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ALTERNATIVE THEORIES OF PRICING, DISTRIBUTION, SAVING, AND INVESTMENT

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#401
100-word summary:

In order to identify similarities and dissimilarities between neo-classical and post-Keynesian distribution models, the present paper specifies both models mathematically, in their stripped form, and in the same notation. Four questions are then asked and answered. First, does saving or investment adjust to a higher propensity to save? Second, within that adjustment what are the roles played by the real wage rate and mark-up pricing? Third, within that adjustment does a Wicksell Effect emerge? Fourth, are the models examined open or closed ones? In conclusion, the relative analytical appeal of the two models is appraised.
"The whole dispute between Keynesian and non-Keynesian theories is whether investment determines savings, or vice versa," Kaldor [3], 301.

The purpose of the present paper is to identify rigorously some similarities and dissimilarities between neoclassical [7] and post-Keynesian [2], [3], [5], [6] distribution models. Four interrelated questions will be asked and answered. First, does saving or investment adjust to a higher propensity to save? Second, within that adjustment what are the roles played by the real wage rate and mark-up pricing? Third, within that adjustment does a Wicksell Effect emerge? Fourth, are the models examined open or closed and if closed, how?

Answers will be facilitated by specifying both models mathematically, in their stripped form, and in the same notation:

Variables

C $\equiv$ consumption

$g_v$ $\equiv$ proportionate rate of growth of variable $v$ $\equiv$ S and X

I $\equiv$ investment

$\kappa$ $\equiv$ physical marginal productivity of capital stock

L $\equiv$ labor employed

m $\equiv$ mark-up pricing factor
I. EQUATIONS COMMON TO BOTH MODELS

We confine ourselves to the stripped form of either model having
one good, an immortal capital stock of that good, and no technological progress in it. Four definitions and one equilibrium condition are common to both models. Define the proportionate rate of growth

\[ g_v = \frac{dv}{dt} \]

Define investment as the time derivative of capital stock:

\[ I = \frac{dS}{dt} \]

Define the wage bill as the money wage rate times employment:

\[ W = wL \]

Define national money income as the sum of wage and profits bills:

\[ Y = W + Z \]

Equilibrium requires output to equal demand for it:

\[ X = C + I \]
II. EQUATIONS PECULIAR TO THE NEOCLASSICAL MODEL

1. Production

Let entrepreneurs apply a Cobb-Douglas production function

\[ X = M L^\alpha S^\beta \]

where \( 0 < \alpha < 1, 0 < \beta < 1, \) and \( \alpha + \beta = 1 \). Profit maximization under pure competition will equalize the real wage rate and the physical marginal productivity of labor:

\[ \frac{w}{P} = \frac{\partial X}{\partial L} = \frac{X}{L} = \alpha \]

Define physical marginal productivity of capital as

\[ \kappa \equiv \frac{\partial X}{\partial S} = \frac{X}{S} = \beta \]

Multiply by value of capital stock \( PS \) and define profits as

\[ Z = \kappa PS = \beta PX \]
Assume full employment:

(10) \[ F = L \]

2. Distributive Shares

Insert (7) into (3) and find the wage bill \( W = \alpha PX \). Insert that and (9) into (4) and find \( Y = PX \), so the distributive shares are \( W/Y = \alpha \) and \( Z/Y = \beta \). Notice that we derived this familiar result before specifying our consumption function. For that result, then, the form of the consumption function is immaterial and could easily be post-Keynesian:

\[ C = c_w W/P + c_z Z/P \]

But if \( W = \alpha PX \) and \( Z = \beta PX \) we may define \( c \equiv \alpha c_w + \beta c_z \) and write it

(11) \[ C = cX \]

where \( 0 < c < 1 \), which is the customary neoclassical form.

Equipped with a Cobb-Douglas production function with \( \alpha + \beta = 1 \), then, our neoclassical model is rich enough to accommodate a consumption
...
function with different propensities to consume real wages and real profits. But such a consumption function is a nice luxury, not a necessity. Considering marginal productivity a meaningful concept, hence capable of deriving distributive shares from it, neoclassicists don't need such a consumption function for that derivation.

3. Does Saving or Investment Adjust to a Higher Propensity to Save?

In the neoclassical model the overall propensity to save \( 1 - c \) is a parameter—or may be written as one, as we just saw. If that parameter were twice as high, how would the model adjust? The neoclassical clue lies on the investment side and begins with a typical neoclassical response of factor proportion to relative factor price. Divide (6) by \( L \) and (7) by \( \alpha \) and set the resulting expressions for \( X/L \) equal. Find

\[
S/L = (\alpha M)^{-1/\beta}(w/P)^{1/\beta}
\]

So a higher real wage rate \( w/P \) will induce a higher capital intensity \( S/L \), and the elasticity of \( S/L \) with respect to \( w/P \) is \( 1/\beta \). Under pure competition individual entrepreneurs take the real wage rate
for granted. But a macroeconomist cannot take it for granted; he must consider it a variable to be solved for before he can see the whole picture.

4. The Real Wage Rate

To solve for the real wage rate divide (6) by $S$, raise both sides to the power $-1$, and find the capital coefficient

$\frac{S}{X} = \frac{1}{M} \frac{S}{L}^\alpha$ \hfill (13)$

Next use (1) and (2) to write $I \equiv g_s S$, insert that and (11) into (5), and find another expression for the capital coefficient

$\frac{S}{X} = \frac{1 - c}{g_s}$

Finally set the right-hand sides of the two expressions for $S/X$ equal, insert the result into (12) and find the real wage rate

$\frac{w}{P} = \alpha M^{1/\alpha} [(1 - c)/g_s]^{\beta/\alpha}$ \hfill (14)
5. Closing the Neoclassical Model

Eq. (14) is not a solution yet. So far it merely expresses one unknown, the real wage rate $w/P$, in terms of another, the rate of growth of capital stock $g_S$. But neoclassicists do close their models. In the absence of technological progress they find proportionate rates of growth to be converging to the steady-state solutions

\begin{equation}
    g_S = g_X = g_F 
\end{equation}

Neoclassicists modify their solutions by allowing for technological progress, ignored here. Insert (15) into (14), and you have a neoclassical solution for the real wage rate $w/P$.

Now we can see the whole picture: According to (14) with (15) inserted, an economy with twice the overall propensity to save will have a $2^{\beta/\alpha}$ times higher real wage rate. According to (12) such a real wage rate will induce a $2^{1/\alpha}$ times higher capital intensity. According to (13) such a capital intensity means a twice as high capital coefficient. Summing up: The economy with twice the overall propensity to save will have a capital coefficient twice as high. In this sense the give of the neoclassical model lies on the investment side.
6. The Wicksell Effect

Raising the proportion of one factor to another raises the marginal productivity of the latter. In our neoclassical model such raising manifests itself as a Wicksell Effect: According to (14) the real wage rate w/P is the higher the higher the propensity to save 1 - c or, as Wicksell [8], 164, put it: "The capitalist saver is thus, fundamentally, the friend of labour."

III. EQUATIONS PECULIAR TO POST-KEYNESIAN MODELS

1. Production

We confine ourselves to the simplest form of a post-Keynesian model having fixed input-output coefficients. Two simultaneous equations will then take the place of a neoclassical production function:

\[
L = aX \\
S = bX
\]
Two conclusions follow at once. First, there can be no response of factor proportion to anything, relative factor price or otherwise, for according to (16) and (17) the factor proportion $S/L = b/a$ is a parameter. Second, because the system (16) and (17) is a simultaneous one, any variation of $X$ implies simultaneous variation of $L$ and $S$. Consequently partial derivatives of $X$ with respect to $L$ or $S$ are meaningless. Marginal productivity is defined as such a partial derivative. Considering marginal productivity a meaningless concept, hence incapable of deriving distributive shares from it, post-Keynesians need something else, i.e., different propensities to consume wages and profits, deployed as follows.

2. Distributive Shares

We confine ourselves to the simplest form of a post-Keynesian model having a propensity to consume wages equalling one. In that case the consumption function is

\[(18) \quad C = W/P + c_Z Z/P \]

With immortal capital stock the entire value of output represents
value added, i. e., money national income:

(19) \[ PX = Y \]

Insert (4) into (19), divide by P, and write

(20) \[ X = \frac{W}{P} + \frac{Z}{P} \]

Subtract (18) from (20) and insert (5). Use (1) and (2) to write

\[ I = g_S S, \]

insert (17) into that, divide by X, and use (19) to express

the profits share in terms of the capital coefficient, the proportionate rate of growth of physical capital stock, and the propensity to

save real profits:

(21) \[ \frac{Z}{Y} = \frac{b g_S}{(1 - c_Z)} \]

3. **Closing the Post-Keynesian Model**

Eq. (21) is not a solution yet. So far it merely expresses one

unknown, the profits share \( Z/Y \), in terms of another, the rate of

growth of capital stock \( g_S \). How do post-Keynesians close their system?
At this point Kaldorian and Robinsonian ways are parting. Kaldor does consider the rate of growth of capital stock $g_S$ a variable and solves for it by assuming full employment—as neoclassicists do. Insert (10) into (16), take the derivatives of (16) and (17) with respect to time, use (1), and find that in the absence of technological progress steady-state proportionate rates of growth are

$$g_S = g_X = g_F$$

Like neoclassicists, Kaldor modifies his solution by offering a "technical-progress function," ignored here. Insert (15) into (21), and you have a Kaldorian solution for the profits share $Z/Y$.

Joan Robinson doesn't consider the rate of growth of capital stock $g_S$ a variable. To her $g_S$ is autonomously given by the "animal spirits" of non-profit-maximizing and otherwise nonrational entrepreneurs. So (21) is already a Robinsonian solution for the profits share $Z/Y$.

4. **Does Saving or Investment Adjust to a Higher Propensity to Save?**

In the post-Keynesian model the overall propensity to save is not
a parameter but a variable depending on income distribution. The parametric propensity to save is the propensity to save real profits $1 - c_Z$. If that parameter were twice as high, how would the model adjust? A post-Keynesian clue cannot lie in a response of factor proportion to relative factor price, for according to (16) and (17) the factor proportion $S/L = b/a$ is a parameter. The clue must lie elsewhere, and it lies in (21) which we read as follows: If two economies have the same capital coefficient $b$ and are growing at the same proportionate rate $g_s$, but one economy has a propensity to save real profits $1 - c_Z$ twice as high as that of the other economy, then the former economy will have a profits share $Z/Y$ half that of the latter. That allows the overall propensities to save, and with them the capital coefficients $S/X$, to stay equal—and they had better, for the two economies were said to have the same capital coefficient $S/X = b$. In this sense the give of post-Keynesian models lies on the savings side.

5. The Real Wage Rate and Monopoly Pricing

In Sec. II, 3 above we found a typical neoclassical response of factor proportion to relative factor price. In the post-Keynesian model the factor proportion $S/L = b/a$ is a parameter unresponsive to the real wage rate $w/P$. But the real wage rate is still there. It
hides behind the price formula that "in modern manufacturing industry ...prices are formed by adding a margin to prime cost" [6], 179. Now in our one-good version of the post-Keynesian model, "prime cost" is labor cost only, and according to (16) per-unit labor cost is aw. Consequently the formula for price is $P = amw$ or for the real wage rate

(22) $\frac{w}{P} = \frac{1}{am}$

where $m$ is the mark-up factor, and $m > 1$.

Mark-up pricing may be a deviation from neoclassical language but not from neoclassical substance. Under neoclassical pure competition, too, there are overhead costs to be covered, and freedom of entry and exit will see to it that they are, so neoclassical price, too, will exceed "prime cost". The proportion in which it does is easily found: Write (7) as

(7) $P = \frac{w}{\alpha} - \frac{L}{S}$

or, in English: Price $P$ exceeds per-unit labor cost $wL/X$ in the pro-
portion $1/\alpha$. Since we assumed $0 < \alpha < 1$, $1/\alpha > 1$, as it should. That proportion could well be labelled a "mark-up factor". So far, post-Keynesian substance doesn't seem to deviate from a neoclassical one.

But a deviation will emerge once we ask whether the mark-up factor is a parameter or not. The neoclassical one $1/\alpha$ clearly is. Joan Robinson's less than rigorous style may give the impression that so is the post-Keynesian one $m$. Could $m$ perhaps be an interesting structural parameter reflecting such things as Kalecki's "degree of monopoly" or at least business convention? It cannot. If it were, Joan Robinson's system would be overdetermined: Divide (4) by $Y$, insert (3), (15), (19), and (22), and find another expression for the profits share:

(23) \[ Z/Y = 1 - 1/m \]

Consider our two expressions for the profits share (21)—if Kaldorian, with (15) inserted—and (23). If $m$ were a parameter those expressions would be two equations in one unknown $Z/Y$, hence would be an overdetermined system. If $m$ were a variable those expressions would be two equations in the two unknowns $Z/Y$ and $m$. Set the right-hand sides of (21) and (23) equal and solve for $m$: 
Joan Robinson's rate of growth of capital stock \( g_S \) was given exogenously by the "animal spirits" of entrepreneurs. Thus (21) was a Robinsonian solution for the profits share \( Z/Y \), and (22) and (24) are accompanying Robinsonian solutions for the real wage rate \( w/P \) and the mark-up factor \( m \), respectively. Entrepreneurs may take their pick: Choosing a lower \( g_S \) would reduce the profits share (21), would lower the mark-up factor (24) and thereby raise the real wage rate (22). All in one scoop! What would make entrepreneurs choose to do such things? Or refrain from doing them? Since her entrepreneurs are neither profit-maximizing nor otherwise rational, Joan Robinson cannot say.

6. A Wicksell Effect?

In a neoclassical world, raising the proportion of one factor to another raises the marginal productivity of the latter. But in post-Keynesian models, as we saw, in the first place the factor proportion \( S/L = b/a \) is a parameter, hence cannot vary. In the second place, marginal productivity is a meaningless concept. Under such circumstances we should hardly expect a Wicksell Effect in post-Keynesian
models. But accepting (21), (22), and (24) as equations accompanying one another, we do find a Wicksell Effect just the same: Like a lower rate of growth $g_S$, a higher propensity to save real profits $1 - c_Z$ would reduce the profits share (21), would lower the mark-up factor (24) and thereby raise the real wage rate (22). Again in one scoop. In other words, as in neoclassical models the real wage rate is the higher the higher the propensity to save. Even a Robinsonian capitalist is "fundamentally the friend of labour"!

IV. CONCLUSIONS

The present paper has confronted neoclassical and post-Keynesian distribution models and has tried to answer four questions.

First, is Kaldor correct in saying that in neoclassical models savings determine investment whereas in post-Keynesian models investment determines saving? Taken literally, he is wrong: Savings and investment are both variables and as such both determined by the parameters of the system. A correctly asked question would be: How sensitive are they to those parameters? What Kaldor really means is
that neoclassical and post-Keynesian models have very different sensitivities to the parametric propensity to save. In the neoclassical model, doubling that propensity was found to double the capital coefficient, and in that sense the give of the neoclassical model lies on the investment side. In the post-Keynesian model, doubling that propensity was found to halve the profits share, and in that sense the give of the post-Keynesian model lies on the savings side.

Second, is the post-Keynesian mark-up factor an interesting new structural parameter reflecting "the degree of monopoly" or business convention? Or, less dramatically, is it merely a variable inherent in the savings-investment adjustment? The latter interpretation was found necessary to avoid overdetermination of the post-Keynesian model.

Third, does a Wicksell Effect emerge? As expected, it does in the neoclassical model. But accepting the interpretation of the mark-up factor as merely a variable inherent in the savings-investment adjustment, we found the post-Keynesian model to have a Wicksell Effect of its own.

Fourth, are the models examined open or closed ones? Yes and No! Kaldor's profit share as well as his mark-up factor are determined by a full-employment assumption requiring capital stock, output, and
available labor force to be growing at the same rate (15). The Kaldorian system may be unnecessarily rigid. The burden of adjustment it imposes upon the distributive shares may be unnecessarily heavy. The distributive shares may be unlikely to be actually carrying that burden. But at least the Kaldorian system is a closed one.

By contrast, Joan Robinson’s system is an open one. Her profits share as well as her mark-up factor are determined by letting non-profit-maximizing and otherwise nonrational entrepreneurs fix an arbitrary growth rate of capital stock.

One fails to see the analytical appeal of post-Keynesian distribution theory. Neoclassical analysis seems more flexible and has less to fear from confrontation with the real world [1]. Perhaps the very openness of Joan Robinson’s system appeals to interventionists: Control investment, and you control income distribution! Perhaps the post-Keynesian appeal is ideological rather than analytical, as Krelle and Gabisch [4], 203, suggest:

Warum aber nun gerade der Widerstand gegen die neoklassische Wachstumstheorie, die doch aus wenigen, plausiblen Voraussetzungen sehr viel mehr erklären kann als die meisten anderen Wachstumstheorien und
REFERENCES


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