ} \) that is in-between at-rest \( K_{O} \) and active \( K_{A} \) earth pressure provides a better estimate of the side shear resistance and 3D/2D FS ratios that are in agreement with FE and FD analyses. An attempt was made to update the charts provided by Arellano and Stark (2000) that show the influence of shear strength on ratios of 3D/2D FS for various slope inclinations and geometries. Charts developed herein
can be used to estimate the importance of performing a 3D slope stability analysis for a translational failure.

A new 3D LE methodology and program, 3DDEM-Slope, were developed as part of this study to incorporate and/or verify some of the findings of this study. The program options include input of shear strength using a stress dependent failure envelope to capture the stress dependent behavior of soils, Janbu’s (1973) correction factor for 2D and 3D slope stability analysis using Janbu’s (1956) simplified procedure, and improved subroutines for calculation of the vertical column base angles. The column base angles are calculated using a third-order finite difference estimator (Horn 1981) using all eight outer points of a grid node instead of using only two adjacent grid nodes so the base angle corresponds to the angle of an inclined plane instead of a line as occurs in 2D calculations. Although the program uses a 3D DEM file, 3DDEM-Slope can be used to calculate a 2D FS at any desired cross-section along the 3D failure surface. In addition, 3DDEM-Slope compares the 2D FS for a cross-section in the middle of the slide mass with the overall 3D FS. This provides the user with a warning signal that 3D/2D FS ratio is less than the reference values obtained from FD and FE analyses for same width to height ratio and slope inclination. If so, the user can select to apply external side forces that are calculated based on the findings of this study and obtain a corrected 3D FS.
To my parents, who raised me with a freedom to decide my career and supported me in all my pursuits
ACKNOWLEDGMENTS

It is a great pleasure to thank the people who made this thesis possible.

First and foremost, I want to thank Engineer-in-Chief’s Branch, Government of Pakistan for providing me with the funding to pursue my post graduate studies at the University of Illinois at Urbana-Champaign.

It is difficult to overstate my gratitude to my advisor, Professor Timothy D. Stark. It has been an honor to be his Ph.D. student. Throughout my research and thesis-writing, he provided encouragement, sound advice, and good teaching, company, and ideas. The joy and enthusiasm he has for his research was contagious and motivation for me, even during tough times of my Ph.D. pursuit. I am also thankful for the excellent example he has provided me as a successful geotechnical professional and professor.

I would also like to thank Professors James H. Long, Erol Tutumluer, and Scott M. Olson, for agreeing to serve as members of my committee. My special thanks to Professor Oldrich Hungr of the university of British Colombia, who in addition to participating in my committee, provided me with the source code of his slope stability program, CLARA-W. In my later work of continuum mechanics modeling, I am indebted to Dr. Ashok K. Chugh, US Bureau of Reclamation in Denver. He aided my understanding of the mechanics of finite difference modeling. Dr Chugh also performed some 3D finite difference analyses to help verify my limit equilibrium model and its results.

I also acknowledge PLAXIS BV for providing a complementary license of PLAXIS 3D FE programs for use in my research.

I am grateful to Dr. Gulfam Alam, who provided me with the administrative and moral support during my stay at the University of Illinois at Urbana-Champaign. I am also grateful to Dr. Syed Muhammad Jamil, whose guidance was instrumental in selecting this prestigious institution for my Ph.D. studies.
My time at University of Illinois at Urbana-Champaign was made enjoyable in large part due to the many friends and groups that became part of my life. I am grateful for this time spent with friends and their families. My time in Urbana-Champaign was also enriched by the Pakistani Graduate Student Association. I am indebted to my many student colleagues for providing a stimulating and fun environment to learn and grow. I am especially grateful to my friends Fawad Niazi, Abdul Qudoos Khan, Aqeel Ahmad, Manzoor Hussain, Adeel Zafar, Muhammad Kashif, Usman Tariq, Faye Aziz, Salman Arshad, Qazi Aurangzeb, Sarfraz Ahmad, Jason Funk, Navid Jafari, Onur Pekcan, and many others.

I would like to thank my family and relatives for all their love and constant encouragement during my studies: my brother, Imran Akhtar, my sister Humera Shehryar, my brother-in-law Shehryar Haider, and my aunt Nabeela Sufi. Most of all my supportive, encouraging, and patient wife, Fouzia, whose faithful support during the final stages of this Ph.D. is so appreciated.

A special thanks to my children, Nazesh, Zurish, and Zohaib, for their patience during my research.

Lastly, I offer my regards and blessings to all of those who supported me in any aspect of my courses and research.
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<tr>
<td>BNS</td>
<td>Brown Native Soil</td>
</tr>
<tr>
<td>$c$</td>
<td>Cohesion intercept (total stress)</td>
</tr>
<tr>
<td>$c'$</td>
<td>Cohesion intercept (effective stress)</td>
</tr>
<tr>
<td>DEM</td>
<td>Digital Elevation Model</td>
</tr>
<tr>
<td>$E$(kPa)</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>F2</td>
<td>Two-dimensional factor of safety</td>
</tr>
<tr>
<td>F3</td>
<td>Three-dimensional factor of safety</td>
</tr>
<tr>
<td>FD</td>
<td>Finite Difference</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>FS</td>
<td>Factor of Safety</td>
</tr>
<tr>
<td>ft</td>
<td>Foot</td>
</tr>
<tr>
<td>FV</td>
<td>Field Vane</td>
</tr>
<tr>
<td>$G$(kPa)</td>
<td>Shear modulus</td>
</tr>
<tr>
<td>GIS</td>
<td>Geographic Information System</td>
</tr>
<tr>
<td>H</td>
<td>Height of slide mass, i.e., vertical distance between crown and toe of slide</td>
</tr>
<tr>
<td>IAEG</td>
<td>International Association of Engineering Geologists</td>
</tr>
<tr>
<td>$K$(kPa)</td>
<td>Bulk modulus</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$K_\tau$</td>
<td>Coefficient of earth pressure used to estimate side shear resistance</td>
</tr>
<tr>
<td>$K_O$</td>
<td>Coefficient of at-rest earth pressure</td>
</tr>
<tr>
<td>$K_A$</td>
<td>Coefficient of active earth pressure</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of slide mass, i.e., horizontal extent of slide parallel to direction of sliding</td>
</tr>
<tr>
<td>LE</td>
<td>Limit Equilibrium</td>
</tr>
<tr>
<td>m</td>
<td>Meter</td>
</tr>
<tr>
<td>mm</td>
<td>Millimeter</td>
</tr>
<tr>
<td>MSL</td>
<td>Mean Sea Level</td>
</tr>
<tr>
<td>MSW</td>
<td>Municipal Solid Waste</td>
</tr>
<tr>
<td>RSL</td>
<td>Rumpke Sanitary Landfill</td>
</tr>
<tr>
<td>SRF</td>
<td>Strength Reduction Factor</td>
</tr>
<tr>
<td>SRM</td>
<td>Strength Reduction Method</td>
</tr>
<tr>
<td>W</td>
<td>Width of slide mass perpendicular to direction of sliding</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Unit Weight</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Friction Angle (total stress), angle of internal friction, angle of shearing resistance</td>
</tr>
<tr>
<td>$\phi'$</td>
<td>Friction Angle (effective stress), angle of internal friction, angle of shearing resistance</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Dilation angle</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.1 Statement of Problem

Slope stability analyses in practice mostly rely on LE procedures rather than continuum mechanics procedures. In LE analyses, the FS for a particular failure surface is calculated by comparing the shear strength mobilized with the shear stress required for equilibrium assuming a plane strain condition. A plane strain condition corresponds to neglect of resistance along the sides of the slide mass, i.e., an infinitely wide slide mass restrained at both ends. These analyses do not require any information about the stress-strain behavior of the soil. Consequently, they also do not provide any information about the magnitude of movement associated with the calculated FS. An initial review of the continuum method was given by Duncan (1996), which primarily concentrates on the deformational analysis rather than slope stability analysis. However, since then continuum mechanics procedures have emerged as a powerful alternative to conventional LE analyses for slope stability. Present continuum mechanics procedures use identical failure criteria as used in LE method thus making it possible to compare FS results obtained using different methods or between different procedures in the same method. Despite its limitations, LE analyses are widely used in practice.

Presently, most LE procedures are two-dimensional (2D) which assume the failure surface is infinitely wide and therefore three dimensional (3D) shear resistance / forces are negligible when compared to the overall driving and resisting forces. Among these 2D procedures, the procedure of vertical slices (Fellenius 1936; Bishop 1955; Janbu 1957; Morgenstern & Price 1965; Spencer 1967) is the most commonly used procedure because of two useful simplifications; (i) the base of each slice passes through only one type of material and (ii) the slices are narrow enough so the slip surface at the base
of each slice can be modeled by a straight line.

Most, if not all, slope failures are not infinitely wide and have a 3D geometry. Therefore, application of 2D analyses to a 3D problem is not theoretically correct but believed to be conservative/sufficient for engineering practice. Past research (for example, Hutchinson and Sarma 1985; Cavounidis 1987; Hungr 1987) shows that, in general, 3D analyses yield greater FS than those calculated using 2D analyses, all other things being equal (Duncan 1996). 2D analyses are conservative because the shear resistance along the two sides of the slide mass (end effects) are neglected in the analysis. This conservatism may be acceptable for slope designs, but in the case of back-analyses of slope failures, 2D analyses may result in unconservative values of back-calculated shear strength by as much as 30%. (Stark and Eid 1998). In addition, 3D analyses are important in slope failure causation analyses, especially in litigation, to accurately assess the relative effects of slope changes, precipitation, shear resistance and remedial measures.

The situations where 3D effects may be of importance and may affect the magnitude of FS include: (i) slopes that are curved in plan or form ridges or corners (Baligh and Azzouz 1975), (ii) slopes that have asymmetry caused by inclusions, such as a geosynthetic liner system (Stark and Eid 1998), drainage blankets, faults or rock joints, (iii) slopes that have shear strength or piezometric conditions that vary in the direction perpendicular to direction of slide movement (Hungr 1989), (iv) slopes that are surcharged by loads (Baligh and Azzouz 1975) or cut by excavation (Hungr 1989), (v) slopes that form corners or ridges (Giger and Krizek 1975, Hungr et al. 1989), and (vi) dams in narrow or curved valleys (Baligh and Azzouz 1975).

Despite the importance of 3D effects, a widely accepted 3D slope stability method and corresponding software is not available to practitioners. Only a few solutions (computer program) have been developed to perform 3D slope stability analysis where the slip surface is approximated by a predetermined shape (Cornforth 2005). The most common approach is the procedure of columns (Hovland 1977; Chen & Chameau 1982; Hutchinson 1981; Humphrey & Dunne 1982; Hungr 1987), which is analogous to the 2D procedure of vertical slices. However, the recommendations for 3D design FS, applicability of these procedures to field conditions, and shapes of 3D failure surface have not been verified. Therefore, accuracy of the 3D analyses depends on the degree to which the analysis represents the field mechanism.
and model the field slope geometry and engineering properties. In addition, most of these programs are difficult to use and not well suited for practice.

1.2 Research Objectives

The main objectives of this research are to: (1) develop a LE procedure to calculate the 3D FS for natural and manmade slopes, and (2) an accompanying user friendly software package.

1.3 Scope and Outline of Current Study

To accomplish this objective the following main tasks were performed.

- Review existing 2D and 3D stability methods and accompanying literature.

- Study 3D geometry of natural slope failures and associated failure surfaces to determine types of surfaces that should be modeled in 3D procedure and code developed herein.

- Compare different procedures of 2D and 3D slope stability analyses using LE and continuum mechanics method.

- Study 3D modeling and interpolation techniques and their effects on 3D FS.

- Study effects of shear resistance acting along the two sides of a slide mass that parallel the direction of movement on 3D FS.

- Verify magnitude of side resistance using 3D continuum mechanics procedures for inclusion in LE analyses.

- Develop 3D LE analysis procedure.

- Develop a computer code to verify 3D LE analysis procedure developed herein and facilitate its use.

- Verify 3D procedure and code using field case histories.
The remainder of this dissertation is organized as follows.

Chapter 2 provides a review of existing 2D and 3D LE stability procedures.

Chapter 3 identifies 3D shape of field failure surfaces.

Chapter 4 discusses different 2D and 3D analysis methods/procedures and indicates applicability of each method/procedure to field conditions.

Chapter 5 presents a method for modeling 3D field failure surface using the Digital Elevation Model (DEM).

Chapter 6 discusses the effect of side shear resistance in 3D stability analyses.

Chapter 7 provides a description of the 3D computer code.

Chapter 8 summarizes the application of 3D analysis of various case histories involving slope failure.

Chapter 9 summarizes the research contributions and provides directions for future research.
CHAPTER 2
PREVIOUS RESEARCH AND BACKGROUND

2.1 Introduction

LE analysis is the most common method in practice due to its simplicity. In a LE analysis, FS is defined as the minimum factor by which the soil strength must be divided to bring the slide mass to the verge of failure. The soil mass is assumed to be at the verge of sliding failure and the equilibrium equations are solved for the unknown FS.

Since the 1960’s, various researchers have proposed 3D slope stability procedures based on LE (i.e., Hungr 1987; Gens et al. 1988), limit analysis (Chen and Scawthorn 1970; Michalowski 1989), and FE (Lefebvre and Duncan 1973; Chen 1981). Some other methods/procedures e.g., distinct element (Cheng 2002), FD, variational calculus (Leshchinsky 1986; Lam and Fredlund 1993), and discontinuous analysis are also available. Of these methods, LE, which assumes plane strain conditions during slope failure, is the most common. However, after more than four decades the number of widely accepted 3D LE procedures is still relatively few as compared with the 2D LE analyses procedures. The fact that the extension of the analysis to 3D makes the problem more complicated thereby bringing the problem to a higher degree of indeterminacy has hindered research on the subject. The key in forming a workable 3D solution is to increase the number of equations, reduce the number of unknowns, or both so the problem becomes statically determinate. Because the maximum number of equations is limited by the conditions of equilibrium, the practical choice remains to make workable assumptions to remove the difference between the unknowns and number of equations.

An extensive literature search was conducted to review existing 2D and 3D LE slope stability procedures and understand the limitations of each. Most of the 3D procedures available use the assumptions and framework of the
2D procedure and extend them to 3D. In accordance with 2D analyses, the measure of available resistance of a 3D soil mass is the factor of safety (FS). Table 2.1 presents the reference, theoretical basis, assumptions, equilibrium conditions satisfied, and other information pertaining to each 3D procedure. It can be seen that there is quite a range in theoretical basis, assumptions, and failure surface geometries that are incorporated in existing 3D LE based procedures. The following paragraphs provide details of the available 2D and 3D LE procedures.

2.2 Terminology

During review of literature it is noticed that the terms “method” and “procedure” are used interchangeably by previous researchers which needs to be clarified. LE is a method whereas Bishop’s (1955), Janbu’s (1956), Morgenstern and Price’s (1965), and Spencer’s (1967) are procedures within LE method. The differences in procedures within LE method are the assumption they make to render the problem determinate. For example, Morgenstern and Price’s (1965) procedure has variable interslice force inclination whereas Spencer’s (1967) procedure assumes uniform interslice force inclination. Thus in this study, the distinction between method and procedure is maintained for consistency and better understanding.

In addition, different researchers use different notations for coordinate axis and dimensions of a landslide. To maintain constancy in the referenced text and figures presented by previous researchers, notations used by respective authors are maintained in this chapter. However, a description of definitions of landslide features and dimensions used in subsequent chapters is presented in Chapter 3.

2.3 Review of Existing Two Dimensional LE Procedures

Conventional 2D LE procedures can be divided into the procedure of slices, circular procedures, and non-circular procedures. Among the 2D LE procedures, the procedure of vertical slices (Fellenius 1936; Bishop 1955; Janbu...
1957; Morgenstern & Price 1965; Spencer 1967) are the most commonly used because of two useful simplifications; (i) the base of each slice passes through only one type of material and (ii) the slices are narrow enough so the slip surface at the base of each slice can be modeled by a straight line. Circular and non-circular LE procedures consider the equilibrium of the whole failing mass and thus are considered less accurate. Fredlund and Krahn (1977) give a comparison of various 2D LE procedures of slices in terms of consistent procedures for deriving FS equations. All equation are solved for a case of a composite failure surface, partial submergence, line loading, and earthquake loading. Figure 2.1 shows the composite slip surface and the forces/variables associated with each slice.

2.3.1 Bishop’s (1955) Simplified Procedure

Bishop (1955), presents two different procedures for slope stability analysis using the procedure of slices, i.e., a “Bishop complete procedure” and a “Bishop simplified (or Modified) procedure”. Bishop’s complete procedure includes both horizontal and vertical forces acting on the sides of the slice but Bishop (1955) does not clearly specify the assumptions or details to fully satisfy static equilibrium (Duncan and Wright 2005). On the other hand the Bishop simplified procedure neglects the interslice shear forces (i.e., forces on the sides of slices are assumed to be horizontal). In addition, Bishop’s (1955) simplified procedure uses vertical force equilibrium equation of each slice and overall moment equilibrium about a center of rotation to determine the unknown forces. The normal force \( P \) on the base of each slice is derived by summation of forces in the vertical direction as shown below:

\[
\sum F_V = 0 \\
W = P \cos \alpha + S_m \sin \alpha
\]  

(2.1)

The failure criteria in terms of shear stress \( S_m \), FS, and shear strength \( \tau_{max} \) expressed in effective stresses with the Mohr Coulomb strength equation is:

\[
S_m = \frac{\tau_{max}}{FS} = \frac{c/l + (P - ul) \tan \phi'}{FS}
\]

(2.2)

Substituting the failure criteria from Equation (2.2) in Equation (2.1) and
solving for normal force gives:

\[ P = \left[ W - \frac{c'l \sin \alpha}{FS} + \frac{ul \tan \phi' \sin \alpha}{FS} \right] / m_\alpha \]  \tag{2.3}

where, \( m_\alpha = \cos \alpha + (\sin \alpha \tan \phi')/FS \).

The FS is derived from the summation of moments about a common point (a fictitious or real center of rotation for the entire mass):

\[
\sum M_a = 0 \quad (2.4)
\]

\[
\sum S_m R + \sum P_f = \sum W x + \sum kW e \pm Aa + Ld
\]

\[
\sum S_m R = \sum W x - \sum P_f + \sum kW e \pm Aa + Ld
\]

Introducing the failure criteria form Equation (2.2), normal force from Equation (2.3), and solving for FS gives:

\[
FS = \frac{\sum [c'lR + (P - ul)R \tan \phi']}{\sum W x - \sum P_f + \sum kW e \pm Aa + Ld} \]  \tag{2.5}

The original Bishop’s (1955) simplified procedure is only suitable for rotational surfaces. However, Equation (2.5) presented by Fredlund and Krahn (1977) incorporates moment arms of each force in addition to composite failure surface with partial submergence, line loading, and earthquake loading and thus is applicable to non-rotational surfaces as well. For reference, FS equation from original Bishop’s (1955) simplified procedure is shown below:

\[
FS = \frac{\sum [c'l + (P - ul) \tan \phi']}{\sum W \sin \alpha} \]  \tag{2.6}

2.3.2 Janbu’s (1956) Simplified Procedure

Janbu (1954, 1973) presents a Generalized Procedure of Slices (GPS). Because the procedure does not rigorously satisfy moment equilibrium, this procedure may not satisfy all conditions of equilibrium, i.e., only satisfies force equilibrium (Duncan and Wright 2005). The GPS usually produces FS values that are identical to those calculated by more rigorous procedures. However, the GPS does not always produce a solution that converges to an acceptable error. The second procedure i.e., Janbu’s Simplified procedure
(Janbu et al 1956, Janbu 1973), is based on the assumption that the interslice forces are horizontal. The normal force is derived from the summation of forces in vertical direction with interslice shear forces ignored as shown below:

\[ \sum F_V = 0 \]  \hspace{1cm} (2.7)

\[ W = (X_R - X_L) + P \cos \alpha + S_m \sin \alpha \]

Neglecting the vertical shear forces \((X_R - X_L)\) results in normal force that are the same as those in the Bishop’s (1955) Simplified procedure (Equation (2.3)).

The FS is derived from horizontal force equilibrium and is:

\[ \sum F_H = 0 \]  \hspace{1cm} (2.8)

\[ \sum S_m \cos \alpha + L \cos \omega = \sum (E_L - E_R) + \sum P \sin \alpha + \sum kW + A \]

\[ \sum S_m \cos \alpha = \sum (E_L - E_R) + \sum P \sin \alpha + \sum kW + A - L \cos \omega. \]

The sum of interslice force cancels out in this analysis. The FS equation derived from horizontal force equilibrium and introducing the failure criterion in Equation (2.2) and normal force expression from Equation (2.3) becomes:

\[ FS_o = \frac{\sum [c'l \cos \alpha + (P - ul) \tan \phi' \cos \alpha]}{\sum P \sin \alpha + \sum kW + A - L \cos \omega} \]  \hspace{1cm} (2.9)

Where \(F_o\) represents a FS which is uncorrected for the assumption of negligible interslice shear forces, which results in an interslice force angle of zero for the Janbu’s Simplified procedure (1956). To compensate for the neglected interslice shear forces, Janbu (1973) proposes a correction factor, \(f_o\). This correction factor is a function of slide geometry and strength parameters of the soil. The correction factor is obtained by Janbu (1973) based on slope stability calculations using both Janbu GPS and the Janbu simplified procedure. Corrected FS, \(FS_{corrected}\), is obtained by multiplying \(f_o\) by the calculated or uncorrected FS, \(FS_o\):

\[ FS_{corrected} = f_o \times FS_o \]  \hspace{1cm} (2.10)

Figure 2.2 shows the variation of \(f_o\) as a function of slope geometry \((d/L)\) and
soil type. More discussion on Janbu’s (1973) correction factor is presented in Section 4.3.

2.3.3 Spencer’s (1967) Procedure

Spencer’s (1967) procedure satisfies all conditions of equilibrium, i.e., horizontal and vertical force equilibrium, and moment equilibrium. Spencer’s (1967) procedure was initially developed for circular surfaces but this procedure was extended to non-circular surfaces. Spencer’s (1967) procedure is based on the assumption that all of the resultant interslice forces are parallel (i.e., all interslice forces have the same inclination). The specific inclination of interslice forces is unknown and it is computed as part of the solution using the following relationship between magnitude of interslice shear and normal forces:

\[
\lambda f(x) = \tan \theta = \frac{X_L}{E_L} = \frac{X_R}{E_R}
\]  

(2.11)

where \( f(x) = 1 \) (constant function), \( \lambda = \tan \theta \), and \( \theta = \) angle of the resultant interslice force from the horizontal.

The normal force is derived from vertical force equilibrium as in Equation (2.8) for Janbu’s (1956) simplified procedure except the interslice shear forces \( (X_R - X_L) \) are included as shown below:

\[
\sum F_V = 0 \\
W = (X_R - X_L) + P \cos \alpha + S_m \sin \alpha
\]  

(2.12)

Substituting the failure criterion (Equation (2.2)) and interslice force relationship (Equation (2.11)) in Equation (2.12) results in:

\[
P = \left[ W - (E_R - E_L) \tan \theta - \frac{c' l \sin \alpha}{FS} + \frac{u l \tan \phi' \sin \alpha}{FS} \right] / m_\alpha
\]  

(2.13)

The horizontal interslice force is obtained from horizontal force equilibrium as follows:

\[
\sum F_H = 0 \\
P \sin \alpha + kW = (E_R - E_L) + S_m \sin \alpha
\]  

(2.14)
Spencer (1967) derives two FS equations, i.e., one is based on moment equilibrium and the other on horizontal force equilibrium. FS based on moment equilibrium is same as the FS equation used in Bishop’s (1955) simplified procedure (Equation (2.5)):

\[ FS_m = \frac{\sum [c'lR + (P - ul)R \tan \phi']}{\sum Wx - \sum Pf + \sum kWe \pm Aa + Ld} \]  

(2.15)

Similarly, FS based on horizontal force equilibrium is the same as that used in Janbu’s (1956) simplified procedure (Equation (2.9)):

\[ FS_f = \frac{\sum [c'l \cos \alpha + (P - ul) + \tan \phi' \cos \alpha]}{\sum P \sin \alpha + \sum kW \pm A - L \cos \omega} \]  

(2.16)

Spencer’s (1967) procedure yields two values of FS for each assumed angle of interslice force (\(\theta\)). At some angle of resultant side force, the moment and force equilibrium is satisfied resulting in the same FS value as shown in Figure 2.3.

2.3.4 Morgenstern and Price’s (1965) Procedure

Morgenstern and Price (1965) present a rigorous procedure that assumes the shear force between slices is related to the normal force as:

\[ \lambda f(x) = \tan \theta = \frac{X_L}{E_L} = \frac{X_R}{E_R} \]  

(2.17)

where \(f(x)\) is the functional variation of x, \(\lambda = \tan \theta\), and \(\theta\) = angle of the resultant interslice force from the horizontal. Figure 2.4 shows typical functions (\(f(x)\)). Morgenstern and Price’s (1965) procedure is similar to Spencer’s (1967) procedure if the interslice force function is constant (i.e., \(f(x) = 1\)). For a constant \(f(x)\) (see Figure 2.4), Morgenstern and Price’s (1965) yields similar results as to those obtained from Spencer’s (1967) procedure.
2.4 Review of Existing Three Dimensional LE Procedures

Since the 1960's, various researchers have proposed 3D slope stability procedures based on LE. However, after more than four decades the number of widely accepted 3D LE procedures is still relatively few as compared with the 2D LE analyses procedures. Extending 2D LE procedures to 3D necessitates more assumptions to render the problem statically determinate. One procedure differs from another in terms of: (a) the assumptions regarding inter column forces; (b) equilibrium equations; and (c) simplifications regarding the shape of the failure surface. The following sections provide a review of various 3D LE procedures presented in the past by different researchers.

2.4.1 Sherard et al. (1963); Lambe and Whitman (1969)

An initial concept for evaluating 3D effects is the weighted average procedure (Sherard et al. 1963; Lambe and Whitman 1969). This procedure suggests using three parallel cross-sections through the slope and calculating the 2D FS for each cross-section. A weighted 3D FS is then computed using the weight above the failure surface in each cross-section as the weighing factor (Figure 2.5) and the following equation:

\[ F = \frac{F_1 A_1 + F_2 A_2 + F_3 A_3}{A_1 + A_2 + A_3} \]  

(2.18)

Where, \( F \) and \( A \) with subscripts represent FS and weight, respectively for corresponding 2D cross-sections. Because it is desirable for the selected cross-sections to cover an equal area of the slip surface, Cornforth (2005) recommends that for a two cross-section weighted analysis, the cross-section should be selected at the quarter-point widths of the slope. For a three cross-section analysis, the cross-sections should be selected at one-sixth, centerline, and one-sixth widths of the slope.

This procedure is not a true 3D procedure because it neglects the existence of forces between the cross-sections. However, this procedure may provide reasonable results if the failure surface tapers gradually up towards the boundaries on sides of the slide mass so the use of three or more 2D cross-section captures the side forces. This procedure is not suitable for slide...
masses with steep sides, e.g., translational slides, because the shear resistance on a vertical or near vertical side cannot be modeled using a 2D cross-section and the weighted average essentially yields the same FS as central cross-section.

2.4.2 Anagnosti (1969)

Anagnosti (1969) presents an extension of the 2D Morgenstern and Price’s (1965) procedure to 3D. Anagnosti shows that the number of statiscal assumptions needed to satisfy all 3D equilibrium equations is four times as many as in a 2D analysis. As a result, many more assumptions are required to solve a 3D slope stability problem. The main limitation of Anagnosti’s (1969) procedure is that the 3D slide surface is not specified. Therefore, the user must select the critical 3D surface. However, it can be seen in Table 2.1 that there is minimal agreement between the various procedures on the shape of the critical 3D slide surface.

2.4.3 Baligh and Azzouz (1975, 1978); Azzouz, Baligh and Ladd (1981)

Baligh and Azzouz (1975) examine end effects on the stability of homogeneous, cohesive slopes by extending the circular arc failure procedure to 3D. The 2D circular arc procedure assumes that the shear surface consists of an infinitely long cylinder and that the mechanism of failure consists of a rigid-body rotation of the cylinder about its axis. These basic assumptions are retained for the 3D problem, however, the shear surface is taken as a surface of revolution extending a finite length. In addition, all elemental shear forces acting on the slip surface are assumed to be perpendicular to the axis of revolution. The computer program STAB3D, developed to analyze end effects, is used to perform the 3D analysis.

In their analysis, Baligh and Azzouz (1975) consider two finite failure surfaces. Each failure surface is composed of a central cylindrical section of length $l_c$ with either conical or ellipsoidal ends of length $l_n$ or $l_e$, respectively, attached to it (Figure 2.6). For the problems of stability of a vertical cut and toe failure of a clay slope, the analysis shows that the 3D FS is greater
than the 2D FS. Figure 2.7 presents the ratio of 3D/2D FS plotted vs $l/H$ for different values of $l_c/H$. Baligh and Azzouz (1975) conclude that 3D effects tend to increase the FS, although for long failure lengths, $l_c/H$ greater than four, 3D FS approaches the plane-strain value (2D FS). In addition, FS obtained using elliptical end shear surfaces are consistently lower than those obtained using conical ones. Hence, Baligh and Azzouz (1975) conclude that the elliptical end shear surfaces are more likely to simulate the geometry of actual slope failures but no case histories are used to confirm this point.

Baligh and Azzouz (1975) analyze failure of the I-95 test embankment using the proposed 3D procedure and find that end effects increase the FS by 19 to 34% and provide a better prediction of the actual FS, i.e., unity, for the failed embankment. The following 2D and 3D FS are calculated during their study: 2D FS (Bishop’s procedure) = 0.80; 2D FS (Ordinary Method of Slices) = 0.79; 3D FS (actual failure length) = 1.06; and 3D FS (underestimated failure length; conical end shear surface) = 0.94. Baligh and Azzouz (1975) report that the conical end shear surface produced a lower FS and that the length of failure calculated using their procedure is substantially underestimated. Baligh and Azzouz (1975) attribute these discrepancies to the actual variation in soil properties along the axis of the embankment which their procedure could not accommodate.

Azzouz and Baligh (1978b) present 3D procedures for cohesive and non-cohesive soils ($c$ and $\phi$ soils). The slip surface is again assumed to be a cylinder with conical or ellipsoidal end caps and the direction of the elemental shear resistance over the slide surface is perpendicular to the axis of revolution. In these procedures it is also necessary to assume that the stresses are normal to the slip surface. Determination of the distribution of normal stresses in three dimensions is a major limitation of these procedures.

Azzouz et al. (1981) analyze four field case histories of embankments rapidly loaded to failure on saturated clay foundations using the computer program STAB3D (Baligh and Azzouz 1975; Azzouz and Baligh 1978b). This program is an extension of the circular arc failure procedure to 3D problems.

The analyses are based on two strength models. The first is the uncorrected Field Vane (FV) strength model (Bjerrum 1972). This model allows for the estimation of the in situ undrained shear strength based on field vane measurements. The second is the SHANSEP (Stress History and Normalized Soil Engineering Properties) strength model (Ladd and Foott 1974). This
model accounts for strength anisotropy and strain compatibility along the shear surface of revolution.

The four field case histories analyzed include the following: (1) I-95 embankment failure, Massachusetts (I-95), (2) failure of an experimental test section, New Hampshire (ETS), (3) New Liskeard embankment failure, Ontario (NLE), and (4) failure of Fore River test section, Maine (FRT). The FS for all embankment failures are calculated using the observed failure length but not the observed failure surface. In addition, the FV and SHANSEP strength models predict essentially identical critical shear surfaces for all four field case histories.

These analyses indicate that the ratio of the 3D to 2D FS ranged from 1.07 to 1.30. Hence, end effects tend to increase the 3D FS. The SHANSEP strength model estimated 3D FS for the failed embankments to be 1.06 (I-95), 1.03 (ETS), 1.03 (NLE), and 0.86 (FRT), while the FV strength model estimated 3D FS to be 1.17 (I-95), 1.00 (ETS), 1.24 (NLE), and 2.05 (FRT). Additionally, the ratios of 3D/2D FS estimated by both strength models are equal for all but the NLE field case history, where the estimated ratios differed by 3%. Therefore, for routine investigations, the FV test can be used to evaluate the likely magnitude of 3D effects.

2.4.4 Hovland (1977)

Hovland (1977) presents a general 3D stability analysis procedure that satisfies moment equilibrium. This procedure is analogous to the Ordinary Method of Slices (OMS) and is sometimes referred to as the Ordinary Method of Columns. It can be applied to any soil with Mohr-Coulomb strength parameters and any geometrical condition. In this procedure, Hovland (1977) assumes that the vertical sides of the soil columns are frictionless.

Hovland (1977) applies his procedure to two special cases. The first case involves an embankment on soft clay so a comparison could be made to the Baligh and Azzouz (1975) procedure. The results of Hovland’s (1977) analysis are in agreement with those of Baligh and Azzouz (1975) and show that the 3D FS is significantly higher than the 2D FS for all combinations of cohesion and friction angle.

The second case involves a wedge-shaped failure surface. For this analysis,
a closed-form solution is feasible due to the simple geometry of the wedge selected. Results show that when the cohesion approached zero, the lowest ratios of 3D/2D FS are obtained. In addition, Hovland (1977) indicates that sandy soils are more likely to experience wedge-shaped failures and that some extreme situations may exist where 3D FS is less than 2D FS. This conclusion is made even though neither case is a field case history where the FS is known.

In closing, Hovland (1977) states that 3D FS is usually significantly higher than 2D FS for a cohesive soil but 3D FS may be less than 2D FS in cohesionless soils. He also states that additional studies are necessary to determine the shape of the 3D shear surface as a function of Mohr-Coulomb strength parameters.

Steiner (1978) shows that Hovland (1977) “implicitly assumes there are no horizontal forces” and that horizontal forces are significant in 3D analyses. Azzouz and Baligh (1978a) and Steiner (1978) also show that Hovland’s (1977) procedure produces erroneous results for cohesionless soils.

2.4.5 Chen (1981); Chen and Chameau (1982)

Chen (1981) and Chen and Chameau (1982) propose a general 3D stability analysis procedure which utilizes the LE concept and can be applied to any sliding surface geometry. The failure mass, which is assumed to be homogeneous and symmetrical, is divided into vertical columns. Horizontal and vertical force and moment equilibrium are satisfied for each column as well as for the entire failure mass. The authors assume that the inter-column shear forces are parallel to the base of the column and are a function of column position in the failure mass. The interslice forces have the same inclination throughout the slide mass. Their analysis is applicable to any soil type, pore pressure condition, and slope angle. The computer program LEMIX was developed to perform their analysis.

The primary failure mass studied by Chen (1981) is a cylinder with semi-ellipsoidal ends (Figure 2.8). This symmetrical failure mass is analyzed using LEMIX. Comparisons of LEMIX with Hovland’s (1977) Ordinary Method of Columns show that the Hovland’s (1977) procedure yields lower FS values. It is also found that Chen (1981) procedure may not always be conservative.
when pore pressures are considered.

The LEMIX results are also compared to Spencer’s (1967) 2D procedure. Typical results show the following: (1) as the length of the failure surface increases, the value of 3D/2D FS decreases and the problem eventually approaches the plane-strain condition, (2) the steeper the slope, the lower the value of 3D/2D FS, and (3) soils with a high cohesion and low angle of internal friction, pore pressures may cause end effects to be even greater. In addition, Chen and Chameau (1982) state that for a soil with cohesion, 3D FS is greater than 2D FS because the end effects caused by the ellipsoidal ends result in a more stable slope. Also, 3D FS may be less than 2D FS for cohesionless soils. However, they admit further study is required to evaluate the implications of the findings.

Chen and Chameau (1982) also develop a FE program to analyze the stability of embankments. Comparisons between the LE (LEMIX) and FE (FE-SPON) procedures show close agreement. The FE procedure yield slightly higher FS with differences of 1.8 % in 2D cases and 5.5 % in 3D cases. Chen and Chameau (1982) did not use their procedure to analyze any field case histories.

Hutchinson and Sarma (1985) question the validity of the Chen and Chameau (1982) statement “in certain circumstances the 3D FS obtained for cohesionless soils may be slightly less than that for the 2D case”. Hutchinson and Sarma (1985) state that the “2D FS for the cross-sections within the ellipsoidal end sections will be higher than that for the central, critical 2D cross-section”. Moreover, they state that the inter-column shear forces contribute to the resisting forces. Based on these statements, they conclude that the 3D FS should be higher than the 2D FS. Hutchinson and Sarma (1985) also question other aspects of the Chen and Chameau (1982) analysis ranging from basic assumptions for some key terms.

2.4.6 Lovell (1984); Thomaz and Lovell (1988)

Lovell (1984) presents a 3D limiting equilibrium stability analysis which is an extension of the 2D procedure of slices to 3D. This procedure of columns satisfies horizontal and vertical force, and moment equilibrium and can be applied to any symmetrical failure mass with any combination of Mohr-
Coulomb strength parameters. This procedure assumes that the resultant shear forces on the inter-column faces are parallel to the base of the column and the resultant forces have the same inclination throughout a given row of columns.

Lovell (1984) proposes that the ratio of 3D/2D FS should be considered as an analysis factor, which is analogous to the load and resistance factor design (LRFD) strategy. He states that high ratios imply an excessively conservative 2D analysis. Conversely, ratios which approach unity imply that the 2D analysis is not conservative.

Two different failure surface geometries are considered. The first is a largely translational slide represented by a block surface (Figure 2.9), which is analyzed using the computer program BLOCK3, which can only consider translational movement. Results from an analysis of an embankment over a foundation that contains a thin weak layer showed the following (from Chen, 1981): (1) 3D/2D FS is typically greater than unity, (2) for small values of L/H, i.e., the ratio of length of failure surface to vertical slope height, the 3D effect is more pronounced for cohesive soils, (3) a lower strength of the weak layer may produce higher 3D/2D FS ratios, and (4) as the failure mass approaches a wedge shape, 3D/2D FS may be less than unity.

The second failure surface geometry examined consists of a rotational failure surface represented by a central cylindrical section with ellipsoidal ends (Figure 2.10). The LE computer program LEMIX (Chen and Chameau 1982) is used to analyze this surface. The central cylindrical cross-section is used because it is the critical surface identified using the 2D program STABL (Lovell and Sharma 1984). Lovell (1984) indicates that Chen (1981) came to the following conclusions for small slope inclinations and a rotational failure surface: (1) 3D/2D FS is higher for cohesive soils, (2) higher pore water pressure ratios may cause a higher value of 3D/2D FS in cohesive soils, (3) 3D/2D FS decreases only slightly as the length of the cylinder increases, and (4) 3D/2D FS is usually greater than unity but may be less than unity for cohesionless soils. Lovell did not use any field case histories with either of the computer programs described above to verify these conclusions.

Thomaz and Lovell (1988) propose a 3D slope stability analysis procedure that is a generalization of the procedure of columns developed by Chen (1981). The proposed procedure satisfies horizontal and vertical force, and moment equilibrium. The authors developed a computer program, 3D-
PCSTABL, which allows for the generation of random 3D surfaces while searching automatically for the critical failure surface.

To render the problem statically determinant, Thomaz and Lovell (1988) assume the following: (1) failure mass is symmetrical with respect to the main axis of sliding, (2) forces on the sides of the column act along the central vertical line of each side, (3) inter-column shear forces are parallel to the base of the column, and (4) inter-slice forces on the sides of the columns have the same inclination throughout the entire failure mass.

Thomaz and Lovell (1988) perform parametric studies to evaluate the influence of varying strength parameters, slope inclinations and pore water pressures on the ratio of 3D/2D FS and the shape and position of 3D critical surfaces. Thomaz and Lovell (1988) compare the results of their program with the results of PCSTABL5 (Carpenter 1985), which is a 2D analysis program based on the procedure of slices.

The results of parametric studies by Thomaz and Lovell (1988) show that the ratio of 3D/2D FS is always greater than unity for cohesive soils and it decreases as the slope becomes steeper. However, the ratio of 3D/2D FS can be less than unity for cohesionless soils and this ratio increases as the slope inclination increases. Results also show that there is good agreement between the 2D and average 3D critical failure surfaces for cohesive soils. For cohesionless soils, the average depth of the 3D critical failure surfaces is always deeper than the critical 2D failure surface. In addition, Thomaz and Lovell (1988) discover that the 3D effects are even greater when pore water pressures exist.

The authors do not use their procedure to analyze any field case histories, nor do they compare their results to those of any other 3D procedure.

### 2.4.7 Dennhardt and Forster (1985)

Dennhardt and Forster (1985) propose a 3D LE procedure for estimating the stability of slopes that is referred to as the Whole-Failure-Body procedure. Their procedure satisfies horizontal and vertical force, and moment equilibrium equations and can be applied to any symmetrical, but arbitrarily shaped, failure mass. The procedure is applicable to any soil type with Mohr-Coulomb strength parameters. Various slope angles, as well as sur-
charges and pore pressures, can also be considered.

Dennhardt and Forster (1985) assume that the mobilized shear stresses act in the direction of the vector tangent to the slip surface. In addition, they introduce a trigonometric function to describe the distribution of normal stresses on the sliding surface as opposed to considering the stress-strain relation of the soils. These assumptions are believed to be as reliable as those made for interslice forces in any procedure of slices.

Analysis of an example using the proposed procedure indicates that 3D FS is greater than 2D FS. The 2D FS are calculated using the Morgenstern and Price’s (1965) and Frohlich’s (1955) procedures. Dennhardt and Forster (1985) state that their procedure is more reliable than those developed by Baligh and Azzouz (1975) and Hovland (1977). However they fail to make any comparison with the results of these or any other 3D analyses, nor do they apply their procedure to any case history.

2.4.8 Leshchinsky, Baker and Silver (1985); Leshchinsky and Baker (1986); Leshchinsky and Huang (1992b)

The 3D slope stability analysis procedure developed by Leshchinsky et al. (1985) is a rigorous mathematical approach based on LE and variational calculus. This procedure may be applied to any slip surface geometry. However, the authors limit their analysis to symmetrical problems for simplicity. Global equilibrium equations are satisfied. In addition, this procedure allows for the analysis of failure masses with varying cohesion, angle of internal friction, and pore water pressure. However, the formulation of this procedure excludes the development of overhang cliffs and deformations of the sliding mass.

Leshchinsky et al. (1985) describe the proposed procedure of analysis as an “improved variational formulation of the 3D slope stability problem introduced by Kopacsy (1957)”. They state that their study focused on the existence of a minimum FS in variational LE problems. However, the authors consider the application of their procedure to a 3D slope stability problem to be justified because their procedure is equivalent to a procedure based on the upper bound theorem of plasticity.

The authors use the proposed procedure to analyze hypothetical cases of
homogeneous slopes. The results of their analysis indicate the following: (1) critical slip surfaces are smooth, (2) FS obtained for a local limited failure surface (3D failure mode) is greater than the FS obtained for a long cylindrical failure surface (2D failure mode), (3) difference between the FS for the 3D and 2D failure modes decrease as the angle of internal friction or the slope inclination increases, (4) 3D FS is independent of the normal stress distribution over the critical slip surface, (5) elementary shear force direction over the slip surface depends on the slip surface geometry, but not on the normal stress distribution, and (6) deep slip surfaces result when the angle of internal friction and the slope inclination are small.

The critical and symmetrical slip surfaces used by Leshchinsky et al. (1985) are represented by extremals contained in the solution of a first-order partial differential equation developed by the authors. These slip surfaces are continuous and smooth (i.e., they have continuous first derivatives). In addition, the authors’ study show that there are two potential slip surfaces represented by two possible modes of failure. The first failure mode is 3D (i.e., local limited failure surface) and is critical when local loading conditions exist, the second failure mode is 2D (i.e., long cylindrical failure surface) and is critical for plane strain conditions.

Leshchinsky et al. (1985) do not use their procedure to analyze any field case histories nor do they compare their results to those of any other 3D procedure.

Leshchinsky and Baker (1986) modify the 3D slope stability analysis procedure based on variational calculus (Leshchinsky et al. 1985) by limiting the scope of the analysis to symmetrical problems (Figure 2.11). They analyze one-half of a symmetrical and homogeneous sliding mass, whereas in the previous general procedure they analyzed the entire sliding mass. In addition, the failure surface consisting of a central cylindrical section and spherical end caps is analyzed to investigate the significance of end effects.

The results of the analysis of hypothetical problems show that smaller cohesion results in shallower slip surfaces, and for cohesionless soils the potential slip surface coincides with slope surface and thus there are no end effects. Therefore, the end effects are negligible in this case. The end effects are most pronounced for cohesive soils.

The authors compare their results with those of other 3D procedures. Baligh and Azzouz (1975) and Azzouz and Baligh (1978) results coincide with
those of the authors for a vertical cut in cohesive soil. However, Ugai’s (1985) variational procedure applied to cohesive slopes yields slightly smaller FS. The authors believe that limiting the problem to cohesive slopes may have produced discontinuous slip surfaces which are not admissible in their analysis. Finally, the results of Hovland (1977) and Chen and Chameau (1982) procedures applied to frictional slopes are contrary to those of the authors. Leshchinsky and Baker (1986) contend that the 3D/2D FS ratio approaches unity as the cohesion approaches zero, whereas Hovland (1977), Chen and Chameau (1982) claim that this ratio is less than unity. Leshchinsky and Baker (1986) again fail to apply their procedure to any field case histories.

Leshchinsky and Huang (1992b) present a mathematically rigorous, generalized 3D LE analysis procedure which may be applied to any symmetrical failure mass with Mohr-Coulomb strength parameters. This procedure is an extension of the 2D procedure developed by Leshchinsky and Huang (1992a). Global equilibrium equations are explicitly satisfied through a “mathematical process in which the normal stress over the specified slip surface is part of the solution”. Therefore, static assumptions are not required. The authors use their procedure to analyze a slip surface consisting of a central cylindrical section with ellipsoidal end caps and an extended log spiral slip surface. Results show that the 3D FS is greater than the 2D FS. Leshchinsky and Huang (1992b) also state that back-calculated shear strengths overestimate field strengths when end effects (i.e., 3D) are ignored. The authors do not analyze any field case histories nor do they compare the results of their procedure with any other 3D procedures.

2.4.9 Ugai (1985, 1988)

Ugai (1985) propose a 3D procedure for analyzing vertical cohesive slopes that is based on 3D LE techniques and variational calculus. This procedure satisfies force and moment equilibrium conditions, allows for the exact determination of the 3D critical failure surface, and allows for the exact evaluation of the 3D effects on the FS.

Ugai (1985) employ 3D LE techniques to prove that “the critical failure surface of any inclined cohesive slope is the surface of a rotational body”. The author then develops his variational calculus procedure for vertical cohesive
slopes. This procedure assumes a failure surface geometry which is composed of a central cylindrical section with curved end caps that terminate with a plane section. Although the author states that this procedure may be applied to slopes with any inclination, he fails to make any provision for this variable in his analysis.

Ugai (1985) used his variational procedure to analyze six other failure surface geometries in addition to the assumed failure surface geometry. The additional failure surface geometries include: a cone, an ellipsoid, a cylinder plus plane ends, a cylinder plus cone ends, a cylinder plus ellipsoidal ends, and a cylinder plus cones with plane ends. The comparisons show that the stability factor ratio, which is the ratio of 3D to 2D stability factors, for the assumed failure surface geometry (i.e central cylindrical section with curved end caps that terminate with a plane section) is the critical surface than the six other failure surface geometries studied. The stability factor is the product of unit weight, $FS$, and vertical slope height divided by the cohesion.

In his conclusions, Ugai (1985) states that the exact shapes of the 3D critical failure surfaces are determined to be split cylinders with curved end caps. He also states that 3D $FS$ is greater than the “2D $FS$ indicated by former researchers”; however, he fails to name any. Lastly, the author states that the failure surface consisting of a cylinder with plane ends approximates the critical 3D failure surfaces and critical $FS$ for vertical cohesive slopes. Ugai (1985) did not apply his procedure to any field case histories.

Ugai (1988) present a 3D slope stability analysis procedure which extends the 2D Fellenius (1936) procedure of slices to 3D. With this procedure, the failure mass is divided into vertical columns and is assumed to be composed of a central cylindrical section with ellipsoidal ends. This procedure is applicable to any soil type with Mohr-Coulomb strength parameters and satisfies moment equilibrium for the sliding mass. In addition, this procedure assumes that the resultant of the inter-column forces acting on the vertical sides of each column is parallel to the base of the column. Ugai (1988) also extends Bishop’s simplified (1955), Janbu’s simplified (1957), and Spencer’s (1967) procedures to 3D.

Ugai (1988) analyzes a hypothetical 3D cohesionless slope failure using the 3D extension of Fellenius (1936) as well as Hovland’s (1977) 3D procedure. The 2D $FS$ are calculated using the Fellenius (1936) 2D procedure. Results show that both Fellenius (1936) and Hovland (1977) 3D procedures produce
FS which are less than the 2D FS calculated using the Fellenius (1936) 2D procedure. The other 3D procedures i.e Bishop (1955), Spencer (1967) and Janbu (1957) did not give such results.

Results of a parametric study (Ugai 1988) indicate the following: (1) as the ratio of failure surface length to vertical slope height decreases, 3D effects increase (i.e., the ratio of 3D/2D FS increases), (2) 3D effects are small for cohesionless slopes and large for cohesive slopes, and (3) the ratio of 3D/2D FS may be less than unity for cohesionless slopes using 3D Fellenius procedure. Ugai (1988) suggests that the longitudinal stress in the slope that acts perpendicular to the sliding direction should be taken into account to avoid this anomaly. Ugai (1988) reports that three field case histories were analyzed and show that the 3D effects increase the FS by 5 to 30% in cohesive soils.

2.4.10 Cavounidis (1987)

Cavounidis (1987) proves that the minimum 3D FS (3D FS\textsubscript{min}) is always greater than the minimum 2D FS (2D FS\textsubscript{min}) for the same slope. The author employs simple algebra in his proof and states that his arguments are solely in terms of LE analysis.

Cavounidis (1987) asserts that results of previous researchers (Hovland 1977; Chen and Chameau 1982), which indicate the ratio of 3D/2D FS is less than unity for cohesionless slopes, were obtained either by comparing inappropriate FS or by using unjustified simplifying assumptions. In addition, Cavounidis (1987) asserts that comparisons between FS for a particular slope are only meaningful when the minimum FS are compared.

2.4.11 Hungr (1987); Hungr, Salgado and Byrne (1989); Hungr (2001)

Hungr (1987) presents a 3D LE slope stability analysis procedure that is an extension of Bishop’s (1955) simplified procedure of slices to 3D. Hungr’s (1987) procedure involves the same assumptions presented in Bishop’s (1955) simplified procedure which are: (1) vertical shear forces acting on both the longitudinal and lateral vertical faces of each column can be neglected in the
vertical equilibrium equation and (2) the vertical force equilibrium equation of each column and the overall moment equilibrium equation of the entire assemblage of columns are sufficient to determine all of the unknown forces.

The proposed analysis, which can be considered a procedure of columns, satisfies vertical force and overall moment equilibrium and may be applied to both frictional and cohesive materials. The normal intercolumn forces and horizontal shear forces are not neglected in the analysis, although they are not used in the formulation of the equation, which is an inherent advantage of the original Bishop’s (1955) simplified procedure. The plan area of sliding body is divided into a series of columns arranged in rows of uniform width (Figure 2.12).

Hungr (1987) use computer program CLARA-3 to perform 3D analyses of the example problem used by Chen and Chameau (1982). Results show that the Hungr’s (1987) procedure estimates values of 3D FS that are greater than those reported by Chen and Chameau (1982). In addition, for materials with zero cohesion, Hungr (1987) shows that 3D/2D FS approaches but does not fall below unity. This particular result is consistent with the critical comments of Hutchinson and Sarma (1985).


Hungr et al. (1989) present a 3D extension of the Bishop’s simplified (1955) procedure which includes modification suggested by Fredlund and Krahn (1977) making Bishop’s (1955) simplified procedure also applicable to non-rotational surfaces. Hungr et al. (1989) indicate that for a rotational surface, the reference axis is also the axis of rotation. However, for a non-rotational failure surface, FS will depend on the location of the reference axis. Hungr et. al. (1989) recommend using a method similar to Fredlund and Krahn (1977) for 2D analysis by using the rotation axis given by the center of a circle fitted to the slide profile (Figure 2.13). Hungr et al. (1989) also derive the FS from horizontal force equilibrium in the direction of sliding
which is equivalent to Janbu’s (1956) simplified procedure in 3D.

Hungr et.al. (1989) procedure implemented in CLARA-3 (Hungr 1987) is compared with other 3D LE solutions and case histories presented in Hungr et al. (1989). Comparison with Baligh and Azzouz (1975), Dennhardt and Forster (1985), Leshchinsky et. at. (1985), Gens et. at. (1988), and Xing’s (1988) procedures indicate that the 3D FS are in close agreement with those calculated using CLARA-3 (Hungr 1987). Investigations of the slide in Lodalen, Norway (Sevaldson 1956) and the 1963 Vaiont Slide (Hendron and Patton 1985) by Hungr et al. (1989) show that the Hungr et al. (1989) procedure accurately predicts the 3D FS for rotational and symmetric sliding surfaces. However, results tend to be conservative for some non-rotational and asymmetric surfaces. The authors state that the latter can be attributed to the lateral force imbalance for asymmetric geometries and neglecting internal shear stresses that arise from strongly non-rotational and asymmetric geometry. In addition, Hungr et al. (1989) indicate cases in which CLARA-3 (Hungr 1987) should not be used.

Hungr (2001) presents an extension of Morgenstern and Price’s (1965) procedure to 3D. The extension uses an approach similar to that proposed by Lam and Fredlund (1993) and Hungr (1997) combined with assuming the resultant of the interslice force on the lateral column surface is parallel to the base of each 3D column. The same iteration scheme used for 3D Bishop’s simplified procedure (Hungr 1987; Hungr et al. 1989) is used. However, both normal and shear force on the column faces are included in the analysis. For an constant interslice force function, \( f(x) = 1 \), Morgenstern and Price’s (1965) procedure produces similar results as Spencer’s (1967) procedure, so the same iteration scheme is used for the 3D extension of Spencer’s (1967) procedure.

The derivations of 3D extensions of Bishop’s (1955) simplified, Janbu’s (1956) simplified, Morgenstern and Price’s (1965), and Spencer’s (1967) procedures presented by Hungr (1987, 2001) and Hungr et al. (1989) are given in Appendix A for reference.
2.4.12 Gens, Hutchinson, and Cavounidis (1988)

Gens et al. (1988) develops a 3D slope stability analysis procedure based on moment equilibrium which can be applied to saturated, homogeneous, isotropic, and purely cohesive slopes with variable slope inclinations. Their procedure is basically the same as that proposed by Baligh and Azzouz (1975) since it is an extension of the circular arc procedure to 3D. However, Gens et al. (1988) procedure considers a wider range of end geometries and utilizes the 2D derivations of Taylor (1937).

Computer program F3SLOP, which allows for variable end geometries, is developed to calculate the 3D FS for a potential sliding surface. Additionally the computer code DEEPCYL is developed to calculate 3D FS and the critical slip surface for a cylindrical failure surface with plane ends. According to the authors, the analysis of this simple model is quite useful if an approximate assessment of 3D stability is required.

Gens et al. (1988) find that minimum FS are calculated when the end sections are produced using a family of power curves. The authors also state that ellipsoidal end sections, as opposed to hyperbolic, parabolic, conical or exponential end sections, give good estimates of 3D FS in most practical cases. Results also show that the 3D critical failure surface is shallower than in the corresponding 2D case, and that there exists critical value of depth factor (ratio of the maximum vertical slide depth to the height of slope) beyond which the critical failure surface will not penetrate. Furthermore, the authors conclude that the slide length decreases with increasing slope inclination, and the failure length increases with increasing depth factor.

Gens et al. (1988) use their stability procedure to analyze eleven field case histories of first-time, short-term failures of cuts in soft, saturated clays. They state that the field soil conditions and geometries closely approximated saturated, homogeneous, isotropic, and purely cohesive slopes. The resultant 3D/2D FS ratios range from 1.03 to 1.30 for the eleven field case histories. Results also show that the authors’ procedure correctly predicted the mode of failure (i.e., slope, toe, or base) but not the actual slip surface. The use of field case histories is prudent, but Gens et al. (1988) only consider one type of slope stability problem i.e., soft saturated clays. Also, the authors fail to compare the results of their procedure with those of any other 3D stability procedures.
2.4.13 Xing (1988)

Xing (1988) propose a 3D stability analysis procedure for concave slopes in plan view that satisfies horizontal and vertical force, and moment equilibrium. This simple (i.e., non-rigorous), yet practical procedure of analysis is analogous to a 2D Ordinary Method of Slices and can be applied to any homogeneous failure mass with a concave or straight slope in plan view. In addition, this procedure assumes a slip surface of elliptic revolution.

The failure mass, which is assumed to be symmetrical, is divided into vertical columns. Normal and shear forces acting on the base and sides of each column, gravity and an end force caused by the lateral pressure of soil are considered in this procedure. In order to render the problem statically determinate, the author assumes the following: (1) a resisting force, which is caused by the end force, acts on the base of each column; (2) the shear resistance on the base of each column is a function of this resisting force, and (3) forces acting on the sides and end of each column that are perpendicular to the movement of the failure mass should be neglected.

Xing (1988) use his procedure to an axisymmetric homogeneous concave slope in plan view. Xing reports (1988), that in general the stability of concave slopes increases as the relative curvature radius (ratio of curvature radius to vertical slope height) decreases, and that the FS approaches that of a straight slope as the relative curvature radius increases to infinity. Results also indicate that the 3D FS increased as the value of the coefficient of active earth pressure, $K_A$, increased. However, it is discovered that varying the value of $K_A$ has little effect of the stability of straight slopes in plan view.

In order to compare his 3D procedure to some 2D procedures, Xing (1988) analyze a straight slope with circular and composite slip surfaces. Six examples with varying slip surface geometries, soil properties and pore pressures are considered. Results of this analysis indicate that the 3D FS is greater than the 2D FS for all six examples. Xing (1988) compared his procedure to the Bishop’s (1955), Spencer’s (1967), Janbu’s (1957), and Morgenstern and Price’s (1965) 2D procedures. Xing does not use his procedure to analyze any field case histories.

Leshchinsky (1990) questions the validity of Xing (1988) statement that “the FS on the slip surface for an elliptic revolution is smaller than that of the slip surface consisting of a cylinder of finite length with an ellipsoid attached
to it. This is because the former slip surface area is smaller than the latter one when both lengths of failure masses and the sections at the symmetric plane of the two slip surfaces are given.” Leshchinsky (1990) states that the opposite conclusion can be drawn based on the same rationale; that is, that the larger slip surface area may result in a smaller FS since a heavier sliding mass is implied. He also contends that worthwhile comparisons can only be made when critical slip surfaces producing minimum FS are considered. The best way to accomplish this is by analyzing field case histories.

Leshchinsky (1990) suggests that Xing (1988) procedure be used to evaluate a failure surface consisting of a central cylindrical section with half an ellipsoid attached to each end. Although this may complicate his procedure due to the introduction of an additional variable, it may produce a more critical failure surface. In addition, a wide range of strength parameters and slope inclinations should be investigated since 3D stability analysis results are so sensitive to statical assumptions.

2.4.14 Michalowski (1989)

3D slope stability analysis procedure by Michalowski (1989) is based on the upper-bound approach of limit plasticity and is an extension of Drescher (1983) analysis to more complex failure mechanisms. Although this approach only yields the upper estimation of the limit load, Michalowski (1989) state that it is mathematically rigorous and satisfies global force equilibrium equations. This procedure assumes a drained, isotropic, and perfectly plastic soil which obeys the Mohr-Coulomb yield condition and the associated flow rule. Moreover, this procedure assumes that surcharges are vertical and the deformations before the limit state are inconsequential.

Michalowski (1989) consider toe and slope failures consisting of rigid-motion blocks with prismatic shapes. He state that the assumption of a rigid motion boundary may not be true, but the calculated results are upper bounds to the true limit load. Michalowski (1989) conclude that in the absence of a local load, the 3D FS approached that of the 2D analysis. In addition, Michalowski (1989) reports that as the length of failure increases and the local load is kept constant, the ratio of 3D/2D FS approaches unity (i.e., the plane-strain case), and 3D FS for locally loaded slope may be less
than 2D FS for an unloaded slope.

While Michalowski (1989) fails to analyze any field case histories with his analysis, he does state that the analysis could be improved by including the below-the-toe mode of failure.

2.4.15 Seed, Mitchell and Seed (1990)

Seed et al. (1990) develop two 3D stability procedure to analyze the slope failure at the Kettleman Hills Hazardous Waste Landfill in California. The first procedure described is termed a “multiple block analysis - force equilibrium approach.” In this analysis, the failure mass is divided into five blocks whose boundaries are assumed to be vertical. In order to evaluate the potential for sliding of the entire mass, the horizontal and vertical force equilibrium of each block, as well as boundary stresses between blocks, are considered by resolving all forces in the anticipated direction of sliding, however, moment equilibrium is not considered. This analysis is rendered determinate by assuming that lateral forces acting on the block boundaries are horizontal, and only normal forces act on the vertical inter-block boundaries (i.e., no lateral shear forces are applied). Since this procedure does not allow for out-of-plane movement or progressive failures, a second approach is developed.

The second analysis procedure is referred to as a “multiple block analysis allowing for differential movements of the slide mass.” In this analysis, the failure mass is divided into eleven blocks having vertical boundaries. Each block is considered to have either an active driving force or a passive resisting force. While this procedure allows for possible non-uniform directions of potential sliding, it does not satisfy overall force or moment equilibrium. The authors state that this is an acceptable situation in some 2D analyses; therefore, they develop this “exploratory analysis” to discover what results might be obtained for this particular slope failure.

In addition to the 3D analyses, the authors develop a 2D analysis procedure based on “conventional force equilibrium procedures”. The authors analyze ten different failure mass cross-sections which are typified by a driving block on one end and a passive block on the other. It should be noted that vertical boundary locations between the active and passive blocks are based on actual field failure conditions.
The results of the analyses indicate that the 3D effects yield a FS less than 2D analyses. The resultant 2D and 3D FS ranged from 1.1 to 1.25 and 1.01 to 1.08, respectively, for the full range of saturation conditions.

2.4.16 Lam and Fredlund (1993)

Lam and Fredlund (1993) presents a 3D procedure based on satisfaction of vertical and horizontal force equilibrium for each column and overall moment equilibrium. Lam and Fredlund (1993) consider various static conditions, number of available equations, and number of unknowns in these equations. For a sliding mass of \( n \) number of columns in the direction parallel to sliding and \( m \) number of rows in the direction perpendicular to sliding, Lam and Fredlund (1993) show that this procedure of columns is indeterminate. The number of unknown is \( 12 \times n \times m + 2 \), while the number of equations is \( 4 \times n \times m + 2 \), which still requires \( 8 \times n \times m \) assumptions. Their governing equations of force and moment equilibrium for the entire failure mass involve a large number of unknown inter-column forces. Convergence issues can be of concern too as trial and error numerical procedures are used to solve the large-scale, nonlinear equations. The derivation of 3D extension of Morgenstern and Price’s (1965) procedure presented by Lam and Fredlund (1993) is given in Appendix A for reference.

2.4.17 Huang and Tsai (2000), Huang et al. (2002)

Huang and Tsai (2000) propose a procedure for the 3D extension of Bishop’s (1955) procedure for asymmetrical surfaces where the sliding direction enters directly into determination of the FS. The procedure satisfies moment equilibrium in two directions, i.e., direction parallel and transverse to the sliding direction. The two components of the shear force required to resist sliding are calculated using moment equilibrium in both directions. Only vertical force equilibrium is considered and vertical shear forces of all columns are ignored. Because this procedure does not satisfy force equilibrium for the entire slope, the overall moment equation for the entire slide mass is related to the position of the moment axis. Thus, this procedure is not considered a rigorous procedure (Chen et al. 2006). The derivation of 3D extension of
Bishop’s (1965) procedure presented by Huang and Tsai (2000) is given in Appendix A for reference.

The generalized 3D slope stability procedure by Huang et al. (2002) is equivalent to Janbu’s (1954) generalized procedure of slices with some simplifications on transverse shear forces. Janbu’s (1954) generalized procedure of slices frequently encounters convergence problems in completely satisfying the line of thrust constraints. Therefore, the generalized 3D procedure by Huang et al. (2002) is also likely to experience convergence problems and thus this procedure has limited utility to practical problems. In addition, Huang et al. (2002) indicate that their procedure does not incorporate side shear resistance parallel to direction of sliding and thus may result in some error in calculated 3D FS for translational landslides with vertical sides.

Cheng and Lau (2008) indicate that because the sliding directions of soil columns are not unique in Huang and Tsai’s (2000) and Huang et al.’s (2002) 3D procedures and some columns are moving apart, the summation process for calculating FS may not be applicable because some of the columns are separating from others. In addition, Cheng and Yip (2007) demonstrate that under transverse load, the requirement of different sliding directions for different soil columns may lead to convergence problems.

2.4.18 Chang (2002)

Chang (2002) develop a 3D procedure of analysis based on the sliding mechanism observed in the 1988 failure of the Kettleman Hills Landfill slope and the associated model studies. Using a LE concept, the procedure assumes the sliding mass is a block system in which the contacts between blocks are inclined. The lines of intersection of the block contacts are assumed to be parallel, which models the sliding kinematics. In consideration of the differential straining between these slide blocks, the shear stresses on the slip surface and the block contacts are evaluated based on the degree of shear strength mobilization on those contacts. The overall FS is calculated based on force equilibrium of the individual block and the entire block system as well. After comparing the procedure to hypothetical problems of known solution, Chang (2002) concludes that: (a) procedure is generally accurate for slope stability analysis of translational failures, however, it overestimates FS
by as much as 10% for rotational failures, and (b) due to the assumed inter-block boundary pattern, the procedure is not fully applicable for dense sands or overly consolidated materials under drained conditions.

2.4.19 Chen et al. (2001a, 2001b, 2003, and 2006)

Chen et al. (2003) uses the conventional definition of FS that reduces the available shear strength parameters to bring the slope to a limiting state. A comparison of results obtained by their procedure with other 3D procedures (for example Hungr et al. 1989, Lam and Fredlund 1993, Huang and Tsai 2000 and Chen et al. 2000) for an example problem is presented by Chen et al. (2003). However, the FS obtained by their procedure that satisfies “complete overall force equilibrium conditions and moment equilibrium” is about 2% (1-2.3%) different from 3D extensions of Morgenstern and Price’s (1965) and Spencer’s (1967) procedures presented by Lam and Fredlund (1993) and Hungr (2001). Additional details about the Chen et al. (2003) procedure are shown in Appendix A for reference.

2.5 Important Deficiencies in Present State of Knowledge

From this review of existing 3D slope stability procedures, it can be concluded that there is considerable discrepancy between the theories, assumptions, and equilibrium conditions met or satisfied in existing 3D procedures. In addition, the field relevance of the assumptions required to render the problem statically determinate is questionable. Previous research has mainly utilized a mechanics approach, and as a result some of the assumptions and geometries do not represent field conditions. This results in a number of 3D procedures predicting erroneous field behavior. For example, Hovland (1977) concludes that deep slip surfaces are more critical than shallow slip surfaces in cohesionless soils. This result does not reflect field behavior of cohesionless slopes that usually experience shallow failure surfaces.

Table 2.1 shows that only ten of the 23 references utilize field case histories. In these ten references, only field case histories that were applicable to their formulation were studied instead of a wide range of case histories with differ-
ing slide surfaces and slide mass geometries, soil types, shear strength, and pore-water pressure conditions. For example, Azzouz et al. (1981) and Gens et al. (1988) only consider case histories of cut slopes in or embankments on saturated soft clay to verify their 3D procedure for stability of undrained, homogeneous cohesive soils.

Another disturbing feature of many existing 3D LE procedures is that a majority of them are based on the Ordinary Method of Slices, which has been shown to calculate erroneous FS. Duncan and Wright (1980) show that the Ordinary Method of Slices yields total stress FS that are 5 to 10 percent lower than accurate 2D procedures. In addition, effective stress analyses with high pore-water pressures and steep slopes may underestimate the FS by as much as 50 percent. Clearly, extending this procedure to 3D should be re-evaluated. Because the undrained shear strength is now being expressed in terms of the effective stress, i.e., the undrained strength ratio, and use of total stress analyses is declining.

Most importantly, procedures for modeling the field 3D geometry remain ambiguous. Even for the available case histories, it is difficult or tedious to model a 3D geometry that closely represents the actual failure shape in the field. For this reason, practicing engineers prefer not to use 3D procedures for the back-analysis of any failed landslide and limit use of 3D procedures in design where pre-designated failure shapes can be used.

2.6 Review and Summary of Chapter 2

- There is considerable confusion over the applicability and accuracy of existing 3D LE slope stability procedures. This is due to the infancy of 3D slope stability procedures. However, the Kettleman Hills Waste Repository failure (Seed et al. 1990) has forced government agencies and practicing engineers to perform 3D analyses. This has resulted in considerable confusion in the profession and there is a need to clarify the performance of different 3D LE slope stability procedures.

- Because, there is no strong consensus on the accuracy and applicability of different existing 3D slope stability procedures thus there are few user-friendly 3D LE slope stability computer programs available for
Based on the review of existing 3D literature, it is concluded that 3D FS is greater than 2D FS. The only studies that indicate 3D FS less than 2D FS are those by Hovland (1977), Chen and Chameau (1983), Thomaz and Lovell (1988), and Seed et al. (1990). Hovland (1977) analysis is based on OMS, which is inaccurate. Some of the assumptions used by Chen and Chameau (1983) were questioned by Hutchinson and Sarma (1985), Cavounidius (1987), and Hungr (1987) and was shown that 3D FS should always be greater than 2D FS. Thomaz and Lovell (1988) followed the assumption used by Chen and Chameau (1983). Duncan (1996) reports that Seed et al. (1990) compared 3D FS and 2D FS from analysis that did not satisfy all conditions of equilibrium. Horizontal force imbalance in Seed et al. (1990) 3D analysis was 3.7% of the weight of sliding mass. Because of small friction angle (8°-9°) along the slip surface, this could result in as much as 25% difference in calculated FS. Thus, in all of the cases where 3D FS was found to be lower than 2D FS, there are serious inaccuracies involved.

If actual shear strength is used in the design of a slope, the assumption of an infinitely long 2D failure would be on the conservative side. However, the same assumption may lead to an overestimate of the back-calculated shear strength from a 2D analysis. The importance of 3D effects on the FS and back-calculated soil shear strengths needs to be quantified.

The use of field case histories can significantly clarify the important variables in 3D slope stability analyses and provide an insight into the assumptions that should be used in a 3D stability procedure. The use of field case histories, along with sound mechanics, presents a new approach to developing a 3D slope stability procedure.
### 2.7 Tables

Table 2.1: Summary of existing 3D limit equilibrium based slope stability procedures

<table>
<thead>
<tr>
<th>Reference</th>
<th>Theoretical Basis for 3D Procedure</th>
<th>Statical Assumptions</th>
<th>Equilibrium Conditions Satisfied</th>
<th>Applicable Soil Type(s)</th>
<th>Shape of 3D Shear Surface</th>
<th>Computer Program Available</th>
<th>Factor of Safety Histories</th>
<th>Field Case 3D&gt;2D Analyzed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agnanosti (1969)</td>
<td>Morgenstern and Price’s procedure</td>
<td>LE on the sliding sides of each slice</td>
<td>Horizontal &amp; vertical force and moment</td>
<td>Any homogeneous</td>
<td>Unspecified</td>
<td>None</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Baligh &amp; Az-zouz (1975)</td>
<td>Extension of circular arc procedure</td>
<td>Same as ordinary method of slices by Fellenius (1936)</td>
<td>Moment</td>
<td>Undrained homogeneous cohesive</td>
<td>Cylinder w/ conical or ellipsoidal ends</td>
<td>STAB3D</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Hovland (1977)</td>
<td>Ordinary method of slices</td>
<td>Side forces on vertical sides of each column are zero</td>
<td>Moment</td>
<td>Any homogeneous</td>
<td>Cylinder w/ conical ends</td>
<td>None</td>
<td>Yes, for cohesive, No cohesive, No for cohesionless</td>
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Table 2.1 – Continued

<table>
<thead>
<tr>
<th>Reference</th>
<th>Theoretical Basis for 3D Procedure</th>
<th>3D Procedure Satisfied Type(s) Surface</th>
<th>Shape of 3D Shear Program</th>
<th>Computed Field</th>
<th>Factor of Safety</th>
<th>Applicable Conditions Soil</th>
<th>Reference Theoretical Statical Equilibrium</th>
<th>Applicable Shape of Computer</th>
<th>Applicable Shape of Computer Factor of Field Case</th>
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<tbody>
<tr>
<td>Chen &amp; Chameau (1983)</td>
<td>Spencer’s procedure</td>
<td>Inter-slice forces have same inclination throughout mass; inter-column shear forces are parallel to column base</td>
<td>Horizontal &amp; vertical force and moment</td>
<td>Any homogeneous Cylinder w/conical or ellipsoidal ends</td>
<td>LEMIX &amp; FEPSON</td>
<td>Yes, for cohesive, No for cohesionless</td>
<td>Horizontal &amp; vertical force and moment</td>
<td>Any homogeneous Cylinder w/conical or ellipsoidal ends</td>
<td>LEMIX &amp; FEPSON</td>
</tr>
<tr>
<td>Lovell (1984)</td>
<td>Spencer’s procedure</td>
<td>Inter-column shear forces are parallel to column base; resultant forces have same inclination throughout a row of slices</td>
<td>Horizontal &amp; vertical force and moment</td>
<td>Any Homogeneous spoon shape</td>
<td>BLOCK3</td>
<td>Not always</td>
<td>Horizontal &amp; vertical force and moment</td>
<td>Any Homogeneous spoon shape</td>
<td>BLOCK3</td>
</tr>
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<td></td>
<td></td>
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<th>Equilibrium Conditions Satisfied</th>
<th>Applicable Soil Type(s) Surface</th>
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<th>Factor of 3D&gt;2D</th>
<th>Field Case Analyzed</th>
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<tr>
<td>Thomaz &amp; Lovell (1988)</td>
<td>Inter-slice forces have same inclination throughout mass; inter-column shear forces are parallel to column base</td>
<td>Horizontal &amp; vertical forces and moment</td>
<td>Any Homogeneous</td>
<td>Symmetrical 3D-PCSTABL</td>
<td>Yes, for cohesive, Not always for cohesionless</td>
<td></td>
<td></td>
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<tr>
<td>Dennhardt &amp; Forster (1985)</td>
<td>Whole body failure procedure</td>
<td>Equilibrium equations are integrals of acting stresses integrated over whole failure body</td>
<td>Horizontal &amp; vertical forces and moment</td>
<td>Any Symmetrical None</td>
<td>Yes</td>
<td>No</td>
<td></td>
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<tr>
<td>Leshchinsky, Baker &amp; Silver (1985)</td>
<td>None</td>
<td>Horizontal &amp; vertical forces and moment</td>
<td>Any Homogeneous</td>
<td>Symmetrical None</td>
<td>Yes</td>
<td>No</td>
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Table 2.1 – Continued

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<th>Factor of Safety</th>
<th>Field Case Histories</th>
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<tbody>
<tr>
<td>Ugai (1985)</td>
<td>3D LE &amp; variational calculus</td>
<td>Distribution of normal forces is continuous &amp; differentiable on failure surface</td>
<td>Horizontal &amp; vertical forces and moment</td>
<td>Homogeneous cohesive</td>
<td>Cylinder w/ curved ends</td>
<td>None</td>
<td>Yes</td>
<td>No</td>
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<td>Ugai (1988)</td>
<td>Ordinary method of slices &amp; Fellenius</td>
<td>Resultant of inter-column forces acting on vertical sides of column is parallel to the base of column</td>
<td>Moment</td>
<td>Any Homogeneous</td>
<td>Cylinder w/ ellipsoid ends</td>
<td>None</td>
<td>Yes, for cohesive, No for cohesionless</td>
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<tr>
<td>Hungr (1987), Hungr et al (1989)</td>
<td>Bishop’s simplified procedure</td>
<td>Same as Bishop’s simplified procedure; vertical inter-column shear is neglected</td>
<td>Vertical force and moment</td>
<td>Any Homogeneous</td>
<td>Symmetrical</td>
<td>CLARA-3</td>
<td>Yes</td>
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<th>Shape of 3D Shear Surface Available</th>
<th>Computer Program</th>
<th>Factor of Safety</th>
<th>Field Case Histories</th>
<th>3D→2D Analyzed</th>
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<tbody>
<tr>
<td>Hungr et al. (1989)</td>
<td>Janbu’s simplified procedure</td>
<td>Same as above</td>
<td>Horizontal and vertical force</td>
<td>Any Homogeneous</td>
<td>Symmetrical</td>
<td>CLARA</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
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<tr>
<td>Hungr (2001)</td>
<td>Morgenstern and Price’s procedure</td>
<td>Interslice force function as in Morgenstern and Price’s procedure, resultant of side force parallel to column base, and horizontal shear stress transmitted from laterally adjacent columns</td>
<td>Horizontal &amp; vertical forces and moment</td>
<td>Homogeneous, Symmetrical</td>
<td>CLARA-W</td>
<td>Yes</td>
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<th>Applicable Soil Type(s)</th>
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<th>Analyzed</th>
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<tr>
<td>Gens et. al. (1988)</td>
<td>Extension of circular arc</td>
<td>Shear force is parallel to slide surface (similar to Azzouz &amp; Baligh, 1975)</td>
<td>Moment</td>
<td>Homogenous isotropic</td>
<td>Cylinder w/ various end geometries</td>
<td>F3SLOP &amp; DEEPYL</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>Xing (1988)</td>
<td>Ordinary method of slices</td>
<td>Lateral pressure of soil causes an end force which acts on base of each column</td>
<td>Horizontal &amp; vertical forces and moment</td>
<td>Any Homogeneous isotropic</td>
<td>Elliptic revolution</td>
<td>None</td>
<td>Yes</td>
<td>No</td>
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<tr>
<td>Michalowski (1989)</td>
<td>Upper bound approach</td>
<td>Surcharges are vertical; deformations before limit state are negligible; rigid motion boundary</td>
<td>Horizontal &amp; vertical forces</td>
<td>Drained homogenous isotropic</td>
<td>Symmetrical None</td>
<td>Yes</td>
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<th>Equilibrium Conditions Satisfied</th>
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<th>Computer Program</th>
<th>Safety</th>
<th>Field Histories</th>
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</thead>
<tbody>
<tr>
<td>Seed, Mitchell &amp; Seed (1990)</td>
<td>Multiple Block Analysis (MBA)-</td>
<td>Block boundaries are vertical; lateral forces acting on boundaries are horizontal; no lateral shear forces on inter-block boundaries</td>
<td>Horizontal &amp; vertical forces</td>
<td>Composite geosynthetic-compacted clay liner system</td>
<td>Any</td>
<td>None</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>MBA allowing differential movement of slide mass</td>
<td>Block boundaries are vertical; inter-column contact forces are inclined @ 20 degrees to the horizontal</td>
<td>None</td>
<td>Composite geosynthetic-compacted clay liner system</td>
<td>SS3D</td>
<td>Yes</td>
<td>Yes</td>
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<th>Equilibrium Conditions Satisfied</th>
<th>Applicable Soil Type(s)</th>
<th>Shape of 3D Shear Surface Available</th>
<th>Computer Program 3D≥2D</th>
<th>Safety History</th>
<th>Factor of Field Case Analyzed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leshchinsky &amp; Huang (1992)</td>
<td>3D LE &amp; variational calculus procedure of columns</td>
<td>None</td>
<td>Horizontal &amp; vertical forces and moment</td>
<td>Any</td>
<td>Symmetrical SS3D</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Lam &amp; Fredlund (1993)</td>
<td>Morgenstern &amp; Price’s 5 Interslice force functions similar to Morgenstern &amp; Price’s</td>
<td>Horizontal &amp; vertical forces and moment</td>
<td>Any</td>
<td>Generalized 3D-SLOPE</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
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<tr>
<td>Huang &amp; Tsai (2000)</td>
<td>Bishop’s simplified procedure</td>
<td>Similar to Bishop’s simplified procedure, vertical shear neglected</td>
<td>Moment parallel and transverse to sliding and vertical force equilibrium</td>
<td>Any</td>
<td>Asymmetrical No / at least partly spherical</td>
<td>Yes</td>
<td>No</td>
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<table>
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<th>Reference</th>
<th>Theoretical Basis for 3D Procedure</th>
<th>Statical Assumptions</th>
<th>Equilibrium Conditions Satisfied</th>
<th>Applicable Soil Type(s)</th>
<th>Shape of 3D Shear Program</th>
<th>Computer Safety Available 3D≥2D Analyzed</th>
<th>Field Case Histories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Huang et al. (2002)</td>
<td>Janbu’s generalized procedure</td>
<td>Similar to Janbu’s generalized</td>
<td>Horizontal and vertical force and moment equilibrium</td>
<td>Any</td>
<td>Asymmetrical No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Chang (2002)</td>
<td>Sliding block system</td>
<td>contacts between blocks are inclined, lines of intersection of block contacts are parallel</td>
<td>Force equilibrium</td>
<td>Loose sands, NC clays, and materials under undrained loading</td>
<td>Asymmetrical SSA-3D</td>
<td>Yes</td>
<td>No</td>
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<tr>
<td>Chen et al. (2001a, 2003, 2006)</td>
<td>Spencer’s procedure</td>
<td>Parallel inter-slice force, similar to Spencer’s procedure</td>
<td>Horizontal &amp; vertical forces and moment</td>
<td>Any</td>
<td>Asymmetrical STAB-3D</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
2.8 Figures

\( W = \) total weight of the slice of width \( b \) and height \( h \)
\( P = \) total normal force on the base of the slice over a length \( l \)
\( S_m = \) shear force mobilized on the base of the slice. It is a percentage of the shear strength as defined by the Mohr-Coulomb equation. That is, \( S_m = l \left(c' + \frac{|P|}{b} - u \right) \) \( \tan \phi' \) \( F \) where \( c' = \) effective cohesion parameter, \( \phi' = \) effective angle of internal friction, \( F = \) factor of safety, and \( u = \) porewater pressure
\( R = \) radius or the moment arm associated with the mobilized shear force \( S_m \)
\( f = \) perpendicular offset of the normal force from the center of rotation
\( x = \) horizontal distance from the slice to the center of rotation
\( a = \) angle between the tangent to the center of the base of each slice and the horizontal

\( E = \) horizontal interslice forces
\( L = \) subscript designating left side
\( R = \) subscript designating right side
\( X = \) vertical interslice forces
\( k = \) seismic coefficient to account for a dynamic horizontal force
\( e = \) vertical distance from the centroid of each slice to the center of rotation

A uniform load on the surface can be taken into account as a soil layer of suitable unit weight and density. The following variables are required to define a line load:

\( L = \) line load (force per unit width)
\( \omega = \) angle of the line load from the horizontal
d = perpendicular distance from the line load to the center of rotation

The effect of partial submergence of the slope or tension cracks in water requires the definition of additional variables:

\( A = \) resultant water forces
\( a = \) perpendicular distance from the resultant water force to the center of rotation

---

Figure 2.1: Composite sliding surface and the forces/variables associated with each slice (Fredlund and Krahn 1977).
Figure 2.2: Janbu’s correction factor for simplified procedure (modified from Janbu 1973).

Figure 2.3: Variation of FS with respect to moment and force equilibrium vs angle of side forces—Spencer’s procedure (Fredlund and Krahn 1977).
Figure 2.4: Interslice force function $f(x)$ for Morgenstern and Price's procedure (Fredlund and Krahn 1977).

Figure 2.5: Weighted average procedure (after Lambe and Whitman 1969).
Figure 2.6: Central cylinder with conical or ellipsoidal ends (after Baligh and Azzouz 1975).

Figure 2.7: Effects of shear surface geometry on the FS of vertical cuts (after Baligh and Azzouz 1975).
Figure 2.8: Front view of failure surface (after Chen and Chameau 1982).

Figure 2.9: Schematic model of BLOCK3 (after Lovell 1984).
Figure 2.10: Spoon shaped failure divided into columns (after Lovell 1984).

Figure 2.11: Translated coordinate system for the assumed symmetrical problem (after Leshchinsky and Baker 1986).
Figure 2.12: (a) Isometry of rotational sliding body, symmetric with respect to a central vertical plane, divided into series of columns (only the bases of active columns are shown); (b) vertical cross-sections of the sliding body in the plane of the axis of rotation (each figure only represents only one half of the body) (after Hungr 1987).
Figure 2.13: Method used to locate the reference axis for non rotational surfaces: (a) profile; (b) plan of the slide (after Hungr 1987).
CHAPTER 3

FIELD FAILURE SURFACE GEOMETRY

3.1 Introduction

The validity of any analysis depends on the degree to which the analysis captures the field mechanism and the accuracy to which the engineering properties of the materials involved and field geometry are modeled in the analysis. A common shortfall in existing 3D modeling of landslides is some/most of the failure surface geometries do not appear to model field failure surface geometries. For example, actual landslides are not infinitely long and do not have a cylindrical shape as hypothesized by prior researchers (i.e., Baligh and Azzouz 1975, Hovland 1977, and Chen & Chameau 1983). Similarly, natural slides do not appear to have conical ends as used by Baligh and Azzouz 1975, Hovland 1977 and Chang 2002.

Determining the failure surface geometries that resemble field failure surface geometries is important in developing an accurate 3D stability methodology because the shape of the failure surface influences the calculated FS. Therefore, this research utilized only failure surface geometries that match field shapes.

Most landslides exhibit either a rotational (curved) or translational (planar) failure mode/surface. Existing literature and landslide cases were studied along-with continuum mechanics modeling to determine the actual shape of rotational and translational landslides.

3.2 Types of Landslides

A number of criteria exist for naming the types of landslides, e.g., Skempton and Hutchinson (1969), and Varnes (1978). The criteria by Varnes (1978) is
widely accepted criteria and is based on the type of movement and material involved. In this scheme, a landslide is classified and described by two nouns: each describing the material and movement respectively (e.g., earth slide, rock fall, debris flow). The materials are grouped in three categories as either (1) rock, (2) debris (predominantly coarse material), and (3) earth (predominantly soil/fine material). Movements are grouped in the following five categories: (1) falls, (2) topples, (3) landslides, (4) spreads, and (5) flows. The resulting classifications are given in Table 3.1 and examples of landslide occurrences are given in Figure 3.1. This study focuses on the specific area of slides and their field geometries.

The early description of the dimensions and geometry of a landslide are given by Varnes (1978) using a cutaway diagram (Figure 3.2). Subsequently, the International Association of Engineering Geologists (IAEG) Commission on Landslides (1990) produced the revised definitions of landslide features and dimensions shown in Figure 3.3 and Figure 3.4, respectively. The total length of the landslide in IAEG (1990) is denoted by dimension L, which is described as minimum distance from tip of landslide to crown (see number 7 in Figure 3.4). However, this definition is different from common geotechnical engineering practice where length, L, of a slide is usually defined as horizontal extent of the landslide. In addition, the IAEG (1990) definitions does not include the height of the landslide, H, which is also of interest in geotechnical engineering practice.

To avoid confusion, and to include height in the dimensions of a landslide, this study uses the letters W, H, and L to represent width, height, and length respectively, of a landslide. A slope is considered to lie in the \( yz \)-plane and width, W, of the slope is in the \( x \)-direction. Figure 3.5 shows that W is perpendicular (\( xy \)-plane) to the direction of sliding, L is the horizontal extent of slide parallel to direction of sliding (\( yz \)-plane), and H is the vertical distance between crown and toe of slide (\( yz \)-plane).

A landslide is defined as a downward movement of a rock or soil mass that occurs dominantly on a failure surface or a relatively thin zone of intense shear strain (Cruden and Varnes 1996). The movement does not occur simultaneously along the entire failure surface but starts with the enlarging of an area of local failure. The displaced mass may go beyond the failure surface covering the original ground surface. Most of the time, the initial signs of the slide are the cracks that develop on the original ground surface.

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and mark the location of the main scarp. Based on the observed mode of sliding, the slides are classified as rotational, translational, or a combination of both, which is called a compound slide as shown in Figure 3.6. Rotational and translational slides do not involve internal deformations while compound slides have internal deformations.

3.2.1 Rotational Slides

A rotational slide moves along a curved and concave failure surface (Cruden and Varnes 1996). Due to the circular or ellipsoidal shape of the failure surface, the displaced mass may move downslope with less internal deformation, i.e., moving the head of the displaced material almost vertically downward and the upper surface of the displaced material tilting backwards towards the back-scarp. Occasionally, the lateral margins of the failure surface may be high and steep causing the flanks to move down into the depletion zone or area vacated by the slide mass. If the slide has considerable width, then the failure surface may be roughly cylindrical. The axis of the failure surface is parallel to the axis about which the slide rotates. Figure 3.7 shows two examples of rotational slides. The rotational nature of slides is evident by the back-rotated lake deposit beds and trees in Figure 3.7 (a) and (b), respectively. Skempton and Hutchinson (1969) found that rotational slides in soils generally have a ratio of depth (see $D_r$, i.e., number 6 in Figure 3.4) to length of failure surface (see $L_r$, i.e., number 4 in Figure 3.4), $D_r/L_r$, between 0.15 and 0.33. This type of slide occurs mostly in homogeneous materials so, rotational slides frequently occur in manmade slopes.

3.2.2 Translational Slides

A translational slide displaces the slide mass along a planer or slightly undulating failure surface, which results in the slide mass sliding out over the ground surface (Figure 3.1(b)) as a block with little or no internal deformation. The failure surface of a translational slide is generally channel shaped in cross-section from crown to toe (Hutchinson 1988) and it usually follows discontinuities like joints, faults, bedding surfaces, or other weak layers such as a pre-existing shear surface or geosynthetic interface in a landfill liner.
system. These types of slides are more common when a stronger material is underlain by a weaker material(s), such as a cohesive soil and/or geosynthetic interface (Stark and Eid 1998). Figure 3.8 shows two examples of translational slides. Due to the weak nature of the underlying material(s), a translational failure can occur in relatively flat slopes. Stark and Eid (1998) also report that slopes failing in translational mode usually involve either a significantly higher or lower mobilized shear strength along the back scarp and sides of the slide mass than along the base, e.g., upstream slope failure in Waco dam (Beene 1967; Wright and Duncan 1972), and the slope failure in the Kettleman Hills hazardous waste repository (Seed et al. 1990) failure surface, respectively. Translational slides in natural slopes can be shallower than rotational slides. For example, Skempton and Hutchinson (1969) found translational slides in soils generally have a $D_r/L_r$ less than 0.1 versus 0.15 to 0.33 for rotational slides.

### 3.2.3 Compound Slides

A compound slide is intermediate between rotational and translational slides. Compound slides have displacement along a complex curved or non-circular failure surface and usually have internal deformation and shearing along surfaces within the displaced material which results in the formation of intermediate scarps. Often the sudden decrease in downslope dip of the failure surface is marked on the displaced mass by uphill facing scarps and the subsidence of blocks of displaced material to form depressed areas known as graben. A compound slide usually occurs due to presence of a weak layer or the boundary between weathered and un-weathered material. These zones mark the location of the failure surface (Hutchinson, 1988). In single compound slides, the width of the graben may be proportional to depth to the failure surface (Cruden et al. 1991). The example of rotational slide shown in Figure 3.7(b) shows some signs of internal deformation near the back scarp, thus it may be classified as a compound slide instead of a rotational slide as described by Varnes (1978).
3.3 Review of the Existing Idealized 3D Failure Shapes

Each 3D LE slope stability procedure uses an assumption regarding the 3D shape of the slide mass. Prior researchers have proposed various idealized 3D failure shapes (Figure 3.9) to represent the geometry of 3D field failure surfaces. These surfaces include ellipsoidal, two cones attached at bases, and a cylinder of finite length terminating at ends with different possible geometries. A brief description of these idealized shapes is presented below.

3.3.1 Cone

Ugai (1985) uses two cones connected at the cone base in his 3D procedure to analyze vertical cohesive slopes. The procedure uses LE and variational calculus to calculate FS.

3.3.2 Ellipsoidal Shapes

Xing (1988) assumes a failure surface of elliptic revolution for his 3D analysis of concave slopes in plan view. The failure mass, which is assumed to be symmetrical, is divided into vertical columns. Xing (1988) concludes that 3D stability increases as the area of the failure surface increases. Therefore, the FS for ellipsoidal failure surface is smaller than that of the failure surface consisting of a cylinder of finite length with an ellipsoid attached to the ends. This is caused by the former failure surface area being smaller than the latter one when both the lengths of the failure masses and sections at the symmetric plane of the two failure surfaces are given. Xing (1988) also adds that an ellipsoidal shape requires fewer geometric parameters than a cylinder with an ellipsoidal ends so it is easier to use.

Hungr et al. (1989) use a profile of a spherical sliding surface in a cohesive material to compare their 3D algorithm for the extension of Bishop’s (1955) simplified procedure with the closed form solution. The 3D FS obtained for the assumed surface is in agreement with the FS from closed-form solution (Hungr et al. 1989). Similarly the computer program CLARA-W (Hungr 2001) can analyze a slope using an ellipsoidal failure surface.
3.3.3 Cylinder

Of the various failure surface shapes considered by previous researchers, the cylinder, discussed below, appears to be the most common. Failure surfaces are considered to be a cylinder of finite length terminating with ends of different geometry (e.g., plane ends, cones, ellipsoids, etc.). At least two researchers use a cylindrical central part that terminates with plane ends. Ugai (1985) uses a variational calculus method to compare the critical 3D FS to that of a cylinder with plane ends. Gens et al. (1988) derived analytical expressions for the 3D FS for toe and base failures, assuming cylindrical failure surfaces with plane ends.

Baligh and Azzouz (1975) use a cone of height, $l_n$, attached to a cylinder of length, $l_c$, to model the stability of cohesive slopes. 3D FS of the assumed failure geometry was found to be greater than for other failure surface geometries, e.g., cylinder with ellipsoid ends, and is not the critical surface. Hovland (1977) uses a cone shaped shear surface attached to a cylinder for his analysis of vertical cuts in clay or embankments on clay. Hovland (1977) concludes that for conical end geometry, 3D FS is considerably higher (44%) from 2D FS for all values of $c$ and $\phi$. Baligh and Azzouz (1975) also analyzed a 3D failure shape consisting of a central cylinder of length, $l_c$, with ellipsoidal ends having a semiaxis, $l_e$. The results of their computations show that elliptic shear surfaces consistently yield lower 3D FS than conical shaped failure surfaces. Baligh and Azzouz (1975) conclude that a cylinder with ellipsoidal ends may provide a better simulation of the actual failure surface.

Chen and Chameau (1982) assume a 3D failure surface in a homogeneous soil, that consists of a central cylinder attached to two semi-ellipsoids at the ends. The cross-section of the central cylinder is a circle using the 2D computer program STABL2 (Boutrup, 1977). The 3D failure surface is generated from the 2D critical circular failure surface. The cylinder has a length of $2 \times l_c$ and the minor axis of the semi-ellipsoid has a length $l_a$. Other researchers, e.g., Lovell (1984), Ugai (1985), Hungr (1987), and Gens et al (1988), have also considered a central cylinder with ellipsoidal or curved ends in their formulation of 3D procedures.

Chang (2002) performs a 3D analysis of rotational failure in a vertical cut in purely cohesive material. The geometry of the failure surface is assumed to be a surface of revolution with a radius that is the same as the cut height
and its axis of rotation along the cut crest. The slide mass consists of a central cylinder with two conical ends.

In summary, a common shortfall in existing 3D modeling of landslides is most of the failure surface geometries do not appear to model field failure surface geometries. For example, actual landslides are not infinitely long and do not have a cylindrical shape. Similarly, natural slides do not appear to have conical ends. Generally, the 3D field failure surface is ellipsoidal or a block for rotational and translational slides, respectively.

3.4 3D Field Failure Shapes of Rotational and Translational Slides

A number of field case histories were reviewed to determine the 3D shape that best models field failure surfaces for rotational and translational landslides in 3D. In addition, 3D FE analyses of these two slope models were performed to investigate the failure surface geometry generated by FE analysis. Continuum mechanics procedures are believed to achieve better simulation of field conditions than LE so the failure surfaces generated by FE analysis can assist in verifying the 3D failure surface shape for a LE slide model. FE analyses were performed using program PLAXIS 3D (Brinkgreve and Broere 2004). More details about the FE analysis and PLAXIS 3D are presented in Chapter 4.

3.4.1 3D Model for Rotational Landslides

Figure 3.10 shows pictures of nine rotational landslides from around the world. It is evident from the pictures presented in Figure 3.10 that a rotational slide has a 3D geometry that approximates an ellipsoid.

The aspect ratio, \( W_r / L_r \) (see number 2 and 4 in Figure 3.4), of the all the rotational landslides in Figure 3.10 is between 0.8 to 1.2. The aspect ratio is the ratio between ellipsoid semi-axes perpendicular (\( W_r \)) and parallel (\( L_r \)) with the direction of sliding.

Because rotational failures usually occur in homogeneous materials, a 3D analysis of a slope model was performed to evaluate the failure surface generated by the FE procedure, i.e., PLAXIS 3D. Figure 3.11 shows the 3D
FE model slope in a homogeneous material. Figure 3.11 includes the slope model (undeformed mesh), deformed mesh, and displacement vectors. The displacement vectors are helpful in identifying the location and shape of the failure surface because the vectors mark the boundary between stable and unstable portions of the model. Displacement vectors in Figure 3.11 (c) show that the overall failure surface is approximately ellipsoidal. Figure 3.12 shows the maximum displacement contour plots at four different 2D cross-sections across the slope model. Because the slope model is symmetric on both sides of the central cross-section, the cross-sections at the end, one-sixth width, one-third width, and one-half (middle cross-section) width of the slope are shown in Figure 3.12. Figure 3.12 shows the failure surface in all cross-sections is circular (rotational). In addition the failure surface is deepest in the middle cross-section and gradually becomes shallower towards the end cross-sections. This indicates that the 3D failure surface in homogeneous material is ellipsoidal which is in accordance with the pictures presented in Figure 3.10.

Review of field case histories and the FE model suggest that the failure surface of a rotational landslide can be approximated by an ellipsoid. The landslide volume may be estimated by considering half an ellipsoid (Cruden and Varnes 1996) with semi-axes $D_r$, $W_r/2$, $L_r/2$ (see Figure 3.4). Referring to Figure 3.13 (a), the volume of the slide can be estimated by computing the volume of an ellipsoid (Beyer, 1987) as:

$$VOL_{opt} = \frac{4}{3} \pi a b c$$  (3.1)

Where $a$, $b$, and $c$ are semi-major axes. Thus, the volume of a spoon shape corresponding to one half of an ellipsoid is given as:

$$VOL_{1s} = \frac{1}{2} \frac{4}{3} \pi a b c = \frac{4}{6} \pi a b c$$  (3.2)

Comparing $VOL_{1s}$ with landslide dimensions shown in Figure 3.13(b), where, $a = D_r$, $b = \frac{W_r}{2}$, and $c = \frac{L_r}{2}$, the volume of ground displaced by a landslide is:

$$VOL_{1s} = \frac{4}{6} \pi a b c = \frac{4}{6} \pi D_r \frac{W_r}{2} \frac{L_r}{2} = \frac{4}{6} \pi D_r W_r L_r$$  (3.3)

This is the volume of slide mass before the landslide moves. The displaced material usually dilates after the movement so the volume can increase after
movement occurs. Assuming an average dilation of 33 percent (Nicoletti and Sorriso-Valvo 1991), the actual slide volume can be estimated as:

\[ \frac{4}{3} \frac{1}{6} \pi D_r W_r L_r \approx \frac{1}{6} \pi D_d W_d L_d \approx 4D_r W_r L_r \approx 3D_d W_d L_d \]  

(3.4)

3.4.2 3D Model for Translational and Compound Slides

Figure 3.14 shows pictures of three translational and compound landslides. Figure 3.14(a) shows the Sabastopol landslide moving as a block with no internal deformation (trees and houses are unaffected). Translational failures are more common when a stronger material is underlain by a weaker material(s). Because of the weak nature of the underlying material(s), the failure surface remains in the weaker material across the slope and extends towards ends resulting in vertical or nearly vertical sides with little variation in different cross-sections. The near vertical sides minimizes the strength contribution from the stronger material. Stark and Eid (1998) indicate that vertical sides provide the minimum amount of shear resistance because of a minimal area of shear surface. Figure 3.14(a) and (b) show vertical sides in translational failures. The possibility of a graben in a translational slides is shown in Figure 3.15. Figure 3.15(a) shows a distinct graben in a Ohio landfill slope failure. Similarly, the Government Hill School Building in Anchorage, Alaska after the 1964 earthquake (Figure 3.15(a) and (b)) moved vertically down in a graben without tilting backwards as in the case of a rotational slide.

Different translational case histories reported in the published literature were reviewed to study the shape that best models the field failure surface for a translational landslide. Figure 3.16 shows 2D critical cross-sections of ten translational case histories. Because, there is little variation in these cross-sections across a translational slide, a 2D critical cross-sections can be used to develop a 3D model. It is evident from the 2D cross-sections presented in 3.16 that the back scarp is inclined and failure surface follows a weak layer or discontinuity like joints, faults, bedding surfaces, or other weak layers such as a pre-existing shear surface or geosynthetic interface in a landfill liner system.

Because translational failures commonly occur when a stronger material
is underlain by a weaker material(s), a 3D slope model was developed to evaluate the failure surface using the FE procedure. Figure 3.17 shows the 3D FE slope model that incorporates three materials: upper, lower, and bedrock. To simulate the stronger upper and weaker lower materials, linear shear strength material envelopes passing through the origin with friction angles of $30^\circ$ and $8^\circ$ were assigned to these materials, respectively. Bedrock was added to ensure that the failure surface followed the weak layer. In addition, the bedrock was assumed to slope at 3 percent to simulate a natural bedding plane or landfill liner system. Fully fixed (no displacement) boundary conditions were imposed at the ends of the model to incorporate shear resistance along the vertical sides of the slide mass. Figure 3.17 includes the slope model (undeformed mesh), deformed mesh, and resulting displacement vectors. The displacement vectors in Figure 3.17 show that the failure surface is translational.

Figure 3.18 presents the maximum displacement contour plots at the end, one-sixth width, one-third width, and one-half (middle cross-section) width of the slope model. Figure 3.18 shows that except for the end cross-section, the failure surface in the other three internal cross-sections (one half of axis of symmetry) follows the weak layer and has an inclined back scarp. There is less variation in these cross-section which indicate that the failure occurs as a block. This is different from a rotational failure where the failure surface is deepest in the middle cross-section and gradually becomes shallower towards the end cross-sections. The failure surface shape is in accordance with the field failure surface geometry observed for the Sabastopol landslide (see Figure 3.14) which moved as a block.

Review of field case histories and the FE model suggest that the 3D model geometry presented by Arellano and Stark (2000) represents the field failure surface geometry for translational landslides. More details about the 3D model for translational landslide is presented in Chapter 6. The slope model presented by Arellano and Stark (2000) is essentially a rectangle whereas actual landslide masses are more rounded at the head and have other rounded or curved areas. However, considering the modeling limitations in LE slope stability software this appears to be a reasonable approximation of the field failure surface for a translational landslide. Thus, the slope model proposed by Arellano and Stark (2000) was used for analysis of translational landslides in this study.
3.5 Review and Summary of Chapter 3

- Natural landslides are classified as rotational, translational, and compound.

- Rotational and translational slides do not involve internal deformation while compound slides have internal deformations and thus are “compound”.

- A 3D slope must use a model that resembles the field failure surface geometry to obtain accurate values of 3D FS.

- Rotational slides occur most frequently in homogeneous material. The study of field failure surfaces of rotational slides and corresponding FE analyses indicates that the 3D failure surface in homogeneous material is ellipsoidal.

- Translational failures commonly occur when a stronger material is underlain by a weaker material(s). The model of translational landslides presented by Arellano and Stark (2000) reasonably approximates the field failure surface geometry and is in accordance with the failure surface generated by the FE model.
### 3.6 Tables

Table 3.1: Classification of slope movement

<table>
<thead>
<tr>
<th>Type of Movement</th>
<th>Bedrock Type</th>
<th>Type of Material Engineering Soils</th>
<th>Predominantly Coarse</th>
<th>Predominantly Fine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall</td>
<td>Rock fall</td>
<td>Debris fall</td>
<td>Earth fall</td>
<td></td>
</tr>
<tr>
<td>Topple</td>
<td>Rock topple</td>
<td>Debris topple</td>
<td>Earth topple</td>
<td></td>
</tr>
<tr>
<td>Slide</td>
<td>Rock slide</td>
<td>Debris slide</td>
<td>Earth slide</td>
<td></td>
</tr>
<tr>
<td>Rotation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Translational</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spread</td>
<td>Rock spread</td>
<td>Debris spread</td>
<td>Earth spread</td>
<td></td>
</tr>
<tr>
<td>Flow</td>
<td>Rock flow</td>
<td>Debris flow</td>
<td>Earth flow</td>
<td></td>
</tr>
</tbody>
</table>
3.7 Figures

Figure 3.1: Major types of landslide movements (USGS 2004).
Figure 3.2: Description of landslide components (after Varnes 1978 modified by USGS 2004).
### Figure 3.3: Definition of landslide features (IAEG Commission on Landslides 1990).

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Crown</td>
<td>Practically undisplaced material adjacent to highest parts of main scarp</td>
</tr>
<tr>
<td>2</td>
<td>Main scarp</td>
<td>Steep surface on undisturbed ground at upper edge of landslide caused by movement of displaced material (13, stippled area) away from undisturbed ground; it is visible part of surface rupture (10)</td>
</tr>
<tr>
<td>3</td>
<td>Top</td>
<td>Highest point of contact between displaced material (13) and main scarp (2)</td>
</tr>
<tr>
<td>4</td>
<td>Head</td>
<td>Upper parts of landslide along contact between displaced material and main scarp (2)</td>
</tr>
<tr>
<td>5</td>
<td>Minor scarp</td>
<td>Steep surface on displaced material of landslide produced by differential movements within displaced material</td>
</tr>
<tr>
<td>6</td>
<td>Main body</td>
<td>Part of displaced material of landslide that overlies surface of rupture between main scarp (2) and toe of surface of rupture (11)</td>
</tr>
<tr>
<td>7</td>
<td>Foot</td>
<td>Portion of landslide that has moved beyond toe of surface of rupture (11) and overlies original ground surface (20)</td>
</tr>
<tr>
<td>8</td>
<td>Tip</td>
<td>Point on toe (9) farthest from top (3) of landslide</td>
</tr>
<tr>
<td>9</td>
<td>Toe</td>
<td>Lower, usually curved margin of displaced material of a landslide, most distant from main scarp (2)</td>
</tr>
<tr>
<td>10</td>
<td>Surface of rupture</td>
<td>Surface that forms (or that has formed) lower boundary of displaced material (13) below original ground surface (20); mechanical idealization of surface of rupture is called <em>sliding surface</em> in stability analysis</td>
</tr>
<tr>
<td>11</td>
<td>Toe of surface of rupture</td>
<td>Intersection (usually buried) between lower part of surface of rupture (10) of a landslide and original ground surface (20)</td>
</tr>
<tr>
<td>12</td>
<td>Surface of separation</td>
<td>Part of original ground surface (20) now overlain by foot (7) of landslide</td>
</tr>
<tr>
<td>13</td>
<td>Displaced material</td>
<td>Material displaced from its original position on slope by movement in landslide; forms both depleted mass (17) and accumulation (18); it is stippled in Figure 1.6</td>
</tr>
<tr>
<td>14</td>
<td>Zone of depletion</td>
<td>Area of landslide within which displaced material (13) lies below original ground surface (20)</td>
</tr>
<tr>
<td>15</td>
<td>Zone of accumulation depletion</td>
<td>Area of landslide within which displaced material lies above original ground surface (20)</td>
</tr>
<tr>
<td>16</td>
<td>Depletion</td>
<td>Volume bounded by main scarp (2), depleted mass (17), and original ground surface (20)</td>
</tr>
<tr>
<td>17</td>
<td>Depleted mass</td>
<td>Volume of displaced material that overlies surface of rupture (10) but underlies original ground surface (20)</td>
</tr>
<tr>
<td>18</td>
<td>Accumulation</td>
<td>Volume of displaced material (13) that lies above original ground surface (20)</td>
</tr>
<tr>
<td>19</td>
<td>Flank</td>
<td>Undisplaced material adjacent to sides of surface of rupture; compass directions are preferable in describing flanks, but if left and right are used, they refer to flanks as viewed from crown</td>
</tr>
<tr>
<td>20</td>
<td>Original ground surface</td>
<td>Surface of slope that existed before landslide took place</td>
</tr>
<tr>
<td>No.</td>
<td>Name</td>
<td>Definition</td>
</tr>
<tr>
<td>-----</td>
<td>-------------------------------</td>
<td>------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>Width of displaced mass, $W_d$</td>
<td>Maximum breadth of displaced mass perpendicular to length, $L_d$</td>
</tr>
<tr>
<td>2</td>
<td>Width of surface of rupture, $W_t$</td>
<td>Maximum width between flanks of landslide perpendicular to length, $L_t$</td>
</tr>
<tr>
<td>3</td>
<td>Length of displaced mass, $L_d$</td>
<td>Minimum distance from tip to top</td>
</tr>
<tr>
<td>4</td>
<td>Length of surface of rupture, $L_t$</td>
<td>Minimum distance from toe of surface of rupture to crown</td>
</tr>
<tr>
<td>5</td>
<td>Depth of displaced mass, $D_d$</td>
<td>Maximum depth of surface of rupture below original ground surface measured perpendicular to plane containing $W_d$ and $L_d$</td>
</tr>
<tr>
<td>6</td>
<td>Depth of surface of rupture, $D_t$</td>
<td>Maximum depth of surface of rupture below original ground surface measured perpendicular to plane containing $W_t$ and $L_t$</td>
</tr>
<tr>
<td>7</td>
<td>Total length, $L$</td>
<td>Minimum distance from tip of landslide to crown</td>
</tr>
<tr>
<td>8</td>
<td>Length of center line, $L_{cl}$</td>
<td>Distance from crown to tip of landslide through points on original ground surface equidistant from lateral margins of surface of rupture and displaced material</td>
</tr>
</tbody>
</table>

Figure 3.4: Definition of landslide dimensions (IAEG Commission on Landslides 1990).
Figure 3.5: Definition of landslide dimensions used in study (from Arellano and Stark 2000).

Figure 3.6: Modes of sliding (combined from Abramson et al. 1996 and Varnes 1978).
Thin bedded lake deposits above failure surface are rotated.

Trees tilted backwards towards back scarp.

Figure 3.7: Examples of rotational slides: (a) Cut through rotational slide of fine-grained, thin bedded lake deposits (Varnes 1978); (b) deep rotational slide (Kieffer et al. 2006).
Figure 3.8: Examples of translational slides: (a) Distinct graben at top of Fengdian landslide, China (Fan et al. 2009); (b) cross-section of a translational slide, Beatton River, B.C., Canada (Photo by O. Hungr).
Figure 3.9: Example of idealized 3D slide masses.
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CHAPTER 4

ACCURACY OF SLOPE STABILITY ANALYSIS METHODS

4.1 Introduction

Slope stability analyses are probably one of the most important areas in geotechnical engineering. Most engineers encounter projects where a slope stability analysis is required. Considering the vast variety in available methods of slope stability analyses, e.g., limit equilibrium (LE), limit plasticity, continuum mechanics, and variational calculus, there are few guidelines for selecting of one method over another. Duncan (1996) concludes that even though it is difficult to know the correct factor of safety (FS), it is still possible to determine a sufficiently accurate values of FS. This conclusion is based on findings that all methods that are considered to be accurate provide a similar FS.

In practice, either LE or continuum methods are preferred over other methods. The choice between these two methods is more of a personal preference and availability to the user. A comparison of various LE and continuum mechanics procedures of slope stability (2D and 3D) analysis is presented in this chapter. Chapter 2 presents a detailed literature review of available 2D and 3D LE procedures which is used in this chapter. This chapter also presents information about continuum mechanics procedures that are commonly used in 3D FE and FD software. The comparison shows that either of these two methods (LE and continuum) are comparable in most of the cases but, it may be easier to use LE procedures than continuum mechanics procedures.
4.2 Review of Continuum Mechanics Method

The computational power of continuum mechanics is well known in engineering practice (Griffiths and Marquez 2007). Initially, continuum mechanics were used for deformation analyses so its utility for FS computations was not widely used. Present continuum mechanics analyses include similar failure criteria as used in LE making it possible to compute FS values. Continuum mechanics has the ability to model complex problems without simplifying assumptions which is a big advantage over LE method. Within the continuum mechanics method, finite element (FE), finite difference (FD), discrete element (DE), and boundary element (BE) are different procedures. Each procedure differs from the other in terms of solution strategy. Past researchers (Griffiths and Lane 1999; Chugh 2003; Griffiths and Marquez 2007) have performed slope stability analyses using FE and FD procedures and show that these procedures provide comparable results to LE procedures.

4.2.1 Difference between FD and FE Procedures

In a FE procedure, a continuum is divided into a number of (volume) elements consisting of a number of nodes. Each node has a number of degrees of freedom that correspond to discrete values of the unknowns in the boundary value problem to be solved. FE procedure requires that the field quantities (stress, displacement) vary throughout each element in a prescribed fashion, using specific functions controlled by parameters. The FE formulation requires the adjustment of these parameters to minimize energy. In addition, FE programs often combine the element matrices into a large global stiffness matrix and thus implicit matrix solution schemes are used.

FD procedure uses a numerical technique for the solution of differential equation sets for a given initial value and/or boundary values. FD procedure solves differential equation(s) in the form of difference equation using an explicit time marching method. An algebraic expression written in terms of field variables (for example, stress or displacement) at discrete points in space directly replaces every derivative in the set of governing equations. The general calculation procedure in FD analysis invokes the equations of motion to derive new velocities and displacements from calculated stresses and forces. Then strain rates are derived from velocities and new stresses
are derived from these strain rates. This cycle takes place within one time step which is small enough so the information cannot physically pass from one element to another in the interval.

Because FE and FD procedures produce sets of algebraic equations to solve, the resulting equations are identical (in specific cases) for the two procedures even though these equations are derived in entirely different ways.

4.2.2 Advantages of Continuum Mechanics for Slope Stability Analysis

Griffith and Lane (1999) summarize the advantages of the elasto-plastic FE procedure that is also applicable to other continuum mechanics procedures, e.g., FD, as follows:

- The shape and location of the failure surface is not required to be assumed or specified. Failure occurs naturally through the soil zones where shear strength of the soil mass is unable to sustain the shear stresses.

- Soil mass is not divided into slices (2D) or columns (3D) therefore, there is no need for simplifying assumptions about slice or column side forces.

- Deformational results can also be obtained from the analysis.

- Progressive failure may be monitored up to and including overall shear failure.

4.2.3 Boundary Conditions

Boundary conditions are required in a continuum mechanics model to extend the geometry and material properties beyond the slide mass. In addition, the boundary condition needs to be expressed in terms of applied forces or displacements. In general, use of displacement boundary conditions is preferred for slope stability applications.

For a 2D slope model, the slope is extended far away from the region where slope failure is expected to occur. An extension of the model equal to the
slope height beyond the crest and toe of slope are usually sufficient for a meaningful analysis (Griffith and Marquez 2007). In addition, appropriate fixities are applied to the 2D model. To develop a realistic 3D model, Chugh (2003) recommends extending the 3D model past the ends of the slope (in x-direction) to include the presence of material outside the slide mass or abutments, which are areas of no movement. Relative movements between slope and abutments may be allowed using interfaces at the slope-abutment contacts. The model boundary conditions of fixed ends are then placed at the ends of the extended model. In his 3D model, Chugh (2003) uses 6 m wide abutments for a 10 m high slope.

4.2.4 Properties of Soil Model

Continuum mechanics procedures require elastic soil parameters as well as other soil parameters. The other soil parameters are the same as those required in LE method, i.e., unit weight, $\gamma$, and shear strength parameters $\phi'$ and $c'$ (or $\phi_u = 0$ and $c_u$ for an undrained analysis). The elastic parameters vary for different types of software and usually include dilation angle, $\psi$, and two independent elastic parameters, e.g., combination of either Poisson’s ratio, $\nu$, and Young’s modulus, $E$, or Bulk modulus, $K$, and Shear modulus, $G$. The elastic parameters $K$ and $G$ are related to $E$ and $\nu$ via:

\[
K = \frac{E}{3(1 - 2\nu)} \quad (4.1)
\]
\[
G = \frac{E}{2(1 + \nu)} \quad (4.2)
\]
\[
G = \frac{3K(1 - 2\nu)}{2(1 + \nu)} \quad (4.3)
\]

The dilation angle, which is specified in degrees, affects the volume change of the soil during yielding. Apart from heavily overconsolidated layers, clay soils tend to show no dilation angle. The dilation angle of sands depends on both the density and the friction angle, which may range from $\phi' - 30^\circ$ for quartz sand. However, for most cases, the dilation angle is zero for $\phi'$ less than $30^\circ$. Griffiths and Lane (1999) indicate the choice of dilation angle for slope stability analysis is less significant and thus $\psi = 0$ implying no volume
change during yield may be used.

The impact of Poisson’s ratio and Young’s modulus on FS calculations are reported by Hummah et al. (2005). They show for the case of homogeneous soils, different combinations of $\nu$ and $E$ result in only a 2.5% change in FS values even though the magnitude of deformations are significantly affected. On the contrary, using different combinations of deformation properties in a three material slope problem indicate -5% to +11% difference from benchmark FS values. This finding indicates that FS values can be quite different for different stiffness ratios for multiple material cases. Therefore, it is important to properly estimate the values of $\nu$ and $E$, if deformations are required for remedial measures.

Because the elastic parameters mainly affect the deformations and have little effect on the computed FS, Hummah et al. (2005) show to obtain comparable values of FS using Strength Reduction Factor (SRF), described below, to that of LE (at least for unreinforced slopes) it is sufficient to:

- use the same $E$ value for the materials in a multiple-material model
- assume a single value of Poisson’s ratio for all materials involved
- assume a dilation angle of zero, and
- use the elastic-perfectly plastic assumption for post-peak behavior.

### 4.2.5 FS Computations

With advancement in continuum mechanics modeling, present FE and FD software offer a strength reduction technique to model progressive failure. In this technique, $c'$ and $\tan\phi'$ are reduced progressively to bring the slope to a state of a limiting equilibrium. The reduced values of $c_{\text{trial}}'$ and $\phi_{\text{trial}}'$ are defined as:

\[
c_{\text{trial}}' = \frac{c'}{SRF} \tag{4.4}
\]

\[
\phi_{\text{trial}}' = \tan^{-1}\left( \frac{\tan\phi'}{SRF} \right) \tag{4.5}
\]

where SRF is the strength reduction factor. The FS of the slope is the value of SRF to bring the slope to failure. It is evident that the definition of SRF
in Equations (4.4) and (4.5) resembles the LE FS. Material strength model for continuum analysis is also similar to standard Mohr-Coulomb model and therefore the effect of elastic parameters on the computed FS is not significant.

In the strength reduction technique, a series of simulations are performed using trial values of $SRF$ to reduce the cohesion and friction angle values until slope failure occurs. In case of an initially unstable slope, $c'$ and $\phi'$ are increased until the limiting condition is reached. The procedure used in FD program FLAC/Slope (Itasca Consulting Group 2010) uses a bracketing approach to calculate FS as follows. Initially the program estimates a representative number of steps, $N_r$, that characterizes the response time of the system by setting the cohesion to a large value making a large change to the internal stresses. The number of steps required to bring the system to equilibrium are calculated. The $N_r$ steps are executed for a given FS. The system is in equilibrium if the unbalanced force ratio is less than $10^{-3}$. The unbalanced force ratio is the ratio of net force acting on a gridpoint to the mean absolute value of force exerted by each surrounding zone. Another $N_r$ steps are executed if the unbalanced force ratio is greater than $10^{-3}$ until the unbalanced force ratio is less than $10^{-3}$. The mean value of force ratio in current and previous span of $N_r$ steps are compared. The system is in non-equilibrium if this difference is less than 10% and the loop is exited with a new non-equilibrium FS. Blocks of $N_r$ steps are continued until either (1) difference is less than 10%, (2) six such blocs are executed, or (3) force ratio is less than $10^{-3}$. The FS solution stops when the difference between upper and lower bracket is smaller than a tolerance of 0.005.

4.3 Validation Examples

A comparison of different slope stability methods/procedures was performed using 2D and 3D example problems used by earlier researchers (Fredlund and Krahn 1977; Xing 1988). 2D and 3D LE analysis were performed using software package CLARA-W (Hungr 2001). CLARA-W (Hungr 2001) has 2D as well as 3D extensions of Janbu (1954), Bishop (1955), Morgenstern and Price (1965), and Spencer (1967) procedures. 2D and 3D FE analysis were performed using PLAXIS 2D V.9 (Brinkgreve and Broere 2008) and PLAXIS
3D Tunnel V.2 (Brinkgreve and Broere 2004) respectively. Results of 2D LE and FE analysis were verified using 2D LE software XSTABL (Sharma 1996) and 2D FD software FLAC/Slope 6.0 (Itasca 2010).

4.3.1 Validation Procedure

2D analysis procedures in LE are widely accepted in geotechnical community. Therefore initially, continuum mechanics procedures were verified using 2D LE procedures as bench mark. After confirmation that FD and FE provide comparable results in 2D, 3D LE procedures were compared with 3D continuum mechanics results. 2D weighted average FS were also computed using LE method and compared with respective 3D extension in LE. Following validation procedure was adopted for the present study:

- **2D Analysis**
  - Results from LE procedures coded in CLARA-W were compared with reported results.
  - 2D LE results of CLARA-W were also verified using XSTABL.
  - Results from FD and FE were then compared with LE method using Spencer’s (1967) stability procedure in CLARA-W.

- **3D Analysis**
  - 3D LE results using different solution algorithms coded in CLARA-W were compared with the reported LE results.
  - 3D results of CLARA-W were also compared with continuum method.

- **2D Weighted Average Analysis**
  - 2D weighted average analysis were performed for the 3D example problem using different 2D LE procedures.
  - Results were compared with 3D LE and FE results.
4.3.2 2D Analysis

Figures 4.1 and 4.2 show two problems selected for 2D analysis with LE and continuum methods in this study. These two problems have been reported by Fredlund and Krahn (1977) to compare different 2D LE procedures. Subsequently, other researchers used same problems to validate their 3D procedures. Various material properties used in the reanalysis are shown in Table 4.1. The 2D problems are as follows:

- 2D Problem I - Simple slope of 2H:1V, 40 ft high, $\phi' = 20^\circ$ and $c' = 600$ psf, no weak layer, no bedrock.
- 2D problem II - Same as Problem I but with thin weak layer ($\phi' = 10^\circ$ and $c' = 0$) and bedrock.

4.3.2.1 2D Problem I - Homogeneous Material Without Weak Layer

Figure 4.3 shows the result of analysis for 2D problem I (Figure 4.1) indicating that for a similar failure surface, different methods (LE and continuum) yield essentially the same FS. The detailed comparison of results from 2D analysis of problem I are shown in Table 4.2. FS computed by Bishop (1955), Morgenstern and Price (1965) and Spencer (1967) procedures in LE method are in agreement with each other i.e FS $2.07 \pm 0.01$ (i.e less than 0.5%). In this case Bishop’s (1955) simplified procedure is expected to provide comparable results with Morgenstern and Price’s (1965) and Spencer’s (1967) procedure because a rotational failure surface is considered.

Janbu’s (1956) simplified procedure coded in XSTABL gives slightly lower (2%) FS than other LE procedures but is still within tolerable limits of 6% as described by Duncan (1996). In addition, uncorrected Janbu’s (1956) simplified procedure coded in CLARA-W gives about 8-11% lower FS values than FS computed by other procedures. The slope has a D/L ratio (see Figure 2.2) of about 0.33. Therefore Janbu’s (1973) correction factor of 1.09 is computed for a $\phi'$ and $c'$ soil. Manually applying Janbu’s (1973) correction factor to the CLARA-W result gives FS=2.04 which is now in better agreement with FS values obtained from other LE procedures.

Table 4.2 also shows the results of FE and FD procedures are in agreement with LE analysis. However when compared with Spencer’s (1967) procedure,
FS computed by FD and FE are 2% and 5% lower, respectively. Overall, 2D FS computed by LE, FD, and FE are within 5% of each other which is considered acceptable (Duncan 1996b).

4.3.2.2 2D Problem II - Weak Horizontal Layer With Bedrock

2D Problem II has the same slope geometry as 2D Problem I (see Figure 4.2), however it also includes a thin seam of weak material to simulate pre-existing shear surface or weak soil type. A thin layer was modeled in XSTABL and PLAXIS, while a discontinuity and interface was used in CLARA-W and FLAC, respectively.

Figure 4.4 shows the results of 2D analyses using LE (CLARA-W and XSTABL), FD (FLAC), and FE (PLAXIS 2D) procedures. The failure surface by all procedures follows thin weak seam and is similar in shape. Table 4.3 shows a comparison of the various methods (e.g. LE and continuum mechanics) and procedures within LE, i.e., Bishop (1955), Janbu (1956), Morgenstern and Price (1965), and Spencer (1967). Spencer’s (1967) procedure coded in CLARA-W and XSTABL gave FS=1.37 and 1.35, respectively which is in agreement with reported FS of 1.37 (Fredlund and Krahn 1977) using Spencer’s (1967) procedure.

Bishop’s (1955) procedure was originally developed for circular failure surfaces and normal forces on the slice base have no moment arm and therefore it is not included in moment equilibrium equation. Fredlund and Krahn (1977) derived moment equilibrium equation for a non circular sliding surfaces in which each normal force has a moment arm (see Figure 2.1) and moment of each normal force is added to the overall moment equilibrium equation. The FS then becomes dependent on vertical position of the reference axis (center of rotation). For an appropriately selected center of rotation, the modified procedure yields FS values that are comparable to more rigorous procedures.

Hungr (1997) show that for a weaker flat basal plane, Bishop’s (1955) simplified procedure with Fredlund and Krahn’s (1977) modification yield FS values that are similar to those obtained from rigorous procedures. On the contrary, Bishop’s (1955) simplified procedure may yield upto 30% lower FS values if the back scarp is weaker than the basal plane. In practice, the occurrence of weak basal plane is more common then weaker back scarp, for example discontinuities like bedding surfaces, or other weak layers such as a
pre-existing shear surface or geosynthetic interface in a landfill liner system. Therefore Bishop’s (1955) simplified procedure is applicable to 2D problem considered herein.

Because XSTABL does not incorporate the modification for non circular surfaces for Bishop’s (1955) procedure, therefore it was not used for this problem. Bishop’s (1955) procedure coded in CLARA-W gives FS=1.36 which confirms that Bishop’s (1955) procedure also may be used for non circular surfaces with weak basal sliding plane, provided that Fredlund and Krahn’s (1977) modification is incorporated.

FS using Janbu’s (1956) simplified procedure in XSTABL is 6% higher when compared with FS using Spencer’s (1967) procedure coded in CLARA-W. Whereas, Janbu’s (1956) procedure coded in CLARA-W yields a lower FS (FS=1.32). After applying the Janbu’s (1973) correction factor of 1.09 (computed for problem I), the corrected Janbu FS (FS=1.44) is in agreement with the FS computed by XSTABL and is 5% higher than Spencer’s (1967) procedure. Finally, FD and FE results are 1% and 11% lower, respectively, when compared with Spencer’s (1967) procedure in CLARA-W.

4.3.2.3 Comments on 2D Results

All LE procedures give similar results and are in agreement with continuum mechanics analysis for the circular and non-circular failure surfaces considered in this study. The FE analysis is lower by about 11% but still within reasonable agreement. As a result 3D LE analyses can be compared with 3D continuum analysis and also to the 2D LE and continuum procedures for reasonableness.

4.3.3 3D Analysis

Comparison of different methods and procedures within, for 3D slope stability analysis were performed on two 3D problems widely referenced in the literature. The 3D examples used are typically simple, and include homogeneous material properties. It may be noted that the simple geometries are used to observe the computational accuracy of different procedures and to facilitate the comparison with the results published elsewhere as follows:
• 3D Problem I - 2D Problem I extended to 3D.

• 3D Problem II - Spherical failure surface in purely cohesive slope.

4.3.3.1 Effect of Boundary Conditions on 3D Continuum Analysis

As indicated earlier, the FS computed by continuum mechanics procedures is sensitive to location of boundaries and their constraints. Therefore the slide boundaries need to extend beyond the region of the slide mass. In addition, the boundary condition at both ends also need to be fixed in a 3D analysis (Chugh 2003; Griffiths and Marquez 2007). Chugh (2003) extended the soil mass beyond the failure surface using 6 m wide end blocks to represent abutments on each side of the slide mass.

Before performing the 3D analysis with PLAXIS 3D, a parametric study was performed on 2D problem I (Figure 4.1) extended to 3D to determine the effect of boundary conditions and width of side blocks on 3D FS. The parametric study was performed by changing the boundary conditions and varying the width of the end blocks in relation to the width of the slide mass.

Initially, the parametric study was performed for a slide mass width to height ratio (W/H) of one and for different boundary conditions (free, fixed etc). Afterwards end blocks of varying widths were added on both sides of the slide mass. Upon finalization of the boundary conditions and the width of the end blocks, different W/H ratios (i.e W/H=0.5 to 14) were analyzed to determine the 3D end effects (3D/2D FS ratio).

The effect of various boundary conditions are shown in Table 4.4 where 3D FS is sensitive to the boundary conditions at both ends. 3D FS with free ends (can deform in any direction) is 2.03 which is about 3% greater than 2D FS for the same slope (see Table 4.2 for PLAXIS 2D results). Restricting horizontal deformation by providing vertical rollers on the sides (only vertical deformation allowed) increase 3D FS to 2.34, whereas fixed ends (no deformation allowed) increases 3D FS to 2.43. In the field, slope failures have undeformed ends, which mark the extents of the 3D slide mass in the transverse direction. Because fixed ends represent field conditions more realistically, subsequent analyses were performed using fixed ends on both sides of the slide mass.

In the field, the slide mass does not start or end abruptly but is enclosed
by end blocks (or abutments) on either side which have an affect on the stability of the slope. To determine the effect of end blocks width on the ends of the slide, 3D stability analyses width W/H ratios of 1, 2, 4, and 6 were performed for the slope shown in Figure 4.1. Width of end blocks on both sides of the slide were varied by either a multiple of the width of the slide mass (0 to 2) or by increasing the width of the end block from 0 ft to 200 ft. 3D FS obtained from the analyses with end blocks were normalized by 3D FS without end blocks. Fixed boundary conditions were imposed at both ends (top and toe) of the slope as well as sides of the model. Results from the analyses are plotted in Figure 4.5. Figure 4.5(a) shows that effect of the end side blocks is more pronounced for lower W/H ratios than for higher W/H ratios. In addition, the effect of the end block size increases with increasing width of the end block up to a certain limit after which any further increase in end block width does not affect 3D FS. Figure 4.5 shows the highest value of normalized 3D FS (3D FS$_{norm} = 1.24$) is obtained for W/H ratio of one and does not increase if the width of the end block is increased more than half of the width of the slide mass. Similarly Figure 4.5(b) shows that 20 ft (or 6 m) wide end blocks are sufficient for a 3D analysis. This finding supports the procedure used by Chugh (2003) in his 3D FD analysis where 6 m wide end blocks were used for a 10 m high slope. Based on parametric study following boundary conditions appear appropriate for the 3D FE analyses:

- Recommended use of fully fixed displacement conditions ($u = 0$, $v = 0$, and $w = 0$) at the ends of slope model.

- Extend 3D continuum model about 0.5 H past both ends to simulate the sides width should not be larger than 20 ft (6 m).

- Apply model boundary conditions at the ends of the extended continuum model.

- Use a displacement condition of fully fixed ($u = 0$, $v = 0$, and $w = 0$) at the extended model boundaries.

Following the above criteria, 3D effects were analyzed in FE program by using 20 ft wide end blocks on either side of the slide mass and fixed boundary conditions. The width of slide mass was varied to achieve different W/H ratios (i.e W/H=0.5 to 14). Results of FE analysis are shown in Figure 4.6
and indicate that 3D effects are more pronounced for lower W/H ratios and
tend to approach to unity for W/H ratios greater than 6. Figure 4.6 also
shows that for the case analyzed, there is little difference if the width of the
end blocks is increased to 40 ft.

4.3.3.2 2D LE Weighted Average Analysis

Lambe and Whitman (1969) suggest using total weight above the individual
failure surface as the weighing factor. However, the possibility of using the
cross-sectional area, tributary area of the slide mass at ground surface and the
tributary area of the failure surface as weighing factors was also investigated.
A three cross-section 2D weighted analysis of the 2D problem I (Figure 4.1)
extended to 3D with ellipsoidal failure surface of ellipsoidal aspect ratio of two
was performed by selecting cross-sections at one-sixth, centerline and one-
sixth width of the slope. The results of 2D weighted average using Bishop’s
(1955) simplified procedure with different weighing factors is shown in Table
4.5. In this example, the same weighted FS of 2.13 is computed using the
weight or area (cross-sectional and tributary) as the weighing factor. This
2D weighted average FS is 3% higher than the 2D FS computed and about
2% lower than 3D FS computed by other procedures.

Because, no significant difference was noted for the different weighing fac-
tors in this example, additional analyses were performed by adding a layer
at toe level in the soil strata (similar to 2D example 2 shown in Figure 4.2).
The soil properties of the embankment (upper material) and ground (lower
material) were varied. Bishop’s (1955) simplified procedure provides a good
estimate of FS for circular surface so it was used in these analysis and com-
pared with the 3D analysis using same stability procedure. In addition, an
analysis using four cross-sections was performed to investigate the effect of
additional cross-sections on the computed weighted average FS. The results
of this parametric study are shown in Table 4.6. The resulting weighted
2D FS varied from 2 % to 27% from the 3D FS. The variation between 2D
weighted average FS computed using the different weighing factors ranges
from 0% to 6%. Based on these results of this 240 ft wide example slope
problem shown in Table 4.6, it is clear that the weight above the failure sur-
face should be used as the weighing factor instead of the cross-sectional or
tributary area because for the following reasons:-
- A cross-section from layered and homogeneous soils will have the same cross-sectional/tributary area but different weights.

- Using the area of a cross-section as a weighing factor does not consider the effects of varying soil layers (weights). However, using the cross-section weight as the weighing factor takes care of differences in driving forces in a particular problem.

It is concluded from Table 4.6 that for homogeneous soils in a rotational mode (ellipsoidal shapes), a 2D weighted average using three cross-sections can provide a good estimate of 3D FS. Although, four cross-sections (Figure 4.7) selected at, one-eighth, three-eighth, five-eighth and seven-eighth, also works well for layered strata in the example problem but this may not hold good for wider slides where more variation is expected in the $x$-direction (perpendicular to sliding). The results of the 2D weighted average are dependent on the location of the cross-sections, so the more cross-section used the better, especially if the slide is wider than few hundred feet. The number of cross-sections for the 2D weighted average should be selected based on the actual geometry of the slope to ensure that each additional cross-section represents any slope or material variation from the other cross-sections. The requirement of addition cross-sections is illustrated in Figure 4.8 which shows the limits of approximately 380 ft wide, field case history (Hussain et al. 2010). Using four cross-sections that cover equal areas of the failure surface and are marked with solid blue lines. It may be seen that there is an abrupt change in soil profile between the third and fourth cross-section and therefore an additional fifth cross-section (shown in blue dotted line) should be selected for the 2D weighted average procedure.

### 4.3.3.3 3D Problem I - 2D Problem I extended to 3D

Xing (1988) performed a 3D analysis of the geometry shown in Figure 4.1 by extending the slide width in the third dimension. For case-1 (see problem I in Figure 4.1 and a slide width of 73.1 m (240 ft), Xing (1988) reports 3D FS=2.12. Subsequently, other researchers (Chen et al. 2001; Chen et al. 2003 and Griffiths and Marquez 2007) performed a 3D analysis of the same geometry to validate their 3D procedures. In this study, 3D analyses of problem I were performed using LE procedures coded in CLARA-W and
the FE software PLAXIS 3D. A 2D weighted average analysis using 2D LE procedures coded in CLARA-W were also performed for this problem.

Problem I was analyzed using CLARA-W and 3D extensions of Bishop (1955), Janbu (1956), Morgenstern and Price (1965) and Spencer (1967) stability procedures. 3D FS computed using all four LE procedures were compared with 3D FS computed by other researchers. For modeling in CLARA-W, the soil mass was extended in 3D for a width of 73.1 m (240 ft) and an ellipsoidal failure shape was selected in accordance with Xing (1988). The length of the model slide mass is 120 ft (36.6 m). Therefore, an ellipsoidal ratio of two is provided with a width of slide of 240 ft (73.1 m) as reported by Xing (1988).

Figure 4.9 shows the 3D plot of model geometry used in CLARA-W and PLAXIS 3D. The comparison of 3D analyses performed during this study and by other investigators is shown in Table 4.7. It may be noted that 3D FS reported by Xing (1988) is used as reference for this problem because all past researchers compared their computed FS with this value (3D FS=2.12). Table 4.7 shows the FS computed by CLARA-W using a 3D extension of Spencer’s (1967) procedure is 2.17 which is comparable with the FS computed by other researchers using LE and continuum methods. The FS computed by 3D extensions of Bishop’s (1955) and Morgenstern and Price’s (1965) procedures in CLARA-W are also in the same range while a 3D extension of Janbu’s (1956) simplified procedure gives a lower FS=1.99. Interestingly, applying the Janbu correction factor of 1.09, which was computed for 2D case, gives FS=2.17 which is similar to the value computed using the Spencer’s (1967) procedure. 3D FS computed using PLAXIS 3D is 2.12, which is slightly lower (2%) than 3D FS computed by other FE and LE procedures. In addition, the 2D weighted average FS computed for the same problem using the three cross-sections analyses is 2.13.

Table 4.7 also shows that FS obtained by other 3D procedures that claim to satisfy “complete overall force equilibrium conditions and moment equilibrium” (for example Chen et al. 2003) is less than 1% (about 0.3 to 0.8%) different from 3D extensions of Morgenstern and Price’s (1965), Spencer’s (1967), and Bishop (1955) simplified procedure presented by Hungr (1987 and 2001).
4.3.3.4 3D Problem II - Purely Cohesive Slope

3D problem II shown in Figure 4.10(a) is a homogeneous purely cohesive slope (zero friction) with a spherical failure surface. The problem has been used by a number of past researchers (for example, Hungr et al 1989; Lam and Fredlund 1993; Huang and Tsai 2000; and Griffiths and Marquez 2007) to validate their 3D procedures. The material properties of the slope model are given in relation to the radius of the sphere as shown in Figure 4.10(a). 3D analyses of the problem were performed using CLARA-W and a 3D extension of Bishop’s (1955) simplified procedure. An ellipsoidal ratio of one was used to model the spherical failure surface as shown in 4.10(b).

The reference 3D FS for this problem is reported to be 1.40 using a closed form solution (Baligh and Azzouz 1978). During 3D analyses it was noticed that the 3D FS converges to 1.40 if the number of active columns is more than 2000. The analyses performed herein used up to 8000 active columns and the 3D FS value remained unchanged (3D FS=1.40). The comparison of 3D FS obtained from this study with 3D FS values reported by past researchers is shown in Table 4.8. Table 4.8 shows that the FS values from different methods (LE and continuum) are within 3% of the reference closed form solution (3D FS=1.40).

4.3.3.5 Comments on 3D Results

The results from 3D LE procedures presented by different researchers (for example, Hungr et al 1989; Lam and Fredlund 1993; Huang and Tsai 2000; Chen 2003; and Griffiths and Marquez 2007) give similar results and are in agreement with continuum mechanics analysis for 3D ellipsoidal failure surfaces considered in this study. The difference in 3D FS between different 3D methods (and procedures) is less than 12% which is an acceptable error for 2D methods (Duncan 1996).
4.4 Advantages of LE Method over Continuum Mechanics Method

The advantages of continuum mechanics highlighted in Section 4.2.2 does not come without a price. Based on 2D and 3D analyses performed during this study using LE, FE, and FD procedures, the advantages of LE procedures over FD and FE procedures are as follows:

- Modeling in LE method is easier than continuum method and requires less input.
- LE method do not require user to input boundary conditions as in the case of continuum method.
- LE analysis consume significantly less computational time as compared with continuum analysis.
- In LE analysis, a failure surface may be defined so shear strength can be back-calculated.
- In FD and FE analyses, failure surface may be forced to follow a certain shape by using different layers around the desired surface that have the same unit weight but a higher shear strength. Alternatively, an interface may also be use, however, the procedure is complicated and results are sensitive to the geometry used in the analysis.
- CLARA-W (Hungr 2001) can model an interface using its discontinuity option and strength of the interface can be specified.
- It is easy to model 3D geometry in LE software using multiple cross-sections.
- In 3D LE software, different 2D cross-sections are input (or copied, if there is no variation in conditions) and the space between two adjacent cross-sections is interpolated by the software to generate a plane or surface. Alternatively DEM of the various layers (materials and /or piezometric surfaces) may be imported in to LE software to create a 3D model.
• In a manmade slope, e.g., embankments, landfills, and dams, there may be no major variation in the different cross-sections. However in natural slopes, the soil profile usually varies in the transverse direction. Therefore, a meaningful 3D analysis of natural slopes is only possible if the variation in cross-sections is modeled in the software. The 3D geometry input procedure in FD/FE is more difficult and time consuming than the 3D geometry input procedures in LE software.

4.5 Review and Summary of Chapter 4

A comparison of different slope stability methods, i.e., continuum mechanics and LE, for analyzing 2D and 3D slope problems is presented in this chapter. For 2D LE analysis, the procedures of Morgenstern and Price (1965) and Spencer (1967) satisfy all conditions of equilibrium and involve reasonable assumptions. Bishop’s (1955) simplified procedure does not satisfy all conditions of equilibrium but it provides accurate results for circular surfaces. Similarly Janbu’s (1956) simplified procedure yields comparative results provided Janbu’s (1973) correction is applied to the computed FS. Because Janbu’s (1973) correction factor is provided for homogeneous soil conditions, the correction factor may result in overestimation of FS in some cases, thus care should be exercised in applying the correction factor. Based on the results, the procedures of Morgenstern and Price’s (1965) and Spencer’s (1967) procedure yield reasonable estimates of the 2D FS for failure surfaces of any shape. However, because of selection for an appropriate function for inter-slice force inclination with the Morgenstern and Price’s (1965) procedure, Spencer’s (1967) is considered appropriate for general engineering practice. For routine analysis, Bishop’s (1955) simplified procedure and Janbu’s (1956) simplified procedure are also suitable for circular and non-circular (translational) failure surfaces, respectively.

Results of 3D analyses presented in Tables 4.7 and 4.8 show that the accuracy achieved by using a more complex 3D procedure in LE analysis is low (less than 3%). Because of uncertainties involved in geometry, pore water condition, and material properties/shear strength, it appears that existing 3D extensions of Bishop (1955) simplified, Spencer’s (1967), and Janbu’s (1956) simplified procedures provide comparable results with continuum me-
chanics procedures. Ralph B. Peck (Dunnicliff 1993) writes “If one can with sufficient accuracy make a direct visual observation with a graduated scale, then a micrometer should not be used. If one can use a micrometer, a mechanical strain gage should not be used. If one can use a mechanical strain gage, an electrical one should not be used. Mechanical instruments are to be preferred to electrical devices and simple electrical devices depending on simple circuits are to be preferred to more complex electronic equipment. That is, where a choice exists, the simpler equipment is likely to have the best chances of success”. Considering Peck’s statement, it is appropriate to use simple 3D extensions (like Bishop’s and Janbu’s procedure) for practical problems instead of using a more complex 3D procedure, that requires user selection of different intercolumn shear force functions and/or encounters convergence problems.

The following observations are derived from comparison of different 2D and 3D slope stability methods.

- Boundary conditions are important for 2D and 3D FE and FD analyses. Whereas, LE analyses do not require user input of boundary conditions.

- LE procedures are more user friendly than FD and FE procedures.

- 3D extensions of Bishop (Hungr 1989), Morgenstern and Price (Hungr 2001) and Spencer (Hungr 2001) coded in CLARA-W provide comparable results with continuum methods and are within 3% of each other which is less than acceptable error of 12% for 2D analysis (Duncan 1996).

- 3D extension of Janbu’ procedure (Hungr 1989) coded in CLARA-W does not apply Janbu’s (1973) correction factor so it gives lower FS then other 3D extensions. Janbu’s (1973) correction factor for 2D may be applied to 3D FS values.

- Morgenstern and Price’s (1965) and Spencer’s (1967) procedures satisfy all conditions of equilibrium so they are preferred over Bishop’s (1955) and Janbu’s (1956) procedures. Selecting an appropriate function for interslice forces in Morgenstern and Price’s (1965) procedure makes the 3D extension of Spencer’s (1967) procedure more desirable.
• 3D extensions of Bishop’s (1955) and Janbu’s (1956) procedures do not satisfy horizontal force equilibrium and moment equilibrium, respectively but these do not have convergence problems, these are viable alternatives to 3D extension of Spencer (1967) procedure.

• For ellipsoidal failure surfaces, an initial estimate of 3D effects may be obtained by a 2D weighted average analysis using the weight above the failure surface as the weighing factor. However, a minimum of three cross-sections and four cross-sections analysis should be used for homogeneous soils and layered soils, respectively.

• For important projects, results of LE analysis should be checked using continuum analysis.
4.6 Tables

Table 4.1: Material properties for stability analysis of 2D validation examples

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Upper Layer</th>
<th>Weak Layer</th>
<th>Bed Rock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion, $c'\text{(psf)}$</td>
<td>600</td>
<td>0</td>
<td>10000</td>
</tr>
<tr>
<td>Friction Angle, $\phi'$ (°)</td>
<td>20</td>
<td>10</td>
<td>45</td>
</tr>
<tr>
<td>Unit Weight, $\gamma\text{(pcf)}$</td>
<td>120</td>
<td>120</td>
<td>150</td>
</tr>
<tr>
<td>Dilatation Angle, $\psi$ (°)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Young’s Modulus, $E'\text{(psf)}$</td>
<td>$2.1 \times 10^6$</td>
<td>$2.1 \times 10^6$</td>
<td>$2.1 \times 10^6$</td>
</tr>
<tr>
<td>Poisson’s Ratio, $\nu$</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 4.2: Results of 2D slope stability analysis - problem I

<table>
<thead>
<tr>
<th>Limit Equilibrium Procedures</th>
<th>Continuum Procedures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Janbu</td>
</tr>
<tr>
<td>Reported²</td>
<td>2.04</td>
</tr>
<tr>
<td>CLARA-W</td>
<td>1.87³</td>
</tr>
<tr>
<td>XSTABL</td>
<td>2.02</td>
</tr>
<tr>
<td>FLAC 2D</td>
<td>-</td>
</tr>
<tr>
<td>PLAXIS 2D</td>
<td>-</td>
</tr>
</tbody>
</table>

¹f(x)=half sine; ²Fredlund and Krahn (1977); ³Corrected Janbu FS=2.04

Table 4.3: Results of 2D slope stability analysis - problem II

<table>
<thead>
<tr>
<th>Limit Equilibrium Procedures</th>
<th>Continuum Procedures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Janbu</td>
</tr>
<tr>
<td>Reported²</td>
<td>1.45</td>
</tr>
<tr>
<td>CLARA-W</td>
<td>1.33³</td>
</tr>
<tr>
<td>XSTABL</td>
<td>1.41</td>
</tr>
<tr>
<td>FLAC 2D</td>
<td>-</td>
</tr>
<tr>
<td>PLAXIS 2D</td>
<td>-</td>
</tr>
</tbody>
</table>

¹f(x)=half sine; ²Fredlund and Krahn (1977); ³Corrected Janbu FS=1.44

Table 4.4: Effects of boundary conditions on 3D FS

<table>
<thead>
<tr>
<th>W/H</th>
<th>Free Ends</th>
<th>Vertical Rollers</th>
<th>Fixed Ends</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.03</td>
<td>2.34</td>
<td>2.43</td>
</tr>
</tbody>
</table>
Table 4.5: Results of 2D weighted average analysis-problem I

<table>
<thead>
<tr>
<th>Cross-Section Location</th>
<th>Weighing Factor</th>
<th>Location X-sec Weight</th>
<th>X-sec Area Tributary Top (ft)</th>
<th>Tributary Bottom (ft)</th>
<th>Area Tributary (ft³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-sixth (70)</td>
<td>2.19</td>
<td>133902</td>
<td>949</td>
<td>5120</td>
<td>6675</td>
</tr>
<tr>
<td>Centerline (150)</td>
<td>2.07</td>
<td>259706</td>
<td>2164</td>
<td>8960</td>
<td>11682</td>
</tr>
<tr>
<td>One-sixth (230)</td>
<td>2.19</td>
<td>113902</td>
<td>949</td>
<td>5120</td>
<td>6675</td>
</tr>
<tr>
<td>Weighted FS</td>
<td>2.13</td>
<td>2.13</td>
<td>2.13</td>
<td>2.13</td>
<td>2.13</td>
</tr>
</tbody>
</table>

1Computed using Bishop’s simplified procedure

Table 4.6: Percentage variation in FS for 2D weighted average analysis from 3D analysis - problem I

<table>
<thead>
<tr>
<th>Material Strengths Upper Layer</th>
<th>Material Strengths Lower Layer</th>
<th>Difference in FS for Different Weighing Factor Top X-sec Area (%)</th>
<th>Tributary Area (%)</th>
<th>Tributary Area (%)</th>
<th>No. of Cross-Sections Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous (strong)</td>
<td>Homogeneous (weak)</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>Strong</td>
<td>Weak</td>
<td>-2</td>
<td>-2</td>
<td>-4</td>
<td>3</td>
</tr>
<tr>
<td>Weak</td>
<td>Strong</td>
<td>-21</td>
<td>-21</td>
<td>-27</td>
<td>3</td>
</tr>
<tr>
<td>Strong</td>
<td>Weak</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>Weak</td>
<td>Strong</td>
<td>-7</td>
<td>-</td>
<td>-</td>
<td>4</td>
</tr>
</tbody>
</table>

Note: FS computed using 3D extension of Bishop’s simplified procedure is 2.17.
Calculations for 2D weighted average use Bishop’s simplified procedure
Table 4.7: Results of 3D slope stability analysis - Problem I

<table>
<thead>
<tr>
<th></th>
<th>Limit Equilibrium Method</th>
<th>Continuum</th>
<th>Difference</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Janbu</td>
<td>Bishop</td>
<td>Spencer</td>
<td>M &amp; P</td>
</tr>
<tr>
<td>Xing (1988)</td>
<td>2.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chen et al. (2001)</td>
<td>-</td>
<td>-</td>
<td>2.26</td>
<td>-</td>
</tr>
<tr>
<td>Chen et al. (2003)</td>
<td>-</td>
<td>-</td>
<td>2.19</td>
<td>-</td>
</tr>
<tr>
<td>Griffith (2007)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.17</td>
</tr>
<tr>
<td>CLARA-W</td>
<td>1.99(^1)</td>
<td>2.17</td>
<td>2.17</td>
<td>2.18(^2)</td>
</tr>
<tr>
<td>PLAXIS 3D</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CLARA-W (WA(^3))</td>
<td>1.96(^4)</td>
<td>2.13</td>
<td>2.13</td>
<td>2.13(^2)</td>
</tr>
</tbody>
</table>

\(^1\)Corrected Janbu=2.17; \(^2\)f(x)=half sine; \(^3\)Weighted average; \(^4\)Corrected Janbu=2.13
Table 4.8: Results of 3D slope stability analysis - Problem II

<table>
<thead>
<tr>
<th></th>
<th>LE Procedures</th>
<th>Continuum</th>
<th>Difference ( % )</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Janbu</td>
<td>Bishop</td>
<td>Spencer</td>
<td>M &amp; P</td>
</tr>
<tr>
<td>Baligh &amp; Azzouz (1978)</td>
<td>-</td>
<td>1.42</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Hungr et al. (1989)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.42</td>
</tr>
<tr>
<td>Lam and Fredlund (1993)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.39</td>
</tr>
<tr>
<td>Huang and Tsai (2000)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.40</td>
</tr>
<tr>
<td>Chen, Z. et al. (2001)</td>
<td>-</td>
<td>1.42</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Chang (2002)</td>
<td>-</td>
<td>1.42</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Chen, J. et al. (2003)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.44</td>
</tr>
<tr>
<td>Cheng and Yip (2007)</td>
<td>-</td>
<td>1.39</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Griffith (2007)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.39</td>
</tr>
<tr>
<td>CLARA-W</td>
<td>-</td>
<td>1.40</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
4.7 Figures

Figure 4.1: Cross-section used for 2D validation problem I reported by Fredlund and Krahn (1977).

Figure 4.2: Cross-section used for 2D validation problem II reported by Fredlund and Krahn (1977).
Figure 4.3: Comparison of failure surface and results for 2D problem I: (a) CLARA-W, (b) XSTABL, (c) FLAC, (d) PLAXIS2D
Figure 4.4: Comparison of failure surface and results for 2D problem II: (a) CLARA-W, (b) XSTABL, (c) FLAC, (d) PLAXIS2D
Figure 4.5: Effect of width of end blocks
Figure 4.6: 3D end effects for rotational landslide

Figure 4.7: Four cross-sections covering equal areas of a landslide with different material layers
Figure 4.8: Extents of Buck-Center landslide showing cross-section locations for weighted average
Figure 4.9: Slope model for 3D problem I: (a) CLARA-W, (b) PLAXIS 3D
Figure 4.10: Slope model for 3D problem II: (a) 2D geometry, (b) 3D model in CLARA-W
CHAPTER 5

USE OF DIGITAL ELEVATION MODEL (DEM) IN 3D ANALYSIS

5.1 Introduction

Advances in computational power have enabled 3D LE analysis of landslides to be easily conducted on personal computers. So the biggest challenge practicing engineers face with 3D analyses is modeling different slope geometric features in a 3D slope stability program. Conventionally, a number (upto 20) of 2D cross-sections were input into slope stability program to define the spatial variation of different layers, piezometric surface, and failure surface in the 3D slide mass. A 3D mesh is generated by linearly interpolating between the 2D cross-sections to generate a complete 3D model. Because more emphasis is usually placed on slope stability calculations, options for inputting the 3D geometry and interpolation methods in stability software are limited. Existing stability programs use relatively simple linear interpolation for estimating slope geometry between available cross-sections.

Accurately modeling the 3D geometry of a slide mass (ground and failure surface geometry) can be as or more important than the stability procedure for calculating a reliable FS. Therefore, a 3D FS may be affected by a poor estimation of the surfaces by linear interpolation. There are number of commercial 3D surface mapping software packages that use more sophisticated interpolation methods than linear interpolation. As a result, they are able to generate complex 3D surfaces that are not possible with 3D slope stability program. Some of these 3D slope stability software are compatible with different digital elevation model (DEM) file formats used in geographic information system (GIS) surveys. This enables existing geographic surface maps to be used conveniently. Use of these mapping software packages to generate various surfaces can facilitate 3D surface input in a slope stability program.

In this chapter, a general procedure is presented that integrates the use
of gridding software to generate different profile surfaces in a LE stability program. In this procedure, a 3D DEM is generated using available slope boreholes/inclinometer data and then refined interpolation methods are used to generate a realistic 3D slope model. A demonstration version of the gridding software Surfer 9 (Golden software 2010) was used to illustrate the steps in the gridding procedure and generate files that can be used in 3D LE software to input the various 3D surfaces, e.g., piezometric, ground, failure surface etc. Surfer 9 (Golden software 2010) is a general purpose contouring and 3D surface mapping program that converts available data into contour, 3D surface, 3D wire frame, vector, image, shaded relief, and/or post maps.

The objectives of this chapter are to present:

- a description of procedure to generate 3D surfaces that describe different material horizons/properties, water table, and failure surface using available gridding software;
- a general procedure to import the 3D slope model in 3D LE software for computations of the 3D FS;
- a comparison of different interpolation methods to generate a 3D model and their effects of interpolation on 3D FS calculations;
- effects of different surface input methods on 3D FS calculations; and
- illustrate the importance of the interpolation method on 3D FS calculations using hypothetical boreholes/inclinometer data generated from a composite-ellipsoid/wedge failure surface.

5.2 Previous Work

The use of DEM for 3D slope stability problems has been used in geotechnical practice for two decades. However, there is little guidance on how to generate a 3D surface mesh for different profile surfaces, e.g., material horizons/properties, water surfaces, soil layers, etc., for use in a 3D LE program.

Lam and Fredlund (1993) use a DEM for slope geometry, stratigraphy, pore-water pressure conditions, and potential failure surface in their 3D slope stability program 3D-SLOPE. Lam and Fredlund (1993) also suggest using
more robust methods of interpolation e.g., Kriging interpolation, than simple linear interpolation. Kriging interpolation is described in more detail in a subsequent section.

CLARA-W (Hungr 2001) analyzes an assembly of columns of equal rectangular plan dimension known as a mesh. Three different linear interpolation methods can be used to construct a regular spaced 3D column mesh from 2D input cross-sections i.e., orthogonal, oblique, and axisymmetric interpolation. Because the CLARA-W mesh is similar to grid data that is generated by gridding software, any stratigraphic surface, piezometric surface, or specified failure surface can also be imported/exported to a DEM file, using the Golden Software Surfer\textsuperscript{TM} *.GRD format. The *.GRD format corresponds to an ASCII file.

Chugh and Stark (2003) define a simple procedure that can be used in the continuum method, e.g., FLAC (Itasca Consulting Group 2002) software, for automatic generation of a 3D mesh from conventional 2D cross-sections. The procedure was implemented in FLAC\textsuperscript{3D} and uses linear interpolation between 2D cross-sections to develop the spatial variation of geometry and distribution of materials in 3D. To implement the procedure in FLAC\textsuperscript{3D}, available data is organized in \(x\), \(y\) and \(z\)-coordinates and interpolated. The procedure generates the same number of control points in all 2D cross-sections to define elevations of different profile lines in all cross-sections.

Xie et al. (2006a) show that existing column based 3D LE procedures can be integrated with DEM grid-based data and yields the following advantages of using DEM in slope stability analyses by allowing: (1) more sophisticated methods for analyzing and viewing data, (2) large volumes of information to be stored and accessed digitally via DEM which reduces time for searching and retrieving data, and (3) multidisciplinary interaction in projects where data sets can be used by different disciplines e.g., geology, hydrogeology, and mapping, for various analyses.

The present work is an extension of the Chugh and Stark (2003) method and presents a general procedure to generate 3D surfaces from available slope borehole/inclinometer data for use in 3D LE software. The procedure developed herein is different from that proposed by Chugh and Stark (2003) because exclusive use of linear interpolation is not followed. In addition, there is no need to define all profile surfaces using common control points in all cross-sections. The proposed procedure uses refined interpolation methods
available in gridding and mapping software so more realistic 3D modeling is achieved with relative ease. The resulting 3D surfaces are also visually appealing and include more options, e.g., adding contours, printing 3D geometry etc, for better comparison with actual slope surfaces.

5.3 Limitations of Conventional 3D Modeling

Modeling a 3D failure surface for design is not difficult because a pre-defined failure surface e.g., ellipsoid for rotational slides, and multi planer wedge surface for translational, can be selected and varied to locate the critical FS. However for back-analysis of a landslide, a search for the failure surface yielding the lowest FS is not necessary because the actual failure surface must be used with a 3D FS equal to unity. The actual failure surface must be used because the shear strength parameters mobilized along that specific surface are the parameters being sought in a back-analysis and the shear strength parameters will change as the failure surface changes. Therefore, the challenge for practicing engineers is how to model slope geometry and failure surface of an actual landslide in a 3D slope stability program to perform a back-analysis.

For a 2D back-analysis, a critical cross-section of the slide mass is modeled in the slope stability analysis. A number of points are selected to define the field failure surface geometry. Segments of the failure surface between two points are represented by a straight line. Usually, relatively few points are sufficient to define each profile line, i.e., slope geometry, material horizons, piezometric surface, and actual failure surface in a 2D analysis. The number of points used depends on the complexity of the problem but are limited by the software. Going from 2D to 3D geometry increases the number of points to define the failure surface, because all parallel cross-sections (perpendicular to direction of slide movement) are assigned a common set of points to facilitate the 3D model criterion.

The increased number of data points required to define the 3D failure surface and slope geometry complicates the practical application of a 3D back-analysis of a landslide. In practice it is usually not possible to drill enough borings and/or install enough slope inclinometers to generate a mesh defining the entire failure surface and slide mass. If there is less variation
in the material properties and water surfaces in a 3D slide mass, linear interpolation between two adjacent cross-sections may provide a reasonable estimate of the field geometry. However, in some cases, the geometry of various surfaces, e.g., ground surface, failure surface etc, vary significantly between cross-sections and the number of available data points may be less than desired to estimate the 3D geometry. Using linear interpolation between these known adjacent points to develop the 3D geometry may result in a surface that is not representative of the field geometry and may lead to erroneous 3D FS values. In this situation, more advanced interpolation methods can be used to develop a better representation of the field geometry. During this study it was found that the Kriging interpolation method produces surfaces that are in better agreement with field surfaces then other interpolation methods. Surfer 9 offers twelve other interpolation methods that can be used depending on specific project/site requirements.

Kriging is a powerful statistical interpolation method that has proven useful and popular in many fields. The word “Kriging” is synonymous with “optimal prediction” (Journel and Huijbregts 1981). It is a method of interpolation that predicts unknown values from data observed at known locations. This method uses variogram to express the spatial variation and it minimizes the error of predicted values which are estimated by spatial distribution of the predicted values (Liu and Calgary 2008). The difference between Kriging and other linear estimation methods is its aim of minimizing the error variance. This method produces visually appealing maps from irregularly spaced data. The Kriging method attempts to express general trends suggested in the available data not every deviation in the data.

5.4 Conceptual Model for Implementation of DEM Based 3D LE Slope Stability Analysis

3D LE procedures in practice are based on procedure of columns, which is a natural extension of 2D the procedure of vertical slices. The slope stability analysis are performed using a grid of columns of equal rectangular plan area. Figure 5.1 shows the assembly of 3D columns, known as mesh, where a grid point is taken as the center of a respective column. The 3D extension of Bishop’s (1955) simplified procedure presented by Hungr et al. (1989)
derives a FS value iteratively from summation of moments around a common horizontal axis, parallel with the \( x \)-axis (see Chapter 2) as follows:

\[
FS = \frac{\tau_{\text{max}}}{S_m} = \frac{\sum_i \sum_j [cA R + (N - uA) R \tan \phi]}{\sum_i \sum_j W x - \sum_i \sum_j N f \cos \gamma_z / \cos \alpha_y} \tag{5.1}
\]

where \( i \) and \( j \) are the number of rows and columns, respectively ("NX" and "NY"), shown in Figure 5.1.

Hungr et al. (1989) also derive the FS from horizontal force equilibrium in the direction of motion (\( y \)-direction) which is a 3D equivalent to Janbu’s (1956) simplified procedure:

\[
FS = \frac{\sum_i \sum_j [cA \cos \alpha_y + (N - uA) \tan \phi \cos \alpha_y]}{\sum_i \sum_j N \cos \gamma_z \tan \alpha_y} \tag{5.2}
\]

All parameters, including material strengths, unit weights, material surface elevations, pore water pressures, total and effective stresses, and slope angles, are assumed constant along each column base. Because of this assumption, the procedure introduces some degree of error, which needs to be compensated by using a fine grid or mesh. However, the advantage of using column centers to evaluate different properties is that it is in the same format as that of a DEM file produced by gridding/mapping software. Therefore all slope geometry data, i.e., elevations of slope surface, material horizons, piezometric surface, failure surface, etc., can be stored in a DEM file generated by gridding software, while other material parameters for each participating layer, e.g., shear strength, unit weights, etc., can be stored separately in the 3D slope stability program. The general format of an ASCII grid file and description of various lines in *.GRD file is shown in Figure 5.2.

A slope stability program based on 3D extensions of Bishop’s (1955) and Janbu’s (1956) simplified procedures proposed by Hungr et al. (1989) can be integrated with gridding/mapping software to import DEM files of various layer. DEM of various layers are superimposed to define the complete geometry as shown in Figure 5.3. Because each layer is generated independently in gridding software, it is important to ensure that the grid size and spacing is the same for all layers.
5.5 Description of Procedure to Generate DEM for Subsequent Use in LE Program

Mapping and gridding software can generate a DEM from a limited number of available data points by interpolating between given points to generate a continuous surface. Gridding software offers variety of advanced interpolation methods (e.g. Kriging, minimum curvature, triangulation with linear interpolation, etc.) for estimating the elevations of points between the known data points. Each surface (slope surface, material layer, piezometric surface, slip surface, etc.) has equally spaced data points (grids) and it is represented as a separate layer in a DEM. The following procedure is suggested to generate a DEM for defining the slope geometry:-

- Data from boreholes and/or inclinometers is collected to define major changes in slope geometry. Usually there is less variation in the geometry of the slope surface, material layers, and piezometric surface in different 2D cross-sections perpendicular to direction of sliding so even a small number of data points may still provide reasonable accuracy. However, other cross-sections should be drawn to determine if there is any major change in the geometry.

- Collected data is tabulated in a spreadsheet (MS EXCEL, etc.) for each separate layer using $x$, $y$, and $z$-coordinates. If for any layer, there is no major change in different cross-sections then data from the central or main cross-section can be repeated at regular intervals in $x$-direction to define the slope geometry.

- Gridding software is used to generate a DEM with grids of specified size. Gridding software interpolates (user specified interpolation method) between given data points and generates a complete surface having equal spaced grids. The steps in the gridding procedure specific to SURFER 9 (Golden Software 2010) are given below for reference:
  - Open program, go to “Grid” option, select “Data”. Browse using “open” to locate the spread sheet containing data.
  - Select appropriate spread sheet containing layer data.
  - Correct “Data Columns” (if required).
– Select desired gridding method.
– Select mesh refinement using “Grid line geometry” (if desired).
– Finally save as “GRD Surfer 6 Text Grid(*.grd )”.

• Once gridding software has generated a surface, it must be checked to confirm that it represents the field surface (slope surface, material layer, piezometric surface, failure surface, etc.). Same gridding software can be used to verify the accuracy of failure surface as follows:-.

  – Open program, select “Map > New”
  – Select “3D Surface or 3D Wireframe” to view the surface.

• If the surface does not represent the actual surface then either adding more data points to improve the generated a DEM of the surface or a different gridding method (minimum curvature, triangulation with linear interpolation, etc.) should be used to improve the match.

5.6 Effect of Interpolation Method on FS Calculations

To illustrate the effect of interpolation methods on FS calculations, 3D analyses of 2D Problem-II presented in Chapter 4 were performed herein. The 3D reference FS value for the example problem was obtained using a composite-ellipsoid/wedge failure surface in CLARA-W. Comparison of the FS obtained from a DEM generated using different interpolation methods with reference 3D FS values is presented in the following sections.

5.6.1 3D Model Description

The example problem shown in Figure 5.4 uses a 2D slope geometry and failure surface that is rotational with a curved back scarp and passing through the underlying weak layer. Values of 3D FS calculated by previous researchers for this 3D problem geometry range from 1.44-1.76 (Xie et al. 2006a) for composite ellipsoid/wedge failure surfaces with different ellipsoidal ratios of slide mass (to achieve a certain volume of slide mass) and with or without a water surface. Shear strength and unit weights of the different materials
layers used in the problem are shown in Table 5.1. Only four data points were used to define the 2D slope geometry and each subsurface layer, while the piezometric surface was defined using three points in 2D. Location (y-coordinate) and elevation (z-coordinate) of data points defining the ground surface, weak layer, bedrock, and piezometric surface are shown in Table 5.2. The width of the 2D geometry was extended to 100 ft to convert it to a 3D geometry. No variation of material properties or water surface was assumed across the width of the 3D slope geometry. Therefore, the other cross-sections defined in the x-direction use these three and four points to define the various layers in each cross-section. The 3D failure surface was defined by a composite-ellipsoid/wedge in CLARA-W (Hungr 2001) with an ellipsoid ratio of 0.75, a center of rotation at x=50 ft, y=60 ft, and z=90 ft. The failure surface is truncated by the weak layer at an elevation of 17 ft. Figure 5.5 shows the 3D geometry of the failure surface generated by CLARA-W.

5.6.2 3D FS for Composite-Ellipsoid Wedge Failure Surface

The failure surface in this problem is non-circular, therefore, Bishop’s (1955) simplified procedure in its original form may not yield accurate results. However, after applying Fredlund and Krahn’s (1977) modification with a revised center of rotation, Bishop’s (1955) simplified procedure can be used for analysis of this non-circular failure surface (see section 4.3.2.2). Revised center of rotation is the center of a circle fitted to the composite-ellipsoid/wedge surface profile (see Figure 2.13). The analysis of this problem using 3D extensions of Bishop’s (1955) and Janbu’s (1956) simplified procedures yield 3D FS of 1.67 and 1.66, respectively, with a revised center of rotation of y=50.64 ft and z=117.83 ft. Values of 3D FS are within the range of FS values reported by previous researchers (1.44-1.76). Therefore, these 3D FS values (1.67 and 1.66 for 3D Bishop and 3D Janbu, respectively) are taken as reference values for further analysis.
5.6.3 Hypothetical Boreholes/Inclinometer Data for Composite-Ellipsoid/Wedge Failure Surface

To evaluate the effect of interpolation method on 3D FS values, the ellipsoid failure surface shown in Figure 5.5 was exported to a DEM file (*.GRD). The output DEM file of ellipsoid failure surface can be opened in Surfer 9\textsuperscript{TM} grid node editor, and elevations (\(z\)-coordinate) of failure surface at different grid points (\(x\) and \(y\)-coordinates) can be viewed. Figure 5.6 shows location of 6,500 nodes (65 rows and 100 columns) that define the ellipsoid-wedge failure surface. To represent slide mass geometry, elevations of 33 grid points on one half of the failure surface were selected as shown in Figure 5.7. For comparison with orthogonal interpolation, the grid points were selected in a way that they could be also used for data input through 2D cross-sections. Therefore, distribution of selected grid points lie on thirteen cross-sections, i.e., at \(x = 0, 7.16, 14.33, 21.5, 28.67, 39.42, 50.18, 60.93, 71.68, 78.85, 86.02, 93.19, \text{ and } 100.35 \text{ ft, as shown in Figure 5.7. Because the failure surface is symmetric on both sides of central cross-section (i.e., } x=50.18 \text{ ft), data from seven cross-sections can be used to define the failure surface. Table 5.3 shows } x, y, \text{ and } z\text{-coordinates of grid points defining the failure surface for cross-sections seven to thirteen (see Figure 5.7). Table 5.3 also includes the additional grid points from pre-failure geometry of ground surface for input into the LE program for orthogonal interpolation.}

5.6.4 3D FS of Slope Model Generated Through Different Interpolation Methods

Following the procedure presented in Section 5.5, a DEM of bedrock, weak layer, piezometric surface, ground surface, and failure surface were generated using ten different gridding (interpolation) methods in Surfer 9 (Golden software). For illustration, Figure 5.8 shows pre-failure slope geometry of 3D model generated by Kriging interpolation using the data points given in Table 5.2.

Similarly, Figure 5.9 shows a DEM generated from data points of failure surface given in Table 5.3 using different interpolations methods. Figure 5.9 also includes for reference the failure surface for a composite-ellipsoid wedge and failure surface defined using 2D parallel cross-sections with orthogonal
interpolation in CLARA-W (Hungr 2001). The same number of data points as given in Tables 5.2 and 5.3 were used for developing the 3D failure surface using different interpolation (gridding) methods.

Table 5.4 shows a summary of the results obtained from analyses of a DEM generated using different interpolation methods. To study the effect of surface input method on 3D FS, additional analyses using different combination of surfaces generated through various methods were also performed. A summary of the different analyses is given in Table 5.5.

5.6.5 Comments on Results

Figure 5.9 shows that in this particular example, the 3D failure surfaces generated from a limited number of data points by Kriging, radial basis function, triangulation with linear interpolation, orthogonal interpolation, and natural neighbor provide a good representation of the reference failure surface (composite-ellipsoid/wedge). Modified shepard’s method, minimum curvature, local polynomial, nearest neighbor, and inverse distance to power generated failure surfaces that are not representative of the reference failure surface. Table 5.4 shows that 3D analysis of surfaces generated by Kriging interpolation is in excellent agreement with reference 3D FS (3D FS=1.66 for composite-ellipsoid/wedge). On the contrary, 3D FS (3D FS=1.97) obtained from orthogonal interpolation is about 16% greater than the reference 3D FS (3D FS=1.66 for composite-ellipsoid/wedge). Because, the failure surface generated by orthogonal interpolation is different near the ends of the slide mass, an additional cross-section near the ends is required to improve the interpolation.

Table 5.5 shows that FS is affected when a different combination of surfaces generated through different methods is used. For example, about 14% variation in 3D FS is observed when a DEM for the failure surface is generated using Kriging interpolation instead of orthogonal linear interpolation in CLARA-W. This could be caused by difference in estimation of intermediate grid points by different methods and therefore, additional active columns (4956 vs 3634) are added due to elevation differences between the initial ground surface and the failure surface. Thus, it is important that the same method of interpolation be used for generating DEMs of different surfaces.
for a particular analysis.

5.7 Review and Summary of Chapter 5

- 3D LE software can use linear interpolation between the input cross-sections to generate a complete surface (or mesh). Usually there is less variation in the material properties and water surfaces, therefore, linear interpolation between two adjacent known points on a profile line in adjacent cross-sections can provide a reasonable estimate of these geometries.

- In some cases, geometry of various surfaces (e.g., ground surface, failure surface, etc.) varies significantly in different cross-sections but the number of available data points may be less than desired to estimate the elevation of additional equally spaced points on the profile line in a cross-section. Using linear interpolation between known adjacent points to estimate the elevation of the remaining (and/or missing) data points usually generates a surface that is not representative of the actual surface which will impact the FS computation.

- In cases of limited data, more advanced interpolation methods that predict the elevation of intermediate points on the basis of the overall distribution of known elevations in the same profile rather than local interpolation between two adjacent points can be helpful.

- 3D extensions of 2D procedures of columns (Hungr et al 1989) can conveniently use a DEM generated by gridding software because of the use of similar formats.

- In the example problem used herein, using a limited number of data points to generate a 3D surface by linear (orthogonal) interpolation results in a 3D FS which is 16% higher than the reference 3D FS. However, using the same number of data points to generate surfaces by Kriging interpolation yields a FS which is in good agreement with the reference 3D FS.

- Gridding/mapping software provides more options for interpolation to estimate the overall geometry of a surface by looking at overall trends
in the limited available data. Gridding/mapping software also includes more options (like adding contours, etc.) for better comparison with the actual surface.

- Care should be exercised in selecting an appropriate advanced gridding (interpolation) method based on data availability because different methods will generate different surface geometries.

- Use of a DEM in slope stability analysis facilitates consistency between same data sets and these data sets can be used by different disciplines for data analysis.

- It is important that same method of interpolation be used for generating DEMs of different surfaces for a particular analysis.
5.8 Tables

Table 5.1: Material properties for stability analysis of example problem

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Clay Layer</th>
<th>Weak Layer</th>
<th>Bedrock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion, $c'(psf)$</td>
<td>600</td>
<td>0</td>
<td>10000</td>
</tr>
<tr>
<td>Friction Angle, $\phi'(^o)$</td>
<td>20</td>
<td>10</td>
<td>45</td>
</tr>
<tr>
<td>Unit Weight, $\gamma(pcf)$</td>
<td>120</td>
<td>120</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 5.2: Data points for input of slope geometry of example problem

<table>
<thead>
<tr>
<th>Data Point No.</th>
<th>$y$ (ft)</th>
<th>$z$ (ft)</th>
<th>$y$ (ft)</th>
<th>$z$ (ft)</th>
<th>$y$ (ft)</th>
<th>$z$ (ft)</th>
<th>$y$ (ft)</th>
<th>$z$ (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00</td>
<td>20.00</td>
<td>0.00</td>
<td>17.50</td>
<td>0.00</td>
<td>15.50</td>
<td>0.00</td>
<td>20.00</td>
</tr>
<tr>
<td>2</td>
<td>40.00</td>
<td>20.00</td>
<td>40.00</td>
<td>17.50</td>
<td>40.00</td>
<td>15.50</td>
<td>40.00</td>
<td>20.00</td>
</tr>
<tr>
<td>3</td>
<td>120.51</td>
<td>60.16</td>
<td>120.17</td>
<td>17.50</td>
<td>120.00</td>
<td>15.50</td>
<td>154.12</td>
<td>36.30</td>
</tr>
<tr>
<td>4</td>
<td>154.12</td>
<td>60.16</td>
<td>154.12</td>
<td>17.55</td>
<td>154.12</td>
<td>15.55</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: No variation in geometry and material properties in all cross-sections
Table 5.3: Data points for input of failure surface of example problem

<table>
<thead>
<tr>
<th>Data Point</th>
<th>Cross-Sec 1&amp;13</th>
<th>Cross-Sec 2&amp;12</th>
<th>Cross-Sec 3&amp;11</th>
<th>Cross-Sec 4&amp;10</th>
<th>Cross-Sec 5&amp;9</th>
<th>Cross-Sec 6&amp;8</th>
<th>Cross-Sec 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>x=0 (ft)</td>
<td>x=7.16 (ft)</td>
<td>x=14.37 (ft)</td>
<td>x=21.5 (ft)</td>
<td>x=28.67 (ft)</td>
<td>x=39.42 (ft)</td>
<td>x=60.93 (ft)</td>
<td>x=50.18 (ft)</td>
</tr>
<tr>
<td>x=100.35 (ft)</td>
<td>x=93.19 (ft)</td>
<td>x=86.02 (ft)</td>
<td>x=78.85 (ft)</td>
<td>x=71.68 (ft)</td>
<td>x=60.93 (ft)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| No. | x  | y  | z  | x  | y  | z  | x  | y  | z  | x  | y  | z  | x  | y  | z  | x  | y  | z  | x  | y  | z  | x  | y  | z  | x  | y  | z  | x  | y  | z  | x  | y  | z  | x  | y  | z  |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1   | 0.00 | 20.00 | 64.51 | 30.25 | 0.00 | 20.00 | 53.76 | 17.00 | 0.00 | 20.00 | 0.00 | 20.00 | 0.00 | 20.00 | 0.00 | 20.00 |
| 2   | 39.43 | 20.00 | 89.60 | 37.90 | 39.43 | 20.00 | 64.51 | 17.00 | 25.09 | 20.00 | 14.34 | 20.00 | 14.33 | 20.00 |
| 3   | 43.01 | 21.58 | -    | -    | 43.01 | 21.50 | -    | -    | 28.67 | 18.66 | 21.50 | 17.80 | 17.92 | 18.33 |
| 4   | 64.51 | 32.35 | -    | -    | 64.51 | 22.33 | -    | -    | 32.26 | 17.19 | 25.90 | 17.00 | 21.50 | 17.00 |
| 5   | 89.60 | 44.90 | -    | -    | 78.85 | 24.85 | -    | -    | 35.84 | 17.00 | 35.84 | 17.00 | 35.84 | 17.00 |
| 6   | 121.86 | 60.16 | -    | -    | 107.53 | 41.62 | -    | -    | 43.01 | 17.00 | 57.34 | 17.00 | 57.34 | 17.00 |
| 7   | 154.12 | 60.16 | -    | -    | 121.86 | 60.16 | -    | -    | 57.34 | 17.00 | 75.26 | 17.00 | 75.26 | 17.00 |
| 8   | -    | -    | -    | -    | 154.12 | 60.16 | -    | -    | 75.26 | 17.00 | 96.77 | 17.00 | 96.77 | 17.00 |
| 9   | -    | -    | -    | -    | -    | -    | -    | -    | 86.02 | 17.00 | 100.36 | 18.83 | 100.35 | 17.35 |
| 10  | -    | -    | -    | -    | -    | -    | -    | -    | 89.60 | 17.93 | 118.28 | 32.57 | 118.28 | 30.75 |
| 11  | -    | -    | -    | -    | -    | -    | -    | -    | 114.70 | 34.50 | 136.20 | 60.16 | 139.78 | 60.16 |
| 12  | -    | -    | -    | -    | -    | -    | -    | -    | 132.61 | 60.16 | 154.12 | 60.16 | 154.12 | 60.16 |
| 13  | -    | -    | -    | -    | -    | -    | -    | -    | 154.12 | 60.16 | -    | -    | -    | -    |

Note: 33 data points that define failure surface are shown in italics, remaining 23 points are from ground level data to make uniform grid.

Cross-section 8 to 13 are same as cross-sections 6 to 1 respectively.
Table 5.4: Summary of results from different interpolation methods

<table>
<thead>
<tr>
<th>Ser.</th>
<th>Interpolation Method</th>
<th>NAC¹</th>
<th>Center of Rotation</th>
<th>3D FS</th>
<th>Janbu 3D</th>
<th>Difference² (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>y (ft)</td>
<td></td>
<td>z (ft)</td>
<td>Bishop 3D</td>
</tr>
<tr>
<td>1.</td>
<td>Composite Ellipsoid Wedge</td>
<td>3634</td>
<td>50.64</td>
<td>1.67</td>
<td>117.83</td>
<td>1.66</td>
</tr>
<tr>
<td>2.</td>
<td>Modified Shepard's Method</td>
<td>3986</td>
<td>45.21</td>
<td>1.50</td>
<td>162.3</td>
<td>1.49</td>
</tr>
<tr>
<td>3.</td>
<td>Kriging</td>
<td>4184</td>
<td>41.39</td>
<td>1.66</td>
<td>142.46</td>
<td>1.66</td>
</tr>
<tr>
<td>4.</td>
<td>Radial Basis Function</td>
<td>4278</td>
<td>53.07</td>
<td>1.69</td>
<td>121.12</td>
<td>1.68</td>
</tr>
<tr>
<td>5.</td>
<td>Minimum Curvature</td>
<td>3958</td>
<td>45.15</td>
<td>1.71</td>
<td>162.56</td>
<td>1.70</td>
</tr>
<tr>
<td>6.</td>
<td>Triangulation with Linear Interpolation</td>
<td>4118</td>
<td>50.77</td>
<td>1.79</td>
<td>120.19</td>
<td>1.77</td>
</tr>
<tr>
<td>7.</td>
<td>Orthogonal Interpolation</td>
<td>4130</td>
<td>51.18</td>
<td>1.97</td>
<td>119.55</td>
<td>1.97</td>
</tr>
<tr>
<td>8.</td>
<td>Natural Neighbor</td>
<td>4602</td>
<td>63.46</td>
<td>2.06</td>
<td>75.89</td>
<td>2.03</td>
</tr>
<tr>
<td>9.</td>
<td>Local Polynomial</td>
<td>5240</td>
<td>39.65</td>
<td>2.53</td>
<td>167.91</td>
<td>2.47</td>
</tr>
<tr>
<td>10.</td>
<td>Nearest Neighbor</td>
<td>3178</td>
<td>49.77</td>
<td>225.86</td>
<td>NC³</td>
<td>2.89</td>
</tr>
<tr>
<td>11.</td>
<td>Inverse Distance to Power</td>
<td>6307</td>
<td>43.92</td>
<td>3.13</td>
<td>169.50</td>
<td>3.09</td>
</tr>
</tbody>
</table>

¹ No. of active columns
² Percentage difference from 3D FS for composite-ellipsoid/wedge using Janbu’s procedure
³ Not converging
Table 5.5: Summary of results from different surface input methods

<table>
<thead>
<tr>
<th>Ser No.</th>
<th>Slope Geometry</th>
<th>Surface Input Method</th>
<th>No of Columns</th>
<th>Center of Rotation</th>
<th>3D FS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>y (ft)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>z (ft)</td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>2D Cross-Sections</td>
<td>Composite-ellipsoid/wedge</td>
<td>3634</td>
<td>50.64</td>
<td>1.67</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>142.46</td>
<td>1.66</td>
</tr>
</tbody>
</table>

<sup>1</sup> Linear orthogonal interpolation;
<sup>2</sup> Kriging interpolation
5.9 Figures

Figure 5.1: Coordinate system and column assembly (Hungr 2001).

```
DSAA
44  29
0  100.3592
17  60
20  20  20  20  20  20  20  20  20  20  20  20  20  21.50554  23.29766  25.08979
26.88192  28.67405  30.46617  32.2583  34.05043  35.84256  37.63469  39.42681
41.21894  43.01107  44.8032  46.59533  48.38745  50.17958  51.97171  53.76384
55.55596  57.34809  59.14022  60  60  60  60  60  60  60  60  60  60  60  60
...
20  20  20  20  20  20  20  20  20  20  20  20  20  21.50554  23.29766  25.08979
26.88192  28.67405  30.46617  32.2583  34.05043  35.84256  37.63469  39.42681
41.21894  43.01107  44.8032  46.59533  48.38745  50.17958  51.97171  53.76384
55.55596  57.34809  59.14022  60  60  60  60  60  60  60  60  60  60  60  60
```

Line no. | Data                | Description
---------|---------------------|-------------------
1.        | DSAA                | ASCII Grid File identifier string
2.        | 44  29              | No. of columns (NY) ; No. of rows (NX)
3.        | 0  154.123          | Minimum column value (YS), Maximum column value (YE)
4.        | 0  100.3592         | Minimum row value (XS), Maximum row value (XE)
5.        | 17  60              | Minimum elevation (z) in grid, maximum elevation (z) in grid
6. & onwards | 20  20 20 20 20 20 ... 60 | NX rows (XS to XE) of elevation (z values),
             |                     | Each row has NY columns (YS to YE)

Figure 5.2: ASCII file format of * .GRD file in Surfer 9(Golden Software Inc 2010).
Figure 5.3: Superimposed DEM of slope layers.
Figure 5.4: 2D cross-section used for example problem (from Fredlund and Krahn 1977).

Figure 5.5: 3D failure surface generated by ellipsoid-wedge (CLARA-W).
Figure 5.6: Location and elevation of nodes of ellipsoid-wedge surface in grid node editor of Surfer 9 (Golden Software 2010).

Figure 5.7: Location of grid points selected for geometry input.
Figure 5.8: 3D surfaces for slope model generated by Kriging interpolation in Surfer 9 (Golden Software Inc 2010).
Figure 5.9: Failure surfaces generated using different interpolation methods: (a) Composite-ellipsoid/wedge, (b) Modified Shepard’s Method, (c) Kriging, (d) Radial Basis Function, (e) Minimum Curvature, (f) Triangulation with Linear Interpolation, (g) Orthogonal Interpolation, (h) Natural Neighbor, (i) Local Polynomial, (j) Nearest Neighbor, and (k) Inverse Distance to Power.
Figure 5.9: (continued)
Figure 5.9: (continued)
Figure 5.9: (continued)
CHAPTER 6
EFFECT OF SIDE RESISTANCE IN 3D ANALYSIS

6.1 Introduction

2D LE analysis are based on a plane strain condition that assumes the slide mass or cross-sections, do not change in the direction perpendicular to slide movement and therefore 3D effects (end effects) are negligible. This assumption is acceptable if the width of the slide is large compared to its height, i.e., ratio of width(W) to height(H) of slide is greater than six. However, most, if not all, are not infinitely long and varies perpendicular to slide movement. Therefore, application of 2D analyses to a 3D problem is not accurate but believed to be conservative/sufficient for engineering use because the end effects are neglected. Past research (for example, Hutchinson and Sarma 1985; Cavounidis 1987; Hungr 1987) shows that, 3D analyses yield greater FS values than those calculated using 2D analyses for the critical failure surface, all other things being equal. 2D analyses are conservative because the resistance along the out-of-plane faces of the slide mass are neglected in the analysis. This conservatism may be acceptable for slope designs but in the case of back-analyses of landslides, 2D analyses may result in unconservative values of back-calculated shear strength by as much as 30% (Stark and Eid 1998).

For a translational landslide, Stark and Eid (1998) show that 3D LE software does not consider the effects of shear resistance offered by vertical sides parallel to the direction of slide movement. Consequently, the computed 3D FS is underestimated which results in an overestimate of back-calculated shear strength. To overcome this limitation, Stark and Eid (1998), Arellano and Stark (2000), and Eid et al. (2006) suggest different techniques to incorporate the side shear resistance in 3D FS computations in a 3D LE slope stability software. In these three techniques, the magnitude of side shear
force is estimated using at-rest earth pressure \((K_O)\) and Mohr-Coulomb failure criteria. Arellano and Stark (2000) suggest using continuum procedures to investigate the magnitude of side shear forces to be used in LE procedures.

The present study uses FE and FD programs to calculate the magnitude of side shear resistance along vertical sides of a translational slide mass. Results of the parametric study show that use of \(K_O\) for approximating the shear resistance results in an overestimation of the 3D/2D FS ratio. However, use of an earth pressure coefficient \((K_r)\) that is in-between at-rest \((K_O)\) and active \((K_A)\) earth pressure provides a better estimate of the side shear resistance and 3D/2D FS ratios that are in agreement with FE and FD analyses. Based on these findings, the charts provided by Arellano and Stark (2000) showing the influence of shear resistance on 3D/2D FS ratios for various slope inclinations and geometries were updated herein.

6.2 Influence of Side Resistance in 3D Analysis

Neglecting the end effects can severely affect the FS especially in narrow slopes with slope angles steeper than 20 degrees (Lafebvre et al. 1973). Baligh and Azzouz (1973) report that FS may be underestimated by as much as 40% if the end effects are not considered. Stark and Eid (1998) and Arellano and Stark (2000) show that translational slides exhibit a significant difference (\(\sim 40\%\)) between 2D and 3D FS. This difference is less pronounced in slopes that fail in a rotational failure mode. End effects are more pronounced in translational than rotational failures due to following reasons (Stark and Eid 1998, Arellano and Stark 2000):

- Slopes failing in translational mode usually involve either a significantly higher or lower mobilized shear strength along the back scarp and sides of the slide mass than that along the base, e.g., the upstream slope failure in Waco dam (Beene 1967; Wright and Duncan 1972) and the slope failure in Kettleman Hills hazardous waste repository (Seed et al. 1990; Byrne et al. 1992; and Stark and Poeppel 1994), respectively. These situations can result in a significant difference between the 2D and 3D FS. This difference is less pronounced in slopes failing in a rotational failure mode because they usually involve homogeneous materials (Figure 6.1).
A translational failure can occur in relatively flat slopes because of the weak underlying material(s). The flatter the slope, the greater the difference between 2D and 3D FS because of the larger area of the sides of the slide mass (Chen and Chameau 1982; Leshchinsky et al. 1985).

A translational failure often involves a long and nearly horizontal failure surface through a weak underlying soil layer [e.g., Maymont slide (Krahn et al. 1979), Gardiner Dam movement (Jasper and Peters 1979), and Portuguese Bend slide (Ehlig 1992)], or geosynthetic interface, e.g., Kettleman Hills repository.

A translational failure often involves a drained shearing condition. This facilitates estimation of the mobilized shear strength of the materials involved because shear-induced pore-water pressures do not have to be estimated only the hydrostatic pressures.

The above mentioned reasons highly affect the 3D/2D FS ratio for translational slides and therefore should be kept in mind for 2D or 3D slope stability analysis. Also as the inclination of the sides parallel to the direction of motion of the slide mass increases, the shear surface area decreases. Therefore, in translational slides, vertical sides provide the minimum amount of 3D shear resistance because the effective normal stress acting on these sides is only due to lateral earth pressure.

Stark and Eid (1998) studied commercially available 3D software to investigate their ability to calculate 3D FS and include the end effects. Stark and Eid (1998) report that STAB3D (Baligh and Azzouz 1975), LEMIX&FESPON (Chen and Chameau 1983), BLOCK3 (Lovell 194), and F3SLOP&DEEPCYL (Gens et al. 1988) are unsuitable for analysis of translational landslides because they only use a cylindrical 3D shear surface. The software that are able to model a translational failure include 3D-PCSTABL (Thomaz 1986), CLARA 2.31 (Hungr 1988), and TSLOPE3 (Pyke 1991). Stark and Eid (1998) and Arellano and Stark (2000) prefer CLARA 2.31 (Hungr 1998) for performing 3D parametric analyses over 3D-PCSTABL and TSLOPE3 because: (1) It is more user friendly for input of slope geometry and pore-water pressure conditions; (2) it uses Janbu’s (1954) simplified procedure for 2D and 3D analyses, which is suitable for translational mode of failure; (3) external loads may be specified and used to simulate the resistance acting on
the vertical sides; and (4) it is capable of performing a 2D analysis from a 3D data file.

Based on the results of a parametric study using slope model in Figure 6.2, Stark and Eid (1998) show that CLARA 2.31 does not consider the shear resistance along the vertical sides of the slide mass. This leads to underestimation of 3D FS, especially when the material along the vertical sides has a higher shear strength than the material along the base of the slide mass.

6.3 Consideration of Side Forces in 3D LE Slope Stability Software

In 3D slope stability software, a user defines the grid extent in $x$ and $y$-directions (see Figure 5.1). The user also specifies the number of rows and columns, which essentially determines the size of the individual vertical columns. These vertical columns are the 3D equivalent of vertical slices in a 2D analysis. Similar to a 2D analysis, the resisting forces are computed at the base of each column using the shear strength of the material through which the column base passes. The resisting forces due to cohesion and/or friction generated by the earth pressure applied to the vertical sides of the end columns of the slide mass are not computed by the 3D software. To overcome this limitation, different techniques have been suggested to include the shear resistance along the vertical sides of the slide mass which are presented below.

6.3.1 Stark and Eid (1998)

Stark and Eid (1998) suggest a change in the input data so that software uses a shear force equivalent to the side resistance in calculating the 3D FS. This is accomplished by assuming that an “imaginary” material layer surrounds only the sides of the slide mass. The material properties of the imaginary layer only affects the shear strength along the vertical sides (Figure 6.3) and not the base or the back scarp of the slide mass. The soil parameters of the imaginary layer are:
• Unit weight of the imaginary layer which is equal to that of the upper layer, $\gamma_{\text{imaginary}} = \gamma_{\text{upper}}$

• Imaginary layer is frictionless, $\phi_{\text{imaginary}} = 0$

• The cohesion of the imaginary layer is equal to the shear strength due to $K_O$, acting on the vertical sides of the slide mass, $c_{\text{imaginary}} = K_O \sigma_v \tan \phi_{\text{upper}}$, where, $\sigma_v$ is the average vertical effective stress over the depth of the sliding mass side, and $K_O = 1 - \sin \phi_{\text{upper}}$.

In addition, each vertical side of the sliding mass is assigned a slight (less than 5°) outward inclination to include a single row of columns so that the software can calculate the effect of cohesion in its resisting force calculations.

Results of 3D analyses performed by Stark and Eid (1998) are reproduced in Table 6.1 showing that including side shear resistance increases 3D FS by about 12%. If the side resistance is not incorporated in the 3D analysis, 3D FS is approximately equal to the average of 2D FS of three cross-sections (Figure 6.2) through sliding mass, i.e., average for cross-sections A-A’, B-B’ and C-C’. Stark and Eid (1998) also indicate that the 3D effect also increases with the increasing difference between the shear strength of upper and lower materials.

### 6.3.2 Arellano and Stark (2000)

Arellano and Stark (2000) use a rectangular slide mass without rounded or a curved head scarp (Figure 6.4) in their parametric study. To include side resistance in their analysis, an external horizontal and vertical side force equivalent to the shear resistance due to at-rest earth pressure ($K_O$) acting on the vertical sides at the centroid of the two parallel sides (Figure 6.5) is included. The technique for calculating the shear resistance acting on the vertical sides is the same as Stark and Eid (1998), i.e., $c' = K_O \sigma_v \tan \phi_{\text{upper}}$ and $\phi' = 0$.

The side shear force, $S'$, acting on the vertical sides is estimated by multiplying $c'$ by the cross-sectional area of the vertical side. For simplicity, the side resistance of only upper layer is used and the small side area between the interface of upper material and lower material and the base of failure surface is not included for estimating the cross-section centroid (Figure 6.5).
Additionally, it is assumed that \( S' \) acts parallel to the base of failure surface at a slope of 3% down slope.

The horizontal component \((S'_y)\) and the vertical component \((S'_z)\) of the side shear force \( S' \) are specified in CLARA 2.31 by its point of application, i.e., \( x, y, \) and \( z \)-coordinates (Figure 6.5). If the vertical force \((S'_z)\) is located within the slide mass the software adds it to the total weight of the column directly below it. Vertical force is not included in column total weight if it is outside the slide mass. Horizontal component \((S'_y)\) of the external load is included in horizontal force equilibrium equation even if the point of application is outside the plan area of slide mass.

Arellano and Stark (2000) investigate effect of side shear resistance on three different slope inclinations, i.e., 1H:1V, 3H:1V, and 5H:1V. For each slope inclination, W/H ratios of 1, 1.5, 2, 4, 6, 8, and 10 are analyzed. Arellano and Stark (2000) indicate that because the slope model has uniform cross-sections across the slope, the 2D FS at all cross-sections is the same. Additionally, 3D FS is equal to 2D FS if no external loads are applied, which confirms the finding of Stark and Eid (1998) that shearing resistance along the parallel vertical sides of a slide mass are not considered in 3D FS calculations by existing LE slope stability software. After including the shear resistance along vertical sides, a relationship between 3D/2D FS and W/H ratio was developed by Arellano and Stark (2000) and is shown in Figure 6.6. For example, a slope of 1V:1H with a W/H ratio of one, has 3D/2D FS ratio of about 1.30 which indicates a 30% increase in 2D FS. On the contrary, a slope of 5V:1H with the same W/H ratio, has a 3D/2D FS ratio of about 3.2. The unusually high 3D/2D FS ratio for a 5H:1V slope is due to the use of at-rest earth pressure and will be revised in this study because a 3D/2D FS ratio of 3.2 does not match field observations and will be discussed in subsequent sections.

6.3.3 Eid et al. (2006); and Eid (2010)

Eid et al. (2006) include the shear resistance along the two vertical side of sliding mass by imposing a ‘group’ of external horizontal and vertical forces \((S_y \text{ and } S_z)\) that are the components of the shear resisting force \((S)\). Calculation of the resisting force is the same as used by Stark and Eid (1998)
and Arellano and Stark (2000) except the forces generated by at-rest earth pressure and pore water pressure are calculated separately and then imposed at the centroids of the their corresponding areas on the vertical sides of slide mass. In Arellano and Stark (2000) the earth pressure forces are approximated using the average vertical effective stress over the depth of the sliding mass and applied at the centroid of the vertical sides of the slide mass.

6.4 Magnitude of Side Resistance

In Chapter 4, a comparison of 2D and 3D LE analyses with continuum analyses is presented which confirms the accuracy of LE procedures for the cases analyzed. The comparison shows for rotational and composite failure surfaces, both methods produce results that are within 12% of each other. To investigate the magnitude of side shear resistance, the slide model used by Arellano and Stark (2000) was reanalyzed in 2D and 3D using LE, FE and FD softwares. 2D and 3D LE analyses were performed using software package CLARA-W (Hungr 2001) and a 3D extension of Janbu’s (1956) procedure. 2D and 3D FE analyses were performed using PLAXIS 3D Tunnel V.2 (Brinkgreve and Broere 2004). 2D and 3D FD analyses were also performed using FLAC (Itasca 2000) and FLAC3D (Itasca 2002), respectively.

The objectives of these analyses are to: (1) determine the magnitude of 3D/2D FS ratios computed by FE and FD procedures; (2) determine magnitude of 3D/2D FS ratios computed using $K_O$ and $K_A$ for the side shear resistance in 3D LE analyses; and (3) develop recommendations for the coefficient of earth pressure that should be used to estimate the side shear resistance in 3D LE analysis of translational landslides with vertical sides.

6.4.1 Parametric Slope Model

As a sequel to Arellano and Stark (2000), Chugh (2003) investigated the 5H:1V slope model using FD procedure and shows the significance of boundary conditions in the continuum method. Chugh (2003) reports 3D/2D FS ratio of 2.05 for W/H ratio of one and a 5H:1V slope, which is lower than 3D/2D FS ratio reported by Arellano and Stark (2000) for the same slope conditions, i.e., 3.2.
To draw a direct comparison with previous research, the slope model used by Arellano and Stark (2000) was reanalyzed using LE, FE and FD procedures herein (see Figures 6.4 and 6.5). Detailed information about the model is given in Arellano and Stark (2000). Slope inclinations of 1H:1V, 3H:1V, and 5H:1V with a height (H) of 10 m were analyzed. For FE and FD analyses, each slope inclination was analyzed with W/H ratios of 1, 2, 5, and 10 with friction angles of 30° and 8° for upper and lower materials, respectively. LE analyses were performed for W/H ratios of 1, 1.5, 2, 4, 5, 6, 8, and 10 with four combinations of $\phi_{upper}/\phi_{lower}$ values. The friction angle of upper material ($\phi_{upper}$) was kept at 30° while the friction angle of the lower material ($\phi_{lower}$) was assigned values of 8°, 10°, 20°, and 30°.

To simulate a natural bedding plane or a landfill liner system, the lower material was assumed to slope at 3% down slope. The groundwater level was placed at a height of H/2 as measured at a distance L from the toe and linearly decreasing to a height of zero at the toe (see Figure 6.5).

2D slope models used in LE, FE, and FD analyses for slope inclinations of 1H:1V, 3H:1V, and 5H:1V are shown in Figures 6.7, 6.8, and 6.9, respectively. In FE and FD analysis, the slope models was extended past the locations where the slope failures is likely to occur. Therefore, slope models in FE and FD analysis are wider than models used for the same inclination in LE analysis. Also, the lower material is represented by a layer of 0.8 m thickness, which is followed by a bottom block. The presence of the bottom block ensures that failure surface remains in the weaker layer to model a translational failure.

LE, FE, and FD are different procedures and use different solution strategy. Therefore, the 3D modeling in each procedure is not identical. For a 3D LE analysis, shear resistance along the parallel sides of the slide mass was incorporated by adding external horizontal and vertical side forces at the centroid of the two parallel sides. 3D slope models used in the FE and FD analyses include 6 m wide end blocks and displacement condition of fully fixed ($u = 0$, $v = 0$, and $w = 0$) at the boundaries as indicated in Chapter 4. In addition, the 3D analysis in FLAC3D uses, side blocks with higher strength and an interface between the slope and end blocks to allow relative movement at the slope-block contact. 3D slope models used in LE, FE, and FD analyses for slope inclinations of 1H:1V, 3H:1V, and 5H:1V are shown in Figures 6.10, 6.11, and 6.12, respectively. The material properties used in
the CLARA-W, PLAXIS, and FLAC analyses are shown in Table 6.2.

6.4.2 Effect of Shear Resistance Along Vertical Sides

To verify the magnitude of shear resistance along the vertical sides of slide mass, 3D/2D FS ratios obtained from FE and FD analysis for three slope inclinations were plotted for different W/H ratios. For the 3D LE analysis, horizontal and vertical side force equivalents were computed using the shear resistance due to at-rest earth pressure \( K_O = 1 - \sin \phi \), active earth pressure \( K_A = \frac{1}{1 + \sin \phi} \), and an earth pressure coefficient \( K_{\tau} \) that is in-between the \( K_O \) and \( K_A \) values. For simplicity/consistency in the analyses, a value of \( K_{\tau} = 0.5(K_O + K_A) \) was used for the in-between case. Thereafter, 3D/2D FS ratios computed for LE analysis were compared with results of FE and FD analysis to determine the optimal earth pressure coefficient to use.

For illustration, 2D model results from FE and FD analyses are shown for 1H:1V slope in Figures 6.13 and 6.14, respectively. 3D FE and FD results for W/H ratio of unity for the same slope inclination, i.e., 1H:1V are shown in Figures 6.15 and 6.16, respectively. Different shading in the figures indicate contours for maximum displacement and maximum shear strain rate at the instant of numerical instability for FE and FD analysis, respectively. The maximum displacement and maximum shear strain rate are helpful in identifying the location and shape of the failure surface because it marks the boundary between stable and unstable portions of the model.

6.4.2.1 Results from FE Analysis

3D/2D FS values obtained from FE analyses are shown in Table 6.3. Figure 6.17 shows the relationships between the ratio of 3D/2D FS and W/H for the three slope inclinations obtained from FE analyses. Figure 6.17 shows that 3D/2D FS ratios for all W/H combinations are greater than unity, i.e., 3D FS is always greater than 2D FS. The highest value of 3D/2D FS ratio corresponds to the 5H:1V slope for W/H=1, and 3D/2D FS is 2.04 instead of 3.2 as reported by Arellano and Stark (2000).
6.4.2.2 Results from FD Analysis

3D/2D FS values obtained from FD analyses are shown in Table 6.4. Figure 6.18 presents the relationships between the ratio of 3D/2D FS and W/H for the three slope inclinations obtained from FD analyses. Again, 3D/2D FS ratios for all W/H combinations are greater than unity and the highest value of 3D/2D FS ratio is 2.05 for 5H:1V slope for W/H=1.

6.4.2.3 Comments on Results from Continuum Analysis

Figures 6.17 and 6.18 show that for all three slope inclinations, 3D/2D FS ratios from continuum procedures (FE and FD) show similar trends. 3D/2D FS ratios increase with decreasing W/H ratios and for a given W/H ratio, flatter slopes have higher 3D/2D FS ratios. These observations are in accordance with the findings reported by Arellano and Stark (2000). 3D/2D FS ratios obtained from the FLAC (FD) analysis are slightly higher than PLAXIS (FE). Therefore, FLAC and PLAXIS analysis are upper and lower bounds, respectively, for 3D/2D FS ratios for each slope inclination of model geometry.

6.4.2.4 Results from LE Analysis

3D/2D FS values for 1H:1V, 3H:1V and 5H:1V slopes obtained using external side forces estimated using $K_O$, $K_A$ and $K_\tau$ in LE are shown in Table 6.5, 6.6, and 6.7, respectively. For comparison, 3D/2D FS values from FE and FD analysis are also included in Tables 6.5, 6.6, and 6.7. 3D/2D FS values of LE, FE and FD analysis for individual slope inclinations, i.e., 1H:1V, 3H:1V, and 5H:1V, are plotted in Figures 6.19, 6.20, and 6.21, respectively.

Figure 6.19, 6.20, and 6.21 also show that using $K_O$ produces 3D/2D FS ratios that are higher than those produced by $K_A$. This is caused by $K_O$ being almost 50% higher than $K_A$, i.e., for a $\phi'_{upper} = 30^\circ$, $K_O=0.5$ and $K_A=0.33$.

Figure 6.20, and 6.21 show that using $K_O$ for estimating side shear results in 3D/2D FS ratios that are higher than the upper limit set by FD analysis for same slope inclination. The 3D/2D FS ratio for the 5H:1V slope and W/H=1 is the highest. This ratio (3D/2D FS=3.2) is significantly higher than the FD
3D/2D ratio because $K_O$ is producing too high of a side resistance. However, using $K_O$ for the 1H:1V slope the 3D/2D FS ratios are in agreement with the FD and FE results (see Figure 6.19). However, using $K_A$ in all three slope inclinations underestimates 3D/2D FS ratios for W/H ratios less than 2. Therefore, the optimal earth pressure to estimate field side resistance is in-between $K_O$ and $K_A$.

In summary, for all three slope inclination studied, 3D/2D FS ratios are within the upper and lower FD and FE bounds when an earth pressure coefficient ($K_\tau$) that is in-between $K_O$ and $K_A$ values was used.

The use of an intermediate value of $K_\tau$ for calculation of side forces is supported by field slide mass observations where, generally the slide mass is cracked near the ground surface and the cracks decreases in width with depth. Therefore, near the surface the side resistance may agree better with $K_A$ and near the base of the slide mass it may agree better with $K_O$. Based on triaxial tests, Lambe and Whitman (1969) report that little horizontal strain, less than 0.5%, is required to change the stresses from at-rest to active earth pressure. Therefore, it is possible that after the slip surface develops and movement begins, the at-rest earth pressure transitions to an active pressure.

It is probably confusing that an earth pressure applied to the side of the slide mass is being used when the slide mass is moving perpendicular to the lateral earth pressure. Therefore at-rest or active deformation is not relevant to the movement of the slide mass. This is correct that cracks near the ground surface are not in the direction of slide movement. However, the earth pressure coefficient is only being used to develop a reasonable approximation of the side shear resistance and not explain the lateral pressure applied to the slide mass because the slide mass eventually moves perpendicular to the applied earth pressure.

Figure 6.22 presents a relationship between the 3D/2D FS ratios and W/H for the three slope inclinations considered in the parametric study. Dotted lines show the 3D/2D FS ratios that were obtained using side shear resistance estimated using $K_O$, while, the solid lines represent 3D/2D FS ratios obtained using $K_\tau$, i.e., in-between $K_O$ and $K_A$. Figure 6.22 shows that for the 1H:1V slope there is little difference between 3D/2D FS ratios obtained using $K_\tau$ or $K_O$. However, the difference in 3D/2D FS ratios is greater for flatter slopes, i.e., 5H:1V slope (W/H=1), using $K_\tau$ and $K_O$ (2.10 and 3.2,
respectively). The maximum value of 3D/2D FS obtained using $K_\tau$ to calculate side resistance is about 2% higher than 3D/2D FS ratio obtained from FE and FD analyses, i.e 2.05 vs 2.10 (see Table 6.7).

Based on these analyses and $K_\tau$, updated relationships between ratio of 3D/2D FS and W/H for the three slope inclinations are presented in Figure 6.23. These relationships are for friction angles of 30° and 8° for upper and lower materials, respectively, and supercede the relationships presented by Arellano and Stark (2000).

6.4.3 Influence of Shear Strength on 3D/2D FS Ratios using $K_\tau$

Based on new LE analyses using $K_\tau$ for calculating side shear resistance, updated figures (see Figures 5, 6, and 7 in Arellano and Stark 2000) that show the influence of various friction angle ratios on 3D/2D FS values are shown in Figures 6.24, 6.25, and 6.26. For these analyses, the friction angle of the upper material ($\phi_{upper}$) of 30° was used for all analyses while friction angle of the lower material ($\phi_{lower}$) was varied as 8°, 10°, 20°, and 30° resulting in four combinations of $\phi_{upper}/\phi_{lower}$, e.g., 1, 1.5, 3, and 3.75. Figure 6.26 shows the influence of shear strength is most pronounced for 5H:1V slope inclination. For example, 5H:1V slope (see Figure 6.26) and a value of W/H of 10 and 1, the difference in 3D/2D FS ratios ranges from 0.9 to 1.04, respectively. These values are lower than the values reported in Arellano and Stark (2000) which ranges from 1.6 to 2.1, respectively, for the same range of $\phi_{upper}/\phi_{lower}$ and a 5H:1V slope inclination.

Finally, Figure 6.27 illustrates the influence of $\phi_{upper}/\phi_{lower}$ on 2D and 3D FS for 5H:1V slope instead of 3D/2D FS ratios. Arellano and Stark (2000) use two separate figures to illustrate the influence of $\phi_{upper}/\phi_{lower}$ on 2D and 3D FS for 5H:1V slope. However, Figure 6.27 produced herein, shows the influence of shear strength on 2D and 3D FS for 5H:1V slope in a single plot.

As highlighted by Arellano and Stark (2000), the effect of varying the friction angle between the upper and lower materials is more significant on 3D than 2D FS values (see Figure 6.27). The effect is greater for lower W/H ratios and the 3D FS values approaches 2D FS values for higher W/H ratios.
6.5 Modified Model Using Side Inclination

All present techniques to include side resistance in 3D LE slope stability calculations rely on estimating shear resistance using the earth pressure acting on vertical sides of the slope mass. Input data in the software is then modified so side shear resistance can be considered in the FS calculation. Application of these techniques to the case studies used by Stark and Eid (2000), and Arellano and Stark (2000) show the results are in agreement with the observed FS in the field. However, the complexity involved in the manual calculation of these side forces limits the practical use of these procedures in routine stability calculations.

As indicated earlier, slope stability software uses the shear strength of the material located at the bottom of each slice (or column in a 3D analysis). For a rotational landslide that is modeled using an ellipsoid, the column bases on the sides are positioned at a higher level than the column bases at the central section, i.e., the columns follow the sides of the ellipsoid. Therefore, if the upper material has a higher strength, it is captured in the 3D calculations because the column bases are located in this material. However, in a translational landslide with vertical sides, the column bases are located in the lower material because the sides are vertical. Therefore, the shear strength of the upper material on both ends is not used in the calculation of resisting forces. If shear strength of the upper material is higher than the lower material, then the 3D/2D FS ratios will be higher if the strength of stronger material is incorporated in the analysis.

With the availability of 3D/2D FS magnitudes determined from FE and FE analyses, method for incorporating side shear resistance through slight modifications in existing LE modeling procedure were also investigated and summarized below:

6.5.1 Modified Model Using Side Inclination

Because LE programs only use the soil parameters that are at the base of the column, a slight modification in modeling the slide mass geometry was studied. The vertical sides were assigned a slight outward inclination from vertical ($\alpha_y$) of about 3°. This was achieved by defining the failure mass at four 2D cross-sections. For a 1H:1V slope with W/H ratio of 1 (Figure 6.28),
four cross-sections were placed at x=0, 0.5, 10.5, and 11. Slope failure at two inner cross-sections (x=0.5, and x=10.5) was defined using 2D cross-sections (Figure 6.7(a)). The failure surface was defined to coincide with ground surface at two end cross-sections (x=0, and x=11). Figure 6.29 shows generated 3D columns between the 2D cross-sections using orthogonal interpolation by CLARA-W. Figure 6.29 shows the column bases on the sides are gradually moving up towards the ends and pass through the upper material. With the modified model, the 3D/2D FS ratio is 1.15 (2D FS remains unchanged) even though no external side force was applied using a earth pressure coefficient. Figure 6.30 shows the higher strength of the upper material was incorporated on the sides in 3D analysis (brown color on the sides of slide mass) using a slight outward tilt of the vertical sides. If the soil mass is extended by 3m to maintain side inclination angles of about 7°, a 3D/2D FS ratio of 1.25 is obtained which is in agreement with the 3D/2D FS ratio of 1.26 obtained using $K_r$ to estimate the side shear forces.

Reanalysis of the all three slope inclinations was performed by varying the side slope inclination, $\alpha_x$, from 3° to 7°. The 3D/2D FS ratios obtained for 1H:1V, 3H:1V, and 5H:1V are tabulated in Table 6.8, 6.9, and 6.10, respectively. The relationship between 3D/2D FS ratios and W/H for the three slope inclination alongside the FE and FD analysis results are shown in Figures 6.31, 6.32, and 6.33. Figures 6.31, 6.32, and 6.31 show using the same $\alpha_x$ for all three slope inclinations does not yield 3D/2D FS ratios that are within upper and lower bounds set by FD and FE analysis. For a 1H:1V slope inclination, an $\alpha_x$ value of 3°, is required while $\alpha_x$ of about 7° may be required for 5H:1V slope.

Based on this parametric study, Figure 6.34 provides suggested side inclination angles, $\alpha_x$, to be used for LE analysis of different slope inclinations that yields 3D/2D FS ratios that are in agreement with continuum analyses. The use of these angles omit the need to use an earth pressure coefficient.

### 6.5.2 Influence of Shear Strength on 3D/2D FS Ratios Using Side Inclination

For comparison purposes, Figures 6.35, 6.36, and 6.37 show the influence of various friction angle ratios on 3D/2D FS values when 3D LE analyses are
performed using side inclinations ($\alpha_x$) of $3^\circ$ to $7^\circ$ instead of applying external side forces estimated using $K_{\tau}$. Comparison of these figures with Figures 6.24, 6.25, and 6.26 shows for the same slope inclination, plots are identical except varying the ratio of $\phi_{upper}/\phi_{lower}$ has more influence (when using $\alpha_x$). For example, for 5H:1V slope (see Figure 6.37) and a value of W/H of 10 and 1, the difference in 3D/2D FS ratios ranges from 0.6 to 1.01, respectively. These values range from 0.9 to 1.04, respectively, for the same range of $\phi_{upper}/\phi_{lower}$ ratios investigated for 5H:1V slope inclination when external side forces on vertical sides (see Figure 6.26) are used. This variation may be caused by the close proximity of two input cross-sections used at the slide mass ends to generate an inclined failure surface near the ends using interpolation which influences the location of the 3D column bases.

In summary, the ratios of 3D/2D FS and W/H for the slope inclinations considered are not significantly affected when either external forces estimated using earth pressure ($K_{\tau}$) or slight outward inclination ($\alpha_x$) of the vertical sides is used. In both cases, the 3D/2D FS ratios remain within the range obtained using FD and FE analyses.

6.6 Review and Summary of Chapter 6

- Earlier research indicates that available 3D LE slope stability software does not incorporate side shear resistance for slides with vertical sides.

- To incorporate the shear resistance along the two vertical sides that parallel the direction of movement in translational slides, external forces can be applied in LE software while calculating 3D FS.

- For translational slides with vertical sides, continuum procedures (FE and FD) may assist in providing better understanding of 3D/2D FS ratios. However, meaningful analyses using continuum mechanics require special attention to slide mass boundary conditions.

- As observed in Chapter 4, FD procedure gave slightly higher FS values than FE procedure. Therefore, 3D/2D FS ratios obtained from FD and FE procedures can be used as upper and lower bounds respectively, to investigate the effects of side resistance along the slide mass.
• Comparison with FD and FE analyses indicates that using $K_O$ for approximating shear resistance along two vertical sides in LE method overestimates 3D/2D FS ratios.

• An earth pressure coefficient ($K_r$) that is in-between $K_O$ and $K_A$ provides a better estimate of shear resistance acting along two vertical sides and results in 3D/2D FS ratios that are in agreement with FE and FD analysis for the cases considered herein.

• The earth pressure concept is only being used to develop a reasonable approximation of the side shear resistance and not explain the lateral pressure applied to the slide mass. The applied earth pressure is really not applicable to the side shear resistance because the resistance is developed along the sides of the slide mass as it moves perpendicular to the applied earth pressure.

• 3D/2D FS ratios for all slope inclinations analyzed using FE, FD, and LE procedures are higher than unity.

• LE, FE, and FD show that effect of side resistance is a function of W/H, and slope inclination. For example, for a given slope inclination, 3D effects are higher for lower W/H ratios, which tends to become constant for greater W/H ratios (e.g., W/H > 6).

• Flatter slopes have higher 3D/2D FS because of larger side area.

• Maximum 3D/2D FS ratio obtained from continuum analyses is 2.05 (5H:1V slope and W/H = 1)

• Modeling of a slope failure, that ensures some of the column bases are founded in the upper material are sufficient to incorporate side shear resistance. This is similar to the analogy of ‘imaginary layer’ used by Stark and Eid (1998).
6.7 Tables

Table 6.1: Results of 3D slope stability analysis (from Stark and Eid 1998)

<table>
<thead>
<tr>
<th>Analysis Type</th>
<th>Input Data</th>
<th>FS(^1)</th>
<th>Difference(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D (Using CLARA 2.31)</td>
<td>No side resistance</td>
<td>0.90</td>
<td>0 %</td>
</tr>
<tr>
<td>3D (Using CLARA 2.31)</td>
<td>With side resistance</td>
<td>1.01</td>
<td>12 %</td>
</tr>
<tr>
<td>2D (Using CLARA 2.31)</td>
<td>Section A-A'</td>
<td>0.92</td>
<td>-</td>
</tr>
<tr>
<td>2D (Using CLARA 2.31)</td>
<td>Section B-B'</td>
<td>0.91</td>
<td>-</td>
</tr>
<tr>
<td>2D (Using CLARA 2.31)</td>
<td>Section C-C'</td>
<td>0.87</td>
<td>-</td>
</tr>
</tbody>
</table>

\(^1\) Janbu’s (1956) simplified procedure;  
\(^2\) Difference in percentage from average 2D FS=0.90

Table 6.2: Material properties for stability analysis of slope model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Upper Material</th>
<th>Lower Material</th>
<th>Bottom Block</th>
<th>End Blocks(^2)</th>
<th>Interface(^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Weight (\gamma (kN/m^3))</td>
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<td>18</td>
<td>18</td>
<td>25</td>
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<tr>
<td>Cohesion, (c'(kPa))</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Friction Angle, (\phi'(\circ))</td>
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<td>8,10,20,30</td>
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<td>45</td>
<td>30</td>
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<tr>
<td>Dilatation Angle, (\psi(\circ))</td>
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<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Young’s Modulus ((kN/m^2))</td>
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<td>(3\times10^3)</td>
<td>(3\times10^5)</td>
<td>(3\times10^6)</td>
<td>-</td>
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<tr>
<td>Poisson’s Ratio, (\nu)</td>
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<td>0.35</td>
<td>0.35</td>
<td>-</td>
</tr>
<tr>
<td>Bulk Modulus ((kN/m^2))</td>
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<td>(3\times10^3)</td>
<td>(3\times10^5)</td>
<td>(3\times10^6)</td>
<td>-</td>
</tr>
<tr>
<td>Shear Modulus ((kN/m^2))</td>
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<td>(1\times10^3)</td>
<td>(1\times10^5)</td>
<td>(1\times10^6)</td>
<td>-</td>
</tr>
<tr>
<td>Normal Stiffness ((kN/m^2))</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>(1\times10^4)</td>
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<tr>
<td>Shear Stiffness ((kN/m^2))</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>(1\times10^3)</td>
</tr>
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\(^1\) Density \(\rho (Kg/m^3) = \text{Unit weight} \times 1000/9.81;\)  
\(^2\) End blocks in PLAXIS analysis use same properties as slope;  
\(^3\) Only used in FLAC analysis
Table 6.3: Summary of 3D/2D FS ratios-PLAXIS

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<thead>
<tr>
<th>W/H</th>
<th>Ratio of 3D/2D FS</th>
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</thead>
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<tr>
<td></td>
<td>1H:1V</td>
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<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
<td>1.10</td>
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<tr>
<td>5</td>
<td>1.03</td>
</tr>
<tr>
<td>10</td>
<td>1.01</td>
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</table>

Table 6.4: Summary of 3D/2D FS ratios-FLAC

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<td>1H:1V</td>
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<td>1.38</td>
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<td>2</td>
<td>1.24</td>
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<td>1.07</td>
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<td>10</td>
<td>1.05</td>
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</table>

Table 6.5: Results of 3D/2D FS ratios from FD, FE, and LE (with side forces) for 1H:1V slope

<table>
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<tr>
<th>W/H</th>
<th>Ratio of 3D/2D FS</th>
<th>FLAC</th>
<th>PLAXIS</th>
<th>$K_O$</th>
<th>$K_T$</th>
<th>$K_A$</th>
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<td>1.13</td>
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<td>-</td>
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<td>1.07</td>
<td>1.06</td>
<td></td>
</tr>
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<td>1.04</td>
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<td>-</td>
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<td>1.04</td>
<td>1.03</td>
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</tr>
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<td>-</td>
<td>1.03</td>
<td>1.03</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
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Table 6.6: Results of 3D/2D FS ratios from FD, FE, and LE (with side forces) for 3H:1V slope

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<tr>
<th>W/H</th>
<th>Ratio of 3D/2D FS</th>
<th>FLAC</th>
<th>PLAXIS</th>
<th>$K_O$</th>
<th>$K_T$</th>
<th>$K_A$</th>
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<td>1.05</td>
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<td>1.05</td>
<td>1.03</td>
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Table 6.7: Results of 3D/2D FS ratios from FD, FE, and LE (with side forces) for 5H:1V slope

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</thead>
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<td>FLAC</td>
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Table 6.8: Results of 3D/2D FS ratios from FD, FE, and LE (with side inclination) for 1H:1V slope

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Table 6.9: Results of 3D/2D FS ratios from FD, FE, and LE (with side inclination) for 3H:1V slope

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<th></th>
<th></th>
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</thead>
<tbody>
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<td>5°</td>
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<td>1.05</td>
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</table>

Table 6.10: Results of 3D/2D FS ratios from FD, FE, and LE (with side inclination) for 5H:1V slope

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<tr>
<th>W/H</th>
<th>Ratio of 3D/2D FS</th>
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</thead>
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<td>4°</td>
<td>5°</td>
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CHAPTER 7

3D COMPUTER CODE

7.1 Introduction

The code for CLARA-W (Hungr 2001) was provided by Professor O. Hungr, University of British Colombia, Canada. To incorporate and/or verify some of the findings of this study, a new 3D limit equilibrium (LE) program, 3DDEM-Slope, was developed as part of this study. 3DDEM-Slope was coded in Visual Basic 6 (VB-6) following the basic framework of CLARA-W (Hungr 2001). 3DDEM-Slope uses a similar graphical user interface (GUI) as CLARA-W (Hungr 2001) to input the material properties. The 3D geometry of the slope, various material layers, piezometric surface, and failure surface are input using a DEM generated using the Surfer 9 software (Golden Software 2010). 3DDEM-Slope uses Surfer 9 (Golden Software 2010) for viewing various surfaces by accessing Surfer-9 (Golden Software 2010) from inside 3DDEM-Slope.

3DDEM-Slope uses 3D extensions of Bishop’s (1955) simplified, Janbu’s (1956) simplified, and Spencer’s (1967) procedures presented by Hungr et al. (1989) and Hungr (2001). Improvements in 3DDEM-Slope include input of shear strength using stress dependent failure envelope to include the stress dependent behavior of soils; 2D and 3D correction factor for Janbu’s (1956) simplified procedure; and a subroutine for incorporating side shear resistance along the two vertical sides of the slide mass that parallel the direction of movement in translational landslides. Although the program uses a 3D DEM file, 3DDEM-Slope can be used to calculate a 2D FS at any desired cross-section in the 3D surface. In addition, 3DDEM-Slope compare the 2D FS for a cross-section in the middle of the slide mass with the overall 3D FS. If the 3D FS is less than (possible if 2D FS central cross-section is not critical) or equal to 2D FS, the user may select to apply external sides forces that
are calculated based on the findings of this study and obtain a corrected 3D FS. 3DDEM-Slope also uses improved subroutines for calculation of the 3D center of rotation and vertical column base angle. The column base angles are calculated using a third-order finite difference estimator (Horn 1981) using all eight outer points of a grid node instead of using only two adjacent grid nodes so the base angle corresponds to the angle of an inclined plane instead of a line as occurs in 2D calculations.

7.2 Improvements Made in 3DDEM-Slope

During the course of this study it was found that to enhance the applicability and accuracy of the present commercial 3D LE software, some additions and/or improvements should be incorporated. Following addition and/or improvements were incorporated in the 3DDEM-Slope software developed during this study:

- Provision for modeling the stress dependent nature of shear strength envelope.
- Janbu’s (1973) correction factor for 2D and 3D slope stability analysis using Janbu’s (1956) simplified procedure.
- Option for calculating 3D FS by incorporating shear resistance from sides for translational slides with vertical sides.
- Calculation of inclination of column bases using technique described by Horn (1981).
- Calculation of 2D FS at a specified cross-section directly while working with 3D input DEM.

7.2.1 Stress Dependent Nature of Shear Strength Envelope

Shear strength parameters are a major input parameter for any slope stability analysis. There are different shear strength models that can be used to relate the available shear strength of a soil as a function of measured properties and associated stress conditions, for example Coulomb isotropic,
Coulomb anisotropic, bilinear, and nonlinear models (Hoek and Brown 1981). The Mohr-Coulomb failure criterion is commonly used to describe the shear strength of soils and states that a combination of normal and shear stress creates a more limiting state than that would be found if only the major principal stress or maximum shear stress are considered individually (Abramson et al. 1996). Figure 7.1 shows the Mohr-Coulomb failure envelope in which normal and shearing stresses are plotted on the horizontal and vertical axes, respectively. For a series of laboratory shear tests, the envelope of the failure circles represents the locus of points associated with the failure of specimens and is known as the rupture line. Figure 7.1 shows that the rupture line is curved (non-linear) but a straight line (linear) is usually used to approximate the envelope within a selected stress range. The resulting expression is known as Coulomb’s equation and is:

\[ s = c' + \sigma' \times \tan \phi' \]

(7.1)

where \( c' \) is the effective stress cohesion intercept on the strength axis, and \( \phi' \) is the effective stress angle of internal friction related to the slope of Mohr-Coulomb rupture line.

Stark and Eid (1998) highlight the importance of the stress-dependent shear strength envelope in slope stability analyses. Stark and Eid (1998) show that for a 2D analysis using a linear failure envelope that passes through the origin and the shear strength corresponding to the average effective normal stress on the slip surface, the FS can be overestimated by as much as 10%. A similar effect is expected for a 3D analysis. In addition, the effect of ignoring the nonlinearity of the failure envelope should increase with an increase in the range of effective normal stress acting on the slip surface. Based on results of 2D analyses, Stark and Eid (1998) recommend including an option for inputting a nonlinear failure envelope in stability analyses to account for the stress dependent nature of soil shear strength.

Jiang et al. (2003) investigate the effect of linear and nonlinear failure envelopes on 2D and 3D slope stability computations and report that neglecting strength envelope nonlinearity is more pronounced under 3D conditions than 2D conditions. Using examples, Jiang et al. (2003) show that a linear approximation of the failure envelope can result in an overestimation of 3D FS by up to 48%.
A nonlinear shear strength model proposed by Hoek and Brown (1981) is a popular approach to model the nonlinearity of a shear strength envelope and uses the following equation:

$$
\tau = AU_c \left( \frac{\sigma'}{U_c} + D \right)^B
$$

(7.2)

where $U_c$ is the uniaxial compressive strength of the intact rock material and $A$, $B$, and $D$ are empirical coefficients. Hoek and Brown (1981) suggest typical values of the coefficients for jointed rocks masses of various types and quality. Other values are determined by curve fitting from test results or back-calculation (Hungr 2001).

A better approach for modeling the stress dependent nature of failure envelope is to use the actual values of normal stress ($\sigma'$) and shear stress ($\tau$) measured from laboratory tests. However, the assessment of the normal stress (or force) at a particular location on the slip surface is difficult (Fredlund 1984). In all LE procedures (except ordinary method of slices), the normal force is derived from vertical force equilibrium and the shear strength parameters are used in the derived equation. Because Ordinary Method of Slices (OMS) allows the normal stress to be computed without using shear strength (Duncan and Wright 2005) therefore an initial estimate of normal stress can be obtained using the normal force equation in the OMS. Thus for a nonlinear shear strength envelope an additional iteration is required to compute the corresponding normal stress and shear strength.

In 3DDEM-Slope, a stress dependent nonlinear shear strength model is implemented for 2D and 3D analyses. The user can select “stress dependent failure envelope” as the desired strength model and input up to fifteen data point sets to define $\sigma'$ and $\tau$ values from available test results as shown in Figure 7.2. In the first iteration, 3DDEM-Slope estimates the normal stress at the column base using the OMS. Based on the initial value of normal stress at the column base, the corresponding slope of the nonlinear failure envelope is computed (see Figure 7.3). The shear strength is represented by a temporary cohesion ($c'_{\text{temp}}$) and friction angle ($\phi'_{\text{temp}}$) representing a failure envelope tangent to the nonlinear envelope at the estimated normal stress. Each column now has a $c'_{\text{temp}}$ and $\phi'_{\text{temp}}$ relative to the effective normal stress at the column base. Using these initial estimated values of $c'_{\text{temp}}$ and
\( \phi'_{\text{temp}} \) the normal force at the column base is calculated which is used in subsequent iterations to estimate revised values of \( c'_{\text{temp}} \) and \( \phi'_{\text{temp}} \). The \( c'_{\text{temp}} \) and \( \phi'_{\text{temp}} \) values in this model should not be considered as a true cohesion or friction angle. The \( c'_{\text{temp}} \) value in this model is simply a cohesion intercept along with a corresponding friction angle \( (\phi'_{\text{temp}}) \) which is defined by tangent at the desired normal stress on the nonlinear failure envelope. Because an apparent cohesion and friction angle is calculated in each iteration based on the effective normal stress at the base of each column, the solution routines are not affected by use of a stress dependent failure envelope and use same FS equations as for linear shear strength envelope.

7.2.2 2D and 3D Correction Factor for Janbu’s (1956) Simplified Procedure

As shown in Chapter 4, 2D Janbu’s (1956) simplified procedure produces a FS that is consistently lower than that of Bishop’s (1955) simplified, Morgenstern and Price’s (1965), and Spencer’s (1967) procedures. This is caused by the assumption of ignoring interslice shear forces, which results in an interslice force angle of zero for the Janbu’s (1956) simplified procedure (1965). To compensate for the neglected interslice shear forces, Janbu (1973) proposed a correction factor, \( f_o \). This correction factor is a function of slide geometry and strength parameters of the soil. Corrected Janbu FS, \( F_S_{\text{corrected}} \) is obtained by multiplying the calculated or uncorrected FS, \( F_S_{\text{uncorrected}} \) with \( f_o \):

\[
F_S_{\text{corrected}} = f_o \times F_S_{\text{uncorrected}} \tag{7.3}
\]

Janbu performed slope stability analyses using his simplified and rigorous procedures for the same slopes with homogeneous soil conditions and developed the correction relationships as shown in Figure 7.4. Figure 7.4 shows the variation of \( f_o \) as a function of slope geometry (\( d/L \)) and soil shear strength parameters \((c', \phi')\). It is added that original figure provided by Janbu (1973) showed \( c \) and \( \phi \) instead of \( c' \) and \( \phi' \). Because Janbu’s (1956) procedure is an effective stress procedure therefore using \( c \) and \( \phi \) in the correction relationship caused a confusion if the soil parameters were total total stress instead of effective stress. However, to illustrate the use of correction relationship, Janbu (1973) use an example having \( c' \) and \( \phi' \). Therefore, in
this study Figure 7.4 has been changed to read $c'$ and $\phi'$. For convenience, $f_o$, may be calculated using the following formula proposed by Abramson et al. (1996):

$$f_o = 1.0 + b_1 \left[ \frac{d}{L} - 1.4 \left( \frac{d}{L} \right)^2 \right]$$  \hspace{1cm} (7.4)$$

where $b_1$ varies according to the soil shear strength parameters as follows:

- $\phi'$ only soils ($c' = 0$) : $b_1 = 0.31$  \hspace{1cm} (7.5)
- $\phi'$ and $c'$ soils : $b_1 = 0.50$  \hspace{1cm} (7.6)
- $c'$ only soils ($\phi' = 0$) : $b_1 = 0.69$  \hspace{1cm} (7.7)

Depending on the soil shear strength parameters (i.e., $c'$ only, $\phi'$ only or both $c'$ and $\phi'$), appropriate values of $b_1$ should be selected for use in Equation 7.4. Janbu (1973) does not provide guidance for selecting, $f_o$ for a failure surface intersecting layered soils with different shear strength parameters, e.g., $c'$ only and $\phi'$ only or both $c'$ and $\phi'$. In mixed soil layers, Abramson et al. (1996) suggest using Equation (7.6) to estimate, $f_o$.

In Chapter 4, it was observed that Janbu’s correction factor for 2D analysis, computed at the center of slide mass is applicable to 3D analyses as well. Therefore, Equation (7.4), has been implemented in the new 3D code developed herein to account for correction factor in 2D and 3D Janbu’s procedure. For verification purposes, the 2D and 3D cases analyzed in Chapter 4 were reevaluated using the corrected Janbu’s stability procedure in the new code. The results are shown in Table 7.1 and show that for the 2D cases, that computed correction factor ranges from 1.08 to 1.09. The corrected FS is 8-9% higher than the uncorrected FS. After applying the correction to the FS calculated using Janbu’s (1956) simplified procedure for problem-I (homogeneous material) and problem-II (weak layer), the corrected FS are similar to the Janbu FS reported by Fredlund and Krahn (1977) and FS computed by XSTABL (Sharma 1996) using Janbu procedure (see Table 7.1). For both cases, the corrected Janbu FS is within 5% of FS computed by Spencer (1967) procedure coded in CLARA-W (Hungr 2001) which indicates that correction factor has been implemented correctly in the new code.
Similarly, for the 3D cases analyzed in Chapter 4, the application of correction factor to two instances of problem-I (3D analysis and 2D weighted average for homogeneous material), yield a computed FS (Janbu’s 3D extension) within 0.5% of FS computed by extension of Spencer’s procedure to 3D (see Table 7.1). For 3D analysis of problem-I with weak layer (similar to 2D problem-II), the corrected 3D FS (Janbu) is less than 3% higher than 3D FS (Spencer) so the Janbu’s correction factor is applicable to 3D.

7.2.3 Effect of Shearing Resistance along Vertical Sides of Slide Mass

In Chapter 6 it was discussed that 3D LE software do not consider the effects of shear resistance offered by vertical sides that parallel the direction of movement of a translational landslide mass. Consequently, the computed 3D FS may be underestimated or back-calculated shear strengths can be overestimated. Based on results of the parametric study using FE and FD analysis, it was found that using an earth pressure coefficient ($K_T$) that is in-between $K_O$ and $K_A$ values provides a reasonable estimate of shear resistance acting along two vertical sides and results in 3D/2D FS ratios that are in agreement with FE and FD analysis.

To overcome this limitation of LE software, a subroutine is added in 3DDEM-Slope that compares the 2D FS for a cross-section in the middle of the slide mass with the overall 3D FS. This provides the user with a warning signal that 3D/2D FS ratio is less than the reference values obtained from FD and FE analyses for same width to height ratio and slope inclination. If so, the user may select to apply external sides forces that are calculated based on the findings of this study and obtain a corrected 3D FS. These external side forces are added at the centroid of the two parallel sides and a corrected 3D FS is reported. Figure 7.5 shows the flow chart of subroutine for incorporating shearing resistance along vertical sides of slide mass in the 3DDEM-Slope.
7.2.4 Calculation of Column Base Angles

One of the inputs in the FS equations in LE procedures is the slope of the failure surface. In 2D analysis, the slope ($\alpha$) is computed as a straight line at the base of a slice, however, a 3D analysis uses a surface represented by a grid. Thus in 3D analysis, slope of column base is required to be calculated in two directions (see $\alpha_x$ and $\alpha_y$ in Figure A.1). For a specified failure surface (like an ellipsoid, wedge or composite surface etc) the column base angles may be calculated directly from the geometry of the ellipsoid or plane. However, column base angles for a gridded surface in a back-analysis must be derived mathematically at each grid node either by computing differences within a square filter or by fitting a polynomial to the data within the filter. There are several methods proposed in the literature that can be used to estimate slope of a gridded plane in two directions. Burrough and McDonell (1998) give a review of different available methods to compute slope of a gridded surface. The slope of the column base is generally computed locally for each grid node on the altitude matrix from a data of $3 \times 3$ grid nodes as shown in the Figure 7.6. The simplest finite difference estimate of the gradient in $x$ and $y$-direction at a grid node $i,j$ is the Maximum Downward Gradient as follows:

$$\left[ \frac{\delta z}{\delta x} \right]_{ij} = \max \left[ \frac{(z_{i+1,j} - z_{i-1,j})/2}{d x} \right]$$

$$\left[ \frac{\delta z}{\delta y} \right]_{ij} = \max \left[ \frac{(z_{i,j+1} - z_{i,j-1})/2}{d y} \right]$$

where $dx$ and $dy$ are the dimensions of column base in $x$ and $y$-direction. Because this method uses only two closest neighboring grid nodes, it has the disadvantage that local errors in terrain elevation contribute quite significantly to errors in slope (Burrough and McDonell 1998). Other higher order finite difference slope estimators use either second order finite difference algorithm fitted to the four neighboring grid node (like Fleming and Hoffer 1979, Ritter 1987, Zavenbergen and Thorne 1987) or a third order finite difference estimator that uses all eight outer grid nodes (Sharpnack and Atkins 1969, Horn 1981).

Jones (1997) has compared eight different algorithms for computing slope using real and synthetic DEM surfaces. Based on the differences between values predicted by an algorithm and the true values of the test surfaces, Jones
(1997) indicate that Horn’s (1981) and Zavenbergen and Thorne’s (1987) algorithms are among the best while the Maximum Downward Gradient (See Equations (7.8) and (7.9)) is consistently one of the worst. Because of their accuracy, Horn’s (1981) and Zavenbergen and Thorne’s (1987) algorithms are used by several commercial Geographical Information System (GIS) software (Burrough and McDonell 1998).

In 3DDEM-Slope the column base angles are calculated using the third-order finite difference estimator (Horn 1981) that uses all eight outer points of a grid node. Referring to Figure 7.6(b), gradient in $x$ and $y$-direction at a central grid node $i,j$ is computed as follows:

\[
\begin{align*}
\frac{\delta z}{\delta x}_{ij} &= \left[\left(z_{i+1,j+1} + 2z_{i+1,j} + z_{i+1,j-1}\right)
- \left(z_{i-1,j+1} + 2z_{i,j-1} + z_{i-1,j-1}\right)\right]/8dx \\
\frac{\delta z}{\delta y}_{ij} &= \left[\left(z_{i+1,j+1} + 2z_{i,j+1} + z_{i-1,j+1}\right)
- \left(z_{i+1,j-1} + 2z_{i,j-1} + z_{i-1,j-1}\right)\right]/8dy
\end{align*}
\]

(7.10) (7.11)

For each grid node, the subroutine for column base angles in 3DDEM-Slope calculates the maximum rate of change in elevation over the distance between that grid node to its eight neighbors. The subroutine fits a plane to the $z$-values of a 3 x 3 cell neighborhood around the center node.

If any of the neighboring node does not contain any data for $z$-value, the $z$-value of the center cell is assigned to that location. For example the columns and/or rows at the problem extremities, will have at least three grid nodes that will contain No Data as their $z$-values. These grid nodes will be assigned the $z$-value of central node. The result is a flattening of the 3 x 3 plane fitted to these edge nodes, which usually leads to a reduction in the slope.

7.2.5 2D Analysis of a Specified Cross-Section from 3D DEM Input

Although 3DDEM-Slope uses a 3D DEM file, it can be used to calculate a 2D FS at any desired cross-section in the 3D surface. 3D extensions of Bishop’s (1955) simplified, Janbu’s (1956) simplified, and Spencer’s (1967) procedures presented by Hungr et al. (1989) and Hungr (2001) revert to standard forms of
respective 2D procedures thus common solver routine is used for 2D and 3D analysis for various procedures. The options of 2D and 3D analysis are added as subroutines under the various slope stability procedures. The advantage of this coding is that a 2D file is not required to be exported each time to calculate a 2D FS and a 2D FS can be calculated at any desired cross-section while working with the 3D DEM input.

7.3 Description of 3DDEM-Slope Software

3DDEM-Slope uses Cartesian coordinate system as shown in Figure 7.7, where $x$-axis is perpendicular and $y$-axis is parallel to direction of sliding, respectively. The FS computations are performed for left facing slopes thus DEM input files should be oriented in the same direction. The input file for 3DDEM-Slope stores the problem description and material properties. Slope geometry, slip surface geometry, piezometric surface, and material layer geometry (maximum layers = 7) are loaded from SURFER-9 (Golden Software 2010) * .grd file to calculate a 3D or 2D FS.

7.3.1 Data Input and Slope Stability Analysis in 3DDEM-Slope Software

Following procedure is used for data input and slope stability analysis in 3DDEM-Slope:

- Installation
  - Unzip folder containing 3DDEM-Slope
  - Double click setup for installation to proceed

- Start up (see Figure 7.8)
  - Browse to folder containing 3DDEM-Slope in the start menu
  - Double click program icon
  - Continue

- Problem Description (see Figure 7.9)
– Select “Create a New File” or “Open an existing File”

– Input problem description as follows:
  * Project details
  * Unit weight of water
  * Number of layers
  * No of piezometric surfaces
  * Note: There is no need to specify grid extent and density as it will be automatically updated after DEM of surfaces are loaded.

– Continue

• Material Properties (see Figure 7.10)
  – Select material strength model for each layer from drop down menu (like Mohr-Coulomb, stress dependent etc)
  – Input material strength properties of each layer
  – Continue.
  – Note: layers are numbered from bottom to up

• Layer Input Method (see Figure 7.11)
  – Select “Surfer(TM) Files (*.grd)” or “Cross-section based”
  – Input material strength properties of each layer
  – Continue.

  – Note: Cross-section based input requires 2D slice data similar to spreadsheet computations described in Duncan and Wright (2005). This is only implemented to verify 2D computation routines for 2D Bishop’s (1955) simplified procedure, thus may not be used.

• Input Surfer(TM) *.GRD Files (see Figure 7.12)
  – Based on total number of surfaces input in problem description, user is presented with individual DEM input tabs for each surface (for example ground surface, slip surface, piezometric surface, and material layers)
Starting from lowest layer (Layer 1), click layer tab and browse to select the corresponding layer *.grd file.

Upload DEM of all surfaces

Continue

- **Graphics (see Figures 7.13 and 7.14)**
  - DEM of surfaces may be checked by clicking Graphics > View DEM which will show DEM (Figure 7.13) in grid node editor of Surfer 9 (Golden Software 2010).
  - If it is desired to see the 3D surfaces or wire mesh etc, Surfer-9 (Golden Software 2010) can be called from within the program by Graphics > open GS Surfer TM (Figure 7.14).

- **Slope Stability Analysis (see Figure 7.15)**
  - Select “2D FS” or “3D FS”.
  - If 2D FS is selected, user is presented with extents of slope model in x-direction with location of central cross-section. User is required to input location (x-coordinate) of 2D cross-section for which 2D FS is required.
  - 2D or 3D FS is computed using 3D extensions of Bishop’s (1955), Janbu’s (1956), and Spencer’s (1967) procedures.

- **Apply External Horizontal and Vertical Side Forces (if required)**
  - 3DDEM-Slope compares the 2D FS for a cross-section in the middle of the 3D slide mass with the overall 3D FS. The program provides the user with a warning that the 3D FS is less than (possible if 2D central cross-section is not critical section) or equal to 2D FS for translational slides with uniform cross-section and vertical sides (see Figure 7.16).
  - If so, the user can select to apply external side forces that are calculated based on the findings of this study and obtain a corrected 3D FS (see Figure 7.17).

- **Troubleshooting**
- 3DDEM-Slope will locate Surfer 9 directory, if default settings for installation of Surfer 9 are followed. Alternatively, copy Surfer 9 folder to root directory of “C” drive, if prompted by program.

- If upon Solve > (any 3D procedure), the software goes into a loop (hour glass is shown). One (or more) surface grids may have problems, due to any of the following reasons:-

  * Some problem in grid size. Check that the grid size (minimum, maximum, spacing and number of lines) is consistent in all the surfaces.

  * 3D Surface generated in Surfer-9 (Golden Software 2010) was not saved as “GRD Surfer 6 Text Grid (*.grd)” file format. Open surface with any text editor (like notepad), to see if all elevation data is readable as a text. If symbols are displayed instead of elevation data then the DEM of surface was saved in default setting as GRD Surfer7 Binary Grid (*.grd) which cannot be read by 3DDEM-Slope.

7.3.2 Application Example

To verify the utility of 3DDEM-Slope and 3D back-analysis methodology, a hypothetical example problem was prepared and assigned to CEE-581 Earth Dams class of FALL 2009. Based on the feedback of students, 3DDEM-Slope was improved and a revised example was provided to the next CEE-581 Earth Dams class in FALL 2010. The assignment involves generation of DEM from limited number of borehole/inclinometer data points using Surfer 9 (Golden Software 2010) and calculating 3D FS using 3DDEM-Slope. The details of the 3D example are provided in Appendix B.

7.4 Review and Summary of Chapter 7

To incorporate and/or verify some of the findings of this study a new 3D LE program, 3DDEM-Slope is developed. 3DDEM-Slope stores the problem description and material properties while geometry input of various layers is imported from 3D DEM files generated from Surfer-9 (Golden Software
The methodology for 3D back-analysis is verified using an hypothetical example problem and shows the utility of the 3D LE program for back-analysis of 3D slope failure.
7.5 Tables

Table 7.1: Results of analysis using Janbu correction factor

<table>
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<tr>
<th>Analysis Type</th>
<th>Case Analyzed</th>
<th>$FS_0$(^1)</th>
<th>$f_o$(^2)</th>
<th>$FS^3$</th>
<th>Reference FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D</td>
<td>Problem-I</td>
<td>1.87</td>
<td>1.09</td>
<td>2.04</td>
<td>2.04</td>
</tr>
<tr>
<td>2D</td>
<td>Problem-II(^6)</td>
<td>1.32</td>
<td>1.08</td>
<td>1.43</td>
<td>1.45</td>
</tr>
<tr>
<td>3D</td>
<td>Problem-I</td>
<td>1.99</td>
<td>1.09</td>
<td>2.17</td>
<td>-</td>
</tr>
<tr>
<td>3D</td>
<td>Problem-II(^6)</td>
<td>1.63</td>
<td>1.08</td>
<td>1.76</td>
<td>-</td>
</tr>
<tr>
<td>2D(WA)</td>
<td>Problem-I</td>
<td>1.96</td>
<td>1.09</td>
<td>2.14</td>
<td>-</td>
</tr>
</tbody>
</table>

\(^1\)Uncorrected Janbu FS;
\(^2\)Correction factor;
\(^3\)Janbu corrected FS;
\(^4\)Reported by Fredlund and Krahn 1977;
\(^5\)CLARA-W;
\(^6\)Same as problem-I but with weak layer;
7.6 Figures

Figure 7.1: Mohr circles of stresses and rupture line (Terzaghi et al. 1996)

Figure 7.2: Data input for stress dependent shear strength envelope
Figure 7.3: Representation of nonlinear Mohr failure envelope by equivalent tangent values of cohesion and friction (Duncan and Wright 2005)

Figure 7.4: Janbu’s correction factor for simplified procedure (modified from Janbu 1973)
PROCEDURE TO APPLY SIDE FORCES IN LE SOFTWARE
1. Compare 3D and 2D FS (at center)
2. If 3D FS ≤ 2D FS (less than 5% higher)
3. Calculate magnitude of side forces based on $K_t$
4. Apply side forces as external loads at both ends and re-calculate 3D FS

LEGEND
- Start and End of Process
- Flow of calculations
- Decision point
- Call input from other functions / subroutines

Calculate cross-section details
1. Locate points of cross-section to define slide mass area
2. Calculate cross-sectional area
3. Find centroid of cross-section area
4. Determine slope inclination

Calculate side forces
1. Determine average effective vertical stress along slide surface
2. Compute $K_t$
3. Determine resultant side shear stress
4. Calculate horizontal and vertical shear forces, $F_y$ and $F_z$ for soil above failure surface

Figure 7.5: Flow chart for incorporating shearing resistance along vertical sides of slide mass
Figure 7.6: $3 \times 3$ grid nodes for estimating column base angles
Figure 7.7: Coordinate axis

Figure 7.8: Startup screen
Figure 7.9: Problem description and control parameters

Figure 7.10: Material properties and shear strength input
Figure 7.11: Layer geometry input method

Figure 7.12: DEM input for various layers
Figure 7.13: Graphics option to view DEM 3D surfaces

Figure 7.14: Graphics option to view 3D surfaces using Surfer 9 (Golden Software 2010)
Figure 7.15: Final report of FS

Figure 7.16: Warning for possible error in 3D FS
Figure 7.17: Corrected 3D FS after applying external side forces estimated using $K_r$ method
CHAPTER 8

CASE HISTORIES

8.1 Introduction

This chapter presents four well documented field case histories that illustrate the importance of performing 3D back-analysis for remedial measures of failed slopes. Two rotational and translational case histories are used to verify findings of the present research to ensure that the research results represent field behavior. Analysis of case histories show that depending on the location of the 2D cross-section, 2D FS values vary due to variations in topography and ground water level over the slide mass. A 3D analysis can accommodate variations in geometry, pore-water pressure, and material properties across a slide mass, better than a 2D analysis.

In addition, the field failure surface and slide mass geometry must be used in back-analysis of a failed slope instead of searching for the failure surface that yields the lowest back-calculated strength (Hussain et al. 2011). Because failed slopes have a well-defined failure surface for which 3D FS is unity, the minimum 3D and 2D FS values can be calculated and compared.

For a meaningful comparison of 3D and 2D FS values for a particular slope, it is important to compare the minimum 3D and 2D FS values (Cavounidis 1987). The units (SI or Imperial) in which the particular case history was reported has been used in the text with equivalent SI or Imperial units in parenthesis.
8.2 Rotational Case Histories

8.2.1 Tianjin, China

8.2.1.1 Brief History

This case history is based on the field investigation (Li et al. 2005a) and 2D analysis (Li et al. 2005b) of a landslide that occurred on September 17, 1997 in submerged and unconsolidated dredged soft slopes in Tianjin, northern China. The field investigations (Li et al. 2005a) revealed that the landslide consists of a number of individual retrogressive, rotational slides that occurred in the reclaimed land at a wharf development project on the western shore line of Bo Hai Bay, China. Figure 8.1 shows the general layout of the wharf design elements and the boundaries of the retrogressive landslides. Direction of north is perpendicular to the shore line and parallel to the landslide direction.

A reclamation dam of stone blocks with 4.4 m height was constructed in 1991 with its base resting on the sea bed at 0 m above mean sea level (AMSL). In 1993, the land reclamation of 4 m thickness was completed using hydraulically transported dredged soil from port navigation channels. The dredged mud, which was in the form of a slurry, was left to dry naturally. In 1997, conventional vacuum pre-loading was used to increase the strength and bearing capacity of the hydraulic fill. During June 10 1997, and July 15 1997, a general fill layer of 2.45 m thickness and a sand cushion of 0.4 m thickness was placed on the reclaimed land. The sand cushion and general fill were not compacted. Due to some design requirements, the lower portion of the dredged slope was steepened from 2.2H:1V to 2H:1V. In addition, a temporary road of 1 m thick residual soil was constructed on the reclaimed land. Because it was observed that dredged slope gradient was unsatisfactory for the barge, therefore, a total of 55 piles were also driven in the dredged slope between September 5-15, 1997. The average groundwater level in the reclaimed land was reported to be at +4.21 m AMSL and the routine tidal level was observed to be above +4.00 m AMSL. At the time of the landslide, the natural tidal elevation was at the lowest level i.e., on September 17, 1997 at 0900 AM, the tide had dropped from +4.11 m AMSL at 0300 AM to +0.99 m AMSL.
8.2.1.2 Description of Failure

The landslide started at 09:00 am on September 17, 1997 and lasted up to 10:00 am. The landslide consists of about nine sequential retrogressive landslides, spreading laterally from the water towards the reclaimed land. The scarp of the first failure was observed in the backfilled area at about 20 m behind the reclamation dam (Figure 8.2a). The first slide (Figure 8.2b) caused a 80 m long (transverse direction) and 30 m wide (parallel to slide direction) soil mass to slip into the bay. The landslide occurred in the central region of the dredged slope and sliding occurred in the north direction.

Subsequently, additional landslides occurred in the remaining reclaimed land behind the first failure surface. The inferred scarps of these slides are shown by dotted lines in Figure 8.1. The slipped soils in front of the slide scarps created a number of terraces (Figure 8.3). The topographic distribution of the terraces and associated tension cracks indicate that the overall slide consists of a number of individual retrogressive landslides which spread laterally in the reclaimed land.

Li et al. (2005b), conclude that the slide is an undrained failure in the recent marine mud deposits underlying the dredged fill. The main cause of the landslide was attributed to the dredge excavation undermining the slope. Other contributing factors to the slide include placement of a 2.84 m surcharge fill, 1 m thick road road behind the reclamation dam, steepening of the dredged slope, pile driving, and lowering of tide. An undrained shear strength of 24 kPa was back-calculated for the recent marine mud by Li et al. (2005b) using total stress analysis, Bishop’s (1955) simplified stability procedure, and the first slide mass.

8.2.1.3 Geologic Setting

The landslide area lies along the western shoreline of Bo Hai Bay, China. This region is a typical mud coastal plain and has experienced continued land depression and sedimentation over the entire Cenozoic era (Hou 1987). The Cenozoic strata is composed of inter layered marine and alluvial deposits.

Li et al. (2005a) provide a soil profile (Figure 8.4) along cross-section C-C (see Figure 8.1) which is at the centerline of the landslide. This cross-section is also the critical cross-section for slope stability analysis. The subsurface
soil profile consists of four main soil layers. The topmost insitu soil layer is a 14 m thick recent marine mud deposit. The recent marine mud is subdivided into three sublayers, i.e., muddy silt 1-1, mud 1-2, and muddy clay 1-3. An interlayer of 1 m thickness, composed of sand and shell mixes, is present between the topmost and next in situ soil layer. A clayey layer (Layer 2) follows this sandy silt interlayer. This layer was also subdivided into three sub layers, i.e., silt, clay, and sandy silt. The last layer is a normally consolidated silty sand having a relatively high shear strength and bearing capacity. Physical properties of the different soil layers are shown in Table 8.1.

Li et al. (2005a) also reports that there was no significant differences between other cross-sections in the slide area. Therefore, cross-section C-C is used as the representative geologic model for the slope stability analysis.

8.2.1.4 Representative Shear Strengths

Li et al. (2005a) performed laboratory and field testing to determine the shear strength parameters of various layers/sublayers. This testing includes direct shear tests, triaxial compression tests, and field vane (FV) shear tests. Because the landslides were determined to be an undrained failure, the undrained shear strength is considered in this study. A FV test profile from before the landslide performed by Li et al. (2005a) is shown in Figure 8.5. The undrained shear strength data was modified to include average shear strength for different layers which are shown by vertical lines (Figure 8.5).

8.2.1.5 2D Analysis of First Slide

For a 2D analysis of the first landslide, cross-section C-C was modeled in CLARA-W (Hungr 2002). The slope was initially modeled with the circular failure surface reported by Li et al., (2005a). The center of rotation of the failure surface was scaled at 5.41 m from the dam and at 27.84 m AMSL using Figure 8.2b. This slope model was reversed or mirrored to create a left facing slope for use in CLARA-W. A tension crack of 0.4 m depth was included because of field observations in the surface sand. The slope model used in CLARA-W is shown in Figure 8.6a. To verify the modeled geometry, a 2D total stress analysis \( (\phi = 0) \) using Bishop’s (1955) Simplified procedure was performed using the reported failure surface. Initially, the reported back-
calculated undrained shear strength ($S_u$) of 24 kPa was used for the hydraulic fill, muddy silt, mud, and muddy clay. The 2D analysis yielded a FS of unity indicating that the geometry was modeled correctly. Another 2D analysis using Spencer’s (1967) stability procedure was performed and $S_u$ of 24 kPa, which also returned a FS of unity so the slope model was assumed to represent field conditions.

8.2.1.6 3D Analysis

After verifying the 2D slope model, a 3D total stress analysis of the slope model was performed using LE (CLARA-W) and FE (PLAXIS-3D) softwares. Because Li et al. (2005a) reports no lateral variations in the soil profile across the slide area, other cross-sections modeled in the lateral directions from C-C (see Figure 8.2) are based on cross-section C-C. For a 3D analysis in CLARA-W (Hungr 2001), a grid of 150 m x 100 m was selected. 2D cross-section C-C was placed at the center of the grid in the transverse direction, i.e., 75 m. Figure 8.1 and Figure 8.2 show that the failure surface is ellipsoidal in 3D and, circular in 2D respectively. Therefore, the 3D failure surface was modeled using an ellipsoid with aspect ratio of 2.67 ($length/width = 80m/30m = 2.67$) with the center of rotation at the same location as 2D analysis (5.41 m from the dam center and at 27.84 AMSL). The 3D slope geometry was modeled in PLAXIS-3D by extending the 2D cross-section (see Figure 8.6b) in Z direction by 150 m and using a coarse mesh. The 3D FE model was also extended by 6 m on both sides to model the soil mass beyond the slide boundaries. Fully fixed boundary displacement condition ($u = 0$, $v = 0$, and $w = 0$) were used at the slope ends and ends of the extended model in FE procedure. The resulting 3D slope geometry used in CLARA-W and PLAXIS-3D are shown in Figure 8.7.

Initial 3D back-analysis in CLARA-W was performed using an extension of Bishop’s (1955) 2D stability procedure to 3D as suggested by Hungr et al. (1989). The recent marine mud was assumed to be homogeneous with no variation in undrained shear strength with depth. The 3D back-calculated undrained shear strength was 20 kPa for 3D FS equal to unity. Extension of Spencer’s (1967) stability procedure to 3D (Hungr 2002) also back-calculated the undrained shear strength of 20 kPa. However, 3D analysis using PLAXIS-3D back-calculated the undrained shear strength of 21.4 kPa for homogeneous
marine mud.

A pseudo 3D analysis using a weighted average of FS values obtained from 2D cross-sections (Lambe and Whitman 1969) was also performed for comparison purposes. Because the recent marine mud was assumed homogeneous with no strength variation with increasing depth, three cross-section were sufficient for a reasonable weighted average FS. Cross-sections were selected at the one-sixth, centerline and one-sixth points of the slide mass. 2D FS at these three cross-sections were computed by varying the undrained shear strength of the recent marine mud to achieve a weighted FS of unity. Using Equation (2.18), 2D weighted average FS values were computed for different values of undrained shear strength. An average undrained shear strength of 20.8 kPa was back-calculated for the whole slide mass to achieve a weighted average FS of unity. The results from 2D weighted average FS values equal to unity are shown in Table 8.2. Table 8.2 shows that 2D FS varies between the cross-sections but the weighted average according to Equation (2.18) is approximately unity.

The undrained shear strength profile from vane shear tests show an increase in average undrained shear strength with increasing depth (Figure 8.5). Therefore, 3D back-analyses were also performed to back-calculate undrained shear strength of mud using measured average undrained shear strengths of 10.9 kPa and 19.2 kPa for hydraulic fill and muddy silt respectively. The back-calculated undrained shear strength of mud using the 3D extensions of Bishop’s (1955) simplified and Spencer’s (1967) procedures were 22.8 kPa and 22.4 kPa respectively. Whereas, back-calculated undrained shear strength of mud using PLAXIS-3D was 24 kPa.

For comparison purposes, the 3D LE software (CLARA-W) required less than 5 seconds to compute 3D FS for any of the analyses performed while the FE software (PLAXIS-3D) required 30-45 minutes to complete a similar 3D analysis.

8.2.1.7 Comparison of 2D and 3D Back-Calculated Shear Strength

Figure 8.8 shows the back-calculated undrained shear strengths and the associated 2D FS and 3D FS values calculated using different procedures assuming the recent marine mud as a single homogeneous layer. Figure 8.8 shows agreement between the 2D Bishop (1955) and Spencer (1967) proce-
dures and agreement between the 3D Bishop and Spencer procedures. The 2D weighted average analyses yielded FS values slightly below the 3D LE values. PLAXIS-3D yields slightly lower FS than 3D LE analysis for a given set of shear strength values. Therefore, 3D FS values from PLAXIS-3D plot slightly lower than 3D LE values. The 3D back-calculated undrained shear strength from LE procedures is 20 kPa, while the 2D value is 24 kPa. This means the 2D back-analysis over-predict the back-calculated undrained shear strength by almost 20% which is in agreement with Stark and Eid (1998).

A summary of the 2D and 3D analyses along with the measured undrained shear strength (FV) is shown in Table 8.3. The Bjerrum (1972, 1973) correction factor has been applied to the FV undrained shear strengths to estimate the mobilized undrained shear strength. The following observations can be made from the comparison of various 2D and 3D analysis:

- Using 2D analyses (Bishop 1955; Spencer 1967), the back-calculated undrained shear strength of the recent marine mud for the first landslide is 24 kPa. While, 3D back-analyses using 3D Bishop (Hungr et al. 1989) and 3D Spencer (Hungr 2002) procedures yield an undrained shear strength of 20 kPa. Back-analysis by PLAXIS-3D yields undrained shear strength of 21.4 kPa.

- 2D and 3D analyses using either Bishop’s (1955) or Spencer’s (1967) procedure yield similar FS values for this case history. For 2D analyses, Fredlund and Krahn (1977) report that Bishop’s (1955) simplified procedure provides comparable results to more rigorous procedures (Spencer 1967) especially for rotational slides. Because an ellipsoid is a natural extension of a circular slip surface in 3D, the 3D Bishop extension is also likely to provide comparable results because the 2D Bishop’s (1955) simplified procedure yields accurate results for circular slip surfaces (Duncan and Wright 1980).

- 3D extensions of LE give comparable results with 3D FE (PLAXIS) procedure while 3D FS computed from 3D FE analyses are slightly lower (about 7%) than 3D LE analyses but within an acceptable tolerance of 12% (Duncan 1996). This results in slightly higher back-calculated shear strength by 3D FE procedure, e.g., 20 kpa (LE procedures) vs 21.4 kPa (FE procedure). In addition, 3D FE analysis are much more
time consuming than 3D LE analysis.

- It is also observed for this case history that 2D analyses overestimate the back-calculated undrained shear shear strength by about 20% compared with a 3D analysis that incorporate the shear resistance along the sides of the slide mass.

- The back-calculated shear strength using a 2D weighted average analyses (Lambe and Whitman 1969) is closer to the results of the 3D analyses than the 2D LE analyses. This is caused by the ellipsoidal shape of the failure surface where the end cross-sections are different from the center cross-section(s). The difference in FS at different cross-sections is accommodated by taking the weighted average of the three cross-sections based on the weight of the slide mass in each cross-section.

- The maximum and minimum undrained shear strengths found at the base of the failure surface (-8 m AMSL) from FV shear tests are 7.2 kPa and 26 kPa (see Figure 8.5). Thus, the undrained shear strength of 24 kPa, back-calculated from the 2D analyses is near the maximum value of undrained shear strength (26 kPa).

- 3D analyses with an ellipsoidal ratio of 2.67 yields a back-calculated undrained shear strength of 20 kPa that is 20% lower than the back-calculated undrained shear strength from 2D analyses and is closer to the average undrained shear strength profile at the base of the failure surface (-8 m AMSL).

- Finally, a 3D analysis with undrained shear strengths increasing with depth are close to the average undrained shear strength obtained from FV tests in various layers (Table 8.3). Therefore, a 3D back-analysis using a shear strength increasing with depth profile yields back-calculated shear strengths that are in agreement with the FV tests.
8.2.2 New Jersey, USA

8.2.2.1 Brief History

This case history involves a rotational slope failure of a municipal solid waste (MSW) landfill in Woodbridge, New Jersey on April 1-2, 1984. The case history is described by Dvirnoff and Munion (1986) and Erdogan et al. (1986). The MSW site covered approximately 220,000 m$^2$ (55 acres) and is surrounded by tidal marsh on three sides (southwest, southeast and northeast) and uplands on the fourth side (northwest). At the highest location, the MSW landfill was about 34.4 m above the underlying tidal marsh. The side slopes of MSW were generally 4H:1V with a toe dike of about 2.4 m to 6 m high. The slope failure occurred on the southeast side of the MSW where the toe dike had the smallest buttressing effect (about 2.4 m - 3.3 m). Before failure, the landfill was in operation for about 15 years. It was filled gradually to the existing height of 34.44 m, except for a small portion on the southeast side where MSW placement was postponed due to the presence of a small stream. This portion was later filled rapidly in 4-5 months in the year prior to the slope failure.

8.2.2.2 Description of Failure

During March 28 to March 30, 1984, approximately 69 mm of rainfall was reported in the vicinity of the landfill, resulting in two high tides of +3.0 m and 3.15 m high in the adjacent marsh. The topography of the landfill and location of the critical cross-section are shown in Figure 8.9. Critical cross-section A-A' is shown in Figure 8.10. The high and low water levels correspond to the range in routine water level and high tides (+0.0 m and +3.15 m AMSL). As the slide mass moved in the southeast direction, it opened a 12 m deep, 18 m wide, and 180 m long steep vertical crack at the top of the landfill. The crack was crescent-shaped in plan view and followed the orientation of original ground contours before the fill was placed. In addition, more cracks also opened down slope of the main scarp. The slide mass lifted and displaced the toe dike and tidal marsh up to 60 m beyond the toe dike. The failure involved approximately 110,000 m$^3$ of MSW.
8.2.2.3 Geologic Setting

The lowest material units in the area belong to Late Cretaceous age formation. These materials include medium dense to very dense stratified sandy silt, silty fine sand, medium to fine sand and stiff to hard silty clay. In the southeast portion of the landfill where failure occurred, the top of the Late Cretaceous deposits was found in the range of -7.5 to -12 m AMSL. Total thickness of the deposit is unknown.

Late Cretaceous deposits were overlaid by tidal marsh deposits. The upper portion of tidal marsh consisted of brown clayey silt, known as “meadow mat” and the lower portion consisted of very plastic gray silty clay/clayey silt. The thickness of the tidal marsh deposit in southeast portion of the landfill was found in the range of 7-12 m. Generally, tidal marsh had very soft consistency, with natural water content of approximately 140% to 146% for meadow mat and 88% to 117% in the gray silty clay/clayey silt.

MSW was deposited on the original surface of the tidal marsh, approximately at elevation of 1.2 m. MSW (Dvirnoff and Munion 1986) consisted of paper, cloth, wood, rubble, and other miscellaneous trash.

8.2.2.4 Representative Shear Strengths

Dvirnoff and Munion (1986) report detailed subsurface conditions and physical properties of the materials in and around the landfill, e.g., tidal marsh, MSW and toe dike. The findings are based on 15 test borings, 5 dutch cone pentrometer probes, and laboratory tests on selected representative soil samples. Because failure was almost tangent to the stiff/dense Late Cretaceous deposits and only involved MSW and tidal marsh materials, shear strength of tidal marsh and MSW will be discussed herein.

Dvirnoff and Munion (1986) performed unconsolidated-undrained triaxial compression tests, consolidated-undrained triaxial compression tests, and dutch cone pentrometer tests to estimate the undrained shear strength of the tidal marsh. Based on laboratory and field data, Dvirnoff and Munion (1986) report an undrained shear strength of 4.8 kPa (100 psf) for the soft tidal marsh deposit.

Dvirnoff and Munion (1986) also report that corresponding data for MSW material was not available, therefore MSW shear strength ($\tau$) of 37.35 kPa.
was back-calculated using MSW unit weight of 7.06 kN/m$^3$ and undrained shear strength of 4.8 kPa for the tidal marsh.

8.2.2.5 Estimation of Waste Shear Strength Parameters for New Jersey Site

Because shear strength of only the tidal marsh was available from laboratory and field tests, strength of the MSW was estimated using back-analysis by Dvirnoff and Munion (1986). In this study, additional 2D and 3D analysis were performed to back-calculate the mobilized shear strength of the MSW and compare it with MSW properties reported in the published literature.

Duncan and Stark (1992) suggest using the best estimate of $\phi'$ to back-calculate a value of $c'$. This approach is more applicable to MSW because, $\phi'$ is less sensitive to the presence or absence of reinforcing material in the MSW than $c'$ (Gray and Ohasha 1983; Maher and Gray 1990). Similarly, laboratory tests on MSW have shown that removal of plastics and other reinforcing materials affect $c'$ more than $\phi'$ (Edincliler et al. 1996; Benson and Khire 1994; Foose et al. 1996).

There is a wide range of effective stress strength parameters for MSW reported in the literature (Stark et al. 2008). Values reported for effective stress friction angle ($\phi'$) range from 10° to 53° while effective stress cohesion ($c'$) ranges from 0 to 67 kPa. For example, Zekkos (2005) and Bray et al. (2009) recommend a strength envelope for MSW with $c' = 15$ kPa and atmospheric pressure, $P_o$, of 1 atm (101.3 kPa or 2115.7 psf) as follows:

$$
\tau = 15kPa + \sigma'_n \tan[36° - 5log(\frac{\sigma'_n}{P_o})]
$$

(8.1)

Eid et al. (2000) and Stark et al. (2008) provide recommendations for the MSW strength to be used in static and seismic slope stability analysis for landfills that have not experienced elevated temperatures. Eid et al. (2000) compiled measured and back-calculated shear strength parameters of MSW for effective normal stresses ($\sigma'_n$) less than 400 kPa and suggest that the shear strength of MSW can be defined by a narrow band with an effective stress friction angle, $\phi'$, of approximately 35°, and cohesion, $c'$, that ranges from 0 to 50 kPa. Eid et al. (2000) also recommend average values of $c'$ and $\phi'$ of 25 kPa and 35°, respectively, for the design of MSW facilities.
that have not experienced elevated temperatures. The relatively high shear strength of MSW is likely caused by the interconnection of plastics and other materials (Eid et al. 2000). The high shear strength recommended for MSW is supported by the fact that nearly vertical cuts and scarps in landfills have been observed to remain stable for months to years (Stark et al. 2008).

In some cases the interconnecting plastics and other reinforcing materials that contribute to the high shear strength of MSW can be consumed, degraded, burnt, and/or decomposed. This results in a reduction of shear strength parameters of the MSW which has been observed in samples obtained during gas well drilling and excavations in MSW landfills experiencing elevated temperatures. Kavazanjian (2008) recommends friction angle of 30° for decomposed waste, while Stark et al. (2010) report $c'$ and $\phi'$ of 0 kPa and 20°, respectively for a site that experienced elevated temperatures and combustion. These shear strength parameters are considerably lower than reported by others for decomposed waste because most of the components of the MSW were reduced to ash including the reinforcing materials.

8.2.2.6 2D Analyses of Slide

A 2D analysis of the slide was performed using the critical cross-section A-A’ (Figure 8.10) in CLARA-W (Hungr 2001). The slope model used in CLARA-W is shown in Figure 8.11a. The center of rotation of the circular failure surface was located at y=81.71 m and z= 116.8 m.

The input geometry and parameters were verified by performing a 2D analysis using Bishop’s (1955) simplified procedure with the material properties reported by Dvirnoff and Munion (1986). The tidal marsh was assigned a unit weight of 16 kN/m$^3$ and undrained shear strength of 4.8 kPa. A 2D FS of unity was archived using an MSW unit weight of 7.06 kN/m$^3$ and shear strength ($\tau$) of 37 kPa, which confirmed the modeled geometry because the MSW strength is in agreement with the value back-calculated by Dvirnoff and Munion (1986). Material properties used for verification of the 2D model are shown in Table 8.4.

Using the verified geometry, a 2D analysis was performed to back-calculate the shear strength of MSW using the material properties of tidal marsh reported by Dvirnoff and Munion (1986), i.e., unit weight ($\gamma$) of 18 kN/m$^3$, and undrained shear strength of 4.8 kPa. A typical unit weight ($\gamma$) of MSW
is 10.2\( kN/m^3 \) to 12.6\( kN/m^3 \) (Eid et al 2000; Stark et al. 2010). Therefore, MSW was assigned a unit weight of 10.2 \( kN/m^3 \) instead of 7.06 \( kN/m^3 \) as used by Dvirnoff and Munion (1986). In accordance with Duncan and Stark (1992), a \( \phi' \) of 35\( ^\circ \) was estimated for MSW using Figure 8.12 from Stark et al. (2008). A back-calculated value of \( c' \) for MSW was computed using Bishop’s (1955) simplified and Spencer’s (1967) stability procedures. The 2D analysis yielded a FS of unity for \( c' \) of 12.5 kPa and 10.3 kPa using Bishop’s (1955) simplified and Spencer’s (1967) stability procedures, respectively.

The finite element (FE) program, PLAXIS-3D, was also used to perform a 2D back-analysis using fully free boundary conditions at the ends of the 3D model. In addition, the 3D model was extended to a width to height ratio (W/H) greater than 6 to simulate a plane strain condition. For a \( \phi' \) of 35\( ^\circ \), the back-calculated \( c' \) of 9 kPa was obtained for a FS of unity.

The back-calculated \( c' \) of MSW by LE procedures (Bishop and Spencer) and FE procedure are in accordance with the reported range of \( c' \) (Eid et al. 2000; Zekkos 2005; Stark et al 2008; Bray et al. 2009) which are based on large scale laboratory and field test results and back-analysis of failed waste slopes.

8.2.2.7 3D Analysis

After verifying the 2D slope model and material properties of the tidal marsh and MSW, a 3D analysis was performed using LE (CLARA-W) and continuum (PLAXIS-3D) methods. No lateral variation in the soil profile across the slide area was assumed, therefore the other cross-sections used in the 3D analysis are based on cross-section A-A' (see Figure 8.10). For a 3D analysis in CLARA-W, a grid of 116 m x 200 m was selected with 50 rows and 50 columns in each direction. 2D cross-section A-A' was placed at the center of the grid in the transverse direction, i.e., 58 m. A 3D failure surface was modeled in accordance with the reported total volume of the slide mass (Dvirnoff and Munion 1986). Using an ellipsoidal aspect ratio of 0.83 with a center of rotation at the same location as the 2D slip surface (x= 66 m, y=81.71 m, and z= 116.8 m) yielded a total slide mass volume of approximately 110,000 m\(^3\) which is in agreement with Dvirnoff and Munion (1986).

The 3D slope geometry was modeled in PLAXIS by extending the 2D cross-section (see Figure 8.11b) in the Z direction by 104 m and using a
coarse mesh. The 3D FE model was also extended by 6 m on both sides to model the soil mass beyond the slide boundaries thereby making the total model width of 116 m. A fully fixed boundary displacement condition \((u = 0, v = 0, \text{and } w = 0)\) was used at the slope ends and ends of the extended model. The resulting 3D slope geometry used in CLARA-W and PLAXIS-3D is shown in Figure 8.13.

The 3D LE back-analysis was performed using an extension of Bishop’s (1955) 2D stability procedure to 3D in CLARA-W (Hungr et al. 1989) with the MSW properties back-calculated from the 2D analyses, i.e., \(c'\) and \(\phi'\) of 12.5 kPa and 35\(^\circ\). The 3D analysis resulted in a 3D FS of 1.74. Because the slope actually failed, the MSW shear strengths are not representative of the actual mobilized shear strength. Further 3D analyses were performed using a \(c'\) of zero and varying \(\phi'\) to achieve a 3D FS of unity. A 3D extension of Spencer’s (1967) stability procedure and Bishop’s (1955) simplified procedure, yielded back-calculated \(\phi'\) values of 25.5\(^\circ\) and 26.0\(^\circ\), respectively with \(c'\) equal to zero. The 3D/2D FS ratio for the different analyses performed was found to be in the range of 1.59 to 1.74.

A 3D back-analysis was performed using PLAXIS-3D with MSW back-calculated properties from 2D FE analysis, i.e., \(c'\) and \(\phi'\) of 9 kPa and 35\(^\circ\) respectively, resulted in a 3D FS of 1.11. The 3D/2D FS ratio obtained from PLAXIS 3D (3D/2D FS=1.11) is less than 3D/2D FS ratio obtained from LE analysis, i.e., 1.59 to 1.74. This difference may be explained by the failure surface not being defined in the FE analysis so failure occurs through soil zones where the shear strength is unable to sustain the applied shear stresses. In this case history, the tidal marsh has lower strength than the MSW, so the failure surface in FE analysis remained in the tidal marsh. In contrast, LE software allows modeling of an ellipsoidal shaped failure surface which tapers upwards towards the ground surface. Therefore, the base of the failure surface passes through the MSW resulting in a higher FS (see Figure 8.13). In the FE analysis, fixity is used for the sides of the slide mass so an ellipsoidal shape does not develop, which means the MSW strength is not included in the FE analysis. Because of the extremely weak layer (tidal marsh) and the fixity, the slide mass extends essentially to the fixed sides of the model resulting in vertical or near vertical sides of the slide mass. The cases where there is not a large difference in shear strength between the upper and lower materials, or the lower material is stronger than the
upper material, a near ellipsoidal failure surface can develop in FEM, e.g., see Tianjin, China case history discussed in Section 8.2.2.10. In addition it was noticed that 3D LE took a fraction of time to complete when compared with 3D LE analysis.

8.2.2.8 2D Weighted Average Analysis

A pseudo 3D analysis using a 2D weighted average of FS (Lambe and Whitman 1969) was also performed for comparison. Because the materials involved in this failure are not homogeneous, four cross-section were used to obtain a reasonable weighted average FS as described in Chapter 4. Cross-sections were selected at the one-eighth, three-eighth, five-eighth, and seventh-eighth points across the slide mass. The 2D FS was computed for these four cross-sections using $c'$ of zero and varying $\phi'$ to achieve a FS of unity. Using Equation 2.18, a 2D weighted average FS was computed for different values of $\phi'$. An average $\phi'$ of 27.5° was back-calculated for the whole slide mass to achieve a weighted average FS of unity. A 2D weighted average of three cross-section back-calculated $\phi'$ of 24.0°. In this case, the slide had an aspect ratio of 0.83, with a total width of approx 104 m. In summary, the use of three or four cross-sections yielded a 3D/2D FS ratio between 1.54 to 1.69.

The back-calculated MSW shear strengths from the 3D analyses are much lesser than those computed by a 2D analysis. However, values still fall in the range of MSW shear strengths reported in the literature. The MSW was reported to consist of paper, wood, cloth, rubble, and other miscellaneous trash. Therefore, it is likely the MSW was decomposed, so the back-calculated shear strength to $c'$ of 0 and $\phi'$ of 24°-27° as back-calculated by 3D analysis may be reasonable.

8.2.2.9 Comparison of 2D and 3D Back-Calculated Shear Strength

A summary of the various 2D and 3D analyses performed is shown in Table 8.5 and shows the 3D FS is higher than 2D FS in all of the analysis. In particular, the 2D and 3D FE analysis show a smaller difference in back-calculated shear strength than the 2D and 3D LE procedures. The following observations can be made from a comparison of the various 2D and 3D analyses performed for the case history:
• 3D/2D FS ratio computed by either LE or continuum method is greater than unity.

• 2D back-analyses yield higher back-calculated shear strength than the 3D back-analyses.

• For circular failure (and ellipsoid) surfaces, LE program captures the 3D effects, however, the failure surface generated by the FE analysis did not model the 3D effects.

• An inherent advantage of the FE analysis is the failure surface does not need to be specified. However in this case, this was problematic because result did not match field observations. In a back-analysis, the field failure surface and slide mass geometry must be used instead of searching for the failure surface that yields the lowest back-calculated strength (Hussain et al. 2011). Because there is no provision for specifying a failure surface, the FE analysis did not provide a good representation of field behavior or back-calculated shear strength.

• 3D FE analysis are significantly more time consuming than 3D LE analysis.

• For ellipsoidal (circular) failure surfaces, a 2D weighted average FS by LE method gives good estimate of the 3D FS.

8.2.2.10 Comparison with Tianjin, China Slope Failure

In the Tianjin, China slope failure analysis discussed in Section 8.2.1, shear strength of problematic marine mud was either assumed to be same for all depths or increase with depth. Because of end fixity in the 3D FE analysis, the failure surface gradually tapered upwards towards the ground surface and a near ellipsoidal surface developed in FE analysis. As a result, FS computed by 3D FE analysis is comparable with FS computed by 3D LE analysis so the back-calculated shear strength values are also closer. Thus, in cases where there is not a large difference in shear strength between the upper and lower materials, or lower material is stronger than upper material, a near ellipsoidal failure surface can develop in FE analysis and result in similar back-calculated strengths as 3D LE method.
In New Jersey MSW slope failure case, FE analyses gave similar 2D FS values but lower 3D FS values when compared with 2D and 3D LE analyses. This results in higher back-calculated shear strength for 3D FE analysis than 3D LE analysis for the New Jersey MSW failure. In this case history, the lower material (tidal marsh) is weaker than the upper material (MSW) so the failure surface in FE analysis remained in the tidal marsh. Because of the extremely weak layer (tidal marsh) and the fixity at ends, the slide mass extends essentially to the fixed sides of the model resulting in vertical or near vertical sides of the slide mass like a translational failure.

Stark and Eid (1998) indicate that vertical sides provide the minimum amount of shear resistance because the effective normal stress acting on the sides equals the lateral earth pressure and a vertical side provides a minimal area of shear surface or shear resistance. Figure 8.14 shows a rock-block slide in Uragara, Japan (Kieffer et al. 2006). The sides of slide mass passing through the strong overlaying rock are vertical. This example of strong material overlaying marsh material resulting in vertical sides may be similar to the weak tidal marsh underlaying the stronger upper material (MSW). The failure surface generated by FE analysis for the New Jersey MSW slope failure extends essentially to the fixed ends of the model resulting in vertical or near vertical sides of the slide mass as shown in Figure 8.14. This suggests the 3D FE analysis results may be less accurate than the 3D LE analysis that utilized an ellipsoidal failure surface and thus incorporated some MSW shear resistance along the sides of the slide mass. In summary, in this case of a landfill, the 3D FE analysis is less accurate than the 3D LE method because the side resistance is omitted. The FE model follows the weak layer and minimizes the MSW shear resistance by using vertical sides of the slide mass.
8.3 Translational Case Histories

8.3.1 Cincinnati, Ohio (Rumpke), USA

8.3.1.1 Brief History

On March 9, 1996, the largest slope failure in a US Municipal Solid Waste (MSW) Landfill, based on volume of waste involved, occurred (Figure 8.15) in the 120 acre Rumpke Sanitary Landfill (RSL) near Cincinnati, Ohio. The failure involved 15,000,000 yd$^3$ (10,000,000 m$^3$) of waste, which was approximately twice the volume of the largest previous waste slope failure that occurred in Maine (Raynolds 1991) involving 750,000 yd$^3$ (500,000 m$^3$). Rumpke landfill failure was described by Schmucker and Hendron (1998), Stark and Eid (1998), and Eid et al. (2000). Schmucker and Hendron (1998) performed 2D LE analysis of the landfill failure, while Stark and Eid (1998) and Stark et al. (2000) performed 2D and 3D LE analyses of the slide. Chugh et al. (2007) performed 2D and 3D finite difference (FD) analyses of the slide. Like other slope failures, there are uncertainties in the leachate level, shear strength of MSW, and mobilized shear strength of the underlying brown native soil (BNS).

Previous studies used estimated shear strength of MSW based on available data in literature (e.g., Kavazanjian et al. 1995 and Eid et al. 2000), and estimated leachate levels to back-calculate the mobilized shear strength of the underlying BNS. The reanalysis of the Rumpke landfill slope failure is presented in this section, and supplements earlier work by incorporating the shear strength of MSW from recent studies (Stark et al. 2008) and corresponding leachate levels. The reanalysis provides a better understanding of the role of uncertainties in the failure and importance of performing a 3D back-analysis for cases that involve complex geometry and varying leachate levels. The project data (cross-sections and contour maps of landfill site) are in Imperial units, and thus Imperial units are used in the text, with SI units presented in parenthesis for reference.
8.3.1.2 Location and Description of Failure

RSL is located about 9 miles (15.3 km) northwest of Cincinnati, Ohio. Landfilling operations in RSL began in 1955 with little excavation or compaction of waste and relied on the in-situ native brown clayey soil as a natural low permeability liner. In 1970, the RSL was retrofitted with a perimeter “toe drain” leachate collection system. By January 1996, the landfill reached an elevation of 1100 ft (335 m) above mean sea level (MSL), and depth of MSW was approximately 350 ft (107 m) under the center of landfill.

Figure 8.16 shows a plan view of the landfill site, showing the extent of the sliding mass. The contour lines of bedrock (contour interval: 50 ft) and, surface of MSW (contour interval=10 ft) prior to failure are shown in Figure 8.16. Original ground surface (before placement of MSW) consists of approximately 10-15 ft (3-5 m) of BNS underlain by bedrock. Locations of sixteen cross-sections that are used to define the geometry of slope are included in Figure 8.16.

Five days prior to the slope failure, i.e., March 4, 1996, some cracks were observed in the recently placed cover soil at the top of the landfill. The cracks were about 3-5 inch (75-125 mm) wide, extending 50-100 ft (15-30 m) across the crest of slope. The landfill owner/operator personnel inspected the entire area for any additional signs of slope distress but no other cracks were identified (Kenter et al. 1996, 1997a, 1997b). Owner/operator concluded that the cracks were caused by waste settlement and filled/covered the cracks with soil to prevent surface water infiltration into the municipal waste. No additional instrumentation was deemed necessary to observe or monitor the movement/displacement.

The cracks continued to reappear at the same location everyday until the landslide occurred on Saturday, March 9, 1996. At about 7:00 a.m. on Saturday, March 9, the cracks at the top appeared again and widened substantially. The cracks were estimated to be at least 10 ft (3 m) deep, and steam was emanating from the cracks. In addition, 1.5 to 2.5 ft (0.45-0.75 m) vertical offset occurred (King 1998) at the northern edge of the vehicle turnaround area (see Figure 8.15) . Between 8:00 and 8:30 a.m., the toe of the slope started moving towards access road (see Figure 8.15). Field investigations after the slide indicated that the movement occurred in the BNS underlying the waste (Stark et al. 2000). The cracking and movement continued
until about noon on March 9, then a large slide block accelerated towards the deep excavation, and the landslide was complete in less than 2-3 minutes (King 1998, Strachchan 1998, Schmucker and Hendron 1998). As the large block moved, a graben formed just behind the slide block because of the compound nature of the slide. The back scarp resulting from slope failure was semi-circular in plan view, with nearly vertical back (in cross-section) of a height up to 200 ft (60 m).

Based on field observation, and results of a subsurface investigation, the failure surface was estimated to have passed at a steep inclination through the MSW and to the underlying BNS, then along the BNS, daylighting at the vertical face of excavation at the slope toe.

Figure 8.17 shows details of critical 2D cross-section (Geosyntec Consultants 1996) along with the location of some of the boreholes. Details of the sixteen cross-sections used to create the 3D geometry are shown in Figure 8.18. The critical 2D cross-section is located at station 453 (see Figure 8.18(h)).

8.3.1.3 Geological Setting

The regional geology of Cincinnati area has been described by Ford (1967), Fleming and Johnson (1994) and Baum and Johnson (1996). Local geology of the landfill site is described in detail by Eid et al. (2000). The BNS underlying the MSW consists of about 15 ft (5 m) thick layer of colluvial and residual soils; the residual soil is a heterogeneous mixture of fine grain soil with rock fragments derived from local bedrock; however, it is not transported like colluvium. BNS is underlain by slightly dipping (1-2 m/km) bedrock shale and inter bedded limestone of Ordovician age (425-500 million years).

8.3.1.4 Representative Material Properties

In all the previous analyses of the Rumpke landfill failure (for example, Schmucker and Hendron 1998, Stark and Eid (1998), Stark et al. 2000 etc), there is a consensus to use an estimated shear strength of MSW and back-calculate shear strength of BNS. Figure 8.19 shows laboratory ring shear test results (Eid et al. 2000) for drained peak and residual failure envelopes on samples of the BNS and gray shale. The peak or fully softened failure
envelope for BNS corresponds to an effective stress friction angle ($\phi'$) of approximately 23 degrees. Similarly, drained residual friction angle ($\phi'$) of BNS is between 10 and 12 degrees. Using index properties of the BNS from boring G (see Figure 8.19) and the empirical correlation proposed by Stark and Hussian (2010), the fully softened and residual $\phi'$ is estimated to be 19.8$^\circ$-26.8$^\circ$ and 9$^\circ$-15.4$^\circ$, respectively, depending on the effective normal stress.

Schmucker and Hendron (1998) use MSW properties suggested by Kavazanjian et al. (1995) which consists of a bilinear shear strength envelope. The initial portion of the envelope is a purely cohesive material with a shear strength of 500 psf (24 kPa) up to an effective normal stress of 770 psf (37 kPa), and then a purely frictional material with a friction angle of 33 degrees thereafter. On the other hand, Stark and Eid (1998), Stark et al. (2000), and Chugh et al. (2007) use recommendations by Eid et al. (2000) for MSW shear strength. Based on their analysis, Eid et al. (2000) estimate site specific values of effective stress cohesion of 835 psf (40 kPa) and effective stress friction angle of 35 degrees.

Based on a number of landfill case histories, Stark et al. (2008) recommend using a stress dependent strength envelope that captures the stress dependency of MSW at different effective normal stresses. For normal stresses less than 4000 psf (200 kPa), $c'$=125 psf (6 kPa) and $\phi'$ =35 degrees is recommended and for effective normal stress equal to or higher than 4000 psf (200 kpa), $c'$=625 psf (30 kPa) and $\phi'$ =30 degrees is recommended (see Figure 8.12) to model the stress dependent envelope.

Because, the average effective stress along the failure surface at RSL is 1980 psf (90 kpa), the present analysis use $c'$=125 psf (6 kPa) and $\phi'$ =35 degrees as suggested by Stark et al. (2008).

All previous reported analyses of the RSL slope failure (for example, Schmucker and Hendron 1998, Stark et al. 2000, Chugh et al. 2007) use unit weights of 65pcf (10.2 kN/m$^3$) and 125 (19.7 kN/m$^3$) for MSW and BNS, respectively. Same unit weights are also used in the present analysis. Table 8.6 shows the material properties used in the 2D and 3D limit equilibrium analysis performed in this study.
8.3.1.5 Estimated Piezometric Levels

The leachate levels in the landfill at the time of failure were not measured. Therefore, all previous studies (Schmucker and Hendron 1998, Stark et al 2000 etc) estimated piezometric levels based on observations from piezometers installed outside the failure zone after the landslide and/or a piezometric sensitivity analysis. Following information provided by Schmucker and Hendron (1998) and Stark et al. (2000) was used in deciding the piezometric levels to be used in the present 2D and 3D slope stability analyses:

- Leachate was observed seeping from the toe of the northwest slope of the landfill before the winter of 1995/1996. This seepage required construction of a collection trench about 30-40 ft (9-12 m) below the access road at the slope toe.

- At the time of failure, the site had started experiencing spring time rainfall.

- Four piezometers were installed in the landfill away from the failure zone which were believed to be representative of the landfill failure area.

- Fluid in the jointed bedrock underlying the brown native soil was also believed to be an additional source of piezometric pressure acting on the failure surface.

- Several months after the slope failure, a pool of water formed beneath the back scarp in the graben area. The elevation of fluid was approximately 50 ft (15 m) above BNS.

From leachate elevations observed in piezometers and elevation of fluid beneath the vertical scarp, Schmucker and Hendron (1998) estimate a leachate level of 50 ft (15 m) above the BNS at a point 330 ft (100 m) away from toe of the landfill (see Figure 8.17). Based on observations from piezometer L/F-B (see Figure 8.15) and natural topography, Stark et al. (2000) report a maximum piezometric level of 80 ft (24.5 m) near the back scarp in the critical 2D cross-section. This height corresponds to a leachate level of 75 ft (23 m) and 5 ft (1.5 m) of piezometric head in the foundation soil/bedrock. In both analyses (Schmucker and Hendron 1998, Stark et al. 2000), the leachate was assumed to decrease linearly to the top of BNS at the slope toe.
Based on available data, the 3D shape and height of the total piezometric levels was re-estimated for each of the sixteen cross-sections. At the critical 2D cross-section (see Figure 8.18(h)) height of piezometric level at the back scarp was assumed to be 75 ft (23 m) above top of BNS which corresponds to elevation of 910 ft (277 m). Because leachate seepages were observed at the toe of critical cross-section, leachate level was modeled as linearly decreasing from back scarp to mid height of the vertical cut at the slope toe (i.e., 10 ft). Similarly, piezometric level behind the back scarp was extended increasing linearly to a maximum elevation of 925 ft (282 m), as scaled from the 2D cross-section (see Figure 8.17) developed by Schmucker and Hendron (1998). Piezometric levels in the other fifteen cross-sections were modeled based on the piezometric level of the critical 2D cross-section. The estimated piezometric level for the sixteen 2D cross-sections are shown in Figure 8.18.

8.3.1.6 Back-Analyses of Landfill Slope Failure

A 2D and 3D analysis of the RSL slope failure were performed using Janbu’s (1954) simplified procedure coded in CLARA-W (Hungr 2001). The 2D analysis was performed using the critical 2D cross-section located at station 453 (see Figure 8.18(h)). For the 3D analysis, all sixteen cross-sections were modeled in CLARA-W, eleven of which i.e., cross-sections at station 68 to 882 (Figure 8.18(c) to (m)) are within the slide mass. Five cross-sections are outside the failure area and are used to model the 3D slope geometry. Two additional cross-sections were also added on outer sides of the first and last cross-section in the failure area (active cross-sections). These two cross-sections are similar to the pre-failure geometry of the adjacent cross-section within the failure zone, except that the failure surface was modeled to start/end at these cross-sections. The distance of these cross-sections from first/last active cross-sections were adjusted to model a 3D analysis with vertical sides (side resistance ignored) and a 3D analysis with inclined sides instead of vertical sides (considering side resistance) as described in Chapter 6. For the 3D analysis with vertical sides (ignoring side resistance), the additional two cross-sections were placed 1 foot from the first and last active cross-sections. Stark et al. (2000) report secant slope inclination (inclination of straight line from toe to crest) of critical 2D cross-section of about 21 degrees (2.6H:1V). Therefore, a side inclination of four degrees was
estimated from Figure 6.34 in Chapter 6 to incorporate shear resistance along parallel sides of the slide mass. Figure 8.20 shows the 3D Digital Elevation Model (DEM) of the slide mass used in 3D analysis.

8.3.1.7 Effect of Column Size on FS Calculations

While performing a 2D and 3D back-analysis, the effect of number of columns and rows used to discretize the 3D slope model on the back-calculated shear strength (\( \phi'_{mob} \)) of the BNS was observed. For a 2D analysis in CLARA-W, the default setting is to discretize a cross-section into 300 columns and the program warns the user if the number of active columns is less than 40. Similarly, the 3D slide mass is discretized into 50 columns and 50 rows in a 3D analysis by default and a warning is given if the number of active columns is less than 400. The CLARA-W user manual also recommends columns in excess of 1000 to improve accuracy. In the present study, the number of columns in 2D analysis and number of columns and rows in 3D analysis were increased in steps until no significant change in the back-calculated \( \phi'_{mob} \) of the BNS was achieved.

A summary of the sensitivity analysis is shown in Table 8.7 and shows that in the 2D analysis using default number of columns (300) provides sufficient accuracy in the back-calculated \( \phi'_{mob} \). The number of active columns is 235 which is greater than the minimum recommended number of active columns (40) for a 2D analysis. However in the 3D analysis, using the default setting of 50 columns and 50 rows does not provide sufficient accuracy in the back-calculated \( \phi'_{mob} \). The number of active columns resulting from a grid of 50 columns and 50 rows are 1416 and 1445 for a 3D analyses with vertical sides and inclined sides, respectively. Even though the number of active columns are greater than the recommended number of active columns for a 3D analysis, the back-calculated \( \phi'_{mob} \) is overestimated by 11%-15%.

Figure 8.21 presents a sensitivity analysis of the relationship between \( \phi'_{mob} \) of BNS and the number of columns used to discretize the 3D slope model as a percentage of total dimensions of the slope model. Figure 8.21 shows that the number of columns has a significant effect on the back-calculated \( \phi'_{mob} \) of the BNS. Figure 8.21 shows, that depending on the number of columns used to the discretize slope, the back-calculated \( \phi'_{mob} \) of BNS may be overestimated by 2-5 degrees. This over estimation may be caused by the discretization of
total model space into columns of equal dimension (same rectangular plan area) based on user input. Because all calculations are performed at the center of columns referred as grid points, when fewer columns are used, the grid points may not capture a significant change in geometry/material property and thus reducing the accuracy of results.

In conventional 2D LE software, user input is not necessary to determine slice (equivalent to columns in 3D) width. The 2D cross-section is divided into slices of vertical sides based on the specific geometry of the problem and failure surface. The location of 2D vertical slices will occur based on the following criteria (Sharma 1996):

- each specified horizontal coordinate (parallel to direction of sliding) used to define slope surface, subsurface boundaries, and water surface located above the failure surface,
- each interaction of a water surface and subsurface boundary,
- each interaction of failure surface with subsurface boundary and/or water surface, and
- each lateral boundary used to define a surface surcharge.

This results in the number of slices that are different in width and thus include effects of different material layers and water surfaces. The user can increase the number of slices by reducing the failure segment length and/or increase the number of points to define subsurface layers and water surfaces. Sharma (1996) indicates that a maximum of 100 slices may be used in XSTABL (Sharma 1996), however, usually twelve slices provide sufficient accuracy.

Because 3D software uses columns of uniform dimensions, the user must ensure that the mesh is fine enough to capture any significant change in slide geometry, material layers, water surfaces, and failure surface. For the RSL case history analyzed, it appears that sufficient accuracy in the back-calculated $\phi'_m \sigma_b$ is achieved when the number of columns and/or rows are around 30% of the slope model dimensions. Therefore, all subsequent analyses were performed by discretization of the 3D slope model into 365 columns and 323 rows (approximately 70000 active columns).
8.3.1.8 Comparison of 2D and 3D Back-Calculated Shear Strength of BNS

Figure 8.22 presents the results of 2D and 3D LE analyses. The 2D analysis for the critical cross-section yields a back-calculated or mobilized friction angle of 15°. This value is in agreement with the mobilized friction angle reported by Schmucker and Hendron (1998) for a 2D analysis of the same cross-section. A 3D analysis with side resistance using side inclination of 4° (described in Chapter 6), the back-calculated $\phi_{mob}'$ of the BNS is 10°. This value is also in agreement with the drained residual friction angle reported by Stark and Eid (1998) and Eid et al. (2000) obtained from ring shear testing of representative samples from boring G (see Figure 8.19).

Figure 8.22 also presents the back-calculated $\phi_{mob}'$ of the BNS if side resistance is ignored in the 3D analysis. As a result, this analysis yields $\phi_{mob}'$ of 12° which is approximately 25% less than the back-calculated $\phi_{mob}'$ from 2D analysis and about 20% higher than the back-calculated $\phi_{mob}'$ obtained from 3D analysis when side resistance is considered. The sample tested from boring G (Figure 8.19) has a liquid limit and clay size fraction of 69 and 55% respectively. Using these index properties of BNS sample from boring G and the empirical correlation proposed by Hussain (2010), a residual friction angle of 9-15.4° is estimated for effective normal stresses of 50-700 kPa. The back-calculated $\phi_{mob}'$ using different LE analysis is within the range of estimated residual friction friction angle which indicates a post peak shear strength of the BNS was mobilized.

In summary, the mobilized friction angle of BNS estimated from 2D analysis and 3D analysis (ignoring side resistance) results in overestimation of BNS shear strength by 50% and 20%, respectively.

8.3.1.9 Comments

The following conclusions can be discerned from 2D and 3D analyses of RSL slope failure:

- The number of columns (equivalent to 2D vertical slices) and rows used to discretize the 3D slope model has a significant effect on the back-calculated $\phi_{mob}'$ of BNS. CLARA-W user manual recommends the use of higher number of columns for reasonable accuracy in a 3D slope stability analysis. Same observation was made regarding the number
of columns in 3D analysis by Lam and Fredlund (1993). However, in 2D limit equilibrium software, e.g., XSATBL, the user is not required to specify the width of slices and slices are generated by the software to account for changes in the geometry. In contrary in 3D software, columns of equal dimensions are used and user input is required to decide mesh sizing, which affects FS calculations. The default or recommended values for number of columns and rows or minimum number of active columns may not provide sufficient accuracy. Therefore, the user must check the results using different numbers of columns.

- The back-calculated shear strength of BNS from 3D analysis considering side resistance is in agreement with the drained residual friction angle reported by Stark and Eid (1998) and Eid et al. (2000) obtained from ring shear testing of representative samples from boring G (see Figure 8.19).

- 2D and 3D analyses (ignoring side resistance) results in overestimation of BNS shear strength by 50% and 20%, respectively.

8.3.2 Oceanside Manor Santiago, USA

8.3.2.1 Brief History

The 1979 Oceanside Manor landslide occurred in San Diego County, California at a site underlain by the Santiago Formation. The case history is described by Stark and Eid (1992, 1998). A 2D analysis of the landslide was performed by Stark and Eid (1992), while 3D analyses of the slide were performed by Stark and Eid (1998) and Arellano and Stark (2000). The landslide occurred along a bluff approximately 18 to 20 m high (60-65 ft). The scarp is approximately 130 m (430 ft) long and the slide involves approximately 122,000 m$^3$ (160,000 cu yd) of soil. Figure 8.23 shows a plan view of the landslide area before the failure. The extent of the landslide area is shown with dotted lines in Figure 8.23. The location and associated 2D FS at five different locations across the landslide are also shown in Figure 8.23. At the mid point of the slide mass, the bluff turns from a northerly direction to an easterly direction at an angle of 90 to 100°.
8.23). Figure 8.24 shows subsurface conditions reported by Stark and Eid (1998) through the middle of slide mass (cross-section D-D’ in Figure 8.23) where the bluff line changes orientation. Cross-section D-D’ (Figure 8.24) is not the 2D critical cross-section. The slide surface was located using slope inclinometers and extensive borings and trenches. In addition, the groundwater levels were monitored using piezometers and water levels in borings and trenches shortly after slide movement. Figure 8.25 shows a 3D Digital Elevation Model (DEM) of the slide mass.

8.3.2.2 Geological Setting

At this site, the Santiago Formation is comprised of a claystone and sandstone. The sandstone is fine to medium grained and overlies the greenish to bluish-gray claystone. The remolded claystone classifies as a clay or silty clay of high plasticity, CH-MH according to the unified Soil Classification system. The liquid limit, plasticity index, and clay-size fraction of the claystone are 89, 45, and 57%, respectively (Stark and Eid 1992). Field investigations indicate that the clay beds and clay seams are usually horizontal and consist of sheared claystone. In addition, claystone is commonly fissured and display slickensided and shiny parting surfaces.

8.3.2.3 Representative Material Properties

Stark and Eid (1992) report the site has undergone at least three episodes of landsliding prior to the 1979 landslide event and it has undergone displacements greater than 2 m (6 ft) in recent geologic history. Because 1 -2 m (3-6 ft) of field displacement is required to mobilize the residual strength condition (Skempton 1977), the claystone probably developed a residual strength condition along the base of the sliding surface. The majority of the slide plane is approximately horizontal through the Santiago Formation indicating that the slide may have occurred along a weak claystone layer or seam. This allows the shear strength of this layer to be approximated by one set of shear strength parameters. As a result, Stark and Eid (1998) assumed that during failure, fully softened and residual shear strengths were mobilized along the scarp and the base in the Santiago Formation, respectively. Using a ring shear test procedure (Stark and Eid 1993, 1997) and represen-
tative samples of Santiago claystone, a fully softened and residual friction angle of 25 degrees and 7.5 degrees, respectively, were reported by Stark and Eid (1998).

The cohesion and friction angle of the compacted fill were measured using direct shear tests to be zero and 26 degrees, respectively. The moist unit weight of the Santiago claystone and the compacted fill were measured to be 19.6 kN/m$^3$ (125 pcf).

8.3.2.4 2D and 3D Analyses of Slide

Stark and Eid (1998), performed 2D and 3D slope stability analyses of the landslide mass using Janbu’s (1956) simplified procedure and the microcomputer program CLARA 2.31 (Hungr 1988a). For the different 2D cross-sections, the overestimation of FS by ignoring the stress dependency of the failure envelopes approximately cancels the underestimation of FS due to using the uncorrected Janbu’s (1956) simplified procedure.

The parallel sides of the slide mass in 3D analyses were assumed to be nearly vertical. To simulate an active earth pressure condition, the back scarp was taken to be inclined 60 degrees from the horizontal. These failure surface conditions result in a minimum sliding resistance along the back scarp and sides during failure. Based on the analysis of forty-four different cross-sections Stark and Eid (1998) report an average 2D FS of 0.92. A 3D FS of 0.94 is reported if side resistance is not considered. The 3D FS is approximately equal to the average 2D FS for forty-four cross-sections and the 3D FS of 0.94 is less than unity which is expected for a slope at incipient failure. The slope was reanalyzed (Stark and Eid 1998) to include the shear resistance along the vertical sides using at-rest earth pressure and an imaginary layer to account for the side resistance as outlined in Chapter 6. The reanalysis yielded, a 3D FS of 1.02 and a ratio between the minimum 3D and 2D FS values of approximately 1.6.

Arellano and Stark (2000) used the Oceanside Manor landslide data to compare the ratio of 3D/2D FS computed from charts prepared for incorporating side shear resistance along vertical sides that parallel the slide direction. Arellano and Stark (2000) estimate 3D FS of 1.01 using their technique of external side forces and at rest-earth pressure.
8.3.2.5 Influence of Shear Strength on 3D/2D FS Ratio

The Oceanside Manor, San Diego, CA case history is used herein to demonstrate the use of updated 3D FS charts (see Figures 6.24 - 6.26 in Chapter 6) that present the influence of shear strength on the ratio of 3D/2D FS. Prior to failure the slope had an average slope inclination of 3.5H:1V. The width and height ratio (W/H) of the landslide is 6.5 (130 m/20 m). The ratio of fully softened and residual friction angle, $\phi'_u/\phi'_l$ is 3.3 ($25^\circ/7.5^\circ$) for Oceanside Manor. Using Figures 6.25 and 6.26, a W/H ratio of 6.5, $\phi'_u/\phi'_l$ of 3.3, a 3D/2D FS ratio of 1.07 and 1.11 is obtained for 3H:1V and 5H:1V slopes, respectively, as shown in Figure 8.26. Because the landslide slope has an inclination of 3.5H:1V, a 3D/2D FS ratio of 1.09 was interpolated for inclination of 3.5H:1V from Figure 8.26. Based on an average 2D FS of 0.92 from forty-four different cross-sections reported by Stark and Eid (1998), a 3D FS of 1.00 is estimated using a 3D/2D FS ratio of 1.09. In comparison with 3D FS of 1.02 and 1.01 calculated by Stark and Eid (1998) and Arellano and Stark (2000) respectively, the 3D FS calculated using the updated 3D FS charts is in better agreement with a FS of unity.

In summary, this case history shows that the updated 3D FS charts which use an earth pressure coefficient ($K_\tau$) that is in-between $K_O$ and $K_A$ values to estimate the influence of shear strength on ratio of 3D/2D FS provides a better estimate of shear resistance acting along two vertical sides and results in 3D/2D FS ratios that are in agreement with 3D FS in the field. However, estimating side shear resistance using the at-rest earth pressure procedure described by Stark and Eid (1998), Arellano and Stark (2000), and Eid et al (2006), provides a slightly unconservative but reasonable estimate of the 3D/2D FS ratios.

The use of an intermediate value of $K_\tau$ for calculation of side forces is supported by field slide mass observations where, generally the slide mass is cracked near the ground surface and the cracks decreases in width with depth. Therefore, near the surface the side resistance may agree better with $K_A$ and near the base of the slide mass it may agree better with $K_O$. Based on triaxial tests, Lambe and Whitman (1969) report that little horizontal strain, less than 0.5%, is required to change the stresses from at-rest to active earth pressure. Therefore, it is possible that after the slip surface develops and movement begins, the at-rest earth pressure transitions to an
8.4 Verification of 3DDEM-Slope

To verify 3DDEM-Slope code, 2D and 3D back-analyses of three case histories, i.e., Tianjin (China), New Jersey (USA), and Ohio (USA) were performed using 3DDEM-Slope. Comparison of back-calculated shear strengths obtained from CLARA-W and 3DDEM-Slope are presented in Table 8.8. All 3D surfaces, e.g., material layers, piezometric surface, slope geometry, and failure surface, were input using a DEM in 3DDEM-Slope. Because 3DDEM-Slope uses 3D extensions of Bishop’s (1955), Janbu’s (1956), and Spencer’s (1967) procedures derived by Hungr et al. (1989) and Hungr (2001), the back-calculated shear strengths are generally in agreement with those computed using CLARA-W. However, some variation in back-calculated shear strengths was observed because different methods are used for calculation of column base angles in 3DDEM-Slope as discussed in Chapter 7.

8.5 Review and Summary of Chapter 8

- Depending on the location of the cross-section, 2D factors of safety can vary significantly due to variations in topography and ground water level over the sliding area (see Figure 8.23) so modeling the 3D geometry can be important in landslides.

- A 3D analysis can accommodate variations in geometry, pore-water pressure, and material properties across a slide mass, better than a 2D analysis and even a weighted 2D average analysis.

- Back-calculated shear strength from 3D LE analysis is more representative of field/laboratory testing

- 2D back-analyses yield higher back-calculated shear strength than 3D back-analyses because side resistance is not included.

- 3D/2D FS ratio computed by either a LE or continuum method is greater than unity not less than unity as reported by Hovland (1977),

- 3D extensions of Bishop’ (1955) simplified and Spencer’s (1967) procedures presented by Hungr (1987, 2001) yield similar 3D FS values for a rotational slide mass.

- 3D extensions of LE give comparable results with 3D FE (PLAXIS) procedure while 3D FS computed from 3D FE analyses are slightly lower (about 7%) than 3D LE analyses but within an acceptable tolerance (12% reported by Duncan 1996). This results in slightly higher back-calculated shear strengths using a 3D FE procedure, which is believed to be the correct answer, than 3D LE procedures.

- 3D FE analysis is more time consuming than 3D LE analysis. For comparison purposes, a 3D FE slope analysis that takes 40-45 minutes can be analyzed by LE in less than one minute.

- In cases where there is not a large difference in shear strength between the upper and lower materials or the lower material is stronger than the upper material, a near ellipsoidal failure surface can develop in a FE analysis and result in similar back-calculated strengths as an acceptable 3D LE method.

- An inherent advantage of the FE analysis is the failure surface does not need to be specified. However in the case of New Jersey landfill case history, this was problematic because the FE failure surface did not match field observations. In this case history, the lower material (tidal marsh) was weaker than the upper material (MSW) so the failure surface in the FE analysis remained in the tidal marsh resulting in vertical or near vertical sides of the slide mass like a translational failure instead of the observed rotational failure surface (ellipsoidal shape). As result, this case history suggests a 3D FE back-analysis is less accurate than a 3D LE method because the side resistance is omitted. The FE model follows the weak layer and minimizes the MSW shear resistance by using vertical sides of the slide mass.

- The present FD and FE analysis may not be appropriate for back-analysis of a failed slope because the observed failure surface cannot
be modeled and thus LE equilibrium analysis should be used for back-analysis.

- A 3D back-analysis of the Tianjin case history yields back-calculated shear strengths that are in agreement with the FV tests because the observed failure surface was modeled.

- The back-calculated shear strength of the brown native soil in the RSL slope failure using a 3D analysis considered side resistance and is in agreement with the drained residual friction angle reported by Stark and Eid (1998) and Eid et al. (2000) obtained from ring shear testing of representative samples from boring.

- 2D analyses for Tianjin, China case history overestimated the back-calculated undrained shear shear strength by about 20% compared with a 3D analysis that incorporates the shear resistance along the sides of the slide mass. Similarly for the RSL case history, 2D analysis and 3D analysis (ignoring side resistance) results in overestimation of brown native soil shear strength by 50% and 20% respectively.

- The number of columns and rows used to discretize a 3D slope model has a significant effect on the calculated FS. The default or recommended value for number of columns and rows or minimum number of active columns may not always provide sufficient accuracy so the user must check results using greater number of column to ensure that the mesh is fine enough to capture any significant change in slide geometry, material layers, water surfaces, and failure surface.

- For ellipsoidal (circular) failure surfaces, a 2D weighted average FS by LE procedures using the weight above failure surface as the weighing factor provides a good estimate of the 3D FS because a 3D ellipsoidal shape of the failure surface with cross-sections at the edges of the the slide mass that are different from the central cross-section(s).

- The number of cross-sections for weighted average should be selected based on the actual geometry of the slope to ensure that each additional cross-section represents any variation from other cross-section.
- Oceanside Manor case history shows that an earth pressure coefficient \( K_r \) that is in-between \( K_O \) and \( K_A \) values for estimating the influence of side shear resistance on the ratio of 3D/2D FS provides a better estimate of shear resistance acting along two vertical sides and results in 3D/2D FS ratios that are in agreement with field FS.

- Charts developed during this study can be used to estimate the importance of performing a 3D slope stability analysis for a translational failure.
### 8.6 Tables

#### Table 8.1: Physical properties of different layers

<table>
<thead>
<tr>
<th>Soil Layer</th>
<th>Layer No.</th>
<th>Thickness (m)</th>
<th>Unit Weight, $\gamma (kN/m^3)$</th>
<th>Plasticity Index, $I_P(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface Layer</td>
<td>-</td>
<td>1.0-2.0</td>
<td>17.9</td>
<td>22.4</td>
</tr>
<tr>
<td>Muddy Silt</td>
<td>1-1</td>
<td>2.0 - 7.0</td>
<td>17.9</td>
<td>14.4</td>
</tr>
<tr>
<td>Mud</td>
<td>1-2</td>
<td>3.0 - 6.0</td>
<td>16.5</td>
<td>25.6</td>
</tr>
<tr>
<td>Muddy Clay</td>
<td>1-3</td>
<td>5.0 - 13.0</td>
<td>17.5</td>
<td>22.0</td>
</tr>
<tr>
<td>Interlayer</td>
<td></td>
<td></td>
<td>19.6</td>
<td>12.7</td>
</tr>
<tr>
<td>Silt</td>
<td>2-1</td>
<td>-</td>
<td>19.7</td>
<td>11.1</td>
</tr>
<tr>
<td>Clay</td>
<td>2-2</td>
<td>2.0 - 7.0</td>
<td>18.7</td>
<td>20.7</td>
</tr>
<tr>
<td>Sandy Silt</td>
<td>2-3</td>
<td>1.0 - 5.0</td>
<td>20.0</td>
<td>6.1</td>
</tr>
<tr>
<td>Silt Sand</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

*Young modulus, $E = 10^5 kN/m^2$ and Poisson’s ratio, $\nu = 0.3$ were used in FE analysis for all soil layers*

#### Table 8.2: Results of 2D weighted average analysis

<table>
<thead>
<tr>
<th>Cross-Section Location</th>
<th>FS</th>
<th>Weight (kN)</th>
<th>Su (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-sixth (75 m)</td>
<td>1.16</td>
<td>3078.21</td>
<td>20.8</td>
</tr>
<tr>
<td>Centerline (125 m)</td>
<td>0.86</td>
<td>7478.78</td>
<td>20.8</td>
</tr>
<tr>
<td>One-sixth (175 m)</td>
<td>1.16</td>
<td>3078.21</td>
<td>20.8</td>
</tr>
</tbody>
</table>
Table 8.3: Comparison of measured and back-calculated undrained shear strengths for recent marine mud deposit-Tianjin, China

<table>
<thead>
<tr>
<th>Layer</th>
<th>Measured$^1$ (kPa)</th>
<th>Back-calculated (kPa)</th>
<th>2D$^2$</th>
<th>2D$^3$</th>
<th>3D$^4$</th>
<th>3D$^5$</th>
<th>3D$^6$</th>
<th>3D$^7$</th>
<th>3D$^8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulic Fill</td>
<td>10.9</td>
<td>24.0</td>
<td>20.8</td>
<td>20.0</td>
<td>21.4</td>
<td>10.9</td>
<td>10.9</td>
<td>10.9</td>
<td></td>
</tr>
<tr>
<td>Muddy Silt</td>
<td>19.2</td>
<td>24.0</td>
<td>20.8</td>
<td>20.0</td>
<td>21.4</td>
<td>19.2</td>
<td>19.2</td>
<td>19.2</td>
<td></td>
</tr>
<tr>
<td>Mud</td>
<td>22.2</td>
<td>24.0</td>
<td>20.8</td>
<td>20.0</td>
<td>21.4</td>
<td>22.8</td>
<td>22.4</td>
<td>24.0</td>
<td></td>
</tr>
</tbody>
</table>

$^1$ Average undrained shear strength measured from FV
$^2$ Li et al (2005b) results using Bishop’s procedure and this study using Bishop’s and Spencer’s procedures
$^3$ 2D Weighted average procedure
$^4$ 3D Bishop and Spencer procedures
$^5$ PLAXIS-3D
$^6$ 3D Bishop’s procedure
$^7$ 3D Spencer’s procedure
$^8$ PLAXIS-3D

Table 8.4: Material properties used to verify 2D model-New Jersey Landslide, USA

<table>
<thead>
<tr>
<th>Layer No.</th>
<th>Material</th>
<th>Unit Weight $\gamma$(kN/m$^3$)</th>
<th>Cohesion $c$ (kPa)</th>
<th>Friction Angle $\phi$ ($^\circ$)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tidal Marsh</td>
<td>16</td>
<td>4.8</td>
<td>0</td>
<td>Dvirnoff (1999)</td>
</tr>
<tr>
<td>2</td>
<td>Waste</td>
<td>7.06</td>
<td>37</td>
<td>0</td>
<td>Dvirnoff (1999)</td>
</tr>
</tbody>
</table>
### Table 8.5: Back-calculated MSW shear strength and associated FS-New Jersey, USA

<table>
<thead>
<tr>
<th>MSW Strength</th>
<th>$c'(kPa)$</th>
<th>$\phi'(^\circ)$</th>
<th>2D FS</th>
<th>3D FS</th>
<th>3D 2D</th>
<th>3D 2D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bishop</td>
<td>Spencer FE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.5</td>
<td>35</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
<td>1.74</td>
<td>-</td>
</tr>
<tr>
<td>10.3</td>
<td>35</td>
<td>-</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
<td>1.74</td>
</tr>
<tr>
<td>9</td>
<td>35</td>
<td>-</td>
<td>-</td>
<td>1.00</td>
<td>-</td>
<td>1.11</td>
</tr>
<tr>
<td>0</td>
<td>42</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
<td>1.73</td>
<td>-</td>
</tr>
<tr>
<td>0</td>
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<td>-</td>
<td>-</td>
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<tr>
<td>0</td>
<td>26.0</td>
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<tr>
<td>0</td>
<td>25.5</td>
<td>0.62</td>
<td>-</td>
<td>-</td>
<td>1.00</td>
<td>-</td>
</tr>
<tr>
<td>0</td>
<td>27.5</td>
<td>0.65</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.00^1</td>
</tr>
<tr>
<td>0</td>
<td>24</td>
<td>0.59</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.00^2</td>
</tr>
</tbody>
</table>

1. Weighted average of four cross-sections
2. Weighted average of three cross-sections

### Table 8.6: Material properties used in 2D and 3D analysis-Cincinnati, OH USA

<table>
<thead>
<tr>
<th>Layer No.</th>
<th>Material</th>
<th>Unit Weight</th>
<th>Cohesion $c'$ ($kN/m^3$)</th>
<th>Friction Angle $\phi'$ ($^\circ$)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BNS^1</td>
<td>125 (19.7)</td>
<td>0</td>
<td>back-calculated</td>
<td>Stark et al. (2000)</td>
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<tr>
<td>2</td>
<td>Waste</td>
<td>65 (10.2)</td>
<td>125 (6)</td>
<td>35</td>
<td>Stark et al. (2008)</td>
</tr>
</tbody>
</table>

^1 Brown native soil
Table 8.7: Summary of number of columns used in 2D and 3D analysis-Cincinnati, OH USA

<table>
<thead>
<tr>
<th>Analysis Type</th>
<th>Width x (ft)</th>
<th>Length y (ft)</th>
<th>NX (No.)</th>
<th>NY (No.)</th>
<th>Total Columns</th>
<th>Friction Active Columns</th>
<th>Percentage</th>
<th>Angle φ′mob</th>
<th>Active Columns (NAC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D</td>
<td>1</td>
<td>1142</td>
<td>1</td>
<td>23</td>
<td>2</td>
<td>18.5</td>
<td>17</td>
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<tr>
<td></td>
<td>1</td>
<td>1142</td>
<td>1</td>
<td>91</td>
<td>8</td>
<td>15.3</td>
<td>71</td>
<td></td>
<td></td>
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<tr>
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<td>1</td>
<td>1142</td>
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<td>183</td>
<td>16</td>
<td>15.1</td>
<td>142</td>
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<tr>
<td></td>
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<td>1</td>
<td>300†</td>
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<td>235</td>
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<td>1</td>
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<td>1</td>
<td>742</td>
<td>65</td>
<td>15</td>
<td>582</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3D</td>
<td>1010</td>
<td>1142</td>
<td>20</td>
<td>23</td>
<td>2</td>
<td>15</td>
<td>240</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Ignoring Side Resistance)</td>
<td>1011</td>
<td>1143</td>
<td>50²</td>
<td>50²</td>
<td>5</td>
<td>13.1</td>
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<td>3D</td>
<td>1010</td>
<td>1142</td>
<td>81</td>
<td>91</td>
<td>8</td>
<td>12.3</td>
<td>4281</td>
<td></td>
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<tr>
<td>(Considering Side Resistance)</td>
<td>1010</td>
<td>1142</td>
<td>162</td>
<td>183</td>
<td>16</td>
<td>12.2</td>
<td>17543</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>1010</td>
<td>1142</td>
<td>323</td>
<td>365</td>
<td>32</td>
<td>12</td>
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<td>1010</td>
<td>1142</td>
<td>657</td>
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<tr>
<td></td>
<td>1010</td>
<td>1142</td>
<td>1010</td>
<td>1142</td>
<td>100</td>
<td>11.8</td>
<td>688916</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 Default for 2D analysis in CLARA-W

2 Default for 3D analysis in CLARA-W

Table 8.8: Comparison of back-calculated shear strengths using CLARA-W and 3DDEM-Slope

<table>
<thead>
<tr>
<th></th>
<th>Tianjin, China</th>
<th>New Jersey, USA</th>
<th>Cincinnati, USA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Recent Marine Mud</td>
<td>MSW</td>
<td>MSW</td>
</tr>
<tr>
<td>Su (kPa)</td>
<td>2D</td>
<td>3D</td>
<td>2D</td>
</tr>
<tr>
<td>CLARA-W</td>
<td>23.9</td>
<td>20.0</td>
<td>42.0</td>
</tr>
<tr>
<td>3DDEM-Slope</td>
<td>23.6</td>
<td>19.7</td>
<td>41.0</td>
</tr>
</tbody>
</table>

1 Ignoring side resistance

2 Considering side resistance
8.7 Figures

Figure 8.1: Location of the landslide with retrogressive extension into the reclaimed land (from Li et al. 2005b)

Figure 8.2: Soil profile along cross-section C-C shown in Figure 8.1 (from Li et al. 2005b): (a) before failure; (b) after first failure
Figure 8.3: Scarps in reclaimed land after sliding (from Li et al. 2005a)

Figure 8.4: Soil profile in the reclaimed land along cross-section C-C (after Li et al. 2005a)
Figure 8.5: Undrained shear strength profile (after Li et al. 2005a)

Figure 8.6: 2D slope model for cross-section C-C of Tianjin, China: (a) CLARA-W; (b) PLAXIS-3D
Figure 8.7: 3D model of Tianjin, China slide: (a) CLARA-W; (b) PLAXIS-3D
Figure 8.8: Relationship between undrained shear strength and FS for 2D and 3D analysis-Tianjin, China
Figure 8.9: Topography of New Jersey landfill slope failure (after Dvirnoff and Munion 1986)

Figure 8.10: Critical 2D cross-section (A-A’) of New Jersey landfill slope failure (after Dvirnoff and Munion 1986)
Figure 8.11: 2D slope model for cross-section A-A’ of New Jersey landfill slope failure, USA: (a) CLARA-W; (b) PLAXIS
Figure 8.12: MSW shear strength envelope for normal stresses less than 500 kPa (from Stark et al. 2008).

Recommended:

- $\sigma' \leq 200$ kPa, $c' = 6$ kPa, $\phi' = 35^\circ$
- $\sigma' = 200$ kPa, $c' = 30$ kPa, $\phi' = 30^\circ$

Eid et al. (2000)
Zekkos (2005)
Van Impe (1998)
Kavazanjian et al. (2000)
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Figure 8.18: (continued)
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Figure 8.18: (continued)
Figure 8.18: (continued)
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<table>
<thead>
<tr>
<th>Sample</th>
<th>LL</th>
<th>PL</th>
<th>CF</th>
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</thead>
<tbody>
<tr>
<td>Gray shale</td>
<td>36</td>
<td>20</td>
<td>44</td>
</tr>
<tr>
<td>Brown native soil (NS)</td>
<td>63</td>
<td>27</td>
<td>53</td>
</tr>
<tr>
<td>Brown native soil (NS)</td>
<td>69</td>
<td>28</td>
<td>55</td>
</tr>
</tbody>
</table>

LL - Liquid limit; PL - Plastic limit; CF - Clay fraction

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CHAPTER 9

SUMMARY AND CONCLUSIONS

9.1 Summary

This study presents a LE methodology for calculating the 3D FS for natural and manmade slopes. The methodology is coded in an accompanying user friendly software package called 3DDEM-Slope. A comparison of different 2D and 3D slope stability methods e.g., LE and continuum methods, is also presented to verify the methodology. Using known slope stability solutions from published literature and field case histories, 2D and 3D slope stability analyses were performed using LE method and then verified by continuum method to investigate the applicability and/or limitations of each method to different slope stability problems and geometries.

Initially, continuum mechanics were used for deformation analyses so its utility for FS computations was not widely used. Present continuum mechanics analyses include similar failure criteria as used in LE making it possible to compute FS values. Continuum mechanics has the ability to model complex problems without simplifying assumptions which is a big advantage over LE method. Another advantage of the continuum mechanics analysis is that the failure surface geometry, i.e., rotational or translational, does not need to be specified and it is located as part of the solution for lowest FS. However in back-analysis of a slope failure, the field failure surface and slide mass geometry must be used instead of searching for the failure surface that yields the lowest back-calculated strength (Hussain et al. 2011). Because there is no provision for specifying a failure surface, the FE analysis does not provide a good representation of field behavior or back-calculated shear strength of a case history. One useful conclusion from this study is that continuum mechanics procedures can be used for design of slopes but possibly not for a back-analysis. In addition, LE procedures are more user friendly than FD
and FE procedures, require less computational time, and are usually preferred for routine analyses. For important projects, results of LE analysis can be verified using FD or FE procedures.

Existing literature on 3D LE slope stability procedures was reviewed and summarized to develop the 3D slope stability methodology presented herein. The review of available 3D LE slope stability procedures revealed a wide variety of theories, assumptions, and equilibrium conditions incorporated in existing 3D LE procedures. In addition, field relevance of the assumptions required to render the procedure statically determinant are questionable because only a few 3D formulations have been verified using field case histories. Verification of most 3D formulations is based on hypothetical example problems and not field case histories. This has resulted in considerable confusion about selection and performance of existing 3D slope stability procedures.

One interesting conclusion from the 3D LE literature review is the 3D FS obtained from different procedures do not differ by more than 1-2% from each other using the hypothetical examples presented by various authors (For example Lam and Fredlund 1993, Hungr et al 1989, Huang and Tsai 2000, Chen et al. 2001, Chen et al 2003 and Cheng and Yip 2007). This difference is significantly less than the acceptable variation of 12% for 2D methods (Duncan 1996).

The actual field conditions during failure involve uncertainties like failure surface geometry, slope geometry, pore water conditions, mobilized shear strength, etc. Therefore, the 3D FS is likely to be more affected by modeling of field geometry and material properties used for the failed slope than a 3D formulation that makes reasonable approximations. Based on comparison of known solutions, the 3D extensions of Bishop’s (1955) simplified, Janbu’s (1956) simplified, Morgenstern and Price’s (1965), and Spencer’s (1967) procedures proposed by Hungr et al. (1989) and Hungr (2001) provide similar 3D FS values as those obtained by 3D formulations presented by other researchers (for example Lam and Fredlund 1993, Huang and Tsai 2000, Chang 2002, Chen et al 2003, Cheng and Yip 2007).

During this study, 3D back-analysis of rotational slope failures resulted in back-calculated shear strengths that are in agreement with laboratory shear strengths measured using representative samples. However, 3D formulations of Morgenstern and Price’s (1965) and Spencer’s (1967) procedures by Hungr (2001) encountered convergence problems during analysis of translational
failures with external side forces or inclined sides. Therefore, 3D extension of Janbu’s (1956) simplified procedure is preferred over these procedures for back-analysis of translational failures.

For a translational landslide, Stark and Eid (1998) show that 3D LE software does not consider the effects of shear resistance offered by vertical sides that parallel the direction of movement. Consequently, the computed 3D FS are underestimated which results in back-calculated shear strengths to be overestimated. To overcome this limitation, Stark and Eid (1998) propose using an “imaginary” material layer that surrounds the sides of the slide mass with a shear resistance derived using the at-rest earth pressure. Similarly, Arellano and Stark (2000) incorporate the side resistance by adding an external horizontal and vertical side force equivalent to the shear resistance corresponding to the at-rest earth pressure acting on the two vertical sides. The present study uses FE and FD programs to calculate the magnitude of the side shear resistance along vertical sides of a translational slide mass. Results of the parametric study show that use of $K_O$ for approximating the shear resistance results in overestimation of the 3D/2D FS ratio. However, use of an earth pressure coefficient ($K_r$) that is in-between at-rest ($K_O$) and active ($K_A$) earth pressures values provides a better estimate of the side shear resistance and 3D/2D FS ratios are in agreement with FE and FD analyses. The findings were also verified using two translational case histories. Based on the findings, the charts provided by Arellano and Stark (2000) showing the influence of shear resistance on 3D/2D FS ratios for various slope inclinations and geometries were updated.

For a rotational (ellipsoidal) failure surface, importance of 3D effects on the FS and back-calculated soil shear strengths can be estimated using a weighted average analysis using the weight above the failure surface as the weighing factor. Regarding the use of weighted average analysis for estimating 3D FS from 2D analyses results for translational landslides with vertical sides, there is less variation in different cross-sections and a weighted average FS is the same as the 2D FS obtained for the central cross-section. Charts developed during this study can be used to determine the importance of performing a 3D slope stability analysis for a translational failure.

Based on a review of existing 3D literature, and LE, FE, and FD analyses performed in the present study, it is concluded that the 3D FS is greater than 2D FS for all conditions considered herein. The findings that 2D analyses
yield lower FS values than 3D analyses is significant for design of slopes. For example, MSW landfill design is regulated by state and federal codes that require a minimum static FS of 1.5. These codes do not specify whether this is a 2D or 3D FS. However, the intent of the state and federal codes is to require a minimum 2D FS greater than 1.5. With the increasing use of 3D stability analyses in practice, some designers have used a 3D FS to satisfy the state or federal codes even though a 3D FS of 1.5 implies relatively less stable than a 2D FS of 1.5. This can result in a less stable landfill slope but more airspace for the facility. Design practices use material properties and leachate levels under normal operating condition which may be significantly different under certain conditions (e.g., combustion or leachate recirculation). Therefore, it is recommended that regulatory codes specify a “minimum 2D FS of 1.5” to achieve the desired level of risk for manmade slopes.

A new 3D LE program, 3DDEM-Slope, was developed to incorporate some of the findings of this study. The 3D geometry of the slope, various material layers, piezometric surface, and failure surface is input in the form of a DEM generated using Surfer 9 (Golden Software 2010). Material properties are directly input and stored in the software. 3DDEM-Slope uses 3D extensions of Bishop’s (1955) simplified, Janbu’s (1956) simplified, and Spencer’s (1967) procedures presented by Hungr et al. (1989) and Hungr (2001). The 2D and 3D correction factor for Janbu’s (1956) simplified procedure is incorporated in the program. The program options include input of shear strength using a stress dependent failure envelope. Although the program uses a 3D DEM file, 3DDEM-Slope can be used to calculate 2D FS at any desired cross-section in the 3D geometry file. In addition, 3DDEM-Slope compares the 2D FS for a cross-section in the middle of the 3D slide mass with the computed 3D FS. The program also provides the user with a warning signal that 3D/2D FS ratio is less than the reference values obtained from FD and FE analyses for same width to height ratio and slope inclination. The user can select to apply external side forces that are calculated based on the findings of this study and obtain a corrected 3D FS.

3DDEM-Slope uses improved subroutines for calculation of the 3D center of rotation and vertical column base angles. The column base angles are calculated using a third-order finite difference estimator (Horn 1981) using all eight outer points of a 3D grid node instead of using only two adjacent grid nodes so the base angle corresponds to the angle of an inclined plane.
instead of a line as occurs in 2D calculations.

9.2 Conclusions

Based on this study, the following conclusions are drawn concerning 3D slope stability analyses of natural and manmade slopes:

9.2.1 Computational Accuracy of Different 3D Slope Stability Methods

A comparison of different slope stability methods, i.e., continuum mechanics and LE, for analyzing 2D and 3D slope problems is presented in this study. This comparison was accomplished using 2D and 3D example problems from prior studies (Fredlund and Krahn 1977; Xing 1988), a parametric study, and field case histories. The following conclusions are derived for the 3D slope stability methods:

- 3D FS obtained from different LE procedures that satisfy all conditions of equilibrium do not differ by more than 1-2%.

- Lowest FS calculated by 3D LE procedures that satisfy all conditions of equilibrium are essentially similar to the values obtained from 3D FD and 3D FE analysis. The LE critical failure surface is generally in agreement with the critical failure surface found by FE and FD procedures.

- FD and FE methods can be used for design of slopes; however their use for back-analysis is limited because failure surface cannot be specified.

- Boundary conditions are important; for 3D FE and FD computer programs, boundary conditions are input explicitly whereas, in 3D LE computer programs, boundary conditions are implied and are not input in the data set.

- 3D LE procedures are more user friendly than 3D FD and FE procedures.
3D extensions of Morgenstern and Price’s (1965) and Spencer’s (1967) procedures satisfy all conditions of equilibrium so their use is preferred over Bishop’s (1955) and Janbu’s (1956) simplified procedures. Difficulties in selecting an appropriate 3D force function for the Morgenstern and Price’s (1965) procedure makes the 3D extension of Spencer’s (1967) procedure more desirable. However, for translational slides, 3D extension of Spencer’s (1967) procedure frequently has convergence problems so Janbu’s (1956) simplified procedure (with correction factor) is preferred.

3D extensions of Bishop’s (1955) and Janbu’s (1956) simplified procedures do not satisfy horizontal force equilibrium and moment equilibrium, respectively. However, these procedures do not have convergence problems and are viable alternatives to the 3D extension of Spencer (1967) procedure for rotational and translational slides, respectively.

3D extensions of Bishop (Hungr 1989), Morgenstern and Price (Hungr 2001), and Spencer (Hungr 2001) coded in CLARA-W provide comparable results with continuum method and are within 3% of each other.

3D extension of Janbu’ simplified procedure (Hungr 1989) coded in CLARA-W does not apply Janbu’s (1973) correction factor so it gives lower FS than other 3D extensions. Janbu’s (1973) correction factor for 2D may be applied to 3D FS values from 3D extensions of Janbu’s (1956) simplified procedure.

9.2.2 Field Failure Surface Geometry

The validity of any analysis depends on the degree to which the analysis can match the field failure mechanism and the accuracy to which the engineering properties of the materials involved and field geometry are modeled in the analysis. A common shortfall found in existing 3D modeling of landslides is their inability to model field failure surface geometries. Most slopes fail in two basic modes, rotational and translational. A number of field case histories were reviewed to determine the 3D failure surface shapes that best represent field failure surfaces for rotational and translational landslides. In
addition, 3D FE analyses of two slope models were performed to investigate the failure surfaces generated by FE analysis.

- Rotational slides occur most frequently in homogeneous material. The study of field failure surfaces of rotational slides and corresponding FE analyses indicates that the 3D failure surface in homogeneous material is ellipsoidal.

- The ellipsoidal aspect ratio for the eleven rotational failures considered in present study range between 0.8 to 2.67 and can be used as a guide for specifying a reasonable ellipsoid.

- Translational failures commonly occur when a stronger material is underlain by a weaker material(s). The model of translational landslides presented by Arellano and Stark (2000) reasonably approximates the field failure surface geometry and is in accordance with the failure surface generated by the FE model.

9.2.3 Comparison of 2D and 3D Factor of Safety

A number of researchers have compared 3D and 2D FS for different slide geometries and various strength combinations. For cohesive soils, there appears to be consensus that 3D FS is always greater than 2D FS. However, some researchers indicate possible situations where 3D FS could be lower than 2D FS (Hovland 1977, Chen and Chameau 1983, Thomaz and Lovell 1988, and Seed et al. 1990). The parametric analyses conducted herein using FE, FD and LE procedures were performed based on the assumption that materials along the vertical sides of the slide mass consist of cohesionless material. In all analyses, 3D FS values are greater than 2D FS, all else being equal, e.g., for the critical failure surface. These findings support the conclusion that all of the cases where 3D FS was found to be lower than 2D FS, appear to involve serious inaccuracies (Duncan 1996) or did not compare minimum 3D and 2D FS as highlighted by Cavouinidis (1987).

9.2.4 Recommended Use of 3D Slope Stability Analysis

Based on the results of this study, the following is recommended:
- Design of slopes should be performed using 2D analysis to maintain the current conservatism inherent in a 2D analysis.

- State and federal regulatory codes for design of MSW landfills should specify a “minimum 2D FS of 1.5” to achieve the desired level of stability for man-made slopes instead of just requiring a FS of 1.5 or greater.

- Back-analysis should be performed using 3D analysis so the back-calculated shear strength is not overestimated.

- The back-calculated shear strength from a 3D analysis using the field geometry and failure surface is more representative of the field and/or laboratory testing. Thus, the back-calculated shear strength from 3D analysis can be used for design of remedial measure for the failed slope.

9.2.5 Effects of Modeling and Interpolation Techniques on 3D FS

3D LE software uses linear interpolation between input cross-sections to generate a complete 3D surface (or mesh). Usually there is less variation in the material properties and water surfaces, therefore, linear interpolation between two adjacent known points on a profile line in adjacent cross-sections can provide a reasonable estimate of the geometry. In some cases, geometry of various surfaces (e.g., ground surface, failure surface, etc.) varies significantly in different cross-sections but the number of available data points may be less than desired to estimate the elevation of additional equally spaced points on the profile line in a cross-section. In cases of limited data, a DEM generated by gridding/mapping software can be used by the 3D procedure of columns (Hungr et al 1989, Hungr 2001). Gridding/mapping software provides more options for interpolation (like kriging, radial basis function, etc.) between cross-sections or data points to estimate the overall 3D geometry of a material type or geometry by considering trends in available data points. Use of DEM in 3D slope stability analysis allows use of GIS datasets from other disciplines like geology, hydrogeology and mapping for data analysis.
9.2.6 Effects of Number of Columns on 3D FS

The CLARA-W (Hungr 2001) user’s manual recommends the number of columns used should be greater than 1000 for reasonable accuracy in the 3D slope stability analysis; the default values for 3D slope geometry discretization is set at 2500 columns (50 columns and 50 rows). Similar suggestions were made regarding the importance of number of columns in 3D analysis by Lam and Fredlund (1993). However, in 2D limit equilibrium software, e.g., XSATBL (Sharma 1996), the user is not required to specify the width of slices, and slices are created by the software to account for changes in geometry. In the 3D procedure of columns analyses, all 3D columns have equal dimensions, and the user is required to decide mesh sizing, which can affect the 3D FS calculation. The default or recommended value for number of columns and rows or minimum number of active columns may not always provide sufficient accuracy. Therefore, user must check the results using as high number of column as possible. This study suggests that the number of columns and/or rows should exceed 30% of the slope model dimensions to ensure accurate results.

9.2.7 Effects of Side Shear Resistance on 3D FS

Prior research (Stark and Eid 1998) indicates that current 3D LE slope stability software does not incorporate shear resistance from the sides of the slide mass for translational slides with vertical sides. A parametric study was performed using 2D and 3D FE and FD procedures to investigate the magnitude of side resistance along vertical sides of a translational slide mass. Based on these results, the following two methods are recommended to incorporate the shear resistance along the two vertical sides that parallel the direction of movement in a 3D LE program:

- Apply an external horizontal and vertical force equivalent to the shear resistance derived using an earth pressure coefficient \( K_T \) that is in-between \( K_O \) and \( K_A \) values acting on the centroid of the sides parallel to the direction of movement of the slide mass.

- Assign slight outward inclination (3°-7°) to the sides of slide mass to ensure that some of the column bases at the ends of the slide pass
through the upper material, and the side surface is considered as part of the failure surface. This study makes recommendations about the side inclinations for different slope inclinations.

9.2.8 Initial Estimate of 3D FS

3D analyses are important in landslide causation analyses to accurately assess the relative effects, changes in FS due to slope changes, precipitation, toe excavation, fill placement, shear resistance, and remedial measures. It is preferable to perform an actual 3D slope stability analysis with site specific geometry, pore-water pressure conditions, and material properties to accurately calculate the side resistance. Using the parametric study and case histories, it is shown herein that importance of performing a 3D analysis can be determined by performing a 2D weighted average analysis or by using the 3D FS charts derived as follows:

- For ellipsoidal failure surfaces, an initial estimate of 3D effects may be obtained by a 2D weighted average analyses using the weight above the failure surface as the weighting factor. However, a minimum of three cross-sections for homogeneous deposits, and four cross-sections for layered deposits should be used.

- In a translational landslide with vertical sides, there is less variation in material properties in different cross-sections through the slope so the weighted average analyses essentially yields a FS that is similar to the 2D FS for the central cross-section. Thus, charts provided in this study should be used to determine the importance of performing a 3D slope stability analysis for a translational slide.

9.2.9 Improvements Made to Existing 3D LE Software

During the course of this study, it was found that some additions were required to improve the applicability and accuracy of existing 3D LE software. The following additions and/or improvements were incorporated in the 3DDEM-Slope software developed during this study:-

- Option for modeling stress dependent nature of soil shear strength.
• Janbu’s (1973) correction factor for 2D and 3D slope stability analysis results using Janbu’s (1956) simplified procedure.

• Options for calculating 3D FS with shear resistance for translational slide mass with vertical sides.

• Horn’s (1981) method to calculate slope of column bases using which is used in GIS software to better model the base angles of 3D columns.

• Option for calculation of 2D FS at a specified cross-section from DEM input.
9.3 Recommendations for Future Research

The present study makes following recommendations for future research:

- Include advanced interpolation techniques in the limit equilibrium slope stability program.

- Investigate effective and efficient use of FE and FD programs for back-analysis of failed slopes.

- Add FD and/or FD subroutine in LE program.

- Verify correction factor for 3D extension of Janbu’s (1956) simplified procedure with additional case histories.

- Analyze additional 3D case histories and document results.
APPENDIX A

DERIVATION OF COMMON 3D LE PROCEDURES

A.1 3D LE Slope Stability Procedures

A detailed literature review of available 3D LE procedures is presented in Chapter 2. Thus, in this appendix only specific information/derivation of representative 3D LE algorithms based on the procedure of columns that are commonly used in 3D computer software is presented for reference. Assumptions used in the various 3D slope stability analysis procedures and equilibrium conditions satisfied are presented in Table A.1.

A.2 Bishop’s (1955) Simplified Procedure in 3D

Hungr (1987) and Hungr et al. (1989) presents 3D extension of Bishop’s (1955) simplified procedure. The 3D procedure proposed by Hungr (1987) is a direct extension of Bishop’s (1955) simplified procedure to 3D because it uses the same assumptions as Bishop’s (1955) simplified procedure. The 3D extension presented in Hungr et al. (1989) includes the modification suggested by Fredlund and Krahn (1977), thus it is generally applicable to non-rotational surfaces. However, as it neglects internal strength thus it may yield conservative FS when used for some non-rotational and asymmetric surfaces (Hungr et al. 1989). Figure A.1(a) shows the forces acting on a single column when vertical interslice forces are neglected. The total normal force acting on the base of a column can be computed by:

\[ \sum F_V = 0 \]

\[ W = N_z + S_z W = N \cos \gamma_z + S_m \sin \alpha_y \]  

(A.1)
where $\alpha_y$ is base angle (same as $\alpha$ in 2D) and $\gamma_z$ is the local dip of the sliding surface. The failure criteria in terms of shear stress, $FS$, and shear strength expressed in effective stresses with the Mohr Coulomb strength equation is:

$$ S_m = \frac{\tau_{\text{max}}}{FS} = \frac{c'A + (N - uA)\tan\phi'}{FS} \quad (A.2) $$

where

$$ A = \Delta x \Delta y \left(1 - \sin^2\alpha_x \sin^2\alpha_y \right) \frac{\cos\alpha_x \cos\alpha_y}{\cos\alpha_y} $$

and

$$ \cos\gamma_z = \left(\frac{1}{\tan^2\alpha_x + \tan^2\alpha_y + 1}\right)^{1/2} $$

$\Delta x$ and $\Delta y$ are the column widths and length and $\alpha_x$ is the inclination of sliding surface in the direction of the x axis (transverse direction).

Substituting the failure criteria (Equation (A.2)) and solving for the normal force gives:

$$ N = \left[ W - \frac{c'A \sin\alpha_y}{FS} + \frac{uA \tan\phi' \sin\alpha_y}{FS} \right] / m_\alpha \quad (A.3) $$

where, $m_\alpha = \cos\gamma_z + (\sin\alpha_y \tan\phi')/FS$.

The $FS$ is derived from the summation of moments about a common point (center of rotation for the entire mass):

$$ \sum M_o = 0 \quad (A.4) $$

$$ \sum S_m R + \sum N f \cos\gamma_z / \cos\alpha_y = \sum W x + \sum kWe \pm + Ed $$

$$ \sum S_m R = \sum W x - \sum N f \cos\gamma_z / \cos\alpha_y $$

$$ + \sum kWe + Ed $$

In accordance with modification suggested by Fredlund and Krahn (1977), $R$, $x$, $f$, $e$, and $d$ are moment arms of the resisting force ($S_m$), column weight ($W$), normal force ($N$), horizontal earthquake force ($kW$), and resultant of horizontal component of applied point loads ($E$), respectively. Vertical load components are included in the column weights.

Introducing the failure criteria from Equation (A.2), normal force from
Equation (A.3), and solving for FS gives:

\[
FS = \frac{\sum [c'AR + (N - uA)R \tan \phi']}{\sum Wx - \sum N_f \cos \gamma_z \cos \alpha_y + \sum kW_e} \tag{A.5}
\]

A.3 Janbu’s (1956) Simplified Procedure in 3D

Janbu’s (1956) simplified procedure ignores vertical interslice forces and thus the normal force calculated from vertical force equilibrium is the same as the Bishop’s (1955) simplified procedure. Therefore, using same assumptions as those for 3D extension of Bishop’s (1955) simplified procedure, Hungr et al. (1989) also derives 3D FS from horizontal force equilibrium in the direction of motion (y-direction) as:

\[
FS_o = \frac{\sum [c'A \cos \alpha_y + (N - uA) \tan \phi' \cos \alpha_y]}{\sum N \cos \gamma_z \tan \alpha_y + \sum kW + E} \tag{A.6}
\]

Equation (A.6) is a 3D extension of Janbu’s (1956) simplified procedure without a correction factor.

A.4 Morgenstern and Price’s (1965) and Spencer’s (1967) Procedure in 3D by Hungr (2001)

Hungr (2001) presents an extension of Morgenstern and Price’s (1965) procedure to 3D. The extension uses an approach similar to that proposed by Lam and Fredlund (1993) and Hungr (1997), combined with an assumption that the resultant of the interslice force \((X_x, and Y_z)\) on the lateral column surface is parallel to the base of the column. The same iteration scheme as used for 3D extension of Bishop’s simplified procedure (Hungr 1987; Hungr et al. 1989) is carried out. In addition to forces on a vertical column shown in Figure A.1(a), both normal and shear force on the column faces (shown in Figure A.1(b)) are included in the analysis. Extension of Morgenstern and Price’s (1965) procedure to 3D is based on the following three assumptions:

- The relationship between vertical shear force, \(X_y\), and normal force, \(E_y\), is given by the interslice force function (Morgenstern and Price 1965) as follows:
\[ \lambda f(x) = \frac{X_y}{E_y} = \frac{X'_y}{E'_y} \]
\[ X_y - X'_y = (E_y - E'_y) \tan \theta_i \] (A.7)

where \( f(x) \) is the interslice force function and \( \theta_i \) = angle of the resultant interslice force from the horizontal.

- The resultant of the interslice force \((X_x \text{ and } Y_x)\) on the lateral column surface is parallel to the base of column:

\[ \frac{X_x}{Y_x} = \frac{X'_x}{Y'_x} = \tan \alpha_y \]
\[ \frac{X_x - X'_x}{Y_x - Y'_x} = \frac{\Delta X_x}{\Delta Y_x} \]
\[ X_x - X'_x = (Y_x - Y'_x) \tan \alpha_y \] (A.8)

- Horizontal shear stress between adjacent rows of columns is transmitted in proportion to the weight of all of the columns in a row. This results in a horizontal force which acts on each column and is transmitted from the adjacent lateral column. This horizontal force is equal to the column weight times a constant \( a_c \), and is similar to horizontal acceleration:

\[ (Y_x - Y'_x) = a_c W \] (A.9)

where \( a_c \) is constant for any given row of columns.

The normal force is derived from vertical and horizontal force equilibrium as shown below:

\[ \sum F_z = 0 \]
\[ W + (X_y - X'_y) + (X_x - X'_x) - N \cos \gamma_z - T \sin \alpha_y = 0 \] (A.10)

Substituting the Mohr-Coulomb failure criteria \((T = S_m \text{ in Equation (A.2)})\), interslice force relationship (Equation (A.7)), and interslice force function on the lateral column surface (Equation (A.8))
\[ W + (E_y - E'_y) \tan \theta_i + (Y_x - Y'_x) \tan \alpha_y - N \cos \gamma_z \]

\[ - \left[ \frac{c'A + (N - uA \tan \phi')}{FS} \right] \sin \alpha_y = 0 \]

\[ N = \frac{W + (E_y - E'_y) \tan \theta_i + (Y_x - Y'_x) \tan \alpha_y + \frac{uA \tan \phi' \sin \alpha_y}{FS} \left( \frac{c'A \sin \alpha_y}{FS} \right) }{m_\alpha} \]  
(A.11)

Similarly,

\[ \sum F_y = 0 \]
\[ aW + (E_y - E'_y) + (Y_x - Y'_x) + N \cos \gamma_y - T \cos \alpha_y = 0 \]  
(A.12)

Substituting the Mohr-Coulomb failure criteria (Equation (A.2)) in Equation (A.12)

\[ aW + (E_y - E'_y) + (Y_x - Y'_x) + N \cos \gamma_y - T \cos \alpha_y = 0 \]

\[ N = \frac{-aW - (E_y - E'_y) - (Y_x - Y'_x) + \frac{c'A \cos \alpha_y}{FS} - \frac{uA \tan \phi' \cos \alpha_y}{FS} }{m_\beta} \]  
(A.13)

\[ m_\alpha \] and \( m_\beta \) in Equation (A.11) and (A.13) are:

\[ m_\alpha = \cos \gamma_z + \frac{\tan \phi' \sin \alpha_y}{FS} \]  
(A.14)

\[ m_\beta = \cos \gamma_y - \frac{\tan \phi' \cos \alpha_y}{FS} \]  
(A.15)
Eliminating \( N \) by equating Equation (A.11) and (A.13):

\[
-aW - (E_y - E'_y) - (Y_x - Y'_x) + \frac{c'A \cos \alpha_y}{FS} - \frac{uA \tan \phi' \cos \alpha_y}{FS} - m_\beta \]

\[
W + (E_y - E'_y) \tan \phi_i + (Y_x - Y'_x) \tan \alpha_y + \frac{uA \tan \phi' \sin \alpha_y}{FS} - \frac{c'A \sin \alpha_y}{FS} - m_\alpha
\]

\[
WS_1 + (E_y - E'_y) S_2 + (Y_x - Y'_x) S_3 + (u \tan \phi' - c') \frac{A}{FS} S_4 = 0
\]

(A.16)

where

\[
S_1 = \frac{1}{m_\alpha} + \frac{a}{m_\beta} \quad (A.17)
\]

\[
S_2 = \frac{\tan \phi_i}{m_\alpha} + \frac{1}{m_\beta} \quad (A.18)
\]

\[
S_3 = \frac{\tan \alpha_y}{m_\alpha} + \frac{1}{m_\beta} \quad (A.19)
\]

\[
S_4 = \frac{\sin \alpha_y}{m_\alpha} + \frac{\cos \alpha_y}{m_\beta} \quad (A.20)
\]

Substituting value of \((Y_x - Y'_x) = a_cW \) (Equation (A.9)) in Equation (A.16):

\[
WS_1 + (E_y - E'_y) S_2 + a_cW S_3 + (u \tan \phi' - c') \frac{A}{FS} S_4 = 0
\]

(A.21)

For the \( n^{th} \) column in each longitudinal row, (taking into account that in the absence of toe submergence external force, \( E'_y \) is zero for the Column 1):

\[
E_n = \sum_{1}^{n} \left( W \frac{S_1}{S_2} + a_cW \frac{S_3}{S_2} + (u \tan \phi' - c') \frac{A}{FS} \frac{S_4}{S_2} \right) = 0
\]

(A.22)

Solving \( a_c \) for each row:
\[ \sum a_c W \frac{S_3}{S_2} = -\sum W \frac{S_1}{S_2} - \sum (u \tan \phi' - c') \frac{A S_4}{FS S_2} \]

\[ a_c = \frac{-\sum W \frac{S_1}{S_2} - \sum (u \tan \phi' - c') \frac{A S_4}{FS S_2}}{\sum W \frac{S_3}{S_2}} \]  

\[ \text{(A.23)} \]

To satisfy overall horizontal force equilibrium for the slide as a whole:

\[ \sum a_c W - \sum F_n = 0 \]  

\[ \text{(A.24)} \]

where \( \sum F_n \) is the sum of the horizontal forces and \( \sum a_c W \) is applied to all of the rows.

The vertical shear force, \( X_y \) can be calculated from interslice force function

\[ X_y = E_y f(x) \lambda \]  

\[ \text{(A.25)} \]

The calculations are performed as follows (Hungr 2001):

- Assume a value of \( \lambda \),
- Calculate value of \( a_c \) for each row of columns from Equation (A.23),
- Calculate value of normal force \( E_y \) from Equation (A.21),
- Calculate value of shear force \( X_y \) from Equation (A.25),
- Solve iteratively as for Bishop’s simplified procedure, adding the \( X_y \) force resultants to the weights of the columns, and
- Change value of \( \lambda \) iteratively to satisfy (A.24)

For a constant interslice force function, e.g., \( f(x) = 1 \), Morgenstern and Price’s (1965) procedure is the same as Spencer’s (1967) procedure, thus the same iteration scheme may be used for a 3D extension of Spencer’s (1967) procedure.

Procedure by Lam and Fredlund (1993) considers various static conditions, a number of available equations, and number of unknowns in these equations. For a sliding mass of \( n \) number of columns in the direction parallel to sliding and \( m \) number of rows in transverse direction the procedure of columns is indeterminate (Lam and Fredlund 1993). The number of unknown is \( 12 \times n \times m + 2 \), while the number of equations is \( 4 \times n \times m + 2 \), still requiring \( 8 \times n \times m \) assumptions. Figure A.2 shows a free body diagram of a vertical column before and after simplifying assumptions presented by Lam and Fredlund (1993). The assumptions are as follows:

- Point of application of normal force is assumed to be at the middle of the bottom of each column, which reduces the number of unknowns to \( 9 \times n \times m + 2 \).

- All intercolumn shear forces acting on various column faces have certain relationships with corresponding normal forces, i.e., intercolumn force function similar to Morgenstern and Price’s (1965) procedure. These intercolumn force functions are as follows:

\[
\begin{align*}
\frac{X}{E} &= \lambda_1 f(1) \quad (A.26) \\
\frac{H}{E} &= \lambda_2 f(2) \quad (A.27) \\
\frac{V}{Q} &= \lambda_3 f(3) \quad (A.28) \\
\frac{Q}{P} &= \lambda_4 f(4) \quad (A.29) \\
\frac{T}{N} &= \lambda_5 f(5) \quad (A.30)
\end{align*}
\]

The intercolumn shear forces \( X, H, V, Q \) and \( T \) can be calculated for known normal forces. The number of unknowns is reduced by \( 5 \times n \times m \) but five new unknowns, \( \lambda_1, \lambda_2, \lambda_3, \lambda_4, \) and \( \lambda_5 \) are added resulting in \( 4 \times n \times m + 7 \) unknowns.

- Using a finite element program ANSYS, for different slope problems, Lam and Fredlund (1993) report that \( \lambda_2, \lambda_4, \) and \( \lambda_5 \) have a negligible

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effect on 3D FS so they may be assumed equal to zero. The number of unknowns is then reduced to $4 \times n \times m + 4$.

- To determine $\lambda_1$ and $\lambda_3$ an iterative procedure is used by assuming: (a) static equilibrium of complete mass will be satisfied when FS with respect to moment equilibrium $F_{S_m}$, is equal to FS with respect to force equilibrium $F_{S_f}$, and (b) $\lambda_1$ and $\lambda_3$ should yield the lowest FS.

With all of these assumptions, the number of unknowns is reduced to $4 \times m \times n + 2$, and the problem becomes determinate. Figure A.2(b) shows the forces acting on a single column after applying the above assumptions.

The normal force is derived from vertical and horizontal force equilibrium as shown below:

$$\sum F_v = 0$$

$$W - (X_L - X_R) - (V_L - V_R) - N \cos \theta_y - S_m \sin \alpha_x = 0 \quad (A.31)$$

Substituting Mohr-Coulomb failure criteria (Equation (A.2)):

$$W - (X_L - X_R) - (V_L - V_R) - N \cos \theta_y - \frac{c' A + (N - U) \tan \phi'}{FS} \sin \alpha_x = 0$$

$$N = \frac{W - (X_L - X_R) - (V_L - V_R) - \frac{c' A \sin \alpha_x}{FS} + \frac{u A \tan \phi' \sin \alpha_x}{FS}}{m_\alpha} \quad (A.32)$$

where

$$\left( \cos \theta_y + \frac{\tan \phi' \sin \alpha_x}{FS} \right) = m_\alpha$$

FS from moment equilibrium ($F_{S_m}$):

$$\sum M_o = 0$$

$$S_m \cos \alpha_x d_y + S_m \sin \alpha_x d_x = W d_x + N \cos \theta_x d_y - N \cos \theta_y d_x$$

$$S_m = \frac{N \cos \theta_x d_y - N \cos \theta_y d_x + W d_x}{\cos \alpha_x d_y + \sin \alpha_x d_x} \quad (A.33)$$

Substituting value of $S_m$ from Equation (A.33) in Mohr-Coulomb failure criteria (Equation (A.2)):
\[
FS_m = \frac{Ac' + N \tan \phi' - uA \tan \phi'}{N \cos \theta_x d_y - N \cos \theta_y d_x + W d_x} \\
\times \frac{\cos \alpha_x d_y + \sin \alpha_x d_x}{N \cos \theta_x d_y - N \cos \theta_y d_x + W d_x}
\]  
(A.34)

FS from force equilibrium (\(FS_f\)):

\[
\sum F_H = 0 \\
S_m \cos \alpha_x = N \cos \theta_x \\
S_m = \frac{N \cos \theta_x}{\cos \alpha_x}
\]  
(A.35)

Again substituting value of \(S_m\) from Equation (A.33) in Mohr-Coulomb failure criteria (Equation (A.2)):

\[
F_f = \frac{Ac' + N \tan \phi' - uA \tan \phi'}{N \cos \theta_x} \\
= \frac{\sum (Ac' + N \tan \phi' - uA \tan \phi') \cos \alpha_x}{\sum N \cos \theta_x}
\]  
(A.36)

This procedure makes reasonable assumptions to the problem statically determinate. However, \(\lambda_1\), and \(\lambda_3\) are still two unknowns that need to be assumed. In addition, this procedure needs to solve a large number of non-linear equations and thus convergence issues limit practical application of the procedure (Chen et al. 2006).

A.6 Procedure by Huang and Tsai (2000)

Huang and Tsai (2000) define a series of FS according to Mohr-Coulomb shear strength criteria as shown in Figure A.3. FS for each column, \(FS_{s,i}\) is defined as:

\[
FS_{s,i} = \frac{T_{f,i}}{T_i} = \frac{c_i A_i + N_i' \tan \phi_i'}{T_i}
\]  
(A.37)
where shear resistance $T_i$ consists of two parts, i.e., $T_{x,y}$ parallel to the the xy-plane and $T_{y,z}$ parallel to yz-plane. Two FS are defined as $F_{sx}$ (along x axis) and $F_{sz}$ (along z axis) as follows:

$$F_{sx} = \frac{T_{f,i}}{T_{x,y}}$$  \hspace{1cm} (A.38)

$$F_{sz} = \frac{T_{f,i}}{T_{y,z}}$$  \hspace{1cm} (A.39)

From the force diagram shown in Figure A.3:

$$F_{sx} = \frac{\sin(\theta_i - \alpha_i)}{\sin \alpha_i}$$  \hspace{1cm} (A.40)

$$F_{s,i} = \frac{F_{sx} \sin(\theta_i - \alpha_i)}{\sin \theta_i}$$  \hspace{1cm} (A.41)

$$F_{s,i} = \frac{F_{sx} \sin \alpha_i}{\sin \theta_i}$$  \hspace{1cm} (A.42)

The FS for the complete slope is defined as $F_s$:

$$F_s = \frac{\sum T_{f,i}}{\sum T_i} = \frac{\sum c_i' A_i + N_i' \tan \phi_i'}{\sum T_i}$$  \hspace{1cm} (A.43)

Huang and Tsai (2000) assume $F_{sx}$ and $F_{sz}$ are equal but $F_{s,i}$ of each column is calculated with $\theta_i$ and $\alpha_i$. This procedure considers force equilibrium in y direction only and ignores shear forces of all columns in the same axis. Because this procedure does not satisfy the force equilibrium for the entire slope in x direction, the moment equation of the complete slide mass is related to the position of the moment axis. Thus, this procedure is not considered a rigorous procedure (Chen et al. 2006).

### A.7 Procedure by Chen et al. (2003)

Chen et al. (2003) present a procedure that uses the conventional definition of FS that reduces the available shear strength parameters to bring the slope
to a limiting state as follows:

\[ c_e' = \frac{c'_e}{FS} \]  
(A.44)

\[ \tan_e' = \frac{\tan'_{FS}}{FS} \]  
(A.45)

where subscript \( e \) denotes the variables that are determined on the basis of reduced shear strength parameters, \( c'_e \) and \( \phi'_e \).

The authors make the following assumptions with regards to force and moment equilibrium (see Figure A.4):

- The horizontal shear force, \( H \), on the row interfaces (see ABFE and DCGH in Figure A.4(a)) are neglected, i.e., the inter column force with inclination of \( \beta \) to the x-axis (direction of sliding) and designated \( G \), are parallel to the xy plane, and \( \beta \) is constant for all columns.

- Shear forces \( P \) and \( V \) on the column interfaces (ADHE, and BCGF) are neglected.

- The shear force \( T \) applied to any column base is inclined at an angle of \( \rho \) measured from the xy plane to the positive z-axis (transverse to sliding).

The normal force is derived by projecting all of the forces to a column in the \( S' \) direction which is perpendicular to the inter column force \( G \) as shown in Figure A.4(b).

\[ N_i = \frac{W_i \cos \beta + (uA_i \tan \phi'_e - c'_e A_i)(-mx \sin \beta + my \cos \beta)}{-nx \sin \beta + ny \cos \beta + \tan \phi'_e (-mx \sin \beta + my \cos \beta)} \]  
(A.46)

where \( m_x \), \( m_y \),and \( m_z \) are direction cosines of the shear force \( T \), and \( n_x \), \( n_y \),and \( n_z \) are direction cosines of the normal to the column base.

Establishing the force equilibrium in the y and z directions and moment equilibrium around z axis yields:

\[ S = \sum [N_i(n_x \cos \beta + n_y \sin \beta) + T_i(m_x \cos \beta + m_y \sin \beta) - W_i \sin \beta] = 0 \]  \hspace{1cm} (A.47)

\[ Z = \sum (N_i.n_z + T_i.m_z) = 0 \]  \hspace{1cm} (A.48)

\[ M = \sum [-W_i.x - N_i.n_x.y + N_i.n_y.x - T_i.m_x.y + T_i.m_y.x] = 0 \]  \hspace{1cm} (A.49)

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The above equations involve three unknowns, i.e., $F$, $\beta$, and $\rho$, which are solved by the Newton-Raphson method. Chen et al. (2003) claim that their procedure satisfies overall force equilibrium and the moment equilibrium about the main axis of rotation. A comparison of results obtained by their procedure with other 3D procedures (for example Hungr et al. 1989, Lam and Fredlund 1993, Huang and Tsai 2000 and Chen et al 2000) is also presented by the authors. The FS obtained by their procedure that satisfies “complete overall force equilibrium conditions and moment equilibrium” is about 2% (1-2.3%) higher from 3D extensions of Morgenstern and Price’s (1965) and Spencer’s (1967) procedures presented by Lam and Fredlund (1993) and Hungr (2001).
### Table A.1: Assumptions and equilibrium conditions satisfied in various 3D LE procedures

<table>
<thead>
<tr>
<th>Reference</th>
<th>Procedure</th>
<th>Column base in x-axis</th>
<th>Assumptions Regarding Internal Shear Forces</th>
<th>Force Equilibrium</th>
<th>Moment Equilibrium</th>
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<td>Bishop</td>
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<tr>
<td>Hungr et al. (1989)</td>
<td>Janbu</td>
<td>Neglected</td>
<td>Neglected Neglected Neglected Neglected</td>
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<td>Yes Yes No No No</td>
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<td>Hungr (2001)</td>
<td>M &amp; P</td>
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<td>Neglected Included Neglected Included</td>
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\(^1\) Implemneted as lateral force balance
A.9 Figures

Figure A.1: Forces acting on a single column in a 3D sliding surface:
(a) column without vertical inter-column shear forces (Hungr et al. 1989),
(b) normal and shear forces on the column faces (Hungr 2001).
Figure A.2: Free body diagram of a column (Lam and Fredlund 1993): (a) before using simplifying assumptions, (b) after using simplifying assumptions for movement in the x direction.

Figure A.3: Shear resistance on bottom of a column (Huang and Tsai 2000).
Figure A.4: Free body diagram of a column (Chen et al. 2003): (a) assumptions for column forces, (b) projection of all forces to a column in $S'$ direction.
APPENDIX B

APPLICATION EXAMPLE

UNIVERSITY OF ILLINOIS                 Dr. T.D. Stark
Urbana-Champaign              Civil Engineering Department

CEE 581 – Earth Dams and Related Problems
Assignment No. 9 – Three Dimensional Slope Stability Analyses

Generate a three dimensional (3D) slope model and failure surface using an appropriate interpolation method from the borehole / inclinometer data shown in Table 1.1 and 1.2. Calculate three dimensional factor of safety (FS_{3D}) for the failure surface shown in Figure 1. The analysis should use the following three steps: (1) generate Digital Elevation Models (DEM) of the material layers, piezometric surface, and failure surface, (2) using the DEM of the various surfaces, calculate FS_{3D}, and (3) compare FS_{3D} with two dimensional factor of safety (FS_{2D}).

Use SURFER™ to generate a Digital Elevation Model (DEM) of the 3D surfaces. After verifying the failure surface, calculate FS_{3D} using the 3D extension of Bishop’s (1955) simplified, Janbu’s (1956) simplified, and Spencer’s (1967) procedures proposed by Hungr (1989, 2001) in the computer program 3DDEM-Slope. The 2D slope geometry at mid-cross section and the 3D failure surface are shown in Figure 1. Figure 2 shows the location of thirty three boreholes / inclinometers used to locate the failure surface on one side of the axis of symmetry and four boreholes to define the subsurface layers. Assume no variation in the different cross-section for the pre-failure geometry of slope. Use thirteen cross-sections to extend the model in Figure 1 and 2 to 3D.

Material properties for the different layers are given in the Table. 1.3. The procedure for generating DEM and use of 3DDEM-Slope is given in the pdf file titled “instructions.pdf”.

Bonus points will be given for comparison of results with different gridding methods and interpretation of the results.

Note: 3DDEM-Slope code is under development and may have some “bugs” so please be patient. Your suggestions for improving the software will be greatly appreciated. If you have any questions about the assignment, please contact Kamran Akhtar (2206 NCEL, kakhtar2@illinois.edu).

Download links

References
Hungr, O. (2001), CLARA-W, Slope Stability Analysis in Two or Three Dimensions for Microcomputers, O. Hungr Geotechnical Research Inc, West Vancouver B.C., Canada
Table 1.1: Data points for input of pre-failure slope geometry

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<th>Weak layer</th>
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<th>Ground</th>
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Table 1.2: Data points for input of slope failure geometry

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Note: Borehole/inclinometer data points are shown in bold. Other points are from surface survey.
Table 1.3: Material properties for stability analysis

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<th>Material</th>
<th>Unit Weight (pcf)</th>
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</tbody>
</table>

Figure 1. Slope geometry of problem, (a) 2D slope geometry, (b) 3D failure surface

Figure 2. Location of boreholes/inclinometers
REFERENCES


thesis, University of Illinois at Urbana-Champaign.


Varnes, D. J. (1978). “Slope movement types and processes.” Landslides:


ronmental Engineering, University of California, Berkeley.