Faculty Working Papers

Nature and Neoclassical Growth

Hans Brems
University of Illinois

#55

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

FACULTY WORKING PAPERS

College of Commerce and Business Administration

May 10, 1972

Nature and Neoclassical Growth

Hans Brems
University of Illinois

#55

SUMMARY

111 Economic growth theory and models
720 Natural resources

Brems, H.—Nature and neoclassical growth

In the one-output, two-input neoclassical growth model output has been shown to be converging to steady-state growth defined as a stationary proportionate rate of growth. The article includes the services of irreproducible and stationary nature as a third input in the production function and shows that steady-state growth of capital stock, output, physical marginal productivity of capital, and the real wage rate still results. The article also shows that even a highly developed economy with plenty of capital accumulation could generate a decaying real wage rate.

May 10, 1972

HANS BREMS
Box 99 Commerce West
University of Illinois
Urbana, Illinois 61801

NATURE AND NEOCLASSICAL GROWTH

By HANS BREMS*

In the one-output, two-input neoclassical growth model [2], [3] output has been shown to be converging to steady-state growth defined [1] as a stationary proportionate rate of growth. Would such steady-state growth be in jeopardy if the production function were to include as a third input the services of irreproducible and stationary nature? Even if not, could the real wage rate in a highly developed economy be displaying steady-state decay?

I. NOTATION

$C \equiv$ consumption

$g_v \equiv$ proportionate rate of growth of the variable v where $v \equiv \kappa$, P, S, and X

g_{gv} ≡ proportionate rate of acceleration of the variable v where v ≡ S

I ≡ investment

κ ≡ physical marginal productivity of capital stock

L ≡ labor employed

P ≡ price of good

S ≡ physical capital stock

X ≡ physical output

Parameters

α, β, γ ≡ exponents of production function

c ≡ propensity to consume

F ≡ available labor force

g_p ≡ proportionate rate of growth of parameter p where p ≡ F, M, and w

M ≡ multiplicative factor of production function

N ≡ nature

w ≡ money wage rate

The parameters listed are stationary except F, M, and w, whose growth rates g_F, g_M, and g_w are stationary.

II. THE EQUATIONS OF THE MODEL

Let capitalist-entrepreneurs produce a single good from labor, an immortal capital stock of that good, and nature, hence investment is the act of setting aside part of output for installation as capital stock. Write the equations of such a system as follows. To the four variable growth rates listed in Section I apply the definition

(1) through (4) $$g_v \equiv \frac{dv}{dt}\frac{1}{v}$$

Define investment as the derivative of capital stock with respect to time

(5) $$I \equiv \frac{dS}{dt}$$

Let the capitalist-entrepreneurs apply the Cobb-Douglas production function

(6) $$X = ML^{\alpha}S^{\beta}N^{\gamma}$$

where $0 < \alpha < 1$; $0 < \beta < 1$; $0 < \gamma < 1$; $\alpha + \beta + \gamma = 1$; $M > 0$; and N is stationary. Let profit maximization under pure competition equalize real wage rate and physical marginal productivity of labor:

(7) $$\frac{w}{P} = \frac{\partial X}{\partial L} = \alpha \frac{X}{L}$$

Physical marginal productivity of capital is

(8) $$\kappa \equiv \frac{\partial X}{\partial S} = \beta \frac{X}{S}$$

Under full employment, available labor force must equal labor employed:

(9) $$F = L$$

Let consumption be a fixed proportion of output:

(10) $$C = cX$$

where $0 < c < 1$. Output equilibrium requires output to equal the sum of consumption and investment demand for it, or inventory would either accumulate or be depleted:

(11) $$X = C + I$$

III. SOLUTIONS FOR PROPORTIONATE RATES OF GROWTH

Despite the diminishing returns imposed by stationary nature, our system (1) through (11) can be shown to possess a set of steady-state solutions for the proportionate rates of growth of

capital stock, output, physical marginal productivity of capital, and the real wage rate. To find that set, let us begin by inserting (9) into (6) and differentiate the latter with respect to time:

(12) $$g_X = g_M + \alpha g_F + \beta g_S$$

From (10), (11), and (1) through (5) find

(13) $$g_S = (1 - c)X/S$$

Differentiate (13) with respect to time, use (1) through (4), and find the proportionate rate of acceleration

(14) $$g_{gS} \equiv \frac{dg_S}{dt}\,\frac{1}{g_S} = (\alpha + \gamma)[(g_M + \alpha g_F)/(\alpha + \gamma) - g_S]$$

In (14) there are only three possibilities: If $g_S >$ $(g_M + \alpha g_F)/(\alpha + \gamma)$, then $g_{gS} < 0$. If

(15) $$g_S = (g_M + \alpha g_F)/(\alpha + \gamma)$$

then $g_{gS} = 0$. Finally, if $g_S < (g_M + \alpha g_F)/(\alpha + \gamma)$, then $g_{gS} > 0$. Consequently, if greater than (15) g_S is falling; if equal to (15) g_S is stationary; and if less than (15) g_S is rising. Now g_S cannot converge toward anything else than (15), for if it did, then by letting enough time elapse we could make the left-hand side of (14) less than any arbitrarily assignable positive constant ε, however small, without the same being possible for the right-hand side. So g_S converges toward (15). Insert (15) into (12) and find

(16) $$g_X = (g_M + \alpha g_F)/(\alpha + \gamma)$$

Output and capital stock, then, will eventually be growing at the same steady-state proportionate rate (15) and (16). That may be lower than when nature was ignored, but it is still a steady-state rate!

What is happening to the physical marginal productivity of capital? Take the derivative of (8) with respect to time, insert (1) through (4), and find

(17) $$g_{\kappa} = g_X - g_S$$

Insert (12) into (17) and find

(18) $$g_{\kappa} = (\alpha + \gamma)[(g_M + \alpha g_F)/(\alpha + \gamma) - g_S]$$

Comparing (18) to (14) we find that the rate of growth of physical marginal productivity equals the rate of acceleration of capital stock. Since the latter converges to zero, so must the former: Physical marginal productivity of capital eventually grows at the steady-state rate of zero, i. e., becomes stationary.

What is happening to the real wage rate? Take the derivative of (7) with respect to time, insert (1) through (4) and (9), and find

(19) $$g_w - g_P = g_X - g_F$$

Insert (12) into (19) and find

$$g_W - g_P = g_M - (1 - \alpha)g_F + \beta g_S \tag{20}$$

But we have just seen that g_S will converge to (15). Insert (15) into (20), write $1 - \alpha = \beta + \gamma$, and find the rate of growth of the real wage rate converging to

$$g_W - g_P = (g_M - \gamma g_F)/(\alpha + \gamma) \tag{21}$$

We conclude that if the natural-resource elasticity of output γ is high enough, if labor-force growth g_F is high enough, or if technological progress g_M is low enough, then even a highly developed economy with plenty of capital accumulation could generate a decaying real wage rate.

*

F O O T N O T E

*Professor of economics at the University of Illinois at Urbana.

R E F E R E N C E S

[1] F. H. Hahn and R. C. O. Matthews, "The Theory of Economic Growth: A Survey," Econ. Jour., Dec. 1964, 74, 779-902.

[2] R. M. Solow, "A Contribution to the Theory of Economic Growth," Quart. Jour. Econ., Feb. 1956, 70, 65-94.

[3] J. Tinbergen, "Zur Theorie der langfristigen Wirtschaftsentwicklung," Weltw. Archiv, May 1942, 55, 511-549.

*