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L162
AN OPTIMUM CONDITION OF ACQUIRING A COMPUTER IN THIRD-PARTY LEASE

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An Optimum Condition of Acquiring a Computer in Third-Party Lease

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Abstract

The third party lease is an economic way to acquire a computer if the organization's needs for computer time is dynamic and the computer is expected to satisfy the needs for several future years. In this case, the ability of the organization to estimate the daily computer time requirement is essential for making an optimum lease contract. This paper proposes a methodology to measure the efficiency of a lease contract in terms of the equivalent annual cost amortizing the sum of the discounted present values of lease expenses incurred over the period of actual use. The methodology is based on two assumptions: one is the ability to estimate the probability of replacing the computer in each future year due to the excess computing requirement over the CPU capacity and the other the inclusion of a schedule of penalties for premature cancellations in the lease contract.
Introduction

A great number of organizations are currently using computers acquired under leasing arrangements. One of the important questions they have to answer in making a lease contract is "what should be the lease period?" Because of intangibles and uncertainties involved, the question will never have a perfect answer. However, this paper discusses one feasible approach to answer the above question with regard to the third-party lease.

The acquisition of a computer may be made by one of the three methods: the manufacturer's rental, the purchase, and the lease. The selection of a particular one out of the three methods of acquisition primarily is an economic decision based on expected computing requirements and the possible obsolescence of current equipment. Obsolescence of an existing computer is caused by the appearance of technologically superior or more economic computers and usually difficult for the regular user to predict. On the other hand, the future requirement for computing capacity may be estimated by the user on the basis of the amount of jobs run by each application program expected to be in use in future years.

Of the three methods, the manufacturer's rental is the costliest in terms of the average annual cost. But it gives the user advantages such as no obligation to pay insurance expenses and property taxes, a minimum financial commitment, and a maximum flexibility when the existing computer must be replaced by a new one because of obsolescence or capacity shortage. Maintenance is included in the basic monthly rental charge, but the user is usually required to pay an additional charge for extra shifts or overtime use. Generally, the termination of a rental contract requires a written notice of a minimum of 90 days.
If the manufacturer's rental is the best in terms of freedom and flexibility available to the user, the purchase is the worst. The purchase requires the purchaser to pay the full price of the computer upon delivery and insurance expenses and property taxes each year, and to make an arrangement for maintenance service. It may cause a great loss to the purchaser if the computer must be replaced within a few years of acquisition.

The lease is a compromise between the manufacturer's rental and the purchase, and a popular arrangement for organizations that prefer to retain flexibility under dynamic requirements for computing capacity or, more specifically, central processing unit (CPU) capacity. A lease contract belongs to one of the following three types:

1. the manufacturer's lease
2. the long-term payout lease
3. the short-term non-payout lease

The manufacturer's lease is a short-term lease contract between the maker of the computer and its user. For example, IBM's plan for leasing a CPU is called a term lease plan (TLP) and written for a four-year period. It charges the user the same monthly rate as that of the rental plan, provides for a change in the monthly rate once a year, charges no extra shift charge for maintenance, and provides purchase options and contract extensions. The manufacturer pays all risk insurance and property taxes on the equipment.

The short-term non-payout lease and the long-term payout lease are third-party lease plans. In these plans, the leasing company purchases a computer from its manufacturer and leases it to the user. In general, the lease contract provides clauses for the lease period, monthly lease payments, renewal lease rates, and penalties on premature termination.
Further, the contract clauses may cover a maintenance arrangement, depreciation, investment tax credit, property taxes and insurance, and purchase options available at the end of the contract period. However, there is a wide latitude for negotiation between two contracting parties.

In the long-term payout lease, the lessee or the user is normally responsible for all risk insurance, property taxes, and maintenance. Under the present tax law, the lessor is allowed to transfer to the lessee the investment tax credit and depreciation declared on the equipment. The long-term lease may not state a specific payment as a penalty for premature termination, but it usually binds the lessee for the lessor's full recovery of the original price and cost of money required for the purchase of the equipment. At the end of a lease term, the lessee is usually given a purchase option or a lease renewal.

The short-term non-payout lease requires a minimum commitment of two years, but it may have a contract period as long as 10 years. Its monthly payments are usually 10 percent to 30 percent less than the manufacturer's rental price. Penalty payments for a premature termination of the lease contract are specifically stated and applicable for each month that the system is in use according to the terms of the contract before the termination. The lessor is almost always responsible for risk insurance expenses and property taxes on the equipment, normally declaring its investment tax credit and depreciation. Maintenance is normally performed by the lessor without extra shift charge.

**Future Computing Requirements**

Given a mix of jobs to be processed, the capacity of a computer system depends on both the CPU and peripheral equipment in the system. As requirements increase, the capacities of the peripheral equipment
may be enhanced maintaining an optimum balance with the CPU. Eventually, however, the CPU becomes the bottleneck of the system and needs to be replaced by a bigger one if the system is to have a single CPU.

Knowledge on future requirements for CPU capacity is essential to the user for making an optimum lease contract that requires a minimum annual cost. But normally the user is uncertain about such requirements. In these circumstances, it is suggested that the user estimate two items for each year of the future period over which a given CPU may be used.

First, the user must estimate the maximum possible number of CPU hours available for productive use. It may be determined by the achievable utilization rate of the CPU over an extended period based on the past experience and the expected number of daily shifts used in the future year. Let us call the maximum available CPU time the operational capacity.

Second, the user must estimate for each future year the probability that the expected daily CPU requirement exceeds the operational capacity estimated above and therefore the CPU needs to be replaced by a larger one. This probability denoted by \( g_i \) for year \( i \), is the cumulative probability reflecting the effects of increasing CPU requirements from the present through the future year. The incrementary annual contribution to the cumulative probability is given by the following \( P_i \) for year \( i \):

\[
P_i = g_i - g_{i-1} \quad i = 1, \ldots, n
\]

where \( g_0 \) is assumed to be zero.

**Expected Lease Cost**

The lease contract analyzed here is of the short-term non-payout type that specifies a schedule of penalties for premature cancellation.
The lease cost will be determined by taking into consideration the lease period, annual lease payment, penalty for premature cancellation of the lease, probability of cancellation, and discount rate. Since variable lease periods are considered, their merits are compared in terms of the expected equivalent annual cost, the annual cost amortizing the sum of the discounted present values of the expected lease expenses over the period.

The expected equivalent annual lease cost is formulated with the following symbols:

- \( n \) = maximum possible lease period in years.
- \( m \) = lease period in years, \( m = 2, \ldots, n \).
- \( s \) = year at the end of which the lease is terminated.
- \( a_{mi} \) = annual lease expense in the \( i \)th year under a \( m \)-year contract.
- \( b_{mi} \) = annual lease extension expense in the \( i \)th year for a \( m \)-year contract.
- \( c_{ms} \) = penalty payment when a \( m \)-year contract is prematurely cancelled at the end of the \( s \)th year.
- \( d \) = annual maintenance cost.
- \( r \) = discount rate in fraction.
- \( t \) = income tax rate in fraction.
- \( f \) = amortization factor to distribute the present cost evenly over \( s \) years, given by
  \[
  f_s = \frac{r(1+r)^s}{(1+r)^s-1}
  \]
- \( p_i \) = probability with which the CPU needs to be replaced in the \( i \)th year.
Following the usual convention, all expenses are assumed to be paid at the end of each year. Depending on whether the lease contract is terminated prematurely on the stipulated date or extended beyond the date, the expected equivalent annual cost of the discounted present value of lease expenses, net of cash saving due to income tax, is given by the following $E_1$, $E_2$, and $E_3$:

(1) Termination before the stipulated date ($s < m$)

$$
E_1 = \sum_{s=1}^{m-1} \frac{c_{s-1} + c_s}{(1+r)^s} \left( \frac{\sum_{i=1}^{s} \frac{a_i + d_i}{(1+r)^i}}{s} + \frac{c_{s+1}}{(1+r)^{s+1}} \right) \quad s = 1, \ldots, m-1
$$

(2) Termination on the stipulated date ($s = m$)

$$
E_2 = \frac{m}{m} \left(1-t\right) \sum_{i=1}^{m} \frac{c_{m-1} + c_m}{(1+r)^i} \left( \frac{\sum_{i=1}^{m} \frac{a_i + d_i}{(1+r)^i}}{m} + \frac{b_m}{(1+r)^m} \right)
$$

(3) Termination after the stipulated date ($s > m$)

$$
E_3 = \sum_{s=m+1}^{n} \frac{c_{s-1} + c_s}{(1+r)^s} \left( \frac{\sum_{i=1}^{s} \frac{a_i + d_i}{(1+r)^i}}{s} + \frac{c_{s+1}}{(1+r)^{s+1}} \right)
$$

The expected equivalent annual lease cost for a $m$-year contract, $F_m$, is the sum of $E_1$, $E_2$, and $E_3$ weighted by the annual probability $P_s$:

$$
F_m = \sum_{s=1}^{m-1} P_s \frac{m-1}{s} \left(1-t\right) \left\{ \frac{\sum_{i=1}^{s} \frac{a_i + d_i}{(1+r)^i}}{s} + \frac{c_{s+1}}{(1+r)^{s+1}} \right\} \\
+ \frac{m}{m} \left(1-t\right) \left( \frac{\sum_{i=1}^{m} \frac{a_i + d_i}{(1+r)^i}}{m} + \frac{b_m}{(1+r)^m} \right)
$$

$$
+ \frac{n}{n} \left(1-t\right) \left( \frac{\sum_{i=1}^{n} \frac{a_i + d_i}{(1+r)^i}}{n} + \frac{b_n}{(1+r)^n} \right)
$$

$$
= \sum_{s=m+1}^{n} P_s \frac{n}{s} \left(1-t\right) \left\{ \frac{\sum_{i=1}^{s} \frac{a_i + d_i}{(1+r)^i}}{s} + \frac{c_{s+1}}{(1+r)^{s+1}} \right\} \\
= \sum_{s=m+1}^{n} \frac{n}{s} \left(1-t\right) \left( \frac{\sum_{i=1}^{s} \frac{a_i + d_i}{(1+r)^i}}{s} + \frac{c_{s+1}}{(1+r)^{s+1}} \right)
$$
The expected annual lease cost $F_m$ in (5) is computed with alternative lengths of the lease period $m$, and the length that gives a minimum value of $F_m$ is selected. This methodology of determining the lease period is illustrated through a numerical example.

**An Example**

We consider to lease a particular CPU from a third party leaser. The leaser will pay insurance and property taxes and claim investment tax credit on the computer. The conditions of lease regarding monthly lease payments, penalty payments in lease cancellation, and lease payments in case of a lease extension vary with the lease period and the length of actual use. These conditions are given in Table 1.

To determine future CPU requirements, we have estimated the probability of daily need for CPU time by one hour interval over the range of CPU hours expected to be used. As an illustration, the estimated probabilities for the third year are listed in the second column of Table 2. If we assume a 24-hour operation and 85% as the expected maximum use of the CPU capacity, the expected operational capacity of the CPU is approximately 20 hours a day.

From the third column of Table 2 showing the cumulative probability of daily CPU requirement through each of the time intervals, we have found 0.15 to be the probability that the daily requirement exceeds 20 hours in the third year. Similarly, we have obtained the same probability for each future year, as is listed in Table 3. These probabilities represent the cumulative probability, $e_1$, previously introduced. From then, we have determined $P_1$, the annual incremental contribution through equation (1).

Using the conditions in Table 1 and the probabilities in Table 3, we have computed $F_m$, the expected equivalent annual cost of leasing the CPU.
for each of the alternative numbers of years, \( m (m = 2, \ldots, 9) \), as is shown in the righthand column of Table 3. The result indicates that the lease period of 5 years gives a minimum annual cost of $24,807, indicating that it is an optimum lease period. Further, to compare the cost of each of other lease periods with the minimum cost, we have computed the excess cost as a percentage of the minimum cost.

**Conclusion**

When an organization acquires a CPU under a third-party lease it should have some idea about its daily requirement for the CPU in the future. By estimating various daily requirements and assigning probabilities to them, it is possible to determine the expected equivalent annual cost of leasing the CPU for each of the possible lease periods. Then, it is possible to make a rational decision on the lease period that minimizes the annual cost. This paper has presented a methodology of performing such computations and applied the methodology to a numerical example. Although conditions assumed for the example are fictitious, they are not totally unrealistic. The substantial excess cost required for a contract with a non-optimum lease period over the cost of an optimum period may justify a systematic approach such as suggested in this paper to be used in determining the lease period.
Bibliography


Table 1. Conditions of Lease Contracts
(in thousand dollars)

<table>
<thead>
<tr>
<th>Lease Period</th>
<th>Annual Lease Payment</th>
<th>Annual Lease Extension Payment</th>
<th>Total Penalty Payment for Lease Cancellation after s years, C_{ms}</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>a_{mi}</td>
<td>b_{mi}</td>
<td>s=1</td>
</tr>
<tr>
<td>2 yrs.</td>
<td>$500</td>
<td>$260</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>370</td>
<td>230</td>
<td>657</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
<td>210</td>
<td>810</td>
</tr>
<tr>
<td>5</td>
<td>260</td>
<td>190</td>
<td>936</td>
</tr>
<tr>
<td>6</td>
<td>240</td>
<td>170</td>
<td>1050</td>
</tr>
<tr>
<td>7</td>
<td>215</td>
<td>154</td>
<td>1160</td>
</tr>
<tr>
<td>8</td>
<td>200</td>
<td>138</td>
<td>1260</td>
</tr>
<tr>
<td>9</td>
<td>190</td>
<td></td>
<td>1360</td>
</tr>
</tbody>
</table>

Discount rate r=15%, Annual maintenance cost=$25,000, Income tax rate=48%
Table 2. Estimated Daily Requirement of CPU Time in the Third Year

<table>
<thead>
<tr>
<th>CPU Time Requirement in One Hour Interval</th>
<th>Probability of Having This Requirement</th>
<th>Cumulative Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.1-16.0 hours</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>16.1-17.0</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>17.1-18.0</td>
<td>0.27</td>
<td>0.35</td>
</tr>
<tr>
<td>18.1-19.0</td>
<td>0.30</td>
<td>0.65</td>
</tr>
<tr>
<td>19.1-20.0</td>
<td>0.20</td>
<td>0.85</td>
</tr>
<tr>
<td>20.1-21.0</td>
<td>0.10</td>
<td>0.95</td>
</tr>
<tr>
<td>21.1-22.0</td>
<td>0.05</td>
<td>1.00</td>
</tr>
<tr>
<td>22.1-23.0</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>23.0-24.0</td>
<td>0.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Table 3. Annual Replacement Probabilities and Expected Equivalent Annual Lease Costs

<table>
<thead>
<tr>
<th>Year</th>
<th>Cumulative Probability of Replacement $g_i$</th>
<th>Annual Contribution to Replacement Probability $p_i$</th>
<th>Lease Period</th>
<th>Annual Lease Cost $F_m$</th>
<th>Comparison of $F_m$ with Minimum Cost $(F_m - F_5)/F_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>0.05</td>
<td>2 years</td>
<td>$55,413</td>
<td>+123%</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
<td>0.10</td>
<td>3</td>
<td>38,837</td>
<td>+57</td>
</tr>
<tr>
<td>4</td>
<td>0.35</td>
<td>0.20</td>
<td>4</td>
<td>27,474</td>
<td>+11</td>
</tr>
<tr>
<td>5</td>
<td>0.65</td>
<td>0.30</td>
<td>5</td>
<td>24,807*</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0.80</td>
<td>0.15</td>
<td>6</td>
<td>30,359</td>
<td>+22</td>
</tr>
<tr>
<td>7</td>
<td>0.90</td>
<td>0.10</td>
<td>7</td>
<td>34,472</td>
<td>+39</td>
</tr>
<tr>
<td>8</td>
<td>0.97</td>
<td>0.07</td>
<td>8</td>
<td>38,644</td>
<td>+56</td>
</tr>
<tr>
<td>9</td>
<td>1.00</td>
<td>0.03</td>
<td>9</td>
<td>42,505</td>
<td>+71</td>
</tr>
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</table>

*This is the minimum cost.*