Inflation and Capital Asset Pricing Determination: A Theoretical and Empirical Investigation

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ABSTRACT

Based upon responsive coefficient concepts, this study has theoretically re-examined the impacts of inflation on capital asset pricing. The relationship between the real parameters of CAPM and the nominal parameters of CAPM is derived. A multi-index model with the change in purchasing power as an additional variable is derived to test whether the real parameters are significantly different from the nominal parameters.

Using the data of 464 securities to test the model derived in this paper, it is found that the estimated real parameters of CAPM are generally not significantly different from the estimated nominal parameters if the real risk free rates are assumed to be nonstochastic. However, the multi-index model derived in this paper does show the importance of inflation variable in estimating the related parameters in capital asset pricing determining process.
I. Introduction

Kennedy (1960), Brian (1969), Johnson, Reilly and Smith (1971), Hendershott and Van Horne (1973), Ondet (1973), Reilly, Smith and Johnson (1975) and others have shown that inflation significantly influences common stock values and bond rates. However, the capital asset pricing model (CAPM) developed by Sharpe (1964), Linter (1965) and Mossin (1966) (SML) does not take into account the impacts of inflation; and therefore, the parameters of SML type CAPM are generally estimated and interpreted in terms of the nominal rates of return without further justification.

Most recently, Roll (1973), Merton (1973), Long (1974), Chen and Bones (1975) and Hagerman and Kim (1976) (HK) have studied the possible impacts of inflation on the capital asset pricing model. However, they have not explicitly taken into account the degree of inflation hedge associated with the individual company (or industry). In testing the multi-period, two-parameter model, Fama and Macbeth (1974) (FM) have found that beta coefficients of the CAPM estimated by observed real rates of return are not statistically significantly different from those estimated by nominal rates. However, they provide no explanation of their empirical results. The main purposes of this study are to derive a CAPM that allows us to estimate all the parameters associated with the impacts of inflation on the capital asset pricing model; and to analyze the possible differences between the nominal and the real systematic risk and Jensen's measure of performance. Results of this study also are compared with those obtained by Roll (1973) and Hagerman and Kim (1976).
In the second section the CAPM with price level changes developed by Roll and HK is reviewed and criticized by using the concept of rates of return without inflation. In the third section, the relationship between real parameters of CAPM and nominal parameters of CAPM is investigated in accordance with the multivariate normal assumption. In the fourth section, a generalized CAPM is derived to allow the change of purchasing power to be an additional explanatory variable in the capital asset pricing model. Impacts of inflation on capital asset pricing also is analyzed in some detail. In the fifth section, data of 464 securities selected from NYSE are employed to test the impacts of inflation on both the estimated systematic risk and the estimated Jensen performance measure. The same data also are used to test the importance of including the change in purchasing power as an additional explanatory variable in capital asset pricing. Both the nominal and real Jensen measure, systematic risk, total risk and nonsystematic risk are also used to test the bias of the Jensen measure. Finally, results of this paper are summarized.

II. Real CAPM and Its Implications

To derive the CAPM in terms of real rates of return, Roll and HK have assumed that (i) all investors are risk averse single-period maxmizers of expected utility of real terminal wealth and have quadratic utility functions; (ii) all investors act in terms of identical joint probability distributions of end-of-period outcomes; (iii) markets are perfect; (iv) there are no transaction costs and (v) borrowing and lending is risk free in nominal but not in real terms.
To review and criticize the real CAPM development by Roll and HK, three alternative rates of return concepts, i.e., rates of return without inflation, nominal rates of return, and observed real rates of return are used to explain the capital asset pricing process. The relationship among these three alternative rates of return is defined as

\[
\begin{align*}
\text{(a)} & \quad R_{j}^{O} = R_{j}^{e} - 1 \\
\text{(b)} & \quad R_{m}^{O} = R_{m}^{e} - 1 \\
\text{(c)} & \quad R_{f}^{O} = R_{f}^{e} - 1
\end{align*}
\]

where \( R_{j}^{O} \) = 1 + observed real rates of return on the \( j^{th} \) asset.
\( R_{j}^{T} \) = 1 + rates of return on \( j^{th} \) asset without inflation.
\( R_{m}^{O} \) = 1 + observed real market rates of return.
\( R_{m}^{T} \) = 1 + market rates of return without inflation.
\( R_{f}^{O} \) = 1 + observed real rates of return on investor borrowing or lending.
\( R_{f}^{T} \) = 1 + rates of return on investor borrowing and lending without inflation.
\( R_{I} \) = one plus the change of general price level.

\( e_{j} \), \( e_{m} \) and \( e_{f} \) are defined as inflation response coefficients which measure the impacts of inflation on rates of return on \( j^{th} \) risky asset, market rate of return, and borrowing (or lending) rate respectively. \( R_{j}^{T}, R_{m}^{T} \) and \( R_{f}^{T} \) are holding period nominal rates of return and \( R_{j}^{O}, R_{m}^{O} \) and \( R_{f}^{O} \) are observed real rates of return as defined by Roll and KH.

Equations (2A), (2B), and (2C) can be used to estimate the response coefficients, \( e_{j}, e_{m}, \) and \( e_{f} \).
(A) \( \log_{e} R_{jt}^N = \log_{e} R_{j}^T + e_j \log R_{It} + \varepsilon_{jt} \)

(B) \( \log_{e} R_{mt}^N = \log_{e} R_{m}^T + e_m \log R_{It} + \varepsilon_{mt} \)

(C) \( \log_{e} R_{ft}^N = \log_{e} R_{f}^T + e_f \log R_{It} + \varepsilon_{ft} \)

where

- \( R_{jt}^N = R_{j}^T R_{I}^e = 1 + \text{nominal rates of return for } j^{th} \text{ security} \)
- \( R_{mt}^N = R_{m}^T R_{I}^e = 1 + \text{nominal market rates of return} \)
- \( R_{ft}^N = R_{f}^T R_{I}^e = 1 + \text{nominal risk-free rate} \)

The estimated \( e_j \), \( e_m \), and \( e_f \) can be used to test whether \( R_{j}^o = R_{j}^T \), \( R_{m}^o = R_{m}^T \) and \( R_{f}^o = R_{f}^T \) or not. If these estimated response coefficients are not significantly different from one, then observed real rates of return are not significantly different from real rates of return without inflation. These analyses and their implications will be explored further in the empirical sections (Section V).

Since the borrowing and lending rate is not risk-free in real term, Hagerman and Kim have derived a real CAPM as

\[
E(R_{j}^o) = E(R_{f}^o) + \frac{\text{Cov}(R_{m}^o, R_{j}^o - R_{f}^o)}{\text{Cov}(R_{m}^o, R_{m}^o - R_{f}^o)} [E(R_{m}^o) - E(R_{f}^o)]
\]

(HK have shown that their real CAPM will reduce to Roll's real CAPM (p. 915, equation 26) when either \( R_{f}^o \) is nonstochastic or uncorrelated with the observed real market rate of return. Using the notations of (3), Roll's real CAPM is written as

\[
E(R_{j}^o) = R_{f}^o + \frac{\text{Cov}(R_{m}^o, R_{j}^o)}{\text{Var}(R_{m}^o)} [E(R_{m}^o) - R_{f}^o]
\]
Merton (1973) has shown that S-M-L type CAPM has implicitly assumed that the investment opportunity is constant over time. He also argued that the changes in the interest rate and purchasing power are main factors affecting the change in the investment opportunity set. These findings imply that rates of return without inflation rather than nominal rates of return should be used to test S-M-L type CAPM. However, Roll, KH, FM and others have used the observed real rates of return rather than the rates of return without inflation to analyze as well as test the impacts of inflation on the capital asset pricing.

Next, the advantage of using rates of return without inflation rather than observed real rates of return concepts are analyzed. From equations (1) and (2), it is obvious that the observed real rates of return will not be equal to rates of return without inflation unless all of inflation response coefficients, $e_j$, $e_m$ and $e_f$, are equal to one. If all response coefficients are equal to one, then the nominal rates of return can be expressed in terms of observed real rates of return and inflation rate as:

\[
\begin{align*}
(a) \quad R_{j}^N &= R_{j}^O + \Delta I + \Delta I \cdot R_{j}^O \\
(b) \quad R_{m}^N &= R_{m}^O + \Delta I + \Delta I \cdot R_{m}^O \\
(c) \quad R_{f}^N &= R_{f}^O + \Delta I + \Delta I \cdot R_{f}^O
\end{align*}
\]

where $\Delta I$ represents the inflation rate.

Gibson (1970), Jaffee and Mandelker (1976) and others have regarded these relationships as a summarization of price expectation (or Fisher effect). Fisher (1930), Fama (1975) and others have shown that the
relationship of (4c) approximately hold. However, the relationships of both (4a) and (4b) generally do not hold. Empirically, Jaffee and Mandelker (1976) have shown that the "Fisher effect" does not hold for market index.

Finally, one of HK's main results as indicated in (2) will be analyzed. Equation (3) implies that the slope of regressing \( \tilde{R}_j^O - \tilde{R}_j^O \) on \( \tilde{R}_m^O - \tilde{R}_f^O \) should be defined as

\[
b_j = \frac{\text{Cov}(\tilde{R}_j^O, \tilde{R}_m^O) - \text{Cov}(\tilde{R}_m^O, \tilde{R}_f^O)}{\text{Var}(\tilde{R}_m^O) - \text{Cov}(\tilde{R}_m^O, \tilde{R}_f^O)}
\]

If the real market return is uncorrelated with price level change, HK has shown that equation (6) can be rewritten as

\[
\beta_j = \frac{\text{Cov}(\tilde{R}_j^O, \tilde{R}_m^O)}{\text{Var}(\tilde{R}_m^O)}
\]

The difference between \( b_j \) and \( \beta_j \) can be written as

\[
b_j = \beta_j - (1-\beta_j)(c/l-c)
\]

where

\[
c = \frac{\text{Cov}(\tilde{R}_m^O, \tilde{R}_f^O)}{\text{Var}(\tilde{R}_m^O)}
\]

c is the regression coefficient of regressing \( \tilde{R}_f^O \) on \( \tilde{R}_m^O \); it can be used to measure the relationship between observed real market rate of return associated with the observed real interest rate. Since Merton (1973) has shown that the change of interest rate is one of the important factors affecting the change of the investment opportunity set, it seems reasonable to conclude that the degree of stability of estimated \( b_j \)
instead of estimated $\beta_j$ has taken shift of investment opportunity into account.

It can be argued that the observed real beta estimate $\beta_j$ is only a proxy of the observed real estimate $b_j$ and $(1-\beta_j)(c/l-c)$ is the related proxy error. Empirically, the estimated $b_j$ can be indirectly obtained from the regressions defined as

$$\bar{R}_o^{o} - \bar{R}_t^{o} = \alpha_j + \beta_j (\bar{R}_{mt}^{o} - \bar{R}_t^{o}) + \epsilon_{jt}$$

$$\bar{R}_t^{o} = a + c \bar{R}_{mt}^{o} + I_t$$

In the following section, the difference between the real parameters of CAPM and nominal parameters of CAPM is analyzed in a multivariate normal distribution framework.

III. Relationship Between Nominal and Real Parameters Under Multivariate Normal Assumption

If $R_f^o$ is nonstochastic in equation (4), a regression relationship in terms of observed real rates of return is defined as

$$(\bar{R}_j^tP_t - R_f^o) = \alpha_j + \beta_j (\bar{R}_{mt}^tP_t - R_f^o) + \epsilon_{jt}$$

(8)

Where $\bar{R}_j$ and $\bar{R}_m$ are nominal rates of return for $j^{th}$ security and nominal market rate of return respectively, $\epsilon_{jt} \sim N(0, \sigma_j^2)$ and $P_t = R_f^{-1}$. From equation (7), it is obvious that the observed real systematic risk and Jensen performance measure can be defined as

(a) $\beta_j = \text{Cov}(\bar{R}_j^tP_t, \bar{R}_m^tP_t)/\text{Var}(\bar{R}_m^tP_t)$

(9)

(b) $\alpha_j = E(\bar{R}_j^tP_t - R_f^o) - \beta_j [E(\bar{R}_m^tP_t) - R_f^o]$
where \( E(\cdot) \) is the expectation operator. By using the trivariate normal moment generating function and the definition of covariance, it can be shown that:

\[
\begin{align*}
(a) \quad \text{Cov}(&\tilde{R}_{jt}, \tilde{R}_{mt}) = \text{Cov}(\tilde{R}_{jt}, \tilde{P}_t, \tilde{R}_{mt}, \tilde{P}_t) - \text{Var}(\tilde{P}_t)E(\tilde{R}_{jt} \tilde{R}_{mt}) \\
&- \text{Cov}(\tilde{R}_{jt}, \tilde{P}_t)\text{Cov}(\tilde{R}_{mt}, \tilde{P}_t) - \frac{\text{PR}_m \text{Cov}(\tilde{R}_{jt}, \tilde{P}_t)}{\text{P}} \\
&- \frac{\text{PR}_j \text{Cov}(\tilde{R}_{mt}, \tilde{P}_t)}{\text{P}^2} \\
\end{align*}
\]

\[(10)\]

\[
(b) \quad \text{Var}(\tilde{R}_{mt}) = \left[ \text{Var}(\tilde{R}_{mt}, \tilde{P}_t) - \text{Var}(\tilde{P}_t)E(\tilde{R}_{mt}^2) - \text{Cov}^2(\tilde{R}_{mt}, \tilde{P}_t) \right] - \frac{\text{PR}_m \text{Cov}(\tilde{R}_{mt}, \tilde{P}_t)}{\text{P}^2}
\]

where \( \bar{R}_m = E(\tilde{R}_{mt}), \bar{P} = E(\tilde{P}_t) \) and \( \bar{R}_j = E(\tilde{R}_{jt}) \)

From (9a), (9b), (10a) and (10b), the relationship between real parameters and nominal parameters is defined as

\[
\begin{align*}
(a) \quad &\beta_j' = \beta_j + S_j \\
(b) \quad &\alpha_j' = \frac{\alpha_j}{\bar{P}} - E(\tilde{R}_{mt})(\beta_j + S_j),
\end{align*}
\]

where: \( \beta_j \) = nominal systematic risk, \( \alpha_j \) = nominal Jensen performance measure, \( S_j = \left\{ \text{Var}(\tilde{P}_t)[\beta_j E(\tilde{R}_{mt}^2) - \tilde{E}(\tilde{R}_{jt} \tilde{R}_{mt}) + \text{Cov}(\tilde{R}_{mt}, \tilde{P}_t)] \right\} \)

\[
\begin{align*}
\left[ \beta_j \text{Cov}(\tilde{R}_{mt}, \tilde{P}_t) + 2\beta_j \text{PR}_m - \text{Cov}(\tilde{R}_{jt}, \tilde{P}_t) - \text{PR}_j \right] - \frac{\text{PR}_m \text{Cov}(\tilde{R}_{jt}, \tilde{P}_t)}{\text{P}^2}
\end{align*}
\]

\[
/ \left[ \text{Var}(\tilde{R}_{mt}, \tilde{P}_t) - \text{Var}(\tilde{P}_t)E(\tilde{R}_{mt}^2) - \text{Cov}^2(\tilde{R}_{mt}, \tilde{P}_t) - 2\text{PR}_m \text{Cov}(\tilde{R}_{mt}, \tilde{P}_t) \right]
\]

If \( \tilde{P}_t \) is independent of both \( \tilde{R}_{jt} \) and \( \tilde{R}_{mt} \), then equation (11a) is reduced to
Under this circumstance, the condition of nominal systematic risk being reduced to the real systematic risk are: (i) inflation is certain, (ii) the real intercept is zero and (iii) the average real market rate of return is zero. Incidentally, these conclusions are similar to Casson's (1973) results in investigating the problem of linear regression with errors in the deflating variable. However, the assumption of independence between \( P_t \) and nominal rates of return is relatively strong; hence, other alternative assumptions are needed for investigating the relationship between real parameters and nominal parameters. In the following section, a multi-index model based upon our generalized assumptions as defined in equation (1) is derived and the impacts of inflation on capital asset pricing are investigated.

IV. A Model For Testing the Impacts of Inflation on Capital Asset Pricing

Using Taylor's expansion, we can linearize the nominal rates of return in terms of rates of return without inflation, \( R_i \) and response coefficients as

\[
\begin{align*}
\beta_j &= \beta_j + \frac{\operatorname{Var}(P_t)[\beta_j E(R^2_{mt}) - \operatorname{E}(R_{jt} R^2_{mt})]}{\operatorname{Var}(R^2_{mt} P_t) - \operatorname{Var}(P_t) \operatorname{E}(R^2_{mt})} \\
&= \beta_j - \frac{\operatorname{Var}(P_t) E(P_t R_{mt})(\alpha_j)}{E(P^2_t)[\operatorname{Var}(R^2_{mt} P_t) - \operatorname{Var}(P_t) \operatorname{E}(R^2_{mt})]^2}
\end{align*}
\] (12)

\[
\begin{align*}
R^N_{ij} &= R^T_{ij} + e_j R_I \\
R^N_m &= R^T_m + e_m R_I \\
R^N_f &= R^T_f + e_f R_I
\end{align*}
\] (13)
If \( R^T_f \) is nonstochastic and equal to \( R^O_f \), then a CAPM in terms of rates of return without inflation can be defined as

\[
(R_{jt} - e_j R_{It} - R_f) = \alpha_j + \beta_j (R_{mt} - e_m R_{It} - R_f) + \epsilon_{jt} \tag{14}
\]

The systematic risk and Jensen performance measure estimated from equation (14) are not necessarily free from the impacts of inflation. From equation (14) real systematic risk and real Jensen performance measure is defined as

\[
(a) \quad \beta_j = \frac{[\text{Cov}(R_{jt}, R_{mt}) - e_j \text{Cov}(R_{jt}, R_{It}) - e_j \text{Cov}(R_{mt}, R_{It}) + (e_j e_m) \text{Var}(R_{It})]}{[\text{Var}(R_{mt}) + e^2_m \text{Var}(R_{It}) - 2e_m \text{Cov}(R_{mt}, R_{It})]} \\
(b) \quad \alpha_j = E(R_{jt} - R_{It} - R^O_f) - \beta_j E(R_{mt} - e_m R_{It} - R^O_f) 
\]

If we define \( e_j = \text{Cov}(R_{jt}, R_{It})/\text{Var}(R_{It}) \)
\[
e_m = \text{Cov}(R_{mt}, R_{It})/\text{Var}(R_{It})
\]

then the relationship between nominal parameters and real parameters can be defined as

\[
(a) \quad \beta_j' = \beta_j - (e_j - \beta_j e_m)(d - 2e_m)/(1te_m^2 - 2e_m^2) \\
(b) \quad \alpha_j' = \alpha_j - (e_j - \beta_j e_m)[(d - 2e_m) (R_{mt} - e_m R_{It} - R^O_f)]/(1te_m^2 - 2e_m^2 - R_{It}) \tag{16}
\]

where \( d = \text{Var}(R_{It})/\text{Var}(R_{mt}) \)

\( \beta_j' \) is nominal systematic risk and \( \alpha_j' \) is nominal Jensen performance measure. Equation (16a) implies that the nominal systematic risk is equal
to the real systematic risk when either (i) inflation is certain or
(ii) $\beta_j = e_j / e_m$. Equation (16b) implies that the nominal Jensen performance measure will reduce to the real Jensen performance measure if and only if $\beta_j = e_j / e_m$. Now a model derived from equation (14) is proposed to test whether $\beta_j$ is equal to $e_j / e_m$ as

$$ (R_{jt} - R_f) = \alpha_j + \beta_j[R_{mt} - R_f] + (e_j - e_m \beta_j)R_{It} + \epsilon_{jt} \quad (17) $$

Equation (17) can be used to empirically test the impact of inflation on the systematic risk estimate. From the specification analysis, it is well-known that the regression coefficient associated with $R_{It}$ can be decomposed into two components, i.e.,

(i) the regression coefficient of regressing $R_{jt}$ on $R_{It}$

and

(ii) the auxiliary regression coefficient of regressing $R_{mt}$ on $R_{It}$ times the systematic risk $(e_m \beta_j)$. If $e_j$ is equal to $e_m \beta_j$, then the systematic risk obtained from nominal rates of return will not be different from the estimate obtained from the real rates of return. In other words, if the ratio among every risky asset's response coefficient $(e_j)$ is equal to the ratios among every risky asset's systematic risk $(\beta_j)$, then inflation will not affect the capital market pricing and therefore, the portfolio efficient frontier constructed from nominal rates of return will not be different from that obtained from the real rates of return. \(^7\)

The method derived in this section is different from the last section in two important aspects, i.e.,
(i) an additive rather than a multiplicative assumption is used to investigate the impact of inflation on capital asset pricing

and

(ii) the response coefficients \( e_j \) and \( e_m \) are introduced to take care of consequences of anticipated inflation on different companies' rates of return. Reilly (1975) has classified companies into (i) complete inflation hedge companies, (ii) superior inflation hedge companies and (iii) inferior inflation hedge companies. Our response coefficients, \( e_j \), \( e_m \) and \( e^f \) defined in equation (1) can be used explicitly as a criterion for classifying companies according to degree of inflation hedge.

Now, our equation (17) is compared with either Roll's equation (27) or HK's equation (16). In terms of our notations, Roll's equation (27) can be defined as

\[
E(R_j) = R_f^0 + [E(R_m^0 - R_f^0)] \beta_j - [\text{Cov}(R_j, \bar{P}) - \beta_j \text{Cov}(\bar{R}_m, \bar{P})]/E(\bar{P}) \tag{18}
\]

Equation (18) implies that the coefficient associated with \( E(R_I) \) will be zero if

\[
\beta_j = \frac{\text{Cov}(R_j, \bar{R}_I)}{\text{Cov}(R_m, \bar{R}_I)} = \frac{e_j}{e_m} \tag{19}
\]

The implication of this equation is similar to our implication described in equation (17). Hence, the results of this section have given Roll's results a useful empirical interpretation. In the following section, data from NYSE are used to test both equations (8) and (17).

V. Some Empirical Results

For investigating the impacts of inflation on the estimated systematic risk and Jensen performance measure, 464 monthly individual stock
rates of return selected from the NYSE during the period of January 1966 to March 1979 are employed to estimate the $\beta_j$, $\hat{\beta}_j$, $\alpha_j$, and $\hat{\alpha}_j$. The Fisher index with dividends is used to calculate the market rate of return, the monthly 90 days treasury bill rate is used as a measurement of risk free rate, and the monthly consumer price index is used to measure inflation rate in estimating the real systematic risk and Jensen performance measure. The results of averaged systematic risk and the Jensen performance measure in both nominal terms and real terms for two sub-periods are listed in Columns 1 and 2 of Table I respectively. The first sub-period is from January 1966-December 1972 and the second sub-period is from January 1973-December 1979.

The results indicate that the estimated real parameters are not significantly different from the estimated nominal parameters. To test the specification of equation (17), 464 multiple regressions are estimated by using the data described in this section. The regression coefficient associated with $R_{jt}$ in equation (17) can be used to test whether the estimated $\beta_j$ is significantly different from estimated $e_j/e_m$ or not.

Results are listed in Table II-a. All significant estimated $\hat{\beta}_2$'s have negative sign. These imply that inflation generally has negative impacts on the security rates of return. To obtain some further insights about the impacts of inflation on security rates of return, Equation (2) is estimated by using 464 individual firms' data and the data of market rates of return. The results associated with individual firms all listed in Table II-b. It is also found that the inflation has significant negative impact on the market rates of return. Our empirical results indicate that the inflation will generally have some effects on the
magnitude of estimated systematic risk and Jensen's measure of performance. This conclusion is essentially based upon the fact that some of the ratios between estimated response coefficient for the \( j \)th security and the response coefficient for the market portfolio is significantly different from the estimated systematic risk for \( j \)th security.

Friend and Blume (1970) have regressed the performance measures on both total risk and systematic risk to determine the possible bias associated with performance measure estimates. We now use the models indicated in equations (20-21) to do further empirical analysis.

\[
\alpha_j = \alpha_0 + \alpha_1 \beta_j + \varepsilon_j \quad (20)
\]

\[
\alpha_j = \beta_0 + \beta_1 \sigma_j^2 + \varepsilon_j \quad (21)
\]

where \( \alpha_j \) is the estimate Jensen measure for \( j \)th security; \( \beta_j \) is the estimated systematic risk for \( j \)th security and \( \sigma_j^2 \) is the total risk for \( j \)th security. Empirical results for two-subperiods are listed in Tables II-a and II-b. For the first subperiod, the Jensen measure obtained from real rates of return, which obtained by \( \frac{R}{1+r} \), does not show the bias as found by Friend and Blume (1970). However, the results of second subperiod do show the bias which was found by Friend and Blume.

VI. Summary

Based upon responsive coefficient concepts, this study has theoretically re-examined the impacts of inflation on capital asset pricing. The relationship between the real parameters of CAPM and the nominal parameters of CAPM is derived. A multi-index model with the change in purchasing power as an additional variable is derived to test whether
the real parameters are significantly different from the nominal parameters.

Using the data of 464 securities to test the model derived in this paper, it is found that the estimated real parameters of CAPM are generally not significantly different from the estimated nominal parameters if the real risk free rates are assumed to be nonstochastic. However, the multi-index model derived in this paper does show the importance of inflation variable in estimating the related parameters in capital asset pricing determining process.
Merton (1980) has argued that the ex ante instead of the ex post measure should be used to do the theoretical test. Rate of return without inflation is an ex ante measure and observed real rate of return is an ex post measure.

See Fama (1975) or Jaffee and Mandelker (1976, p. 455).

See section IV for detail.

See appendix A.

Based upon the assumption that real market return is uncorrelated with the price level change, HK (equation on 14b) have also derived the relationship between nominal systematic risk and systematic risk. However, they do not obtain the interesting implications as developed in this section.

Chen and Bones (1975, p. 471) have used this kind of technique to derive some similar relationships. In addition, these results are similar to those defined in equation (2).

Biger (1976) has found that there exist some differences between the efficient portfolio frontier obtained from real rates of return and those obtained from nominal rates of return.

It can be shown that Roll's equation (27) is identical to HK's equation (16).

The specification of (8) with and without adjusting for the purchasing power change is used to estimate the related parameters.
APPENDIX (A)

The derivation of (10):

If $V_{jt}$, $V_{mt}$ and $P_t$ are trivariate normally distributed, then from Hogg and Craig (1969, Chapter 13), the moment generating function of this trivariate normal distribution can be written as:

\[
(c) \quad (t_1, t_2, t_3) = \exp(t_1\overline{R}_j + t_2\overline{R}_m + t_3\overline{P} + 1/2[t_1^2\text{Var}(R_{jt}) + t_2^2\text{Var}(R_{mt}) + t_3^2\text{Var}(P_t) + 2t_1t_2\text{Cov}(R_{jt}, R_{mt}) + 2\text{Cov}(R_{jt}, P_t) + 2\text{Cov}(R_{mt}, P_t)])
\]

From (c), it can be shown that

\[
(d) \quad E(R_{jt}R_{mt}P_t^2) = \frac{\partial^4(t_1, t_2, t_3)}{\partial t_1 \partial t_2 \partial t_3} \quad t_1 = t_2 = t_3 = 0
\]

\[
= \text{Cov}(R_{jt}, R_{mt})\overline{R}_j\overline{R}_m + \overline{R}_j\overline{P}\text{Cov}(R_{jt}, R_{mt}) + \overline{R}_m\overline{P}\text{Cov}(R_{jt}, P_t) + 2\text{Cov}(R_{mt}, P_t).
\]

From the definition of covariance, we also have

\[
(e) \quad E(R_{jt}P_t) = \text{Cov}(R_{jt}, P_t) + \overline{R}_j\overline{P}
\]

\[
(f) \quad E(R_{mt}P_t) = \text{Cov}(R_{mt}, P_t) + \overline{R}_m\overline{P}
\]

From the moment generating function of bi-variate normal distribution, it can be shown that
(g) \( E(R^2_{mt} t) = \frac{\theta(4)(t_1, t_2)}{\theta^2 t_1 a t_2} \quad t_1 = t_2 = 0 \)

\[
= \sigma^2_m P^2 + \sigma^2_m P^2 + \sigma^2_R P^2 + 2Cov^2(R_{mt}, P_t) \\
+ 2R_{m} PCov(R_{mt}, P_t) + \frac{R^2_m}{m}
\]

Substituting (d), (e) and (f) into (b), then we have (10b), and substituting (f) and (g) into (a), then we have (10a).
REFERENCES


Table I

AVERAGE REAL AND NOMINAL PARAMETERS*

<table>
<thead>
<tr>
<th></th>
<th>1966-72</th>
<th></th>
<th></th>
<th>1973-79</th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Division</td>
<td></td>
<td></td>
<td>Subtraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Systematic Risk</td>
<td>1.10362</td>
<td>(.41543)</td>
<td>1.10291</td>
<td>(.41417)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.10467</td>
<td>(.41328)</td>
<td>1.10291</td>
<td>(.41417)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.06219</td>
<td>(.34490)</td>
<td>1.0614</td>
<td>(.34448)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.06158</td>
<td>(.33901)</td>
<td>1.0614</td>
<td>(.34448)</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>1.00090</td>
<td>(.00891)</td>
<td>0.00093</td>
<td>(.00898)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.00137</td>
<td>(.00997)</td>
<td>0.00093</td>
<td>(.00898)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Standard errors appear in parentheses beneath the corresponding coefficients.
Table II-a

Percentage Significant of $a_2$ in $R_{jt} - R_{ft} = a_0 + a_1(R_{mt} - R_{ft}) + a_2R_{It} + e_{jt}$

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>5% LEVEL</th>
<th>10% LEVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No.</td>
<td>%</td>
</tr>
<tr>
<td>1966-72</td>
<td>73</td>
<td>16%</td>
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<tr>
<td>1973-79</td>
<td>42</td>
<td>9%</td>
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</table>

Table II-b

Percentage Significance of $a_1$ in $R_{jt} = a_0 + a_1R_{It} + e_{jt}$

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>5% LEVEL</th>
<th>10% LEVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No.</td>
<td>%</td>
</tr>
<tr>
<td>1966-72</td>
<td>109</td>
<td>23%</td>
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<tr>
<td>1973-79</td>
<td>90</td>
<td>19%</td>
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</table>
Table III-a

REGRESSION RESULTS 1966-1972

Model 1: \( \hat{\alpha}_j = a_0 + a_1 \epsilon_j + \epsilon_j \)

<table>
<thead>
<tr>
<th></th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>Unadj. R(^2)</th>
<th>SEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal  term</td>
<td>-.00542**</td>
<td>.00289**</td>
<td>.02630</td>
<td>.00730</td>
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<tr>
<td></td>
<td>(.0096)</td>
<td>(.00082)</td>
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<td></td>
</tr>
<tr>
<td>Real term</td>
<td>-.00153</td>
<td>.00098</td>
<td>.00315</td>
<td>.00728</td>
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<td></td>
<td>(.00096)</td>
<td>(.00081)</td>
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</tr>
</tbody>
</table>

Model 2: \( \hat{\alpha}_j = b_0 + b_1 \epsilon_j^2 + \epsilon_j \)

<table>
<thead>
<tr>
<th></th>
<th>( b_0 )</th>
<th>( b_1 )</th>
<th>Unadj. R(^2)</th>
<th>SEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal  term</td>
<td>-.00446**</td>
<td>.29794**</td>
<td>.04680</td>
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<tr>
<td></td>
<td>(.00058)</td>
<td>(.06255)</td>
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</tr>
<tr>
<td>Real term</td>
<td>-.00311**</td>
<td>.06592</td>
<td>.00232</td>
<td>.00728</td>
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<tr>
<td></td>
<td>(.00058)</td>
<td>(.06360)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Significant at .01 level
*Significant at .05 level
Table III-b

REGRESSION RESULTS 1973-1979

Model 1: \( a_j = a_0 + a_1 \beta_j + \epsilon_j \)

<table>
<thead>
<tr>
<th></th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>Unadj. ( R^2 )</th>
<th>SEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>-.00768**</td>
<td>.00812**</td>
<td>.09694</td>
<td>.00856</td>
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<tr>
<td></td>
<td>(.00129)</td>
<td>(.00115)</td>
<td></td>
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</tr>
<tr>
<td>Real</td>
<td>-.00762**</td>
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<td>.09711</td>
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<tr>
<td></td>
<td>(.00128)</td>
<td>(.00114)</td>
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</tr>
</tbody>
</table>

Model 2: \( a_j = b_0 + b_1 \sigma_j^2 + \epsilon_j \)

<table>
<thead>
<tr>
<th></th>
<th>( b_0 )</th>
<th>( b_1 )</th>
<th>Unadj. ( R^2 )</th>
<th>SEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>-.00539**</td>
<td>.65844**</td>
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<td>(.00540)</td>
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<tr>
<td>Real</td>
<td>-.00534**</td>
<td>.66223**</td>
<td>.24343</td>
<td>.00770</td>
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<tr>
<td></td>
<td>(.00628)</td>
<td>(.05432)</td>
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</tr>
</tbody>
</table>

**Significant at .01 level  
*Significant at .05 level