Faculty Working Papers

OPTIMAL HARVESTING POLICIES AND REGULATIONS FOR RENEWABLE RESOURCES

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Summary:

Renewable resources constitute an increasingly important source of food and material necessary for meeting the needs for survival of the increasing population of human beings in the world. Continuous, unplanned consumption of these resources could very dangerously lead to their extinction.

In this paper, we develop a dynamic theory for optimally managing such resources in order to prevent their extinction, while at the same time insuring an adequate level of supply for human consumption. We derive a policy for harvesting which meets several requirements: (i) does not exceed a certain level set by technological or conservational considerations, (ii) results in a certain level of population of the resource conserved at end of a given time horizon, and (iii) maximizes the total quantity harvested. A discussion of the properties of this policy and the effects of enforced regulations on it are also presented.

## Introduction

Renewable resources such as fish, whale, deer, forest, etc., constitute an increasingly important class of economic resources for the sustenance and improvement of human welfare on the planet Earth. The common characteristic of these resources are that they are for direct human consumption and that they can reproduce themselves with a specific speed of renewal given a specific environment.

In order to prevent extinction of these resources, regulatory agencies have been set up to regulate and limit consumption. At the same time in order to maintain a certain degree of economic growth, consumption must be allowed to be at an adequate level.

Before any regulations were intorduced, the history of consumption of many such resources (such as whale, deer, etc.) was at a level high enough to make it impossible for sustained growth to take place. When consumption regulations were introduced, the problems that were faced centered around what policies should the regulatory agencies impose in order to insure that an adequate supply of the resources is available at all times. Similarly, from the producer's point of view, the problem remains that of determining production[1] policies in order to maximize production without violating the regulatory agency's requirements.

In this paper we develop a dynamic theory of renewable resources economics that takes these common characteristics into consideration to establish:

a) an adequate policy of consumption, and

b) a principle of conservation for this class of resources.

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[1]We assume that for these resources, the market is such that consumption is always equal to production.

Since this class of resources embraces a large number of species, in this paper we will deal with only one species (for example fish) without loss of substance. The problem that regulatory agencies face could be summarized as that of determining and recommending the maximum allowable intensity of withdrawal (harvesting) that will sustain an adequate level of the species for it to grow. The problem that the producing sector faces could be summarized as that of determining an optimal withdrawal policy that will maximize production without violating the requirements of the regulatory agencies.

Stated differently, renewable resource economists consider the following questions as theoretically and practically important:

a) What is the optimal rate at which the species should be harvested?

b) Why might the maximum sustainable yield not be optimal?

c) Under what conditions will extinction of the species occur? (Peterson and Fisher [1])

d) Is withdrawal regulation necessary and if so at what level?

<u>The Model</u>

While the literature on renewable resources[2] is extensive, in this paper we avoid our own review. However, we should mention that the model considered in this paper is novel in that it allows for a formulation of the problem in its most natural dynamic optimization framework and takes into consideration the interplay, or interdependence, of the policies of both the production sector and the regulatory agencies. Furthermore it is implicitly assumed that this analysis is

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[2] We avoid our own review of the existing literature in this field. The reader is referred to a comprehensive review in [1], and interesting results in [2] and [3].

justified mostly in cases where the resources are limited relative to technology and human demand.

We conceive of a renewable resource, or species, population which follows dynamics over time described by the differential equation:

$$\frac{dx(t)}{dt} \equiv \dot{x}(t) = f(x(t)) - u(t)$$

where

$x(t)$ denotes the recruitable species population at time t, (more clearly, the unit must be expressed in number, pounds or tons of the resource at or older than the recruitable age);

$\dot{x}(t)$ denotes the rate of change of the population at time t;

$u(t)$ denotes the intensity or rate of withdrawal (catch, or harvest) of the resource at time t; and

$t$ denotes the real time over which the population and withdrawal are moving and measured.

We assume that policies are to be determined over a time horizon [0, T] and that at time t = 0, the species population is known and equal to x(0).

The differential equation[3]

$$\dot{x} = f(x) \qquad (2)$$

itself, is called the biological growth law [1]. It represents the law, such as exponential, quadratic, Volterra, etc., that governs growth if no harvesting takes place. The function $f(\cdot)$ naturally depends on the type of species and the environment. From (2), it is easy to see that

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[2]Hereafter, unless otherwise stated or special emphasis needed, the time t in x(t) and u(t) (and other variables that may be introduced later) will be omitted.

the population x(t) at time t satisfies:

$$x(t) = \int_0^t f(x(\tau))\, d\tau + x(0) \tag{3}$$

and is a function of the initial population x(0) and the population history $x(\tau)$ over the interval of time [0, t]. For many species such as fish, deer, etc., growth follows an exponential law. In this case equation (2) is linear and takes the form:

$$\dot{x} = ax \tag{4}$$

The population at time t, (3), can be easily shown to be of the form:

$$x(t) = x(0)\, e^{at} \tag{5}$$

The constant a, assumed to be nonnegative, is called the rate of growth. If withdrawal at the rate u(t) takes place, the population at time t can be computed to be:

$$x(t) = x(0)\, e^{at} - \int_0^t e^{a(t-\tau)}\, u(\tau)\, d\tau \tag{6}$$

and the total quantitity of species harvested over the time interval [0, t] is

$$h(t) = \int_0^t u(\tau)\, d\tau \tag{7}$$

If the producing sector's technology permits a maximum rate of harvesting equal to $u_{max}$, and if no regulations exist to limit harvesting, then it is easy to determine the time $t_{max}$ at which extinction of the species will occur if the producing sector harvests at the rate $u_{max}$. This is:

$$t_{max} = \frac{1}{a} \ln \left[\frac{u_{max}}{u_{max} - ax(0)}\right] \tag{8}$$

The total[4] quantity harvested is obtained as

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[4]Naturally here we assume that $u_{max} - ax(0) \geq 0$.

$$h_{max} = \frac{u_{max}}{a} \ln \left[\frac{u_{max}}{u_{max} - ax(0)}\right] \qquad (9)$$

we note that, from (8) and (9) we have

$$\frac{\partial t_{max}}{\partial u_{max}} = \frac{-x(0)}{u_{max}(u_{max} - ax(0))} < 0 \qquad (10)$$

and

$$\frac{\partial h_{max}}{\partial u_{max}} = \frac{-x(0)}{u_{max} - ax(0)} + \frac{1}{a} \ln \frac{u_{max}}{u_{max} - ax(0)} < 0 \qquad (11)$$

which essentially confirms well known intuitive conclusions that the better the technology (i.e., the higher $u_{max}$), the faster extinction will occur and the smaller the total harvest.

From a conservational point of view, as well as from an economical point of view, this of course is not a desirable policy. Unfortunately, the production industry, which in many cases is interested in short term profit maximization, may not worry about these effects. This will necessitate regulation which can be applied by imposing:

i) a maximum limit of harvesting rate, $\bar{u}$, which is generally smaller than the maximum technologically feasible rate $u_{max}$ (i.e., $\bar{u} < u_{max}$), and

ii) a required species population[5], $\bar{x}$, left over at the end of the harvesting period T.

In short, the problem that the production sector faces is: given that

$$\dot{x} = ax - u, \; x(0) = x_o \text{ given}, \qquad (12)$$

determine a harvesting policy u(t), which meets $0 \leq u(t) \leq \bar{u}$, over the time horizon [0, T], such that the total harvest

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[5]Naturally $\bar{x}$ must not exceed $x(T) = x(0)\, e^{aT}$ which is the species population if no harvesting takes place.

$$h = \int_0^T u(t)\, dt \tag{13}$$

is maximized; and such that the population at T meets the requirement $x(T) = \bar{x}$. In (12), we have chosen a species with exponential growth, ($f(x) = ax$) simply because the analysis in this case is tractable analytically. The results in general, however, are not limited to this case and can account for any other form of growth such as quadratic, Volterra, etc. In subsequent papers we will show how this can be done.

The problem that the regulatory agency faces is how to choose $\bar{u}$ and $\bar{x}$ in order to insure that extinction of the species will not occur while at the same time helping the producing sector to maximize its harvest.

Optimal Harvesting Policy and Properties

In this section we determine an optimal harvesting policy which:

i) satisfies $0 \leq u(t) \leq \bar{u}$

ii) maximizes the harvest (13), and

iii) result in $x(T) = \bar{x}$.

The standard solution procedure, as well known in optimal control theory [4], is to define a Hamiltonian function:

$$H(x,u,\lambda) = u + \lambda(ax - u) \tag{14}$$

where $\lambda$ is a Lagrange multiplier. The necessary conditions for optimality are [4]:

$$\dot{\lambda} = -\frac{\partial H}{\partial x} = -\lambda a,\ \lambda(0) \text{ and } \lambda(T) \text{ free}^{6} \tag{15}$$

and

$$H(x^*,u^*,\lambda^*) \geq H(x^*,u,\lambda^*) \tag{16}$$

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[6] Note that $\lambda(0)$ and $\lambda(T)$ depend implicitly on $x(0)$ and are not parameters that we choose.

If we assume that possibility 2 holds with the switching time $t_s$ satisfying $0 \leq t_s \leq T$, then the other two possibilities will become a special case with $t_s = T$ for possibility 1 and $t_s = 0$ for possibility 3.

In order to meet the population requirement $x(T) = \bar{x}$, we must have

$$\bar{x} = x_o e^{aT} - \int_{t_s}^{T} e^{a(T-t)} \bar{u}\, dt \tag{19}$$

or

$$\bar{x} = x_o e^{aT} + \frac{\bar{u}}{a} (1 - e^{a(T-t_s)}) \tag{20}$$

This gives a harvest period of

$$T - t_s = \frac{1}{a} \ln \left[\frac{\bar{u} + a(x_o e^{aT} - \bar{x})}{\bar{u}}\right] \tag{21}$$

The total harvest resulting from this policy is equal to:

$$h^* = \frac{\bar{u}}{a} \ln \left[\frac{\bar{u} + a(x_o e^{aT} - \bar{x})}{\bar{u}}\right] \tag{22}$$

And the species population over the time horizon [0, T] can be computed to follow:

$$x(t) = \begin{cases} x_o\, e^{at} & \text{for } 0 \leq t \leq t_s \text{ (no harvesting period)} \\ (\bar{x} - \frac{\bar{u}}{a})\, e^{-a(T-t)} + \frac{\bar{u}}{a} & \text{for } t_s \leq t \leq T \text{ (harvesting period)} \end{cases} \tag{23}$$

It is interesting to note that from (21) and (22) we have:

$$\frac{\partial(T-t_s)}{\partial \bar{u}} = \frac{-(x_o e^{aT} - \bar{x})}{\bar{u}(\bar{u} + a(x_o e^{aT} - \bar{x}))} < 0 \tag{24}$$

and

$$\frac{\partial(T-t_s)}{\partial \bar{x}} = \frac{-1}{\bar{u} + a(x_o e^{aT} - \bar{x})} < 0 \tag{25}$$

and that

$$\frac{\partial h^*}{\partial \bar{u}} = \frac{1}{a} \ln \left[\frac{\bar{u} + a(x_o e^{aT} - \bar{x})}{\bar{u}}\right] - \frac{x_o e^{aT} - \bar{x}}{\bar{u} + a(x_o e^{aT} - \bar{x})} > 0 \tag{26}$$

and

$$\frac{\partial h^*}{\partial \bar{x}} = \frac{-\bar{u}}{\bar{u} + a(x_o e^{aT} - \bar{x})} < 0 \tag{27}$$

Thus, the larger $\bar{u}$, the smaller the harvesting period but the larger the total harvest; and the larger $\bar{x}$, the smaller the harvesting period and the smaller the total harvest. Figures 4 and 5 illustrate the population trajectories according to (23) for different values of $\bar{x}$ and $\bar{u}$.

The optimal harvesting policy (17) can be expressed in terms of the population level x(t) (i.e., in feedback form) by making use of (21). This gives:

$$u^*(t) = \bar{u} \;\; St(x_h - x(t)) \tag{28}$$

where

$$x_h = \frac{\bar{u} + a(x_o e^{aT} - \bar{x})}{\bar{u}\, e^{aT}} x_o \tag{29}$$

The interpretation of this law is that in order to maximize the total catch, harvesting should not start until the population reaches a certain level $x_h$ which we will refer to as the harvesting level.

The special case of possibility 1 (Fig. 1) occurs if $\bar{x} = x_o e^{aT}$. In this case harvesting is restricted at zero level throughout [0, T], and the species is left to growth according to its biological growth law (2).

The special case of possibility 3 (Fig. 3) occurs if $\bar{x} = x_o$ and $\bar{u} = ax_o$. In this case harvesting will take place during all of the time horizon and is equal to the quantity of species in excess of

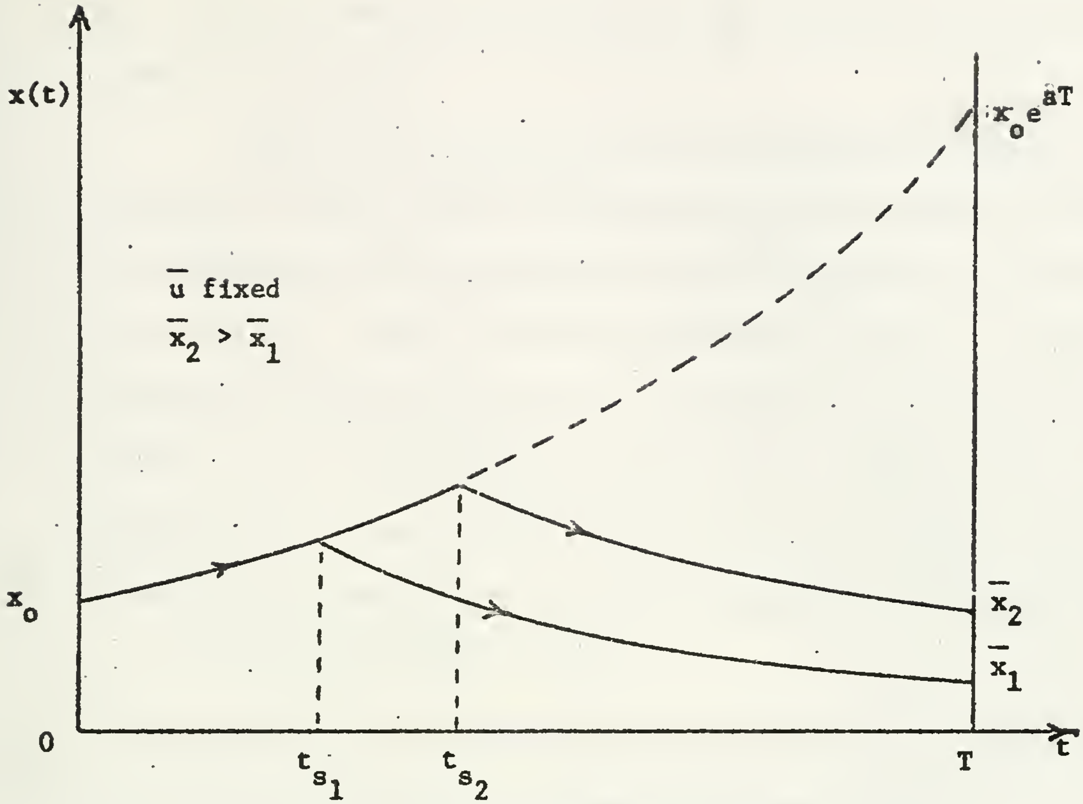


Fig. 4. Population trajectories with $\bar{u}$ fixed and $\bar{x}_2 > \bar{x}_1$.

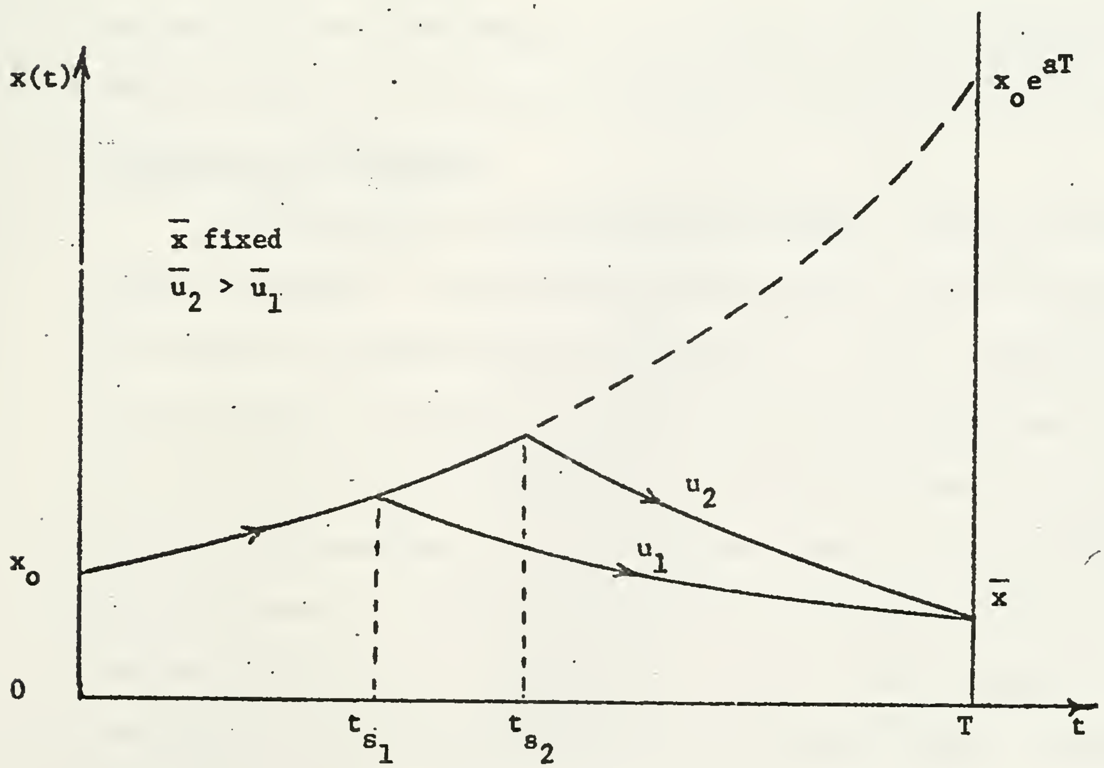


Fig. 5. Population trajectories with $\bar{x}$ fixed and $\bar{u}_2 > \bar{u}_1$.

$x_o$. Thus the species will not grow with time and the total harvest is equal to $h^* = ax_oT$.

Thus from the point of view of the regulatory agency, it would seem that in order to help the producing sector maximize its harvest, it should allow for the maximum possible harvesting rate to take place, i.e., $\bar{u} = u_{max}$. This in essence means that it should not impose any regulation on the harvesting rate and allow the producing sector to harvest at its technologically maximum rate. However, from a conservational point of view, $\bar{x}$ may be selected so as to allow a certain "target" growth rate to take place. In other words, if $\bar{x}$ is chosen according to

$$\bar{x} = x_o e^{\beta T} \text{ (with } \beta < a) \tag{30}$$

it would mean that at the end of the time horizon the population would have grown according to the target growth law (Fig. 6):

$$\dot{x} = \beta x, \; x(0) = x_o \tag{31}$$

This would insure conservation of the species. Stiff penalties however should be imposed if the requirement $x(T) = \bar{x}$ is not met by the producer.

## Conclusions and Comments

In this paper we have developed a dynamic model for renewable resource economics. We have determined an optimal harvesting policy which meets two conservational requirements. First, a maximum allowable rate of harvest and second, a terminal constraint on the species population at the end of the time horizon. It was shown that the optimal policy, which maximizes the total harvest, consists of a no harvest period followed by a period during which harvesting is done at the maximum allowable rate. The length of each of these periods is a function of the maximum allowable rate of harvest $\bar{u}$ and the terminal requirement on the species population $\bar{x}$. The effects of variations

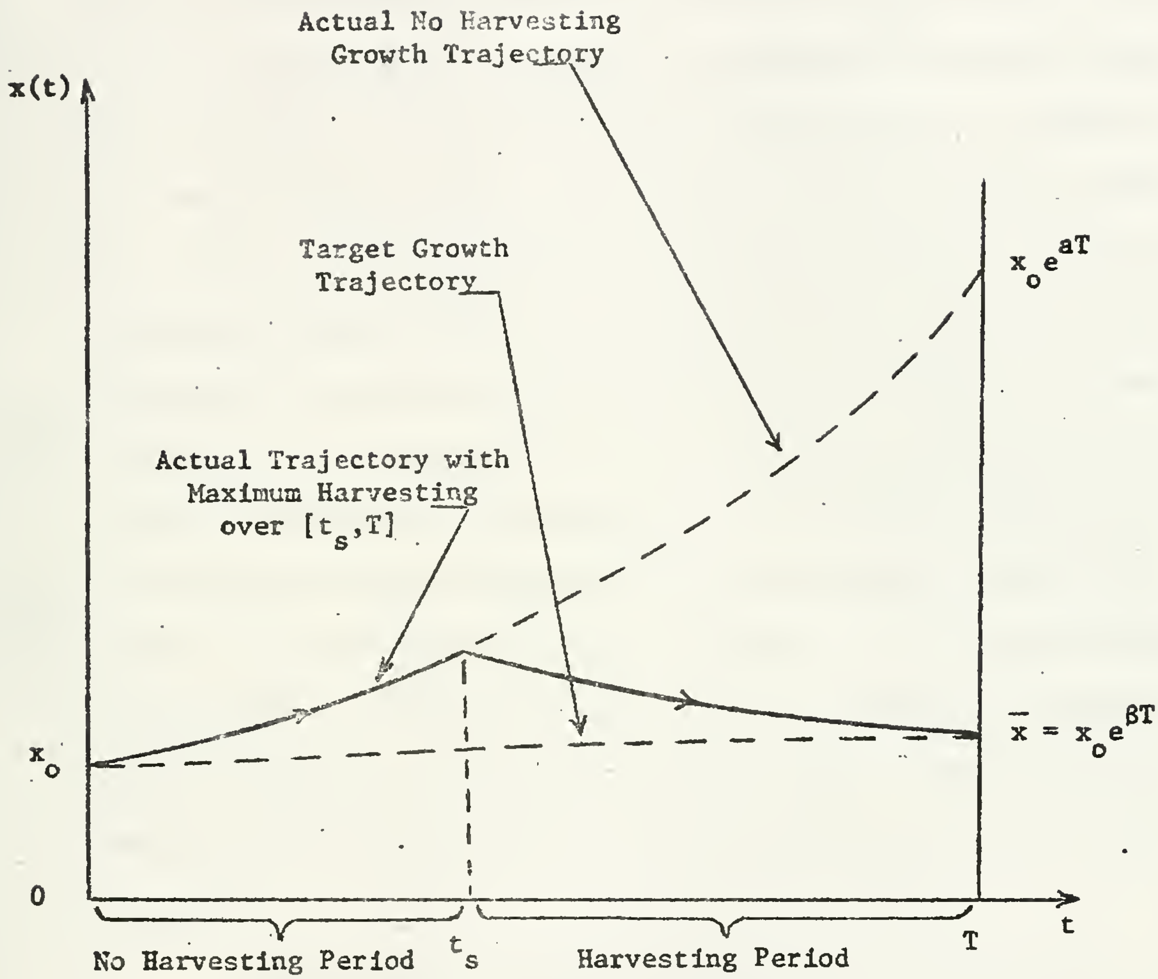


Fig. 6. Actual vs. target growth population trajectories.

of $\bar{u}$ and $\bar{x}$ on the harvesting period and the maximum harvest have been determined. It was shown that a larger $\bar{x}$ will cause a decrease in the length of the harvest period and in the total maximum harvest, and that a larger $\bar{u}$ will cause a decrease in the length of the harvest period but an increase in the total maximum allowable harvest. Answers to the four questions raised in the intorduction have also been given. In summary, the optimal harvesting policy has been given in (28); the maximum sustainable yield is not optimal because it leads to early extinction of the species and does not result in maximum total harvest; Extinction of the species will occur at a time $t_{max}$ given by (8); and finally it was shown that regulation on the rate of withdrawal may not be necessary as long as regulations on the terminal population are imposed. The rate of withdrawal is then set by the maximum technologically feasible rate.[8] This will automatically limit the harvesting period according to the expression given in (21). Thus an easy way of monitoring this policy would be to forbid harvesting over the period $[0, t_s]$.

A final comment, which is of interest, is with regards to cases where the time horizon [0, T] is very large such that the resulting no harvest period may be too long for the optimal policy to be economically desirable. In this case, a long no-harvest period may be undersirable from the consumer point of view. A possible implementation of the optimal policy derived in this paper would be to devide the interval [0, T] into several smaller intervals $[t_i, t_{i+1}]$ for i = 0,...,N-1 with

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[8] If such rate does not exist, or is infinitely large, then naturally a maximum rate $\bar{u}$ should be imposed.

$t_o = 0$ and $t_N = T$. The population requirement at the end of each of these intervals may be imposed to be in the form

$$x_i = x_o e^{\beta t_i}, i = 1,\ldots,N$$

where $\beta$ is the desired "target growth rate". The harvest period is then spread over [0, T], and the species trajectory will follow a path as shown in figure (7). This policy corresponds to a seasonal harvesting law, (such as the case of shrimp harvesting in the Gulf of Mexico), where for instance each year, harvesting is allowed only during a certain season known as the "hunting or fishing" season.

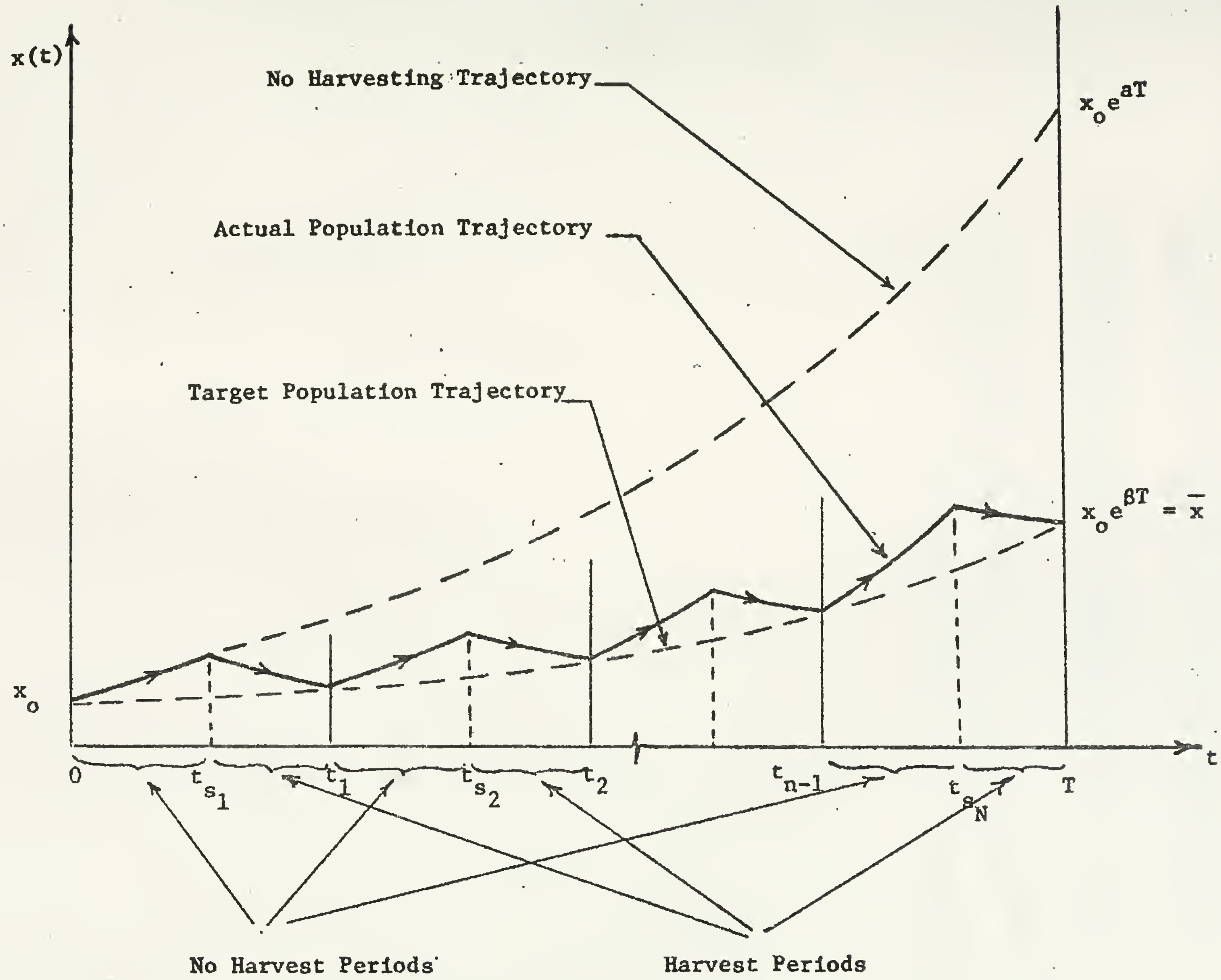


Fig. 7. Implementation of harvesting policy for large time horizons.

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