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Estimating Pecuniary Damages in Lawsuits With a Reasonable Degree of Economic Certainty

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ABSTRACT

The determination of pecuniary damages in personal injury and death actions often requires actuaries and economists to make estimates that encompass expected future experience over extended periods of time. Moreover, such experts must be prepared to assert that their estimates are made with a reasonable degree of economic certainty.

An anomaly plays an integral role in the estimation process. Namely, although neither the actual levels of future interest rates nor future wage growth rates can be predicted with reasonable certainty, the present value of magnitudes emerging from them can be. It is important to understand why that is so.

This paper provides empirical evidence that the present value of a worker's future earnings can be estimated with a reasonable degree of economic certainty. Such certainty emerges by virtue of the predictable covariance between the rates of growth of workers' earnings and interest rates reflecting the time value of money.
I. INTRODUCTION

Typically, the measurement of pecuniary damages is an important aspect of lawsuits relating to wrongful death or disability, and employment termination. Expert economic testimony is frequently placed into evidence to assist the court in establishing a trial-date or present value of a future stream of payments. The question of whether economists and actuaries can make reliable present value estimates of pecuniary damages in personal injury and death actions has been a source of frequent debate in this Journal [5, 8, 9, 10, 13, 15, 16, 22, 23, 26].

Four variables determine the trial date value of a future payments stream: (1) the beginning payment level; (2) the time period over which payments will extend; (3) the anticipated annual rates of growth in the payments; and (4) the investment rates of return at which the expected stream of payments is to be discounted to its present value.¹ The first two variables are readily subject to estimation in most lawsuits. However, economists and actuaries cannot predict with a reasonable degree of economic certainty the year-by-year future investment returns \((r)\) or the annual growth rates \((g)\) of workers' earnings. Even so, because the valuation process revolves around the predictable average differential between \(r\) and \(g\) rather than their actual levels, economists can make reliable present value estimates.

The purpose of this article is to provide empirical evidence that the present value of the average worker's future stream of earnings can be estimated with a reasonable degree of economic certainty.² Section II explains why the \((r-g)\) differential is of controlling importance in
determining the present value of future earnings streams. Section III examines the role of the covariance between discount rates and growth rates in specifying the \((r-g)\) differential. Section IV provides a simulation analysis of the estimation of a worker's earnings using the \((r-g)\) differential technique. The implications of these results are discussed in Section V. Concluding remarks are presented in Section VI.

II. THE \((r-g)\) DIFFERENTIAL AND PRESENT VALUE ESTIMATES

The measurement of pecuniary damages in personal injury and death actions often requires actuaries and economists to make projections over long time periods.\(^3\) It is important to understand how that can be done, in light of the fact that actuaries and economists cannot predict with confidence what the actual level of interest rates and the actual wage growth rates will be over an extended period in the future. Fortunately, it is neither necessary or important to project the time path of discount rates in estimating the present value of a future earnings stream. The purposes of the court are sufficiently served with testimony relating to the \((r-g)\) differential—that is, to the difference between the discount rate \((r)\) and the earnings growth rate \((g)\).\(^4\)

This point is illustrated in Table I. The present values associated with specific differentials are contained in the diagonal vectors. Although there are great differences in the magnitudes of the income streams corresponding to alternative growth rates in income (see bottom tier of Table I), there is virtually no difference in the present values consistent with any specific differential.\(^5\) For example, the present value of a 25-year earnings stream that has an initial $1000 value is
$22,023 if a one percent (r-g) differential occurs at a r = 1.0 percent and g = 0.0 percent; the present value is $22,202 if the one percent differential occurs at r = 8.0 percent and g = 7.0 percent.  

A brief review of present value logic reveals why an economist or an actuary should be more interested in predicting the average (r-g) differential than with predicting the expected average levels of r and g separately. The present value of a worker's future earnings can be represented as

\[
PV_0 = \sum_{t=1}^{n} \left[ \left( E_{t} \right) \prod_{a=1}^{t} (1+g_{a}) \right] / \left[ \prod_{a=1}^{t} (1+r_{a}) \right]
\]

where

- PV\(_{0}\) = present value at t=0 of a stream of payments to replace a worker's future earnings;
- g\(_{\alpha}\) = rate of growth in earnings in period \(\alpha\) (\(\alpha=1,2,3,...,n\));
- E\(_{t}\) = \(E_{t} \prod_{a=1}^{t} (1+g_{a})\) = earnings in period t (t=1,2,3,...,n);
- r\(_{\alpha}\) = discount rate in period \(\alpha\) (\(\alpha=1,2,3,...,n\)); and
- \(\prod\) = multiplication operator.

In estimating the present value of future earnings using an average growth rate (g) and an average discount rate (r), equation (1) converts into

\[
PV_0 = E_{0} \left[ \sum_{t=1}^{n} (1+g)^t/(1+r)^t \right]
\]

\[
= E_{0} \left[ 1-((1+g)^n/(1+r)^n))/((1+r)/(1+g)-1) \right].
\]
If in the interest of expositional convenience, we let \( n \rightarrow \infty \), then the summation term in equation (2) converts into the following geometric expansion

\[
P_{V_o} = P_o \frac{A}{1 - R}
\]

(4)

where

\( A \) = the initial term in the expansion; and

\( R \) = the ratio between successive terms in the expansion.

It turns out that the initial term, \( A \), is given by \((1+g)/(1+r)\), and the ratio between successive terms is also given by \((1+g)/(1+r)\). Hence,

\[
P_{V_o} = P_o \frac{(1+g)/(1+r)}{[1-(1+g)/(1+r)].}
\]

(5)

Letting \( k \) equal the difference between \( r \) and \( g \) (i.e., \( k = (r-g) \)), substituting \((r-k)\) for \( g \), and taking derivatives of (5) with respect to \( r \) and \( k \), we get

\[
\frac{\partial P_{V_o}}{\partial r} = -(1/k) = -[1/(r-g)]
\]

(6)

and

\[
\frac{\partial P_{V_o}}{\partial k} = -[(1+r)/k^2] = -[(1+r)/(r-k)^2].
\]

(7)

It is clear from inspection that (7) is \([1+r]/(r-g)\) times larger than (6). For this reason, an analyst should be more interested in the (\( r-g \)) differential than in the actual likely levels of either \( r \) or \( g \).

Thus, given the initial level of income and the length of the time period, all that is necessary in order to estimate the present value of a worker's anticipated future earnings is knowledge of the average
differential \((r-g)\). The selection of an appropriate differential must be based upon wage growth and discount rates that are consistent with theoretical considerations and bear some relation to experience.

III. COVARIANCE BETWEEN GROWTH RATE AND DISCOUNT RATE

Economic theory indicates that there is a strong covariance between rates of growth in the annual earnings of workers and interest rates. They have common determinants. Growth theory does not, however, specify the steady-state equilibrium values of \(r\) and \(g\), or the \((r-g)\) differential. The differential is, ultimately, an empirical issue.

The determination of the growth in wages and the general level of interest rates are often summarized as follows:

\[
\begin{align*}
\text{annual wage growth rate} &= \text{annual rate of increase in labor productivity} + \text{annual rate of change in the price level} \\
\text{annual time value of money} &= \text{annual real rate of interest (or productivity of capital)} + \text{annual rate of change in the price level}
\end{align*}
\]

Having said this much, however, it is important to move rapidly toward qualifying statements. First, it is important to recognize that there is not a single wage rate. At any point in time there is a collection of wage levels and a collection of growth rates in wages. In a market system, such differences (in wage levels and in growth rates) are the means by which resources are allocated. Even so, as markets accomplish their resource allocation objectives there is a tendency for growth rates attaching to specific occupations to regress toward an economy-wide mean.
Second, it is important to recognize that there is no single unambiguous rate summarizing the return on capital. At any time there is a constellation of interest rates, with rates of return within the constellation reflecting differences in various aspects of risk. Moreover, through time there are changes in the general level of the constellation (i.e., the level of interest rates) as well as shifts in the relationship among rates of return within the constellation (i.e., the term structure of interest rates).

Empirical studies show that the longer-run annual increase in labor productivity in this country has averaged 2.00 to 3.50 percent, and that the real rate of interest is in the 2.00 to 4.00 percent range.8 Thus, over protracted stretches of time the growth in money earnings and the time value of money can be expected to display a high covariance.

It is our view that the one-year U.S. Treasury yield is the most useful proxy for the time value of money. In this study, we measure the one-year rate \( r_1 \) with the one-year U.S. Treasury securities constant maturities data. The use of shorter maturities is questionable because their yields are influenced importantly by money market pressures. The use of longer-term securities as a measure of the time value of money is questionable by virtue of the fact that their yields embody a premium for interest rate risk. Investing in one year maturities, whose yields covary with expected changes in the price level, avoids the substantial risk of investing in long maturity instruments to meet an annual payment need that will also covary with the changing price level. Stated differently, the duration of one-year U.S. securities is consistent with the time horizon of workers' annual compensation reviews.
Data are presented in Table II that depict the relationships between wage growth and the contemporaneous rates of return on equities ($r_e$), one-year ($r_1$) and twenty year ($r_{20}$) U.S. securities. The data show that during the 1953-1984 period the $(r_1-g)$ differential averaged 0.56 percent. The 6.07 variance of the $(r_1-g)$ differential was substantially lower than the 10.68 variance of $r_1$ alone because of the positive covariance between $r_1$ and $g$. The variance of the $(r_1-g)$ differential is

$$\text{Var}(r_1-g) = \text{Var} r_1 + \text{Var} g - 2 \text{Cov}(r_1,g).$$  \hspace{1cm} (8)

A large and positive covariance between $r$ and $g$ obviously contributes to the relative stability of the $(r-g)$ differential and, therefore, to the reliability of present value estimates of workers' future earnings. Of course, if there were perfect covariance between $r$ and $g$ the variance of the differential would be zero.

Over this same period, the differential using 20-year U.S. securities of constant maturity $(r_{20}-g)$ averaged 0.99 percent and the variance was 6.01, somewhat smaller than the variance with one-year securities. The relationships between the growth in wages and contemporaneous interest rates appear to be inconsistent with our working hypothesis that the one-year U.S. Treasury yield is the most useful proxy for the time value of money. On its face, portfolios consisting of 20-year U.S. securities appear to provide a less expensive means of replacing lost wages than do portfolios consisting of one-year U.S. securities. Moreover, according to these data there is less variance in the $(r_{20}-g)$ differential than in the $(r_1-g)$ differential. With equities,
the differential \((r_e - g)\) averaged 6.18 percent but the variance was a whopping 334.20.

Table II about here

Up to this point, we have been discussing relationships between the growth in wages and contemporaneous rates of return on alternative investment media. Such relationships have been the focus of the literature relating to the measurement of pecuniary loss in lawsuits and the annual cost of pension plans. In the remaining portions of this article, we break away from the traditional preoccupation with growth rates and contemporaneous investment returns. As it turns out, those relationships are not of primary importance and can be misleading. It is necessary to recognize that the covariance of ultimate importance emerges from the relationship between the growth in wages and the rate of return on portfolios dedicated toward meeting future wage payments as they arise in the actual course of time. That is, the relationship of chief importance is between \(g\) and the \(r\) from what we refer to as a "dedicated" portfolio.

IV. THE \((r-g)\) DIFFERENTIAL ESTIMATION TECHNIQUE: EVIDENCE FROM SIMULATION ANALYSIS

A desirable valuation technique would be one that generates a present value estimate as of the trial date which proves to be just sufficient to provide the payments stream that actually occurs. Any given payments stream could be valued within the framework of several alternative investment strategies. Presumably, the choice among investment
strategies will be guided by the risk-cost trade-off performance standards of the court. This portion of the paper employs simulation analysis to develop a set of experience which can be used to assess the \((r-g)\) differential estimation technique.

**Simulation Procedures**

Simulation analysis was used to determine the present value or the principal amount of money required at \(t=0\) in a dedicated investment portfolio in order to provide exactly the earnings stream of the average U.S. worker over alternative time spans in the post Federal Reserve-Treasury Accord period (1953-1984). The portfolios were dedicated in the sense that the principal and the associated realized investment income were used to replace the actual annual earnings of the average U.S. worker as that yearly earnings stream emerged. There were thirteen 20-year time frames from 1953 to 1984, eighteen 15-year periods, twenty-three 10-year periods, and twenty-eight 5-year periods.

Alternative investment strategies with dedicated portfolios were specified for each 20-year, 15-year, 10-year and 5-year period. One strategy consisted of investing in one-year U.S. Treasury securities only. Other strategies consisted of investing only in, alternatively, 5-year, 10-year, 15-year and 20-year U.S. Treasury securities. A final strategy consisted of investing entirely in common stocks. Each investment strategy was made operational by means of simulation. According to the terms of the simulations, the actual annual earnings of the average worker were first identified for each year within a time period. Then, given an initial principal investment and portfolio strategy, amounts equal to the actual annual earnings of the average worker were
withdrawn year-by-year from the portfolio. If the investment income earned in any year were less than the earnings of the average worker in that year, then securities were sold at prevailing market prices.\textsuperscript{13} To the extent that the annual income of the portfolio exceeded the annual earnings withdrawal, the excess was reinvested as prevailing market rates.\textsuperscript{14} For each investment strategy, the simulation was performed in iterative fashion until an initial principal amount was identified that was just sufficient to generate the annual earnings during the time span being analyzed. That initial amount constituted the dedicated portfolio at $t=0$, or the present value needed to replace the earnings stream that actually occurred following $t=0$, or in time periods $t=1,2,3,\ldots,n$.

The simulations identified time period specific joint observations of \textit{ex post} growth rates and discount rates. Aggregate \textit{ex post} income payments were known. Given initial income, it was possible to solve for the \textit{ex post} implied growth rate, $g'$, that created those income payments.\textsuperscript{15} As indicated, the simulations found the minimum initial investment for the dedicated portfolios for each investment medium. Given this present value amount, along with the initial payment and the growth rate in income, it was possible to solve for each implied \textit{ex post} return (or discount rate), $r'$, on each dedicated portfolio.\textsuperscript{16} Thus, for each simulation it was possible to calculate the \textit{ex post} differential between the growth rate and the discount rate ($r'-g'$). These simulation data were summarized for each investment strategy and for each time period.
Results

The simulation results are shown in Table III. Summarized there are the means and standard deviations of the \((r' - g')\) differentials for the alternative investment strategies across alternative loss periods. Within investment strategies confined to U.S. securities, visual inspection suggests that the \((r' - g')\) differential grows increasingly negative and the standard deviation gets larger as we move from a 1-year strategy to a 20-year strategy. It is also clear that the standard deviation of the \((r' - g')\) differential associated with an investment strategy is inversely related to the length of the loss period.

Over the entire range of observations, the \((r'_e - g')\) differential for the equities strategy averaged 3.78 percentage points while the \((r' - g')\) differential for U.S. Treasury investment strategies generally were negative (i.e., \(g' > r'\)). Specifically, the average return on dedicated portfolio's comprised of 20-year U.S. Treasury securities was 2.57 percentage points below the earnings growth rate, and the average return on dedicated portfolio's comprised of 1-year Treasury securities was .17 percentage points below the earnings growth rate. Thus, in terms of the present value cost of replacing a worker's future earnings, dedicated equity portfolios were the least costly, dedicated portfolio consisting of 1-year securities were second, and dedicated portfolios of 20-year securities were the most expensive. But risk and return (cost) are inversely related and, as the standard deviation data in Table III suggest, the consistency of performance was much better with U.S. Securities than with the equities strategy.
Regression analysis was conducted with the \((r'-g')\) simulation data in order to estimate the expected \((r'-g')\) differential for each investment strategy. Because we were interested in controlling for potential effects of loss periods on estimates of the \((r'-g')\) differential, we used dummy variable techniques to control for alternative loss periods. Also, because interest rates have generally risen during the 1953-1984 period, it was plausible to believe that an equation that did not take such a trend into account would be misspecified. Hence, a trend variable was introduced. At length, the regression model to be estimated was

\[
_s(r'-g')_t = a_0 + a_1 D_1 + a_2 D_2 + a_3 D_3 + a_4 T_1 \\
+ a_5 T_2 + a_6 T_3 + a_7 T_4 + U_t
\]  

(9)

where

\[
s(r'-g')_t = \text{the (r'-g') differential on dedicated portfolio } t \text{ with} \\
\text{investment strategy } s \text{ (}\ell=1 \text{ through } 6, \text{ corresponding to}} \\
\text{portfolios consisting of } 1,5,10,15 \text{ and } 20\text{-year U.S.} \\
\text{Treasury securities, and equities),}
\]

\[
D_1 = 1 \text{ with } 5\text{-year loss periods and }0 \text{ at all other times,}
\]

\[
D_2 = 1 \text{ with } 10\text{-year loss periods and }0 \text{ at all other times,}
\]

\[
D_3 = 1 \text{ with } 15\text{-year loss periods and }0 \text{ at all other times,}
\]

\[
T_i = \text{time trend (subscripts } i = 1 \text{ through } 4 \text{ relate to } 5\text{-year, } 10\text{-year,} \\
\text{15-year and } 20\text{-year loss periods, respectively),}^{17} \text{ and}
\]

\[
U_t = \text{the stochastic term at time } t.
\]

According to regression results with the simulation data, none of the coefficients associated with loss period dummies—i.e., \(a_1, a_2\) and \(a_3\)—reaches an acceptable level of significance. This means that the average differential does not depend upon the length of the loss period. In regression confined to U.S. securities, the coefficients associated
with the trend variables, \( a_4 \) through \( a_7 \) were not often significant. But these coefficients were significant in tests wherein the investment strategy was confined to equities. 19

Selected results from the regression tests are provided in the Regression Summary in Table III. 20 Shown there are intercepts and standard deviations of the residuals around the regression hyperplane. The intercept values shown in Table III are, of course, the means from 20-year loss periods. Estimated values of \( a_0 \) for 1-year and 5-year securities do not differ from zero. 18 Recall that the loss-period dummies are not statistically significant; hence, we can interpret the intercepts with generality. Results with the regression summaries are consistent with impressions from visual inspection. That is, equities constitute the least-cost dedicated portfolios. Among U.S. Treasury securities, 1-year securities are the least-cost medium; dedicated portfolios consisting of 20-year instruments are the most expensive.

But the consistency of performance was much better with debt instruments than with equity. Figure 1 illustrates this dimension of the problem. Bear in mind that Figure 1 is illustrative only. Each of the three normal curves is calculated from the means and standard deviations of the distribution indicated. With 1-year U.S. Treasury securities the standard deviation of the residuals of the actual \( (r_1' - g') \) differential around the estimated \( (r_1' - g') \) differential was 1.07 percentage points; with 20-year securities the standard deviation was 2.17 percentage points; and with equity it was 4.88 percentage points. Thus, in terms of consistency in least-cost performance, 1-year
securities ranked first; 20-year securities ranked second; and equity was least consistent.

We began with the working hypothesis that investing in one-year maturities, whose yields vary with expected changes in the price level, avoids the substantial risk of investing in long maturity instruments to provide a stream of payments that will be affected by the changing price level. As discussed earlier in Section III, the covariance between the growth in wages and contemporaneous interest rates appeared to be inconsistent with this hypothesis. However, the covariance of ultimate importance is between the effective wage growth (g') and the realized rate of return (r') on portfolios dedicated toward meeting future wage payments as they arise in the actual course of time. The (r'-g') data shown in Table III provide support for the working hypothesis. 21

The point that emerges throughout Table III is that the least-cost means of providing for a future payments stream of uncertain size is to construct the dedicated portfolios with very short-term securities. Moreover, the standard deviation of the (r'-g') differential is, without exception, smaller with dedicated portfolios consisting of short-term maturity securities than with dedicated portfolios consisting of long-term securities.

The substantial stability in the differential between the growth rate in wages and the returns on dedicated portfolios consisting of short-term securities results from the high covariance between growth rates and short-term interest rates coupled with little, if any, offsetting capital depreciation. In contrast, the relatively high
covariance between contemporaneous movements in interest rates on long-term securities and wages has served to increase the costs associated with dedicated portfolios consisting of such securities. Even as long-term interest rates have risen along with wages, these increases have resulted in capital losses on the dedicated portfolio.

V. PERFORMANCE STANDARDS AND THE RISK-RETURN TRADE-OFF

The analysis and results presented above have a direct bearing on issues typically before the court. Specifically, the results can be of use in placing a trial-date value on a lost future income stream, on prospective future medical payments, or on a range of other types of labor-intensive payments.

As suggested, if the court were simply to use the averages from the simulation experience it is clear that equities present the least-cost option. For example, the rate of return on dedicated equity portfolios for a 20-year loss period averaged 4.36 percentage points above the growth rate in wages. Such a growth rate/discount rate relationship results in a substantially lower present value than was the case with either 1-year or longer-maturity U.S. securities. A numerical example may be helpful.

Suppose that the initial payments rate is $1000 and that the payments will continue for 20 years. For computational convenience we can assume a 0.0 percent growth rate and a 4.36 percent discount rate. The differential related to equities implies a present value of $13,167. By comparison, the differential related to 1-year securities (−.28 percent) implies a present value of $20,600, approximately 60 percent more than the cost of an equities portfolio. The present value implied by
the differential relating to 20-year maturity securities (-4.36 percent) is $33,004, well over twice the cost of an equities portfolio. In short, the differences in present values implied by these alternative differentials are by no means trivial.

But there is a problem. Recall that the differentials used above are averages. It is the very nature of measures of central tendency that about one-half the values in a distribution lies above the mean, and one-half lies below. Hence, if the court opts to use the arithmetic mean for valuation purposes it will do so knowing that roughly one-half the claimants will be unable to construct dedicated portfolios that meet their needs.

And there is a problem within a problem. Recall the distributions with alternative dedicated portfolios (see Figure 1 and Table III).

There is a very large dispersion of experience around the average differential associated with dedicated portfolios consisting of equity, \( (r_e' - g') \). Such a result is consistent with what might be expected in light of the relationship between returns and risk. That is, there is a widely-held view that higher returns will be related to higher risk; conversely, lower returns will be related to lower risk. But the consequence of such a risk-return trade-off is that some claimants will fall far short of meeting their needs. As applied to the matter at hand such a consequence constitutes an ironic anomaly. It means that the benefits of the increased risk accrue immediately to the defendant (the wrongdoer) without regard to final outcomes, while the burden of
the increased risk falls upon the claimant (the injured party). Such a state of affairs reverses the risk-return tradeoff logic undergirding investment decisions.

The court has already faced up to at least a portion of these issues. In Jones and Laughlin Steel Corporation v. Pfeifer [U.S. 76 L Ed 2d 768, 103 S Ct (June 15, 1983)] the court held that "... the discount rate should not reflect the market's premium for investors who are willing to accept some risk of default." The court appears to have effectively limited the range of acceptable investment media to U.S. Treasury securities. In any event, because the risk of default is clearly present in equities, it would appear to be quite clear that common stock returns cannot be used for valuation purposes.

But there is another basis upon which equities do not provide a least-cost basis for loss evaluation. That basis relates to the court's reasonable insistence upon high performance standards for dedicated portfolios. It is reasonable to believe that the court would not be satisfied with a decision-making process that results in present-value amounts that are insufficient to meet the needs of roughly one-half the claimants. But suppose the court insists upon a process that implies only a 5 percent error. According to Figure 1, 95 percent of the dedicated portfolios consisting of 1-year U.S. Treasury securities lie above a \( r_1' - g' \) differential of -2.04. For 20-year maturity securities a \( r_{20}' - g' \) differential of -5.68 is required in order to include 95 percent of the sufficient dedicated portfolios. For equities, a \( r_e' - g' \) differential of -3.66 would be required. If the court were to enforce an even higher standard, for example one permitting shortfalls
with only 2 percent of the dedicated portfolios, \((r' - g')s\) of -2.48, -6.57 and -5.66 would be required for, respectively, 1-year securities, 20-year securities, and equities.

In terms of numerical examples, the very high-performance dedicated portfolio (2 percent shortfalls) consisting of 1-year U.S. Treasury securities would cost $26,308. The high-performance portfolio consisting of 20-year securities would cost $44,032; and the portfolio consisting of equities would cost $38,991. In short, within the context of speaking about high-performance dedicated portfolios, 1-year U.S. Treasury securities provide the least-cost investment vehicle. 23

VI. CONCLUDING REMARKS

It is our view that the present value of future payments streams can be estimated with a reasonable degree of economic certainty. There is reasonable certainty by virtue of the relative stability in differentials between growth rates in payments streams and returns on 1-year dedicated portfolios. Moreover, the stability in differentials does not result from fortuitous circumstances. Rather, both wages growth rates and interest rates are determined by a set of common factors. In particular, in the short run each is importantly affected by inflation rates. Over the long run there is a tendency towards a zero differential.

If the court insists on high performance standards it may use the standard deviation of the \((r' - g')\) differential to control for error. One might imagine that the willingness of the court to countenance error will vary from case to case. For example, in those cases in which liability is clear and dependence of claimant upon the award is virtually
complete (e.g., total disability), the court may wish to hold potential shortfalls to a minimum. To accomplish this, the court could adjust the mean value by, say, two standard deviations. In other instances the court might be willing to countenance only average performance.

Perhaps the strongest conclusion from this study is that, for valuation purposes, we may limit our attention to returns on dedicated portfolios consisting of 1-year securities. In part, this conclusion is based on priors, specifically, on the belief that there is no basis in logic for a claimant to carry the burden of increased risk without an entitlement to the benefits associated with risk-taking. But the conclusion is also based on the empirical finding that portfolios consisting of 1-year securities are least-cost portfolios. Their advantage with high performance portfolios is especially pronounced. This advantage reflects the low variance in the \((r' - g')\) differential. In turn, that low variance reflects the high covariance between the wages growth rate and the return on portfolios consisting of 1-year treasury securities. Our study suggests that there is virtually no basis for a belief that the appropriate discount rate is greater than the growth rate. Rather, in rough terms, our results are consistent with the presumption that the average differential is zero.
BIBLIOGRAPHY


FOOTNOTES

1 Other variables that may be important in a specific valuation problem, such as the age-education life cycle of the payments stream, are not discussed in this paper. Issues posed by these variables do not produce a fundamental effect on conclusions presented here.

2 Economists and actuaries cannot predict the economic future of any specific individual with a reasonable degree of economic certainty. The best estimate of the economic future of any individual is the expected economic future of the average worker in the statistical cohort to which the worker in question belongs.

3 Similar long-term projections of wage growth and investment returns are required for annual pension funding decisions.

4 It is interesting that most of the literature on estimating pecuniary damages has addressed the estimation of the nominal levels of r and/or g, which are not predictable with confidence, rather than focusing on the more important and predictable (r-g) differential. For example, see [3, 5, 9, 10, 15, 19, 23, 26].

5 The present values associated with a specific differential would show no differences if the analysis were conducted using continuous compounding for both the wage growth rate and the discount rate.

6 Moreover, it makes virtually no difference whether the average differential is created by a constant r and g set through time (e.g., remaining at g = 6 percent and r = 7 percent), or whether the average differential emerges by virtue of a succession of different sets (e.g., g as a sequence of 6 percent, 8 percent, 5 percent, and 7 percent, etc.; and r as a sequence of 7 percent, 9 percent, 6 percent and 8 percent, etc.).

7 The derivative with respect to g is equal to but with opposite sign of the derivative with respect to r.

8 These studies rely on alternative estimates of labor productivity and alternative measures of interest rates.

9 Interest rate data are obtained from the Federal Reserve constant maturity series published in various issues of the Federal Reserve Bulletin. Equity returns are from the Ibbotsen and Sinquefield study [14, p. 18] of 1926-1984 stock market returns. Wage growth rates are calculated using the index of adjusted (for overtime and interindustry employment shifts) hourly earnings for private nonagricultural workers [4, p. 276]. The wage growth rate data are biased downward inasmuch as the rapid expansion of employer provided fringe benefits (non-money earnings) in the 1953-1984 period is not considered.

10 Cov(r, g) = ρσ_r σ_g where ρ is the correlation coefficient between r and g.
11 Much of the analysis reported here was also done with data relating to the 1926-1984 period. Results with the longer data span were generally consistent with results reported here. But the experience of the 1930s and 1940s differed so markedly from experience since the Accord that we feel justified in focusing our presentation on the more recent data set.

12 The use of overlapping data results in the progressive underweighting of experience as we move toward the beginning and the end of the data set, and a corresponding overweighting of experience as we move toward the middle of the set. To the extent that there are trend dependent variables operating on the data, the results can be systematically biased. This problem is further addressed in footnote 17.

13 The need to sell portfolio assets to meet an annual withdrawal payment is not difficult to accommodate in the simulation with the one-year U.S. Treasury securities investment strategy or with the common stock investment strategy. Investing in one-year U.S. Treasury securities causes the assets of the portfolio (principal and interest) to be available for making a payment at the end of each year. After the required annual withdrawal is made, the remaining balance in the portfolio is reinvested at the prevailing one-year U.S. Treasury securities yield. The situation is similar for the equity investment strategy inasmuch as the annual yield is decomposed into current dividend income and capital appreciation (loss).

Investment strategies that create portfolios composed of longer term U.S. Treasury securities involve added considerations. The market value of a bond changes because of movements in market interest rates and as a result of decreases in the time to maturity. The market values of U.S. Treasury securities sold prior to maturity were determined by calculating the present value of the remaining coupon interest payments and the maturity value. The discount rate used was equal to the current market yield for U.S. securities with a maturity equal to the remaining time to maturity of the bonds being sold.

14 If, from time-to-time, an additional purchase were required, the maturity of those securities would be equal to the lesser of the number of years remaining in the time period or the investment strategy maturity.

15 The \textit{ex-post} growth rate, \( g' \), was obtained by solving the equation

\[
\frac{n}{\sum_{t=1}^{n} \text{Earnings}_t} = \left( \text{Earnings}_{t=0} \right) \sum_{t=1}^{n} (1+g')^t.
\]

16 The \textit{ex-post} investment return or discount rate, \( r' \), was obtained by solving the equation.

\[
\text{Present Value}_{t=0} = \sum_{t=1}^{n} \left( \text{Earnings}_{t=0} (1+g')^t / (1+r')^t \right)
\]

using the previously calculated \( g' \) value and the \( (t=0) \) present value.
For 20-year loss periods, \( t = 1 \) through 13; for 15-year loss periods, \( t = 1 \) through 18; for 10-year loss periods, \( t = 1 \) through 23; and for 5-year loss periods, \( t = 1 \) through 28. In fact, however, the trend variables are expressed as differences from arithmetic means. This is done in order to remove the effects of estimation from the intercepts.

The finding that the \((r_1-g)\) and \((r_5-g)\) differentials do not differ from zero provide support for use of the so-called "Alaska Method" which employs a total offset \((r=g)\) logic.

Because there is autocorrelation in the residuals, the normal tests of significance cannot be trusted. Such a state of affairs is frequently a barrier to useful research because the t-statistic is biased towards enlarged departures from zero, and, therefore, unwarranted findings of statistical significance. In this instance, however, the presence of autocorrelation actually serves to strengthen our findings. As it turns out, our research interests are satisfied if coefficient estimates are not statistically significant. Because serial correlation contributes to an unwarranted finding of statistical significance, our assertions relating to a lack of statistical significance are strengthened.

Regression results are available upon request from the authors.

The wage growth and risky investment (discount) return assumption used in pension planning by the largest U.S. corporations also provide support for the reasonableness of this \((r'-g')\) differential hypothesis [20, pp. 35-37; 25, pp. 27-28]. A recent study [27] by the Wyatt Company, a prominent national actuarial firm, of the 1984 actuarial assumptions of 961 large pension plans covering 1000 or more active participants revealed the average wage growth rate used was 5.9 percent and the average investment (discount) rate was 7.2 percent. Moody's Financial Services, a prominent national investment analysis firm, conducted a similar survey in 1979 which revealed the Aaa, Aa, A and Baa rated U.S. corporations employed an average 5.15 percent wage growth rate and an average 6.45 percent investment rate in pension planning. It is noteworthy that the \((r-g)\) differential was 1.3 percent in both 1979 and 1984 even though the yield on long-term U.S. securities was three percentage points higher in 1984 than in 1979, and the wage growth rate in the U.S. economy was approximately 2.5 percentage points lower in 1984 than in 1979. These \((r-g)\) pension assumptions data support the conclusion expressed in this paper that actuaries and economists faced with estimating the present value of a stream of future payments rely upon the predictable \((r-g)\) differential rather than upon speculative estimates of the average \(r\) and average \(g\) that might prevail during the time period in question.

As shown in Table I, the nominal levels of the growth rate and the discount rate are of little importance.

The specific calculations, based upon the assumption of a normal distribution, are illustrative only.
Table I


(Earnings \(_{t=0}\) = $1000)

<table>
<thead>
<tr>
<th>Discount Rates</th>
<th>0.0%</th>
<th>1.0%</th>
<th>2.0%</th>
<th>3.0%</th>
<th>4.0%</th>
<th>5.0%</th>
<th>6.0%</th>
<th>7.0%</th>
<th>8.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0%</td>
<td>$25,000</td>
<td>$28,526</td>
<td>$32,671</td>
<td>$37,553</td>
<td>$43,312</td>
<td>$50,113</td>
<td>$58,156</td>
<td>$67,676</td>
<td>$78,954</td>
</tr>
<tr>
<td>1.0%</td>
<td>22,023</td>
<td>25,000</td>
<td>28,488</td>
<td>32,582</td>
<td>37,396</td>
<td>43,065</td>
<td>49,749</td>
<td>57,640</td>
<td>66,963</td>
</tr>
<tr>
<td>2.0%</td>
<td>19,523</td>
<td>22,050</td>
<td>25,000</td>
<td>28,451</td>
<td>32,495</td>
<td>37,243</td>
<td>42,825</td>
<td>49,395</td>
<td>57,138</td>
</tr>
<tr>
<td>3.0%</td>
<td>17,413</td>
<td>19,569</td>
<td>22,077</td>
<td>25,000</td>
<td>28,415</td>
<td>32,410</td>
<td>37,094</td>
<td>42,591</td>
<td>49,051</td>
</tr>
<tr>
<td>4.0%</td>
<td>15,622</td>
<td>17,471</td>
<td>19,614</td>
<td>22,103</td>
<td>25,000</td>
<td>28,379</td>
<td>32,328</td>
<td>36,948</td>
<td>42,362</td>
</tr>
<tr>
<td>5.0%</td>
<td>14,094</td>
<td>15,688</td>
<td>17,528</td>
<td>19,658</td>
<td>22,128</td>
<td>25,000</td>
<td>28,345</td>
<td>32,246</td>
<td>36,805</td>
</tr>
<tr>
<td>6.0%</td>
<td>12,783</td>
<td>14,164</td>
<td>15,752</td>
<td>17,584</td>
<td>19,701</td>
<td>22,153</td>
<td>25,000</td>
<td>28,311</td>
<td>32,167</td>
</tr>
<tr>
<td>7.0%</td>
<td>11,654</td>
<td>12,856</td>
<td>14,233</td>
<td>15,816</td>
<td>17,639</td>
<td>19,744</td>
<td>22,178</td>
<td>25,000</td>
<td>28,277</td>
</tr>
<tr>
<td>8.0%</td>
<td>10,674</td>
<td>11,727</td>
<td>12,928</td>
<td>14,302</td>
<td>15,879</td>
<td>17,694</td>
<td>19,785</td>
<td>22,202</td>
<td>25,000</td>
</tr>
</tbody>
</table>

Earnings in Year 25

| $1,000 | $1,282 | $1,641 | $2,094 | $2,666 | $3,386 | $4,292 | $5,427 | $6,848 |

Cumulative Earnings

| 25,000 | 28,526 | 32,671 | 37,553 | 43,312 | 50,113 | 58,156 | 67,676 | 78,954 |
### Table II

Relationships Between Wage Growth and Contemporaneous Rates of Return: 1953-1984

<table>
<thead>
<tr>
<th>Alternative Investments</th>
<th>Mean (%)</th>
<th>Variance</th>
<th>Cov(r,g)*</th>
<th>(r-g) Differential Mean (%)</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. Securities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Year (r₁)</td>
<td>6.07</td>
<td>10.68</td>
<td>4.17</td>
<td>.56</td>
<td>6.07</td>
</tr>
<tr>
<td>20 Year (r₂₀)</td>
<td>6.50</td>
<td>9.35</td>
<td>3.54</td>
<td>.99</td>
<td>6.01</td>
</tr>
<tr>
<td>Equities (rₑ)</td>
<td>11.68</td>
<td>317.51</td>
<td>-6.48</td>
<td>6.18</td>
<td>334.20</td>
</tr>
</tbody>
</table>

*The mean and variance of the annual wage growth rates (g) are 5.51 percent and 3.73, respectively.*
Table III
(r'-g') Differential Simulation Results: 1953-1984

<table>
<thead>
<tr>
<th>Investment Strategy</th>
<th>5 Years (n=28)</th>
<th>10 Years (n=23)</th>
<th>15 Years (n=18)</th>
<th>20 Years (n=13)</th>
<th>Regression Summary (n=82)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Mean</td>
</tr>
<tr>
<td>U.S. Securities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Year</td>
<td>.00099</td>
<td>.01968</td>
<td>-.00305</td>
<td>.00654</td>
<td>-.00323</td>
</tr>
<tr>
<td>5 Year</td>
<td>-.00347</td>
<td>.02199</td>
<td>-.00552</td>
<td>.00929</td>
<td>-.00521</td>
</tr>
<tr>
<td>10 Year</td>
<td>-.01444</td>
<td>.02751</td>
<td>-.01188</td>
<td>.00995</td>
<td>-.00941</td>
</tr>
<tr>
<td>15 Year</td>
<td>-.02297</td>
<td>.03143</td>
<td>-.02070</td>
<td>.01182</td>
<td>-.01660</td>
</tr>
<tr>
<td>20 Year</td>
<td>-.02815</td>
<td>.03334</td>
<td>-.02645</td>
<td>.01208</td>
<td>-.02420</td>
</tr>
<tr>
<td>Equities</td>
<td>.04122</td>
<td>.07717</td>
<td>.03279</td>
<td>.06166</td>
<td>.03469</td>
</tr>
</tbody>
</table>


Figure 1

Distributions of \((r' - g')\) Differential for 1-Year, 20-Year and Equities Investment Strategies

Percent

![Graph showing distributions of (r' - g') differential for 1-Year, 20-Year and Equities Investment Strategies.](image)