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Detection of Rate Changes in Periodic Phenomena

R. A. Avner

UNIVERSITY OF ILLINOIS COMMITTEE ON SCHOOL MATHEMATICS

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Preface

This report is being published simultaneously as a Research Report of the UICSM Mathematics Project and as a report of the Coordinated Science Laboratory. It represents a portion of the results of the UICSM study of physiological correlates of human learning. The material covered in this report supersedes a less general approach to the problem which was treated in an appendix and an addenda to CSL report R-198, Heart Rate Correlates of Insight. A more powerful group of tests is made available by this general approach and limitations and assumptions of the previous methods have been made more explicit.

The most straightforward measure of a periodic phenomenon is the length of time between successive phenomena, i.e. the "period". Given the period and the time of occurrence of any one event of a sequence of periodic events, the exact time of occurrence of any other event is completely determined. Similarly, a change in period is a direct indication of a change in its reciprocal, the rate of events. Unfortunately, in situations where large numbers of events are observed, the detection, recording, and analysis of between-event times for the purpose of determining the presence and magnitude of rate changes may be impractical. An alternative is the counting of the number of events, k_n, happening in the nth of N time intervals of equal duration, t. The expected value of k, E(k), is then estimated by

$$\bar{k} = \frac{\sum_{n=1}^{N} k_n}{N}$$

the rate of events per unit time, ρ, by

$$r = \frac{\sum_{n=1}^{N} k_n}{Nt}$$

and the period, $\frac{1}{\rho}$, by $\frac{1}{r}$. For $t > \frac{1}{\rho}$, $\bar{k} > 1$, that is, when the duration of the examination interval is longer than the time between successive events, the average number of events counted within each examination interval will exceed unity. As a result, fewer measures need be recorded than if each between-event time was measured. This economy in information storage is offset to some extent by a loss in the precision with which the moment of occurrence of a rate change may be detected. An optimum time interval would be long enough to reduce the total number of items of information recorded but short enough to allow points of rate change to be located within desired limits of precision.

An immediate consequence of sufficiently large changes in period (and hence in rate) will be changes in the number of events within time intervals, e.g. if 10 events occur in one interval and 25 in another, a rate change might be assumed to have occurred. The objective of this paper is to show with what probability differences between numbers of observed events within a pair of time intervals may be ascribed to specified changes in rate of a periodic phenomenon. In the example given above it might be required to say with what probability the observed difference of (25 - 10) = 15 events might be ascribed to an actual rate change of 0, 5, 10, 12, etc., events per interval. Such measures are valid for the class of phenomena, such as human heart rate, for which magnitude of rate change is relatively unrelated to base rate (Kaelbling, King, Achenbach, Branson, and Pasamanick (1960), Avner (1964)).

Distribution Model

Two general situations in which pairs of time intervals might be compared will be considered; (I) pairs of nonadjacent intervals, and (II) pairs of adjacent intervals. Basic assumptions which will be made are; (1) all events are of equal duration, (2) events occur at some periodic rate which is uniformly distributed over the range of observed rates, and (3) changes from this rate are instantaneous and always to another periodic rate. The number of events in each interval is then completely determined by; (a) the time of occurrence of at least one event in each interval (or in either interval for the adjacent interval case), (b) the time of occurrence of all rate changes within the two intervals, and (c) the initial and final rates for the first rate change in each interval (or in the first interval alone for the adjacent interval case) and final rates for all changes in both intervals thereafter. Three additional assumptions are made which simplify the situation while not severely limiting the generality of the findings. These additional assumptions are; (4) events are of zero duration, i.e. are impulses, (5) no more

than one rate change is expected in any two intervals chosen, and (6) rate changes occur between the two intervals, i.e. at the boundary between adjacent intervals or after the first and before the second of two nonadjacent intervals. Assumption 5 requires that the duration of intervals be chosen small enough that the probability of a rate change in any given interval will be arbitrarily small. The lower bound of interval duration has, however, been set at $\frac{1}{\rho}$ for reasons of recording economy. Use of the method outlined in this paper is not recommended when rate changes occur with frequencies approaching ρ. Assumption 6 is made since this is the condition under which maximum effects of differences in event rates will be expected. The effects of this assumption will be examined in detail at a later point.

In the general case of periodic events of zero duration occurring within finite intervals of equal duration, the number of events, k, observed in any given interval will be one of the integers A or $A + 1$, such that $A \leq E(k) < A + 1$ where $E(k)$ is the expected value of the number of events per interval. It follows that the probability of a given interval containing A or $A + 1$ events will be given by

(1) $P(A|E(k)) = [1 - E(k) + A]$

(2) $P(A + 1|E(k)) = 1 - P(A|E(k)) = [E(k) - A]$

Except where $E(k) = A$, there will always be cases of adjacent intervals differing by one event when events are actually occurring at a constant rate. Given some initial value, $E(k_1)$, and some new value, $E(k_2)$, of $E(k)$, the probability, $P[(k_2 - k_1)|E(k_1), E(k_2)]$, of the observation of any given differences in number of events between an interval for which $E(k) = E(k_2)$ and an interval for which $E(k) = E(k_1)$ may be easily derived.

I. Nonadjacent interval case. In the case of nonadjacent intervals an unknown number of rate changes can occur at unknown times between the two intervals. An initial level $E(k_1)$ exists in the first interval producing either A_1 or $(A_1 + 1)$ events while a final level $E(k_2)$ exists in the last interval producing either A_2 or $(A_2 + 1)$ events. For given values of $E(k_1)$ and $E(k_2)$, probabilities of actual events are computed by equations (1) and (2). Since $E(k_2)$ may be the resultant of a number of unknown changes between the two intervals, $P(A_1)$ or $P(A_1 + 1)$ are independent of $P(A_2)$ or $P(A_2 + 1)$ and the probability of a given difference would be the sum of the probabilities of all ways of obtaining that difference. For example, if A_1 events occur in the first interval and $(A_2 + 1)$ events occur in the last interval, the observed difference is $(k_2 - k_1) = [(A_2 + 1) - A_1]$. This difference will occur with probability;

$$P[(k_2 - k_1) = (A_2 - A_1 + 1) | E(k_1), E(k_2)] = P(A_1)P(A_2 + 1)$$

Probabilities of all other differences are similarly derived. If $(k_2 - k_1) = D$,

(3)a $P(D < A_2 - A_1 - 1) = 0$

b $P(D = A_2 - A_1 - 1) = P(A_1 + 1)P(A_2)$

c $P(D = A_2 - A_1) = P(A_1)P(A_2) + P(A_1 + 1)P(A_2 + 1)$

d $P(D = A_2 - A_1 + 1) = P(A_1)P(A_2 + 1)$

e $P(D > A_2 - A_1 + 1) = 0$

Note that the range of D is $(A_2 - A_1) \pm 1$. Since a difference measure is used, the parameters $E(k_1)$ and $E(k_2)$ may be replaced respectively by σ and $(\sigma + \Delta')$ where

(4) $\sigma = [E(k_1) - A_1]$, and

(5) $\Delta' = [E(k_2) - E(k_1)]$, the rate change in events per interval,

with equation (3) holding as before. With this substitution the probability of a given difference in events between two nonadjacent intervals is solely a function of (1) the difference $(A_2 - A_1)$ which determines the midpoint of the possible range of D and (2) the relation of $E(k_1)$ to A_1 and $E(k_2)$ to A_2 which determines $P(A_1)$, $P(A_2)$, $P(A_1 + 1)$, and $P(A_2 + 1)$. For this reason all cases for which values of σ and Δ' are identical will have the same distribution of observed differences in events.

II. Adjacent interval case. For the case of adjacent intervals $P(A_1)$ or $P(A_1 + 1)$ is not independent of $P(A_2)$ or $P(A_2 + 1)$. This follows from assumptions (2) and (3) which hold that events, even under conditions of rate changes, will always be related to periodic functions. Another way of stating this assumption is

$$(6) \quad \sum_{j=1}^{J} T_{m1}\rho_{m1} + T_{m2}\rho_{m2} + \cdots + T_{mj}\rho_{mj} + \cdots T_{mJ}\rho_{mJ} = 1$$

where T_{mj} is the jth increment of time on the mth interval of time between two events such that

$$\sum_{j=1}^{J} T_{mj} = T_m$$

is the total time between the two events and ρ_{mj} is the rate of events per unit time during T_{mj}. In the adjacent interval case the result of each new ρ_{mj} is observed as an effect on the total number of events in one of the two intervals. Since by assumption (5) at most one event would be expected in two intervals, equation (6) may be simplified to $T_{m1}\rho_{m1} + T_{m2}\rho_{m2} = 1$ and we make concentrate on the effect of the single rate change. The length of time, T_2, from a point of rate change to the first event following the rate change is then determined as $T_2 = \frac{1 - T_1\rho_1}{\rho_2}$, where T_1 is the length of time from the event immediately preceding the rate change to the point of rate change, and ρ_1 and ρ_2 are respectively the rates of events

preceding and following the rate change. For a given T_1, ρ_1, and ρ_2, the values of k_1 and k_2 are thus completely determined in the adjacent interval case. Under assumption (6), rate changes will occur only at the boundary between the two intervals. When T_1 is allowed to vary over its entire range, from $\frac{1}{\rho_1}$ to zero, T_2 will range from zero to $\frac{1}{\rho_2}$ and if values of $D = (k_2 - k_1)$ are noted the following relations will be found;

(7)a $P(D < A_2 - A_1 - 1) = 0$

b $P(D = A_2 - A_1 - 1) = \inf [P(A_1 + 1), P(A_2)]$

c $P(D = A_2 - A_1) = \sup [P(A_1), P(A_2 + 1)] - \inf [P(A_1), P(A_2 + 1)]$

d $P(D = A_2 - A_1 + 1) = \inf [P(A_1), P(A_2 + 1)]$

e $P(D > A_2 - A_1 + 1) = 0$

where inf [ ] is the smaller of the values in [ ] where they are unequal, or either value if they are equal; sup [ ] is the larger of the values in [ ] where they are unequal, or either value if they are equal; and $P(A)$ and $P(A + 1)$ are determined from equations (1) and (2) after setting $E(k) = \frac{\rho}{t}$, where t is the duration of an observation interval. Since only differences in numbers of events are of interest, σ and $(\sigma + \Delta')$ may again be substituted for $E(k_1)$ and $E(k_2)$ by using the relations in equations (4) and (5). With this substitution, the distribution of differences is once more a function only of the differences $(A_2 - A_1)$ and of the relation of each $E(k)$ to each A.

Generation of Distributions

Using equations (3) and (7), computer programs which determined

$$P(D|\Delta') = \sum_{\sigma=0}^{1} P(D|\sigma, \Delta')$$

were written both for the case of adjacent intervals and nonadjacent intervals. Summation over the full range of σ was performed under the assumption that the range of $E(k)$ is greater than unity and that every value of σ was equally likely.

If the range of E(k) does exceed unity all values of σ will be possible with some probability. Under assumption (2), ρ and hence $(\frac{\rho}{t} - A) = \sigma$ is uniformly distributed. If the range of E(k) does not exceed unity, no $D > 2$ will be possible; thus a check on the validity of this assumption is available for individual cases. An assumption that every value of σ is equally likely would also be defensible whenever the range of E(k) exceeded unity and the distribution of ρ was unknown. The uniform continuous distribution of σ was approximated by a finite uniform distribution. The number of values of σ in the distribution could be varied in the programs. Trial computations indicated that, as the number of terms approached 4,000 for the adjacent interval program and 20,000 for the nonadjacent interval program, the computed $P(D|\Delta')$ approached limiting values to 8 decimal places. Distributions of σ of 0.00000(.00025)0.99975 and 0.00000(.00005)0.99995 were used respectively for the final adjacent and non-adjacent interval programs. Values of Δ' used were 0.0(0.1)2.0. Probabilities of observed differences of magnitude D, given rate shifts of magnitude Δ' are given to four decimal places in Table 1 in the columns headed $P(D|\Delta)$. Since D is restricted to the range $\Delta' \pm 2$ it is possible to construct a general table based on a transformation of Δ', i.e. $\Delta = (\Delta' - D)$. As an example of the use of Table 1 to determine $P(D|\Delta')$ suppose that it is desired to know with what probability a difference of four events between nonadjacent intervals can be expected when an actual rate change of 3.4 events has occurred, that is, $P(D = 4|\Delta' = 3.4)$. In this case $\Delta = (\Delta' - D) = -.6$. Negative values of Δ are given in the right-most column of the table and column titles are read from the bottom row of the table for negative values of Δ. In the case of $P(D|\Delta)$ the distribution is symmetrical about $\Delta = 0$ so $P(D|\Delta = x) = P(D|\Delta = -x)$. For the example given, $P(D = 4|\Delta' = 3.4) = P(D = 4|\Delta = -.6) = .4147$. If the intervals had been adjacent, $P(D = 4|\Delta = -.6) = .4100$.

	Nonadjacent Intervals			Adjacent Intervals			
Δ	$P(D \mid \Delta)$	$P(\Delta^* \geq \Delta \mid D)$	$P(\Delta^* \leq \Delta \mid D)$	$P(D \mid \Delta)$	$P(\Delta^* \geq \Delta \mid D)$	$P(\Delta^* \leq \Delta \mid D)$	
0.0	.6667	.5000	.5000	.5000	.5000	.5000	0.0
0.1	.6572	.4337	.5663	.4975	.4501	.5499	−0.1
0.2	.6307	.3691	.6309	.4900	.4007	.5993	−0.2
0.3	.5902	.3080	.6920	.4775	.3523	.6477	−0.3
0.4	.5387	.2515	.7485	.4600	.3053	.6947	−0.4
0.5	.4792	.2005	.7995	.4375	.2604	.7396	−0.5
0.6	.4147	.1558	.8442	.4100	.2180	.7820	−0.6
0.7	.3482	.1177	.8823	.3775	.1786	.8214	−0.7
0.8	.2827	.0861	.9139	.3400	.1427	.8573	−0.8
0.9	.2212	.0610	.9390	.2975	.1107	.8893	−0.9
1.0	.1667	.0417	.9583	.2500	.0833	.9167	−1.0
1.1	.1215	.0274	.9726	.2025	.0607	.9393	−1.1
1.2	.0853	.0171	.9829	.1600	.0427	.9573	−1.2
1.3	.0572	.0100	.9900	.1225	.0286	.9714	−1.3
1.4	.0360	.0054	.9946	.0900	.0180	.9820	−1.4
1.5	.0208	.0026	.9974	.0625	.0104	.9896	−1.5
1.6	.0107	.0011	.9989	.0400	.0053	.9947	−1.6
1.7	.0045	.0004	.9996	.0225	.0022	.9978	−1.7
1.8	.0013	.0001	.9999	.0100	.0007	.9993	−1.8
1.9	.0002	.0000	1.0000	.0025	.0002	.9998	−1.9
2.0	.0000	.0000	1.0000	.0000	.0000	1.0000	−2.0
	$P(D \mid \Delta)$	$P(\Delta^* \leq \Delta \mid D)$	$P(\Delta^* \geq \Delta \mid D)$	$P(D \mid \Delta)$	$P(\Delta^* \leq \Delta \mid D)$	$P(\Delta^* \geq \Delta \mid D)$	Δ

Table 1

Notes —

1. $\Delta = (\Delta' - D)$ where Δ' is a rate change and D is an observed difference in events between two intervals. For negative values of Δ, use column titles at the bottom of the table, for positive Δ use column titles at the top of the table.
2. Δ^* is the unknown real value of Δ which has produced an observed difference in events, D.
3. $D = (k_2 - k_1)$ where k_1 is the number of events observed in an interval and k_2 is the number of events in some later interval. If k_2 refers to the interval immediately following the interval for which k_1 events are observed, the columns headed "Adjacent Intervals" apply, otherwise use the columns headed "Nonadjacent Intervals".

When D is used as an estimate of Δ', the "posterior probability," $P(\Delta' | D)$, is of more interest than $P(D | \Delta')$. Since Δ' has a continuous distribution the probability of any specific value of Δ' is zero and only the probability of a range of values of Δ' can be non-zero, e.g. $P(\Delta^* \leq \Delta | D)$, where Δ^* is the unknown actual value of Δ and $\Delta = (\Delta' - D)$. By modification of Bayes theorem

$$P(\Delta^* < \Delta | D) = \frac{\int_{c=-\infty}^{\Delta} P(c)P(D|c)dc}{\int_{c=-\infty}^{+\infty} P(c)P(D|c)dc}$$

but $P(c) = P(\Delta) = \frac{1}{4}$ since Δ is uniformly distributed as $-2 \leq \Delta \leq 2$

$$\text{and} \int_{c=-\infty}^{+\infty} P(D|c)dc = 1, \text{ hence}$$

$$P(\Delta^* \leq \Delta | D) = \frac{\frac{1}{4} \int_{c=-\infty}^{\Delta} P(D|c)dc}{\frac{1}{4} \cdot 1} = P(D | \Delta^* \leq \Delta).$$

An integrable expression for $P(D|\Delta)$ is necessary before $P(\Delta^* \leq \Delta | D)$ can be determined. The values of $P(D|\Delta)$ given in Table 1 can be exactly described by the expressions given in Table 2, which may be arrived at by use of orthogonal polynomials (Milne, 1954). These expressions also exactly describe computed values of $P(D|\Delta)$ to the eight decimal places originally determined and can probably be assumed to be adequate to describe $P(D|\Delta)$ for values of Δ not given in Table 1.

	Nonadjacent Intervals	Adjacent Intervals
$\Delta < -2$	$P(D\|\Delta) = 0$	$P(D\|\Delta) = 0$
$-2 \leq \Delta \leq -1$	$= \frac{1}{6}(2 + \Delta)^3$	$= \frac{1}{4}(2 + \Delta)^2$
$-1 \leq \Delta \leq 0$	$= \frac{2}{3} - \Delta^2 - \frac{1}{2}\Delta^3$	$= \frac{1}{2} - \frac{1}{4}\Delta^2$
$0 \leq \Delta \leq 1$	$= \frac{2}{3} - \Delta^2 + \frac{1}{2}\Delta^3$	$= \frac{1}{2} - \frac{1}{4}\Delta^2$
$1 \leq \Delta \leq 2$	$= \frac{1}{6}(2 - \Delta)^3$	$= \frac{1}{4}(2 - \Delta)^2$
$2 < \Delta$	$= 0$	$= 0$

Table 2

Integrable Expressions which Reproduce Values of $P(D|\Delta)$ in Table 1.

Values of $P(\Delta^* < \Delta|D)$ were computed using the expressions given in Table 2 for $\int_{c=-\infty}^{\Delta} P(D|c)dc = P(\Delta^* \leq \Delta|D)$. Values of $P(\Delta^* \geq \Delta|D) = 1 - P(\Delta^* \leq \Delta|D)$ were also computed and both are given in Table 1. As an example of the use of Table 1 to determine posterior probabilities, suppose that 12 events are observed in one interval and 11 events in the interval immediately following. The value of D is then -1 and the distribution of adjacent intervals applies. If it is desired to know with what probability the difference could have resulted from a rate change greater than zero, Δ is determined for $D = -1$ and $\Delta' = 0$, i.e. $\Delta = (\Delta' - D) = 1$. For $\Delta = 1.0$, $P(\Delta^* \geq \Delta|D) = .0833$, that is, the probability that a given Δ' greater than or equal to $\Delta' = 0$ could have caused a $D = -1$ is .0833 to four decimal places. Similarly, $P(\Delta^* \leq \Delta|D) = .9167$ to four places. Note that for posterior probabilities when Δ is negative the column titles at the bottom row of the table must be used.

At this point it will be well to examine one of the initial assumptions made in deriving these distributions, namely the assumption that rate changes occur, if at all, only between the two intervals examined. In reality it is quite likely that rate changes might occur within one of the intervals in question. An example will serve to demonstrate the effect of such an occurrence. Suppose a rate change, Δ,

of +2.0 events per interval occurs in the middle of the third of a series of five intervals, "1", "2", "3", "4", and "5". Since the rate change will be in effect for half of interval 3 it will produce the same effect for the adjacent comparisons 2 with 3 or 3 with 4 as a rate change of 1.0 event per interval. Only for nonadjacent comparisons such as 2 with 4 or 1 with 4 will the full effect of the rate change be evident. The probabilities of observing various values of D for some possible comparisons made between the five different intervals are shown in Table 3. It will be noted that using the interval within

		D			
Compared Intervals	Distribution	0	+1	+2	+3
2, 3	Adjacent	.25	.50	.25	.00
3, 4	Adjacent	.25	.50	.25	.00
2, 4	Nonadjacent	.00	.17	.67	.17
1, 3	Nonadjacent	.17	.67	.17	.00

Table 3

Probabilities of Observing Various Differences in Number of Events When a Change $\Delta = +2.0$ Occurs in the Middle of Interval 3.

which a rate change has occurred as one of the two intervals for a comparison has the effect of decreasing the probability that the larger possible differences will be observed. This has a serious effect on adjacent comparisons since it decreases the possibility that a change in rate will be detected. In the nonadjacent case the effect is quite beneficial for comparisons made across a single intervening interval. There is no decrease in the probability of observing the larger possible differences when the rate change is between the two comparison intervals and, as with the adjacent interval case, when the rate change occurs within one of the comparison intervals its effects are decreased. The overall result for the nonadjacent case is that the interval within which a rate change

actually occurs and the magnitude of this rate change have a maximum liklihood of being correctly identified. When rate changes occur within intervals, D becomes an estimate of effective Δ rather than the actual Δ. For adjacent interval comparisons the effective Δ, Δ_{eff} is equal to the actual Δ only when the rate change occurs between the two intervals. As the rate change occurs further and further from the boundary between the two intervals Δ_{eff} decreases until it equals zero when the rate change occurs at an outside boundary. If a rate change occurs within an interval there are two adjacent interval comparisons which will be affected, that for which the interval containing the rate change is compared with the preceding interval and that for which it is compared with the following interval. The largest D resulting from these comparisons is of most interest since it is the better estimate of Δ. The Δ_{eff} causing the largest D has a minimum value of $\frac{1}{2}\Delta$, occurring when the rate change happens in the middle of an interval, and as a rate change is equally likely to occur at any point within an interval the expected value of Δ_{eff} is $\frac{3}{4}\Delta$. The use of nonadjacent intervals guarantees that for each rate change there will be at least one interval comparison for which $\Delta_{eff} = \Delta$, i.e. the comparison between an interval preceding that in which the rate change occurs and an interval following it. For maximum precision in identifying single points of rate change, only one interval should separate the two nonadjacent comparison intervals. In either the adjacent or nonadjacent interval comparison a value of D may be assigned to each interval and the maximum D in any pair of adjacent intervals will be the best measure of a rate change in the two intervals. Moreover, the interval assigned the maximum D would be the most likely location of a possible rate change if nonadjacent comparisons are made. If adjacent comparisons are made, the interval assigned the maximum D and the following interval are equally likely to contain the possible rate change. In the remainder of this paper when D is referred to it will be understood that a maximum D for some small number of adjacent intervals is implied.

The determination of the magnitude of Δ is the final problem which will be covered. One measure follows from the fact that the range of D is within $\Delta \pm 2$. Since $\Delta_{eff} \geq \Delta$ only the bound which is smaller in magnitude can be determined in this way when a single D is calculated. As an example, if $D = 2$, Δ must be greater than zero and the upper bound will be unknown unless $|D| \leq 2$ is true in the two adjacent intervals. In this case it is possible to say that $0 < \Delta < 4$. Similarly, if $D = -1$, Δ must be less than 1 and, if $|D| \leq 1$ is true in the adjacent intervals, $-3 < \Delta < 1$. A similar approach may be used to identify all rate changes greater than some selected value. A typical objective would be the detection of all true rate changes, i.e. all cases where $\Delta > 0$. This could be attempted with a test of the form $D > c$ by which the hypotheses that $\Delta = 0$ would be rejected whenever a value of D greater than some constant, c, was observed. Table 4 shows the power of three such tests to reject the hypothesis of $\Delta = 0$ when it is false. Values given for adjacent interval tests assume that $\Delta_{eff} = \frac{3}{4}\Delta$. The most powerful tests are those for which $c = 0$ and among these the adjacent interval test is most powerful for $|\Delta| \leq .5294$ while the nonadjacent is most powerful elsewhere. Among tests for which $c = 1$, the adjacent interval test is more powerful for $|\Delta| \leq .8438$ and the nonadjacent for $|\Delta| \geq .8438$. Despite the greater power of tests of the form $D > 0$ there is a major drawback to their use. The probability of rejecting the hypotheses $\Delta = 0$ when it is true is .33 and .50 respectively for the nonadjacent and adjacent interval tests with $c = 0$. For the tests with $c \geq 1$ the probability of making this error is zero but at the cost of a sacrifice in power. If the test criterion is the minimization of both the errors of rejecting a true hypothesis or failing to reject a false hypothesis, the nonadjacent test with $c = 0$ is best for $\Delta < 1.72$ and the nonadjacent test with $c = 1$ is best for $\Delta \geq 1.72$. The lack of a uniformly best test makes it necessary to evaluate each testing situation individually. In some cases

	c = 0		c = 1		c = 2		Randomization
Δ	I	II	I	II	I	II	I
ϵ	.33	.50	.00	.00	.00	.00	.05
.5	.52	.54	.02	.04	.00	.00	.12
1.0	.83	.64	.17	.14	.00	.00	.29
1.5	.98	.81	.50	.31	.02	.04	.57
2.0	1.00	.94	.83	.50	.17	.06	.87
2.5	1.00	1.00	.98	.69	.50	.34	.98
3.0	1.00	1.00	1.00	.86	.83	.52	1.00

Table 4

Power of Four Nonadjacent and Three Adjacent Interval Tests of the Form $D > c$

Notes — 1. Power of nonadjacent interval tests are given in columns headed "I" and power of adjacent interval tests are given in columns headed "II".

2. ϵ is nonzero but extremely small.

3. Δ_{eff} assumed to be $\frac{3}{4}\Delta$ for adjacent interval tests.

4. The randomization test has $\alpha = .05$, see text for details.

a composite null hypothesis may simplify matters by allowing the placing of the bulk of the region in which the $c = 0$ test is best within the null hypothesis region. If this is done, α, the probability of rejecting a true hypothesis, must be recomputed. Another alternative is the use of a randomization test at some selected α level. A .05 level nonadjacent interval test can be performed by rejecting the hypothesis that $\Delta = 0$ whenever a $D > 1$ is observed and, with probability .15, when a $D = 1$ is observed. The power of this test is given in the right column of Table 4. Such a test allows some degree of compromise between the relative disadvantages of both the test for which $c = 0$ and that for which $c = 1$.

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