INTEGRATING WORKING CAPITAL AND CAPITAL INVESTMENT PROCESSES

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Summary:

The objectives of this paper are to create a model that simulates the dynamics and uncertainty in the working capital process and integrates these parameters into the investment decision making process; to capture the affect of forecasting errors and inflationary conditions on working capital components and cash flows; to measure the cost and benefits of short-run investment and financing variables on the cash flows of an investment; and finally to offer an inflation adjusted cost of capital profile for investment decision making.
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Capital investment creates the need for additional investment in inventory, accounts receivable and cash throughout the life of the plant and equipment. It is normally assumed that an investment in working capital assets also causes a comparable expansion in current liabilities. The theoretical models for evaluating capital investment alternatives implicitly assume the costs resulting from changes in the working capital components or the cash benefits flowing from these components are imbedded in the cash flow of the investment, e.g., Bogue and Roll [8], Hertz [21, 22], Hespos and Strassman [23], Myers [40], Van Horne [57, Chapters 5-7, 13], Weingartner [59] and Weston and Brigham [61, Chapters 10, 11]. Under conditions of certainty working capital components may be included implicitly in the capital investment analysis, but when forecasting error occur, having the capacity to measure the explicit contribution of working capital components to cash inflows and outflows provides a powerful tool for management.

There are several reasons for identifying and measuring the costs and benefits created by the working capital components and linking them explicitly into the total investment planning process. More-than-likely, in the early life of an investment it is operating below capacity; while later in the life cycle, there is often an increase in operating capacity. Thus, throughout the life of an investment there is often a continuing growth of investment in working capital. Furthermore, the discounted cash outflows related to cash, receivables and inventories can range
from a modest to a major proportion of the total cost of an investment. The need for additional investment in working capital is dependent on the type and size of investment, the size and growth of the market, the growth of the relative market share and the length of the planning horizon. Also the source of financing these current assets, either long or short-term, can affect the cash flow patterns of an investment.

Forecasting errors can result in a cash flow shortfall, e.g., actual inflow less than forecasted inflow. This shortfall can result in a series of short-term investment and financing decisions that impact significantly on the cash flows of an investment. In a different context, inflation can create a drag on cash flows if the price increases related to labor and materials are always leading the price increase for the products sold by a significant margin. Finally, within many corporations, the management of the working capital components is usually separate from the capital investment and long-run financial planning systems. Continuous communications between operating and strategic management are vital to the long-run success of a firm, but communications are often infrequent or nonexistent. These observations indicate the need for a planning model that explicitly integrates the working capital components into the capital investment decision-making process, and allows for the dynamics of forecasting errors and inflation. These are basic objectives of this article.

There have been many valuable theoretical and operational contributions in managing cash [1, 2, 4, 17, 19, 25, 32, 39, 42, 43, 52, 53, 54, 55], measuring liquidity [6, 13, 29, 33, 56], managing receivables and credit selection [3, 4, 7, 24, 30, 31, 34, 35, 36, 60, 63], and
controlling inventory [5]. Several authors have linked trade credit policy and inventory management [18, 37, 46, 47, 50]. Linear programming (LP) models were used to introduce the dynamic features of the working capital process. Two large scale LP models were designed to link the sources of short term credit to short term uses [44] and to integrate the variables involved in managing short term cash flow [42]. Sartoris and Spruill [45] used goal programming as a tool for the management of working capital. Gentry, Mehta, et al. [16] evaluated management perceptions of the working capital process. Walker [58, Chapters 7,8] and Van Horne [57, Chapter 13] developed general working capital models.

Several observations emerge from the preceding set of articles. Each focused on specific segments of working capital management and each developed sets of relationships among the WC variables. Many of the models did not incorporate the dynamics of uncertainty involved in the short-run investment and financing processes and none of these models were integrated into the capital investment and long-run financing processes of the firm.

The need for integrating the working capital processes into the long-run financial planning processes has been recognized by several authors. A variety of theoretical linkages have been suggested [9, 10, 11, 12, 15, 20, 26, 27, 28, 38, 48, 49, 54, 64]. A primary objective of this article is to offer an integrated model designed for management decision making and to provide a model for testing "what if" policy questions concerning the impact of forecasting errors, inflation and working capital variables on the total profitability of a capital investment alternative.
The model is divided into two parts. The capital investment (CI) module will be presented first followed by the working capital (WC) module.

CAPITAL INVESTMENT MODULE

The financial planning process is composed of many variables and occurs in an uncertain and dynamic environment, therefore, this model will use simulation techniques to represent the interactions among the capital investment and working capital variables. The Hertz model [21, 22] is revised to simulate the capital investment process. The variables are divided into three major categories—market, investment and cost. The market analysis variables are market size, growth rate of market size related to the life cycle of the product, and market share related to the price of the product. The investment analysis variables are life of the investment, on line time, initial and future investment costs excluding working capital costs. The cost analysis variables are the variable and fixed costs. Each variable is assumed to be stochastic and independent. However, it is assumed the parameters specified for each variable by the decision makers reflects their perception of the interrelationships among the variables. If there are well established relationships among variables, these functional relationships can be easily inserted into the model. The variables involved in the capital investment module are presented in Figure 1 and the actual operation of the CI module is presented later.

The program randomly selects in a sequential order a value from the specified distribution for each variable. The uncertain and dynamic characteristics of the CI process are reflected in this random inter-
Figure 1
SIMULATION OF THE WORKING CAPITAL - CAPITAL INVESTMENT PROCESS

PROBABILITY VALUES FOR SIGNIFICANT VARIABLES

RANDOMLY SELECT VALUES FOR EACH VARIABLE IN THE TOTAL SET

COMPUTE RATE OF RETURN, NET PRESENT VALUES AND BENEFIT/COST RATIOS FOR EACH COMBINATION

REPEAT PROCESS TO PROVIDE A SIMULATED CUMULATIVE FREQUENCY DISTRIBUTION OF RATES OF RETURN, NET PRESENT VALUES AND BENEFIT/COST RATIOS.

CHANCES THAT VALUE WILL BE ACHIEVED

RANGE OF VALUES

INFLATION RATE FOR PRICE AND INTEREST RATE

INFLATION RATE FOR PURCHASES

INFLATION RATE FOR LABOR COSTS

INFLATION RATE FOR FIXED COSTS

CHANCES THAT RATE OF RETURN WILL BE ACHIEVED

CHANCES THAT NET PRESENT VALUE WILL BE ACHIEVED

-10 0 5 10 15 20
-4m 0 2m 4m 6m 8m

RATE OF RETURN (%) NET PRESENT VALUE
action of the variables. The selected values are used in the calculation of a net present value (NPV), internal rate of return (IRR), and a benefit/cost ratio (B/C) for each simulation. This process including the working capital module is repeated 100 times and the final outcomes are profitability profiles or cumulative frequency distributions of NPV, IRR and B/C.

WORKING CAPITAL MODULE

An Overview

The module simulates the integration of working capital components into the capital investment (CI) process. Forecasting errors and inflation are introduced into the total investment planning process because they are the primary causes of working capital management problems. The objectives of the module are (1) to identify and measure the benefits and costs of investments in working capital components; (2) to identify the impact of forecasting errors and inflation on cash, inventory, receivables, payables and short-term borrowing; and (3) to measure the sensitivity of net present value (NPV) and internal rate of return (IRR) profiles to changes in WC strategies designed to offset forecasting errors and inflationary conditions.

Historically, WC activities are frequently revised, relatively routine and occur in a relatively short time period. Also the WC process is usually considered to be independent of the CI planning process. The management of short-run cash flows is a continuous and dynamic process occurring in an uncertain environment. However, because the CI model operates on an annual basis it is assumed, for convenience, that
the strategic planning of the WC cycle also operates on a one year horizon. This assumption was made to simplify the calculation of monthly financial statements and hold down the size of an already complex computer program. A shorter time period could be programmed to accommodate the needs of decision makers.

Cash Flow Crises

A cash flow crises often occurs when unexpected events arise, e.g., actual short-run expenditures being greater than forecasted and/or actual short-run cash inflows being less than forecasted. Surprise outflows can be related to a large price increase for raw materials. An unexpected decline in inflows arises when actual sales are less than forecasted or if there is an extension of the normal trade credit payment pattern. In both cases total cash outflows are frequently greater than total cash inflows plus existing cash items.

The standard approach to investment planning is to assume that the forecasted cash flows actually occur. For example, in solving for the NPV or IRR of an investment it is assumed the forecasted distributions of revenues and costs actually occurred. By assuming the profitability profiles generated from the forecasted inputs actually happen, the analysis misses the affect a short-run financial crisis has on cash flows. Because cash flow crises often arise when forecasting errors occur, the analysis of investment opportunities requires an additional step. A dual simulation process is introduced with one assumed to represent the forecasted results and the other the actual outcomes. In simulating these two sets of conditions, the model assumes sales are the key mechanism. It is assumed that the actual sales (S) are generated
by the CI program and a separate forecast of sales (SF) is produced by the WC module. Simulating sales conditions where the actual sales are randomly different from forecasted sales captures the essence of forecasting errors, which incorporates the major cause of WC problems.

If the forecasted sales exceed actual sales there will be a cash flow shortfall. To offset the shortfall, current asset and current liability components are adjusted by management. The cash flow shortfall is the heart of the problem related to a WC crisis. The model provides management a variety of short-run policy alternatives to offset the cash flow shortfall.

New Variables

In the WC module two new sets of probabilistic variables are introduced and combined with the variables in the CI model. One set represents three WC variables. These variables inject the uncertainty existing in the WC system into the total financial planning process. The three probabilistic variables are sales forecast (SF) in year 1, an annual growth rate of forecasted sales (G) related to the life cycle of the product, and trade credit/sales (TC) ratio that is related to the quantity of production. These variables are presented in Figure 1.

The second set of probabilistic variables represent the inflation dimension as it impacts on the firm. There are four separate rates of inflation. The stochastic inflationary variables serve as an adjustment to the values selected for price (P), interest (i_b, i_L), total purchases (PUR), total labor costs (LC), and fixed costs (FC). These four stochastic variables are presented in Figure 1.

Additionally, management determines a single value for each of the following variables:
cash

beginning cash
minimum cash
maximum cash (minimum cash plus marketable securities)

receivables

beginning receivables
bad debt allowance

inventory

beginning inventory
required ending inventory
maximum inventory cushion
cost of excess inventory

operating

gross margin for purchases
percent of marginal sales achieved
gross margin for additional purchases
marginal labor cost on marginal sales

interest rates

short term borrowing rate
short term lending rate

Each of these values are portrayed in Figure 1.

The working capital module is divided into three additional parts beyond the investment information presented in the CI module. The first module comprises the cost and income components. Also, inflation adjustments occur in this module. The second module involves the processes related to production and inventory. Finally, the module that links the total process together is the cash and trade credit system.
OPERATION OF THE WC MODULE

The objectives of each module and an explanation of its operation are presented in the following sections. A numerical example with accompanying comments concerning the operation of the module are found in Table 1. The example of the model in Table 1 integrates all of the modules in the total working capital-capital investment process (WC-CI). The following presentation is an explanation of the sequential operation of the WC module and is based on the data in Table 1.

Investment Information

The first operation of the model is the determination of actual and forecasted sales in both dollars and units. The initial price is provided by the user, which is $12.50 in our example in Table 1. A random market size of 4.81 million units is selected for year 1. Also, a market share of 17.92 percent is selected at random. Sales demand in year 1 is 862,219 units and is determined by multiplying the random market size times the market share. Sales in dollars, $10.77 million, are calculated by multiplying price times sales demanded in units. Sales forecast in dollars is randomly drawn from a distribution and sales forecast in units is calculated by dividing by price.

The difference between sales demand (S) and sales forecast (SF) reflects the forecasting error and is a critical value in the working capital management process. If forecasted sales are greater than the actual sales (SF > S), the firm did not achieve its sales forecast. Therefore, the sales demanded (Q) become the actual sales (AS). However, if sales demand is greater than forecasted sales (S > SF), unforecasted
or potential sales arise and it is assumed management will try to produce as much as possible to close this gap and meet these potential marginal sales. In the example in Table 1 management assumed 40 percent of potential marginal sales would be produced. Whenever $S_t > SF_t$, the marginal sales achieved are added to the sales demand to give actual sales (AS). The final operation of this module is the calculation of forecasted production (FPQ). Table 1 shows FPQ is calculated by adding the change in the amount of required ending inventory (units) to the sales forecast in units.

Previously capital investment theory has assumed that $S_t = SF_t$. A small contribution of the WC-CI model is to include a sales forecast variable and build on the assumptions that (1) $S_t \neq SF_t$, for $t = 1, \ldots, n$, (2) if $S_t > SF$ all marginal sales may not be achievable and there might be a cost premium for marginal sales that should be added to total costs of the investment, (3) also, when marginal sales are achieved, they generate cash inflows that were not anticipated when it was assumed $S_t = SF_t$, (4) if $SF_t > S_t$, there can be a cost of carrying additional inventory that should be added to the total cost of the investment, and cash inflows will be smaller than planned under conditions when $S_t = SF_t$.

**Costs and Income Module**

There are several key operations involved in the costs and income module. The three primary operating costs are purchases (PUR), labor (LC) and fixed (FC). It is assumed these variables are closely related to the forecasted production, i.e., quantity of goods produced. When $S_t > SF_t$, it is possible to have marginal production above the original forecasted level. When this condition occurs, the marginal cost of
purchases and labor are calculated separately and added to their respective forecasted costs for purchase and labor.

Increases in costs as a result of inflationary conditions are introduced in this module. The program randomly selects an inflation rate for purchases and this rate is applied to total purchases. The model also randomly selects and applies an adjustment to labor and fixed costs. Additionally, a random inflationary rate is injected into the price of the product. Thus, as shown in Table 1, actual sales are computed by multiplying the production quantity times the price which has been adjusted for inflationary pressures. When actual sales and costs are calculated, the model calculates the remaining components of the income statement.

The major cost items are calculated in this module and Table 1 provides an example of these calculations. First, for period 1, labor cost per unit of 2.52 was randomly chosen, and multiplied times forecasted production. The labor costs are increased for inflation, which was 5.35 percent in year 1. In this model inflation is assumed to be a random variable. When marginal sales are achieved, marginal labor costs are calculated in units of marginal sales. In Table 1, the marginal labor costs (MLCX) are 10 percent. Thus on the additional marginal sales achieved, the cost of labor was 10 percent higher than under normal operating conditions.

Fixed costs are assumed to be a random variable that is related to the sales demanded. The fixed costs are also increased by a separate inflation rate, 4.45 percent, that was selected randomly.
Purchase costs are calculated by multiplying the gross margin (.5) times the dollar value of forecasted production. The gross margin is a key input variable determined by the user. When $S_t > SF_t$, there are marginal purchases made to accommodate the marginal sales determined earlier. Normally, the gross margin for marginal sales ($GMP$) should be greater than the gross margin. It is .6 in Table 1. Additionally all purchase costs are adjusted by a randomly selected inflation value of 5.35 percent.

In year 1 the initial investment cost of $7.72$ million is randomly selected, and an annual straight line depreciation schedule of $857,778 is determined for nine year horizon. With all receipts and costs determined, earnings before interest and taxes (EBIT) of $758,065 are calculated. Next it is assumed the interest cost on short-term debt is paid the following year. Thus, in year 1, there is no interest cost, and earnings before taxes (EBT) equal EBIT. The assumed tax rate is 40 percent. Table 1 shows after the calculation of $303,226 in taxes, net income equals $454,839.

**Production-Inventory System**

This module focuses on the production-inventory process. Management has at its discretion a set of variables that may be used for controlling inventory limits. In this module management establishes single point estimates for each of the following production-inventory decision variables. (1) An estimate of the beginning inventory quality (BIQ); (2) A required ending inventory (REI) value which is an estimate of management's desired ratio of ending inventory (units)/sales forecast (units). The level of REI is related to the units of production in
period $t$. (3) A maximum inventory cushion (MIC) that management expresses as a percent above the required ending inventory quantity (REI). If the actual ending inventory ($\text{AEIQ}_t$) is greater than the maximum ending inventory ($\text{MEIQ}_t$), there is excess inventory ($\text{EXINV}$). (4) The cost of carrying excess inventory is a cost that arises when $\text{SF} > \text{S}$ thereby causing a cash cost for holding inventory in excess of the forecasted inventory needs. The cost of carrying excess inventory is assumed to be the cost of capital.

**Cash-Trade Credit System**

This module ties together receivables and the cash operations, including short-term borrowing or investing in marketable securities. The module collects all of the cash inflows and outflows which serve as inputs for calculating the net present value of the investment.

The primary decision variables available to management in controlling levels of cash and accounts receivable are: (1) The beginning value of cash ($\text{DC}$) and accounts receivable ($\text{AR}$); (2) A minimum cash level ($\text{CLOW}$) is determined as a ratio of cash/sales forecast ($\text{CMIN}$), which depends on the levels of production; (3) A maximum cash level ($\text{MAXCASH}$) that is expressed as a percent above the minimum cash balance ($\text{CLowe}$). $\text{MAXCASH}$ is a discretionary variable that can range from zero to a large whole number. If ending cash ($\text{EC}$) falls between $\text{CLOW}_t$ and $\text{MAXCASH}_t$, the difference between $\text{EC}_t$ and $\text{CLOW}_t$ is invested in marketable securities ($\text{MS}_t$). (4) The sum of minimum cash balances ($\text{CLOW}_t$) and marketable securities ($\text{MS}_t$) equals the maximum cash position in a specific time period ($\text{CMAX}$). There are three interest rate variables in the model: (5) Interest rate on short term borrow funds ($i_b$); (6) Interest rate
on lending \( (i_L) \); (7) Cost of capital \( (k) \). The cost of capital is the discount rate in the present value equation.

The model assumes there is a controlled stochastic relationship between receivables and sales and between payables and sales. The ratios of receivables/sales \( (AR/S) \) and payables/sales \( (AP/S) \) serve as proxies for these two relationships. These two distributions are crucial management inputs in establishing trade credit policy. It is assumed management perceives a probability distribution of \( AR/S \) and \( AP/S \) ratios, and it wishes to control the level of each one within a specified range of sales. The difference between these two trade credit variables \( (AR/S - AP/S) \) is the input used to calculate the net for accounts receivable. In Table 1 a trade credit/sales \( (TC/S) \) ratio of .3535 was randomly chosen for year 1. Multiplying it times actual sales \( (AS_1) \) gives accounts receivable \( _1 \) of $3.7 million. After deducting for bad debts \( t-1 \) cash receipts \( (CR) \) are determined by adding to actual sales \( t \) the change in accounts receivable between period \( t \) and \( t-1 \). \( CR_1 \) equal $6.78 million.

In Table 1 cash equals zero at the beginning of year 1. For subsequent years beginning cash \( t \) equals ending cash \( t-1 \), which includes cash plus marketable securities. The total cash available \( t \) for meeting the cash payments \( t \) equals cash receipts \( (CR_t) \) plus beginning cash \( (BC_t) \) plus interest earned on marketable securities \( (INT) \) minus any capital investment costs \( FUTCOST \).

Total cash payments equals the sum of all cash outflows which are shown in Table 1. All outflows occur in the year the cost was incurred except for interest costs which are assumed to be paid in the next period.
after the borrowing occurred. If cash payments \( t \) are greater than cash available \( t \), the company will have to borrow short-term to meet the total cash payment. Under these conditions, ending cash equals the lower boundary (CLOW) prescribed by management. Table 1 shows this was the case in year 1. Ending cash is a complex variable that is explored further in the next few paragraphs.

If cash payments \( CPAY_{t} \) were greater than cash available \( CBAL_{t} \), it is necessary to borrow enough short-term funds to cover the cash shortfall in order to maintain the minimum level of cash. When \( CBAL_{t} > CPAY_{t} \), and there is short term (S/T) debt outstanding, the difference between \( CBAL_{t} - CPAY_{t} \) is used to retire debt and also maintain the minimum cash balance. The model will continue to retire debt in future years when \( CBAL > CPAY \). Once the S/T debt is retired and \( CBAL_{t} > CPAY_{t} \), the model will invest the idle cash balances that exist above the cash lower boundary. Thus, the model assumes the investment is either borrowing or lending in any given period. Of course, it is possible that neither could occur, but it is highly unlikely. The model will invest up to CMAX in any year. Any cash above CMAX is included in cash receipts and is assumed to be earning the cost of capital.

When there is cumulative S/T debt outstanding, an interest cost is incurred at the short term rate \( i_b \). The interest costs became a cash outflow the following period. If there are investments in marketable securities, they earn at the lending rate \( i_l \). Any cash earned on marketable securities becomes cash available in the following period. Traditional analysis assumes investment is financed by long term sources and, furthermore, it assumes the debt/equity max is relatively stable
in the long-run. It does not explicitly allow for a cash flow shortfall which results in an increase in short term debt. If a prolonged working capital crisis was financed by short-run financing, one serious outcome could be a radical change in a company's debt/equity mix. The chances of a short-term debt surge is quite plausible in an inflationary environment or a period of high interest rates. Finally the traditional approach does not consider a cash flow overflow, where liquid assets would earn less than the cost of capital. Thus explicitly introducing interest cost on short term borrowing and interest earned on marketable securities is an addition to traditional investment analysis.

PRESENT VALUE MODULE

Traditional Analysis

In traditional analysis cash receipts were equal to the inflow of funds from actual sales, and cash payments included initial and future investment costs, price adjusted labor cost, total purchases and fixed costs, plus taxes. In equation form, the net present value (NPV) of investment proposal \( A \) was:

\[
NPV_A = -IC_0 + \frac{AS_1 - TCO_1}{(1+k)^1} + \frac{AS_2 - TCO_2}{(1+k)^2} + \ldots + \frac{AS_N - TCO_N + SV_N}{(1+k)^N} \tag{1}
\]

Where

- \( IC_0 \) = investment cost at the beginning of the period;
- \( AS_t \) = total cash inflow from actual sales or cash inflow in each period;
- \( TCO_t \) = total cash outflows in each period which included the following:
  - \( TLC_t \) = total price adjusted labor costs, or \( PALC_t + PAMLC_t \)
  - \( TPC_t \) = total price adjusted purchases, or \( PAPUR_t + PAMPUR_t \)
\[ PAFC_t = \text{price adjusted fixed costs}; \]
\[ T_t = \text{taxes}; \]
\[ \text{FUTCOST}_t = \text{future investment costs}; \]

Other variables included are:

\[ SV_n = \text{salvage value of the investment} \]
\[ k = \text{cost of capital} \]

Revised Analysis

An objective of this model is to provide a more comprehensive investment analysis framework by explicitly integrating the working capital operations into the capital investment process. Forecasting errors and inflation may cause cash flows from the revised model to vary markedly from flows generated by the traditional model. Short-term borrowing and/or the sale of marketable securities are used to offset a cash flow shortfall or the retirement of debt. Marketable securities absorb an overflow of cash above the minimum level but below the maximum level. Also changing required inventory levels, minimum cash levels or payment patterns of receivables and payables can alter shortfalls or overflows. Thus an objective of the revised model is to measure the size and sign of the cash flows and thereby make it possible to highlight the cost and benefits related to the working capital strategies. The significance of these working capital strategies on the net present value of an investment are explored in the following paragraphs.

First, declining business conditions and/or a prolonged inflationary environment can change payment behavior and dramatically alter the inflow of funds from receivables. Thus a lengthening of payments on receivables that was not forecast can result in a significant cumulative shortfall.
in the actual cash inflows from an investment. During this period if the delays in receivable payments are not offset by a stretching of payables, the net cash flows will be further reduced. If cash flows are negative short-term borrowing will occur after cash and marketable securities are reduced to their minimums. The borrowing costs are determined in the revised model and are reflected as a cash outflow. Alternatively, an acceleration of receivable inflows without a change in the payment of payables can expand the net cash flow during that period. Excess idle cash balances are invested and generate cash benefits for the investment.

Second, an adjustment in taxes is required because in the traditional models, short-term interests cost or benefits were not permitted. Thus, EBIT is increased when interest income is received which makes EBT higher than in the traditional case. Also, in the revised model, interest costs cause EBT to be lower than in the traditional model. Thus, tax outflows in the revised model differ from taxes in the traditional model.

Third, carrying excess inventory produces a holding cost. This cost enters the revised NPV equation as a cash outflow. This cost only occurs when inventory exceeds the upper control limit.

Fourth, another big actor in the revised model relates to cash management policies and operations. The change in the ending cash account \( EC_t \) between period \( t \) and \( t-1 \), can produce a substantive change in cash inflow or outflow. When \( EC_t > EC_{t-1} \), it indicates an increase in the level of cash held and a reduction in cash available for meeting cash payments, which results in a lower net cash flow. The opposite
effect occurs if $EC_t < EC_{t-1}$. There are several refinements related to ending cash and they are presented in Appendix 1.

Finally, positive cash flows $t$ are used to retire accumulated short-term borrowing $t$ ($SUMCX_t$), thereby reducing cash flows available for reinvestment, as shown in Appendix 1. When negative cash flows $t$ happen, short-term borrowing will occur after liquid assets are reduced to a minimum. The result is $SUMCX_t > SUMCX_{t-1}$. Appendix 1 presents the logic of these two cases.

In comparing the revised cash flow (RCF) model to the traditional cash flow (CF) approach, it is necessary to refer to the equation for each one on the last page of Table 1. The RCFs differ from the CFs for the following reasons. First, the contribution of trade credit policies and bad debts are reflected in cash receipts (CR). Additionally, forecasting errors and inflation cause changes in the level of cash ($EC_t - EC_{t-1}$) and short-term borrowing ($SUMCX_t - SUMCX_{t-1}$). Also the cost of carrying excess inventory because of forecasting errors enter the NPV equation as an outflow. Each of these variables along with the interest return from marketable securities and the interest cost of short-run borrowing are appropriately included in the revised cash flows.

If the sum of the costs $t$ are greater than the receipts $t$, and liquid assets are at a minimum, short term borrowing is employed to offset the amount of the negative cash flow. The rationale for this modeling assumption is that after the initial investment costs (IC) are incurred, any additional costs are related to an investment in working capital components or delayed capital expenditures (PUTCOST$_t$); in the event of a shortfall, short-term borrowing (STD) makes up this exact difference.
Increases in the level of short-term borrowing \((\text{SUMCX}_t - \text{SUMCX}_{t-1})\) to offset a shortfall are included in the RCF equation as an inflow, therefore when \(\text{outflow}_t > \text{inflow}_t\) additions to \(\text{SID}_t\) equal the shortfall and the RCF is recorded as a zero. Table 1 illustrates this concept.

In the traditional model a cash flow (CF) shortfall is recorded as a negative value, e.g., year 1 in Table 1.

By making the working capital components explicit and introducing forecasting errors, the revised model can identify and measure the cost of the shortfall, which was not previously accomplished. When a positive cash flow exists, short-term borrowing is paid off before any positive cash flows are available for discounting. When borrowing occurs the RCFs are limited on the down side to zero; when positive flows occur the RCFs are limited on the top side by the retirement of debt, an increase in the level of cash or an increase in other working capital components. In conclusion, the RCFs operate in a more narrow band than the CF from the traditional model.

For comparative purposes the model calculates the traditional \((\text{CF}_t)\) and revised cash flows \((\text{RCF}_t)\) for each period in the life of the investment. This provides management an invaluable source of information to compare the \(\text{CF}_t\) profile to the \(\text{RCF}_t\) profile. Because of the large number of possible combinations and permutations, interpretation of these data should be done with care. The size of the gap between \(\text{CF}_t\) and \(\text{RCF}_t\) profiles, reflects the affect working capital components and strategies have on the profitability of an investment. For example, when the \(\text{CF}\) profile is positive and greater than the \(\text{RCF}\) profile, a narrow gap shows working capital components have a limited
affect on the value of the investment. However, a large gap indicates the importance of working capital components and strategies in determining the value of the investment. An analysis of these wide differences can aid management in reevaluating the forecasted inputs, the forecasting errors and the working capital strategy that is creating the gap. There are many possible combinations of inputs that cause a gap. The model produces the necessary data to identify the variable(s) causing the problem.

Cost of Capital

In this model it is assumed management provides a single value it prefers to use as the initial cost of capital ($k_0$). However, the impact of inflation must be included in the cost of capital, $k_t$ [41]. The cash inflows and outflows in the numerator of the NPV equation are adjusted for inflation. The price of the goods sold in period 2 ($P_2$) is calculated in the following manner as shown in Table 1: $P_2 = P_1(1 + \tilde{P}_A)$, where $\tilde{P}_A$ is a randomly selected value for inflation drawn from a distribution established by management. The inflation adjustment to the cost of capital is the change in the perceived rate of inflation between the current and most recent time periods, $\tilde{P}_A - \tilde{P}_A_{t-1}$. This adjustment for inflation is presented below in equation form, which reflects the real cost of capital:

$$\begin{align*}
(1 + k) &= (1 + k_t)(1 + IF) \\
where \quad IF &= \tilde{P}_A - \tilde{P}_A_{t-1}
\end{align*}$$
The PA are price adjustment coefficients randomly selected from a distribution created by management. Using the change in the perceived rate of inflation between periods more closely approximates the behavior of financial markets [14, pp. 411-414], than an expected rate of inflation for a long time horizon [41]. The essence of the adjustment assumes a positive change in the rate of inflation will increase the cost of capital from the preceding period; however, if the rate of inflation decreases, the cost of capital will subsequently decrease. This would not occur if the adjustment process assumed a constant rising mean perceived rate of inflation as suggested by Nelson [41].

By allowing the cost of capital to change each year, the inflation adjusted net cash flows in the numerator are discounted with a different $k$ in each period. In this simulation model the cost of capital is a probability distribution, and a cost of capital profile is created. The task of evaluating the profile of NFVs of an investment is complicated by not having a common cost at capital. Management must interpret the statistical properties of the NFVs, especially the mean, standard deviation, skewness and kurtosis when comparing separate simulations of the same alternatives or different alternatives. Furthermore, there is no longer a single cost of capital to serve as a benchmark for which the internal rates of return (IRR) can be compared. Previously if $IRR > k$ the investment was acceptable or rejected if $IRR < k$. In this simulation model, the profile of 100 IRR's are compared to the profile of the 100 $k$'s and the judgment of the user is needed to determine if the investment is acceptable.
CONCLUSION

The WC-CI model extends the traditional capital investment model and provides management a tool to test the sensitivity of an investment's profitability to changes in working capital strategies. Forecasting errors and inflationary conditions are shown to be the primary causes of working capital problems. These working capital strategies are designed to offset forecasting errors and inflation. The model aids management in finding the best possible mix of strategies to generate the highest possible values of an investment.
REFERENCES


56. James E. Townsend, "Abstract - A Multivariate Analysis of Relation-


<table>
<thead>
<tr>
<th>YEAR</th>
<th>VARIABLES</th>
<th>INVESTMENT MODULE</th>
<th>WORKING CAPITAL-CAPITAL INVESTMENT MODEL</th>
<th>AN EXAMPLE OF THE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.50</td>
<td>4.80</td>
<td>PS (PSP)</td>
<td>TABLE 1</td>
</tr>
<tr>
<td>2</td>
<td>13.44</td>
<td>4.91</td>
<td>PS (PSP)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15.33</td>
<td>5.05</td>
<td>PS (PSP)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>17.69</td>
<td>5.20</td>
<td>PS (PSP)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>19.44</td>
<td>5.35</td>
<td>PS (PSP)</td>
<td></td>
</tr>
</tbody>
</table>

PS = $0.00
SP = $0.00

$0.00 = $0.00

Note: The table represents a snapshot of a financial model, likely related to sales forecast and market share calculations. The variables include sales forecast, market share, and other financial metrics.
### TABLE 1

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>YEAR</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Marg Sales Achiev (MSA)</td>
<td>($)</td>
<td>193,344</td>
</tr>
<tr>
<td>Marg Sales Achiev* (MSAQ)</td>
<td>(u)</td>
<td>15,388</td>
</tr>
<tr>
<td>Actual Sales (AS)</td>
<td>($)</td>
<td>10,489,218</td>
</tr>
<tr>
<td>Actual Sales* (ASQ)</td>
<td>(u)</td>
<td>839,137</td>
</tr>
<tr>
<td>Forecast Prod (FPQ)</td>
<td>(u)</td>
<td>922,600</td>
</tr>
</tbody>
</table>

**INVESTMENT MODULE**

If SF > S: MSA = 0; If S > SF: MSA = PS<sub>t</sub>(PSA<sub>t</sub>);

PS<sub>A</sub> = .4, PSA = Potential Sales Achieved.

MSAQ<sub>t</sub> = MSA<sub>t</sub>/P<sub>t</sub>

If SF > S: AS<sub>t</sub> = S<sub>t</sub>; If S<sub>t</sub> > SF<sub>t</sub>;

AS<sub>t</sub> = (SFQ<sub>t</sub> + MSAQ<sub>t</sub>)P<sub>t</sub>

ASQ<sub>t</sub> = AS<sub>t</sub>/P<sub>t</sub>

FPQ<sub>t</sub> = SFQ<sub>t</sub> + (REIQ<sub>t</sub> - AEIQ<sub>t-1</sub>); where

REIQ is required ending inventory and AEIQ<sub>t-1</sub>
is actual required ending inventory given below.

These concepts are presented in the inventory module.

**COST AND INCOME MODULE**

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>YEAR</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($)</td>
<td>2,324,951</td>
</tr>
</tbody>
</table>

LC<sub>1</sub> = LC<sub>1</sub> per unit (FPQ<sub>1</sub>); where LC<sub>1</sub> = 2.52;

where LC<sub>2</sub> = 2.2799 and BIQ = beginning

inventory = 98,850 (u)
### TABLE 1

**COST AND INCOME MODULE**

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>YEAR</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infl Adj Lab Cost (PALC) ($)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>- 2,449,334</td>
<td>1,990,566</td>
<td></td>
</tr>
<tr>
<td>Marg Labor Cost (MLC) ($)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>- 42,654</td>
<td>203,609</td>
<td></td>
</tr>
<tr>
<td>Infl Adjust (PAMLIC) ($)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>- 44,936</td>
<td>216,029</td>
<td></td>
</tr>
<tr>
<td>Fixed Cost (FC) ($)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>- 175,000</td>
<td>215,000</td>
<td></td>
</tr>
<tr>
<td>Infl Adjust (PAFC) ($)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>- 182,787</td>
<td>216,029</td>
<td></td>
</tr>
<tr>
<td>Purchase Cost (PUR) ($)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>- 5,766,248</td>
<td>5,576,087</td>
<td></td>
</tr>
<tr>
<td>Infl Adjust (PAPUR) ($)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>- 6,074,737</td>
<td>6,019,381</td>
<td></td>
</tr>
</tbody>
</table>

PALCₜ = LCₜ(1 + PALₜ); where PAL is inflation rate for labor cost; PAL₁ = .0535; PAL₂ = .061.

If $Sₜ > SFₜ$: MLCₜ = MSAQₜ(LCₜ per unit × 1 + MLĈX); MLĈX = Marginal labor cost per unit above LC per unit. MLĈX = .10. If $Sₜ < SFₜ$: MLC = 0.

If $Sₜ < SFₜ$: PAMLICₜ = 0; If $Sₜ > SFₜ$: PAMLICₜ = MLCₜ(1 + PALₜ)

FCₜ = related to SFₜ.

PAPFCₜ = FCₜ(1 + PAFₜ); where PAFₜ is inflation rate for fixed cost. PAF₁ = .0445; PAF₂ = .064.

PURₜ = GM(FPQₜ x Pₜ) where GM is gross margin. GM = .5.

PAPURₜ = PURₜ(1 + PAPₜ); where PAPₜ is inflation rate for purchases. PAP₁ = .0535; PAP₂ = .0795.
<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>YEAR</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marg Purch Cost (MPUR)</td>
<td>($</td>
<td>115,406 654,667</td>
</tr>
<tr>
<td>- Infl Adj (PAMPUR)</td>
<td>($</td>
<td>121,580 7,076,712</td>
</tr>
<tr>
<td>Operating Income (OI)</td>
<td>($</td>
<td>1,615,843 4,317,355</td>
</tr>
<tr>
<td>Investment Cost (IC)</td>
<td>($</td>
<td>7,720,000 0</td>
</tr>
<tr>
<td>Deprec (DEP)</td>
<td>($</td>
<td>857,778 857,778</td>
</tr>
<tr>
<td>Book Value (BV)</td>
<td>($</td>
<td>6,862,222 6,004,444</td>
</tr>
<tr>
<td>EBIT</td>
<td>($</td>
<td>758,065 3,459,477</td>
</tr>
<tr>
<td>Interest (INT)</td>
<td>($</td>
<td>0 241,050</td>
</tr>
</tbody>
</table>

If $S_t > SF_t$: $\text{MPUR} = (\text{MSAQ}_t \times P_t)_{\text{GMP}}$;

GMP = .6, Gross margin for marginal purchases.

If $SF_t > S_t$: $\text{MPUR} = 0$.

\[\text{PAMPUR}_t = \text{MPUR}_t \times (1 + \text{PAP}).\]

\[\text{OI}_t = \text{AS}_t - \text{PALC}_t - \text{PAMLC}_t - \text{PAFC}_t - \text{PAPUR}_t - \text{PAMPUR}_t.\]

$I_{C_0}$ is a randomly selected variable.

\[\text{DEP}_1 \text{ is straight line depreciation; } \text{DEP}_t = \frac{I_{C_t}}{N}; \text{ where } N = 9 \text{ years.}\]

\[\text{EV}_1 = I_{C_0} - \text{DEP}_1; \text{ EV}_2 = \text{EV}_1 - \text{DEP}_2.\]

\[\text{EBIT}_t = \text{OI}_t - \text{DEP}_t.\]

\[\text{INT}_t = i_b (\text{cumulative } S/T \text{ borrowing}_{t-1}); i_b \text{ is borrowing rate of interest, } i_b = .08;\]

For $t_2, \ldots, t_n$, $i_b = i_{b_{t-1}} (\text{PA}_t)$; cumulative $S/T$ borrowing$_{t-1}$ given on the last page.
<table>
<thead>
<tr>
<th></th>
<th>($)</th>
<th>($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEIO</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>XIC</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MRC</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BGO</td>
<td>110,605</td>
<td>118,620</td>
</tr>
<tr>
<td>SPC</td>
<td>966,266</td>
<td>987,969</td>
</tr>
<tr>
<td>BGD</td>
<td>984,850</td>
<td>0</td>
</tr>
<tr>
<td>REL1</td>
<td>92,711</td>
<td>94,850</td>
</tr>
</tbody>
</table>

**Production-Inventory Model**

\[
\begin{align*}
\text{IN} & = \text{ERL} - \text{IN} \\
\text{ERL} & = \text{TR(ERL)}; \text{where TR is tax rate; TR} = 0.45 \%
\end{align*}
\]

**Net Income (IN)**

\[
\begin{align*}
\text{IN} & = 494,839 \\
\text{TR} & = 1.531,116
\end{align*}
\]

**Sales (I)**

\[
\begin{align*}
\text{I} & = 758,065 \\
\text{TR} & = 3.218,526
\end{align*}
\]

**Comments**

**Variables**

**Table 1**

<table>
<thead>
<tr>
<th>Year</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 1

**CASH-TRADE CREDIT SYSTEM**

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>YEAR</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>MAX Cash + MS Level (CMAX) ($)</td>
<td>741,374</td>
<td>743,268</td>
</tr>
<tr>
<td>Short Term Debt (STD) ($)</td>
<td>3,013,131</td>
<td>0</td>
</tr>
<tr>
<td>Payment S/T Debt (PAY) ($)</td>
<td>0</td>
<td>1,127,981</td>
</tr>
<tr>
<td>Cum S/T Debt (SUMCX) ($)</td>
<td>3,013,131</td>
<td>1,885,150</td>
</tr>
<tr>
<td>- Interest Cost (INT) ($)</td>
<td>241,050</td>
<td>161,160</td>
</tr>
<tr>
<td>S/T LEND (Mkt Sec) (MS) ($)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>- Interest Benefits (BEXC) ($)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cash CT MAX Cash + MS (CEX) ($)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
\[
9_{\text{##}} = \text{AVT (CASH) + INT)}
\]

\[
\text{cash lower bound (CLOW)}
\]

\[
6,175,925.281
\]

\[
(\$)
\]

\[
\text{end cash + NS bal (EC)}
\]

\[
6,175,925.281
\]

\[
(\$)
\]

\[
\text{total cash pay (CPAY)}
\]

\[
11,76,600
\]

\[
(\$)
\]

\[
\text{cash available (CALT)}
\]

\[
6,175,925.281
\]

\[
(\$)
\]

\[
\text{begin cash + NS bal (BC)}
\]

\[
6,175,925.281
\]

\[
(\$)
\]

\[
\text{cash receipts (CR)}
\]

\[
6,175,925.281
\]

\[
(\$)
\]

\[
\text{bad debt allowance (BAD)}
\]

\[
3,707,974
\]

\[
(\$)
\]

\[
\text{net acc. rec. bal. (AR)}
\]

\[
4,467,317
\]

\[
(\$)
\]

\[
\text{trade credit/ sales (TC)}
\]

\[
3,393.5
\]

\[
(\%)
\]

\[
\text{comments}
\]

\[
\text{TC is a random variable and proxies for}
\]

\[
\text{T}
\]

\[
\text{variable}
\]

<table>
<thead>
<tr>
<th>YEAR</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASH</td>
<td>TRADE CREDIT SYSTEM</td>
<td></td>
</tr>
</tbody>
</table>

Table 1
<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>YEAR</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow (CF)</td>
<td>($)</td>
<td>( \text{CF}_t = \text{CR}_t - \text{PAlC}_t - \text{PAMLC}_t - \text{PAPUR}_t - \text{PAMPUR}_t )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \text{PROF}_t - \text{T}^* )</td>
</tr>
<tr>
<td>Revised Cash Flow (RCF)</td>
<td>($)</td>
<td>( \text{RCF}_t = \text{CF}<em>t + \text{T}^* - \text{T}<em>t + \text{REXC}</em>{t-1} - \text{INT}</em>{t-1} - )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \text{EC}<em>t - \text{EC}</em>{t-1} + (\text{SUMCX}<em>t - \text{SUMCX}</em>{t-1}) - \text{CEXINV}_t )</td>
</tr>
<tr>
<td>Cost of Capital (k)</td>
<td>(%)</td>
<td>( k_t = k_{t-1} + (\text{PA}<em>t - \text{PA}</em>{t-1}) )</td>
</tr>
</tbody>
</table>

* = \( T^* \) is taxes on EBIT and arises in the traditional model. It is assumed there is no short-term debt to meet a cash flow shortfall. Thus no interest costs (INT). Also in the traditional model it is assumed there is no short-term investment in marketable securities to accommodate a cash flow overflow, thus no income from short-term lending (MS).
APPENDIX 1

DETERMINING NET CASH FLOW FROM INVESTMENT AFTER ADJUSTING FOR ENDING CASH AND S/T DEBT RETIREMENT OR ADDITION

Legend

CBAL = Total cash inflow
CPAY = Total cash outflow
CEX = Net cash flow in excess of cash minimums
UNDISX = Net cash flow from investment after adjusting for ending cash and s/t debt retirement or addition
EC = ending cash plus m.s. balance
BEXC = Interest benefit from s/t investment
INT = Interest cost on s/t debt
CMAX = Maximum level of cash plus m.s. in $
CMAXir = CMIN × (1.0 + MAXCASH)
CR = Cash receipts
FUTCOST = Any future investment cost
CEINV = Cost of excess inventory
CLOW = Minimum cash balance
SUMCX = Cumulative s/t debt

\[
\text{CBAL}_t = \text{CR}_t - \text{FUTCOST}_t + \text{EC}_{t-1} + \text{BEXC}_{t-1}
\]

\[
\text{CPAY}_t = \text{fixed cost}_t + \text{labor cost}_t + \text{purchase cost}_t + \text{Taxes}_t + \text{CEINV}_t + \text{INT}_{t-1}
\]

\[
\text{CEX}_t = \text{CBAL}_t - \text{CPAY}_t - \text{CMAX}_t
\]

If \( \text{CBAL}_t - \text{CPAY}_t < \text{CLOW}_t \), \( \text{CEX} = 0 \)

When \( \text{CBAL}_t - \text{CPAY}_t < \text{CMAX}_t \), \( \text{UNDISX}_t = 0 \)

\[
\text{UNDISX}_t = \text{CBAL}_t - \text{CPAY}_t - (\text{EC}_t + \text{SUMCX}_{t-1} - \text{SUMCX}_t)
\]

Case 1. When \( \text{CBAL}_t - \text{CPAY}_t < \text{CLOW}_t \)

\[
\text{EC}_t = \text{CLOW}_t
\]

\[
\text{SUMCX}_t = \text{SUMCX}_{t-1} + \text{CLOW}_t - (\text{CBAL}_t - \text{CPAY}_t)
\]
\[ EC_t + SUMCX_{t-1} - SUMCX_t = CBAL_t - CPAY_t \]

\[ UNDISX_t = 0 \]

**Case 2.** When \( CLOW_t < (CBAL_t - CPAY_t) < CMAX_t \)

and \( CBAL_t - CPAY_t < SUMCX_{t-1} \)

\[ EC_t = CLOW_t \]

\[ SUMCX_t = SUMCX_{t-1} - (CBAL_t - CPAY_t - CLOW_t) \]

\[ SUMCX_{t-1} - SUMCX_t = CBAL_t - CPAY_t - CLOW_t \]

\[ EC_t + SUMCX_{t-1} - SUMCX_t = CBAL_t - CPAY_t \]

\[ UNDISX_t = 0 \]

**Case 3.** When \( CLOW_t < (CBAL_t - CPAY_t) < CMAX_t \)

and \( SUMCX_{t-1} < CBAL_t - CPAY_t \)

\[ SUMCX_t = 0 \]

\[ EC_t = CBAL_t - CPAY_t - SUMCX_{t-1} \]

\[ EC_t + SUMCX_{t-1} - SUMCX_t = CBAL_t - CPAY_t \]

\[ UNDISX_t = 0 \]

**Case 4.** When \( CMAX_t < CBAL_t - CPAY_t \),

\[ UNDISX_t = CBAL_t - CPAY_t - CMAX_t = CE_0, \text{ and} \]

when \( CMAX < CBAL_t - CPAY_t \) and \( SUMCX_{t-1} < CBAL_t - CPAY_t \):
\[ EC_t = CMAX_t \quad SUMCX_t = 0 \quad SUMCX_{t-1} = 0 \]

\[ EC_t + SUMCX_{t-1} - SUMCX_t = EC_t \]

\[ UNDISX_t = CBAL_t - CPAY_t - CMAX_t = CEX_t \]