CENTRAL CIRCULATION AND BOOKSTACKS
The person borrowing this material is responsible for its renewal or return before the Latest Date stamped below. You may be charged a minimum fee of $75.00 for each non-returned or lost item.

Theft, mutilation, or defacement of library materials can be causes for student disciplinary action. All materials owned by the University of Illinois Library are the property of the State of Illinois and are protected by Article 16B of Illinois Criminal Law and Procedure.

TO RENEW, CALL (217) 333-8400.
University of Illinois Library at Urbana-Champaign

When renewing by phone, write new due date below previous due date.
Gustav Cassel's Contributions to Economic Theory

Hans J. Brems
BEBR

FACULTY WORKING PAPER NO. 1282

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

September 1986

Gustav Cassel's Contributions to Economic Theory

Hans J. Brems
Professor emeritus
Department of Economics
GUSTAV CASSEL'S CONTRIBUTIONS TO ECONOMIC THEORY

104-WORD ABSTRACT

Before the fiftieth anniversary of the Swedish School, the present paper examines the contributions to economic theory of one of its founding fathers, Gustav Cassel. Cassel broke new ground in at least three areas. First, his microeconomic growth theory was entirely new and became important because it directly inspired John von Neumann. Second, Cassel's macroeconomic growth theory was equally new and fully anticipated Harrod by thirty years. Third, Cassel's theory of the optimal depletion of mines awakened a subject that had remained dormant since Ricardo. Here, Cassel anticipated by sixty years much of what economists were to say in the aftermath of the oil crisis.
Cassel's _Theoretische Sozialökonomie_ (1918) embodies all of his contributions to economic theory, indeed is a massive attempt--our last--to restate, as Mill, Marshall, Walras, and Pareto had done, the entire body of economic theory in one piece. Its lucid style made it an international success. It appeared in five German editions, two different English translations, a French, a Japanese, and, at last, a Swedish translation.

The book broke new ground in several areas of economic theory. The purpose of the present paper is to consider three such areas. First, Cassel's microeconomic growth theory was entirely new and was important because it directly inspired von Neumann [1937 (1968)]. Second, Cassel's macroeconomic growth theory was equally new and fully anticipated Harrod (1948). Third, Cassel's theory of the optimal depletion of mines awakened a subject that had remained dominant for a century. Here, Cassel anticipated much of what economists were to say
about natural resources in the aftermath of the oil crisis of the seventies.

II. MICROECONOMIC GROWTH THEORY

1. Variables

\[ g = \text{rate of growth} \]
\[ P_j = \text{price of } j\text{th output} \]
\[ p_i = \text{price of } i\text{th input} \]
\[ \tau = \text{rate of interest} \]
\[ X_j = \text{jth physical output supplied by industry} \]
\[ X_{jk} = \text{jth physical output demanded by } k\text{th household} \]
\[ x_{ij} = \text{ith physical input demanded by } j\text{th industry} \]
\[ Y_k = \text{money value of income of } k\text{th household} \]

2. Parameters

\[ a_{ij} = \text{ith physical input demanded per physical unit of output of } j\text{th industry, a "technical coefficient"} \]
\[ x_{ki} = \text{endowment of } k\text{th household with ith physical input} \]
3. The Model

Cassel [1923 (1932: 32-41 and 137-155)] was the first to dynamize general equilibrium into his "uniformly progressing state," thus inspiring John von Neumann [1937 (1968)] who, as Weintraub (1983: 4-5) has pointed out, knew the Walras system only in its Cassel version. In neither Cassel nor von Neumann did prices display any growth. But physical quantities did, hence need a time coordinate \( t \).

Let there be \( m \) physical outputs \( X_j \) supplied by industry, demanded by \( s \) households, and priced \( P_j \), on the one hand, and \( n \) primary physical inputs \( X_i \) supplied by \( s \) households, demanded by industry, and priced \( P_i \), on the other. Cassel set out his dynamic system as follows. Input prices will equalize the given supply of any input with the demand for it. Once such prices, assumed to be stationary, are known, so are all incomes:

\[
Y_k(t) = \sum_{i=1}^{n} p_i x_k^i(t) \tag{1}
\]
Wicksell [1919 (1934: 225-226) criticized Cassel's failure to incorporate his treatment of capital and interest into his algebra. Let us help Cassel and incorporate it in the simplest possible way, i.e., in the form of a von Neumann assumption of a universal period of production of one time unit: at time $t+1$ let the $j$th physical output be $X_j(t+1)$ and the $i$th physical input absorbed one time unit earlier be in proportion to it:

$$x_{ij}(t) = a_{ij}X_j(t+1)$$  \hspace{1cm} (2)

All inputs, then, will have to be purchased one time unit before output can be sold, hence will need financing. Let capitalists finance them at the rate of interest $r$. Multiply each input price by the technical coefficient for an industry, add such products for that industry, add interest, assumed to be stationary, and find the price of its output, also assumed to be stationary.

$$P_j = (1 + r) \sum_{i=1}^{n} a_{ij}p_i$$  \hspace{1cm} (3)

Once all incomes and such prices are known, consumer demand follows:
Output prices (3) will equalize the supply of any output with the demand for it:

\[ X_j(t) = \sum_{k=1}^{s} X_{jk}(t) \]  

Cassel's growth of physical outputs and inputs was balanced and steady-state. Balanced growth means that the rates of growth of all physical outputs are equal:

\[ X_j(t + 1) = (1 + g(t))X_j(t) \]  

Steady-state growth means that the rates of growth of all physical outputs are stationary:

\[ g(t + 1) = g(t) \]  

So we may purge \( g \) of its time coordinate \( t \). Once industry supply (5) is known, find that industry's demand for input by inserting (6) into (2):
\[ x_{ij}(t) = a_{ij}(1 + g)X_j(t) \] (8)

Input prices will equalize the given supply of any input with such demand for it:

\[ x_i(t) = \sum_{k=1}^{s} x_{ki}(t) = \sum_{j=1}^{m} x_{ij}(t) \] (9)

As the Austrians had done, Cassel assumed the primary input \( x_{ki} \) supplied by the kth household to be its entire endowment; consequently \( x_i \) is the endowment of the entire economy with the ith primary input, a parameter growing, of course, at the rate \( g \) if with fixed input-output coefficients \( a_{ij} \) physical inputs \( x_{ij} \) and outputs \( X_j \) are to be growing at that rate.

Like von Neumann, Cassel could have related the rate of growth \( g \) to the rate of interest \( r \) and assumed income \( Y_k(t) \) from primary inputs to be never saved and income from interest to be never consumed. He neither did so nor prove the existence of a solution. Instead Cassel merely counted equations and unknowns and was satisfied [1923 (1932: 140, 145)] that equal numbers of them would "in general" suffice to determine the unknowns—with one reservation.
4. The Walras-Cassel Dichotomy that Came, Went, and Came Back

Cassel's system, like that of Walras, was homogeneous of degree zero in its prices, money expenditures, and money incomes. In this sense the system was indeterminate. The job of determining absolute prices, money expenditure, or money incomes would be left, Cassel [1923 (1932: 154-155)] said, to monetary policy: doubling the money supply would double all prices and money incomes but leave relative prices and real incomes unchanged, hence leave all physical quantities of goods unchanged. In short: monetary policy could affect nominal variables but never real ones.

The Walras-Cassel dichotomy between nominal and real variables came, went, and came back. Keynes removed it: under frozen prices and idle resources a larger money supply would shift the LM curve to the right, thus reducing the rate of interest and raising physical output. By introducing their real-balance effect, Pigou (1943), (1945) and Patinkin (1956) not only restored the Walras-Cassel dichotomy but showed that monetary equilibrium was stable. Their argument was this. Under flexible prices and money wage rates, a negative excess demand in the goods and labor markets would make prices and money wages fall, hence raise the real value of whatever cash balances held. Include such real cash balances in the consumption function.
Then real cash balances would keep rising and encouraging consumption until the initial negative excess demand had vanished. Indeed monetary equilibrium was stable in both directions: a positive excess demand in the goods and labor markets would make prices and money wages rise, hence lower the real value of whatever cash balances held. Then real cash balances would keep falling and discouraging consumption until the initial positive excess demand had vanished. In short, as Hansen (1957: 92-93) pointed out, any money supply would do for full employment, and the quantity theory had been revived! If Keynes thought otherwise, it was because he had frozen his prices and lacked the real balances in his consumption function. For good measure, Friedman (1968) and Phelps (1970) restored the Walras-Cassel dichotomy by an alternative argument: there exists a natural rate of unemployment unaffected by the money supply.

5. Cassel and Walras

So at the dichotomy between nominal and real variables Cassel and Walras looked eye to eye. Other agreements were readily apparent.

Like Walras, Cassel allowed for substitution in consumption but not in production: Walras's "coefficients de fabrication" became Cassel's "technical coefficients." Like Walras, Cassel failed to
treat the distinction between free and economic goods as endogeneous although, inconsistent as he rarely was, he himself [1923 (1932: 148)] offered an example of a good which under one technology would be free but under another economic: when used to generate mechanical power the waterfalls of Scandinavia were abundant and their power "very cheap...if not valueless." Used to generate electric power they had become scarce, which "raised the price of natural waterpower."

Was Cassel no more and no less than Walras, then? He was at the same time more and less.

Walras asked how a stationary economy would allocate inputs among outputs and outputs among households. Cassel asked how a growing economy would do those things and showed [1923 (1932: 153)], in our (8), that in a growing economy the current physical input required per physical unit of current output was a new coefficient \( a_{ij} (1 + g) \) that would "contain, in addition to the elements of the old 'technical coefficients' \( a_{ij} \), only the rate of progress \( g \)." In this sense, Cassel was indeed more than Walras.

Walras thought of utility as a measure of human sensation. Pareto [1906 (1971: 105-133)] abandoned the meaning of utility as such a measure and replaced it by a utility index which "must satisfy the following two conditions, and ... is arbitrary in other respects: (1)
Two combinations between which the choice is indifferent must have the same index; (2) of two combinations, the one which is preferred to the other must have the larger index." Infinitely many indices would serve equally well as long as any of them was a monotonic transformation of any other. Here we may ask two questions. First, given a utility function using such a Paretian index, can a demand function always be found by maximizing the utility function subject to a budget constraint? The answer is yes provided the utility function is differentiable and strictly quasi-concave. But let us go the opposite direction: given an observed demand function, can a utility function always be found whose maximization subject to a budget constraint will deliver the given demand function? Here the answer is: not necessarily. Antonelli (1886) and Fisher (1892: 86-89) were the first to see this so-called integrability problem.

Cassel may never have heard of the integrability problem; at least he never mentioned it. What he did say was that demand is observable and that utility is not. As a quantitative science economics must deal with observables only, so Cassel [1899, 1923 (1932)] purged his system of all references to utility. In this sense he was less than Walras and the first to use revealed preference—anticipating Samuelson (1938) by 20 years.
So Cassel was at the same time more and less than Walras. Either way his debt to Walras is apparent. Cassel (1899) did mention Walras but merely to scold him for his utility concept. Nowhere in Cassel [1923 (1932)] can the name Walras be found. In his autobiography Cassel (1940: 435) says: "When [after 1899] I continued developing economic theory on the foundation I had chosen, I found it unnecessary to occupy myself with Walras and actually never had time to open his works."

III. MACROECONOMIC GROWTH THEORY

1. Variables

\[ C \equiv \text{physical consumption} \]
\[ g \equiv \text{rate of growth (Cassel's p/100)} \]
\[ I \equiv \text{physical investment} \]
\[ S \equiv \text{physical capital stock (Cassel's C)} \]
\[ X \equiv \text{physical output produced and sold (Cassel's I)} \]
2. Parameters

\[ b = \text{capital coefficient (Cassel's C/I)} \]
\[ c = \text{propensity to consume (Cassel's } 1 - 1/s) \]

3. The Model

Thus Cassel had given us a microeconomic growth model. But later in the same volume he [1923 (1932: 61-62)] also gave us a macroeconomic one, fully set out in hard algebra identical except for notation \(^1\) to that of Harrod (1948) 30 years later. We set it out in our own notation as follows. Define the rate of growth of output as

\[
g = \frac{\frac{dX}{dt}}{X} \quad \text{(10)}
\]

Define investment as the derivative of capital stock with respect to time:
\[
\frac{dS}{dt} = I \quad \text{(11)}
\]

Let physical capital stock be in proportion to output:

\[ S = bX \quad \text{(12)} \]

Let physical consumption be a fixed proportion of output:

\[ C = cX \quad \text{(13)} \]

where \(0 < c < 1\).

Finally, let the system be in equilibrium. Goods-market equilibrium requires the supply of goods to equal the demand for them:

\[ X = C + I \quad \text{(14)} \]

We may now solve our Cassel system.

Insert (12) into (11) and write the pure accelerator \[ I = b\frac{dX}{dt} \].

Use (10) to write it \[ I = bgX \]. Insert the accelerator along with the consumption function (13) into the goods-market equilibrium condition.
(14), divide by \( X \), and find the solution for the steady-state equilibrium rate of growth of physical output:

\[
g = \frac{(1 - c)}{b} \tag{15}
\]

Exactly as in Harrod, then, the rate of growth of output equals the propensity to save divided by the capital coefficient. Since both \( 1 - c \) and \( b \) are stationary parameters, the rate of growth of output is stationary: growth is steady-state and balanced or, in Cassel's [1923 (1932: 62)] own words: "We ... come to the conclusion that, in the uniformly progressive exchange economy, the total income as well as both its parts—consumption and capital accumulation—increases in the same percentage as the capital."

In a Cassel model a higher propensity to save will permit more investment and hence more rapid growth; indeed our solution (15) shows the steady-state equilibrium rate of growth to be in direct proportion to the propensity to save. Saving is a Good Thing! Writing in 1914, Cassel had no Keynesian savings paradox to unlearn and observed [1923 (1932: 61-62)] that "saving is the chief element in progress."

Cassel saw his uniformly progressive economy merely as a first, but important, approximation—many other possible patterns were to be
found at Stockholm by Lundberg (1937). Empirically, would such a first approximation be a good one?

Cassel [1923 (1932: 62)] was sure that "the total income I ... stands in an invariable ratio to the total capital C." As always, Cassel was looking for statistical estimates and found one for the year 1908 done by a Swedish commission for national defense using tax and insurance valuations of real capital. The result was a Swedish capital coefficient \( b = 6^{2/3} \). As for saving [1923 (1932: 61)] "the degree of saving \( 1/s \), the relative 'thriftiness' of the people, may be assumed to be constant." Without referring to sources, Cassel estimated it to be one-fifth. Now insert Cassel's estimates into (15) and find an equilibrium rate of growth of 3 percent, delightfully close, Cassel [1923 (1932: 63)] observed, to the national defense commission estimate of a Swedish growth rate for the period 1885-1908 of 3.18 percent.
IV. OPTIMAL DEPLETION OF MINES

1. **Variables**

\[ c = \text{cost of extraction per annum} \]
\[ J = \text{present net worth of mine} \]
\[ q = \text{physical quantity of mineral extracted per annum} \]
\[ R = \text{rent of mine per annum} \]
\[ u = \text{useful life of mine} \]

2. **Parameters**

\[ a = \text{cost of extraction per ton} \]
\[ g = \text{rate at which price and cost per ton are inflating} \]
\[ p = \text{price of mineral per ton} \]
\[ Q = \text{initial physical mass of mineral contained in mine} \]
\[ r = \text{rate of interest} \]
3. A Practical Issue

Sweden was traditionally a major exporter of iron ore mined at Kiruna-Gällivare in her Far North. Sweden had traditionally applied a conservationist public policy imposing a maximum export quota. In 1906 Cassel attacked that policy in the daily press and observed (1940: 113-114) that with annual compound interest at five percent, one krona now would become more than 130 kronor a century hence. But unmined ore carried no interest; consequently it would be more than 130 times better to extract a ton of ore now than to wait a century before doing so. It would be much better, Cassel continued, if the ore deposit had been a coal deposit or, even better, many smaller coal deposits near the major ports of Sweden. Wishful thinking? By no means. "So powerful is the giant power called international trade that it is literally capable of moving mountains!"

Cassel's advice was to do away with the export quota and let the market decide what the optimal depletion of mines should be. What should it be, then? Ricardo [1817 (1951: ch. 2)] had observed that the rent of mines "is paid for the value of the coal or stone which can be removed from them, and has no connection with the original and indestructible powers of the land." In other words, such rent was not income but depletion allowance. Perhaps Ricardo's negative conclusion
made the subject look uninteresting. However that may be, the subject remained virtually dormant for a century until Cassel [1923 (1932: 289-297)] took it up and showed that in a free market optimal depletion will depend on the rate of interest and the future price of the mineral.

To show how, we simulate Cassel's words by simple algebra.

4. Solution for Optimal Depletion

At time \( t \) let an entrepreneur own a mine containing the physical mass \( Q \) of a mineral. He cannot enhance the mass \( Q \). His option is a large annual extraction with a short useful life of his mine **versus** a small annual extraction with a long useful life. Whatever he chooses he is up against the fact that

\[ qu = Q \quad (16) \]

Whatever he chooses assume, as Cassel [1923 (1932: 292)] did "for simplicity's sake," the annual extraction \( q \) to remain uniform throughout the useful life of the mine. Let the cost of extraction be in direct proportion to extraction:
\[
\begin{align*}
\text{c} &= aq \quad (17)
\end{align*}
\]

where \( a \) is the cost of extraction per ton and is inflating at the rate \( g \) per annum

\[
\begin{align*}
a(t) &= e^{g(t-v)}a(v) \quad (18)
\end{align*}
\]

Let the mineral sell at the price \( p \) also inflating at the rate \( g \) per annum:

\[
\begin{align*}
p(t) &= e^{g(t-v)}p(v) \quad (19)
\end{align*}
\]

Defined as revenue minus cost of extraction, the rent of the mine will also be inflating at the rate \( g \) per annum:

\[
\begin{align*}
R(t) &= p(t)q - c(t) = [p(t) - a(t)]q = [p(v) - a(v)]qe^{g(t-v)} \\
R(t) &= \int_{v}^{t+
u} e^{r(t-v)}R(v)dt, \quad \text{then. As seen from time } v \text{ its present worth is } e^{-r(t-v)}R(t)dt = e^{-(r-g)(t-v)}R(v)dt, \quad \text{and the present worth of all future rent will be the integral}
\end{align*}
\]
\[
J \equiv \int_{v}^{v+u} e^{-r(t-v)} R(t) dt = \frac{1 - e^{-(r-g)u}}{r-g} R(v) \quad (21)
\]

Now how will the entrepreneur choose between a short and a long useful life of his mine? He will maximize his present worth \( J \) with respect to his useful life \( u \). Remembering that via (16) and (20) \( R(v) \) is a function of \( u \), take the derivative of (21) with respect to \( u \), set it equal to zero, and find a transcendental first-order condition

\[
e^{-(r-g)u} - 1 + e^{-(r-g)u}(r-g)u = 0 \quad (22)
\]

To find the sensitivity of useful life \( u \) to \( r-g \), consider both \( u \) and \( r-g \) variables, differentiate (22) implicitly with respect to \( r-g \), and find the remarkably simple elasticity

\[
\frac{r-g}{u} \frac{du}{d(r-g)} = -1 \quad (23)
\]
or, which is the same thing, optimal useful life is always in inverse proportion to the factor \( r - g \). As a result, given the rate at which price and cost per ton are inflating, optimal useful life will be the shorter the higher the rate of interest. And given the rate of interest, optimal useful life will be the longer the higher the rate at which price and cost per ton are inflating. Those were exactly Cassel's [1923 (1932: 293)] own conclusions, arrived at without the benefit of algebra or our unitary elasticity (23).

V. CASSEL AND WICKSELL

Cassel (1866-1945) was one of the founding fathers of the Swedish School; Wicksell (1851-1926) was the other, and a comparison suggests itself.

Both men came to economics from mathematics. Thus both had a head start, but Wicksell made more operational, and therefore more effective, use of his mathematics. Both men had a remarkable ability to reduce a problem to its essence; both wrote a terse and lucid German. Both were original thinkers, but Wicksell thought deeper. Cassel's comparative advantage was his ease with data. Long before the days of national income accounting, Cassel managed to find and effectively use
the data he needed. We have seen his estimate of the capital coefficient and the propensity to save. Another example is the massive use of data in his business-cycle theory to which Wicksell [1919 (1934: 255)] paid tribute: "it is in my opinion incomparably the best part of his work. Professor Cassel's great gifts for concrete description based on facts and figures here show to advantage."

What did, by the way, Cassel and Wicksell think of one another? Surprisingly enough, Wicksell was particularly critical of what we might consider Cassel's greatest contribution, i.e., his growth theory. Was Wicksell carried away by his own Neomalthusianism when he [1919 (1934: 241)] wrote about "Professor Cassel's irrational inclination to regard as normal what is from a quantitative point of view a violently progressive society"? Equally surprising, Cassel (1940: 36) was particularly critical of what we might consider Wicksell's greatest contribution, i.e., his models of circulating capital [1893 (1954)] and [1901 (1934)], his model of fixed capital [1923 (1934)], and his integration of capital with monetary theory [1898 (1936)]. In 1901 Cassel (1940: 36) could comment only on Wicksell's early work: "Wicksell consistently refers to Böhm-Bawerk as 'the master' and sees a mission in clothing the master's theory in mathematical formulae." Cassel's objections were that the period of production was not
generally measurable and that circulating capital was practically unimportant anyway.

In character Cassel and Wicksell were as different as night and day. A writer more generous to others than Wicksell would be hard to find. By contrast, Cassel followed Walras and Pareto, mentioned neither, and never paid tribute to anybody. Indeed if Cassel's autobiography (1940-1941) and the successive editions and translations of *Theoretische Sozialökonomie* were marred by a unifying theme it was his lack of generosity to others and his conviction of his own infallibility, so irritating to his reader—and so redundant: his work could well have spoken for itself!
FOOTNOTE

Cassel's capital coefficient $C/I \equiv$ Harrod's $C$; Cassel's propensity to save $1/s \equiv$ Harrod's $s$; Cassel's proportionate rate of growth $p/100 \equiv$ Harrod's $G$; and Cassel's physical output $I \equiv$ Harrod's $Y$. Cassel's physical capital stock $C$ had no explicit counterpart in Harrod. Explicitly Harrod merely used his capital coefficient $C$. 
REFERENCES


Cassel, Karl Gustav, "Grundriss einer elementaren Preislehre,"
*Zeitschrift für die gesamte Staatswissenschaft*, 1899, 55, 395-458.


END