(A1.5) \[ \frac{(1 - \phi' B)}{(1 - \theta B^4)} y_t = a + (1 - \phi B)w_0x_t + \varepsilon_t \]

where \(a\) and \(w\) have been added to correct for the fact that \(a_t\) in (A1.4) and \(a_t\) in (A1.3) might be of different scale and correlated. Next multiplying both sides of A1.5 thru by \(\frac{(1 - \theta B^4)}{(1 - \phi' B)}\) we obtain

(A1.6) \[ y_t = a' + \frac{(1 - \theta B^4)}{(1 - \phi' B)} (1 - \phi B)w_0x_t + \frac{(1 - \theta B^4)}{(1 - \phi' B)} \varepsilon_t \]

and assuming \((1 - \phi' B)\) cancels with \((1 - \phi B)\) (empirically we found these factors to be approximately equal\(^9\)) we obtain the final model

(A1.7) \[ y_t = a' + (1 - \theta B^4)w_0x_t + \frac{(1 - \theta B^4)}{(1 - \phi' B)} \varepsilon_t \]

which can be written in more conventional form

(A1.8) \[ y_t = a' + w_0x_t + \theta w_0x_{t-4} + \phi' B n_{t-1} + \phi a_{t-4} + \varepsilon_t \]

where \(n_t\) is the noise series.

The result is identical to AMI but the term \(\theta w_0x_{t-4}\) is added to the model.
MORE ON THE USE OF BETA IN REGULATORY PROCEEDINGS

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Summary

This paper analyzes the process by which security prices adjust to new information and shows that the adjustment process itself can lead to temporary non-stationarity of security return distributions. The paper illustrates the serious effect this can have on security returns and argues that the price adjustment process can have a similar effect on ex-post measurements of beta. Some implications for utility rate regulation are discussed.
1. Introduction

Modern portfolio theory is sometimes used to measure the risk perceptions of equity investors and thereby to determine an appropriate allowed rate of return for a utility. Recent articles in this Journal (Breen and Lerner, 1972; Myers, 1972a, 1972b; and Pettway, 1978) have questioned whether utility betas display sufficient stationarity for the return estimation logic of modern portfolio theory to be operationally useful in rate regulatory proceedings.

Blume (1971, 1975), Levy (1971), Porter and Ezzel (1974), Pettway (1978), Francis (1980), and others have reported significant non-stationarity in measured beta. These authors have generally argued that the non-stationarity of measured beta is due to changes in the underlying "true" beta. This hypothesized non-stationarity of "true" beta has been shown to generate unusual measured betas at the individual firm level. Brigham and Crum (1977) have used simulated data to demonstrate that under extreme conditions a drop in the price of a security resulting from an increase in true beta could actually cause measured beta to decline temporarily. Instability in measured beta as well as the possibility of such divergent movements between "true" and measured beta have caused Pettway (1978), Breen and Lerner (1972), Brigham and Crum (1977), Carleton (1978), and others to suggest the E framework may not provide a feasible approach to rate regulation.

This paper will point out that the process by which securities adjust to dramatic, unanticipated, new information can produce misleading estimates of systematic risk. Section 2 presents a theoretical
description of the phenomena. Section 3 suggests this phenomena as a possible explanation for the beta non-stationarity in the utility industry observed by Pettway and the beta non-stationarity surrounding stock splits observed by Bar-Yosef and Brown. Concluding observations comprise the final section.

2. Measured Beta and A Variability Phenomenon

Modern portfolio theory does not require the β measure of security risk be stable over time. Indeed, a firm's perceived systematic risk characteristics and hence its β can be expected to vary with strategic and tactical management decisions made in response to changing product and factor market conditions.

This paper will point out that the price adjustment process by which securities adapt to new information can produce misleading beta measurements.

The potentially bizarre nature of this adjustment phenomena is best illustrated by its effect on security returns. Consider a firm that experiences a shift in its systematic risk from .5 to .75. If the risk free rate, \( R_f \), is 6.0% and the expected return on the market, \( E(R_m) \), is 12%, then investors pre-shift required return is 9.0%, while the post-shift required return is 10.5%. Assume for convenience this firm earns 9.0% on its $100 book value per share, pays out all earnings, and the pre-β shift market price of the stock equals book value. The equilibrium holding period return, \( R_{e_{pre}} \), in the β pre-shift period would be

\[
R_{e_{pre}} = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} = \frac{$100 - $100 + $9}{$100} = 9.0\%
\]
where $P_t$ is price in time $t$, and $D_t$ is dividend in time $t$. Assuming no change in expected return on book value, the only way the $9$ annual dividends can provide the required $10.5\%$ post shift return is for the price to decline to $85.71$. This will produce an observed annual rate of return, $R_o$, during the adjustment period of

$$R_o = \frac{85.71 - 100.00 + 9.00}{100.00} = -5.29\%.$$  

An increase in required return has produced a temporary decrease in return! Such price adjustments are also triggered by changes in earnings expectations or the prevailing risk-return tradeoff.

The effect of the adjustment process on beta is more subtle. The effect can be analyzed by expressing the total observable return, $R_o$, as the sum of an equilibrium return, $R_e$, and an adjustment return, $R_a$, or

$$R_o = R_e + R_a.$$  

Beta is equal to

$$\beta_o = \frac{\text{Cov}(R_o,R_M)}{\sigma_M^2}$$  

$$= \frac{[E(R_o R_M) - E(R_o)E(R_M)]}{\sigma_M^2}.$$  

Expressing $R_a$ as a linear function of $R_e$ yields

$$R_a = a_0 + a_1 R_e + e$$  

where $e$ is the error term. Substituting equation (4) into (3) reveals

$$\beta_o = \frac{E[(a_0 + a_1 R_e + e + R_e) R_M] - E(a_0 + a_1 R_e + e + R_e)E(R_M)}{\sigma_M^2}.$$
\[
(1 + a_1)[E(R_e R_M) - E(R_e)E(R_M)]
\]
\[
= \frac{\sigma^2}{\sigma_m}
\]
\[
= (1 + a_1)(\beta_e)
\]

where

\[
a_1 = \frac{\rho_{a,e} \sigma_a}{\sigma_e}.
\]

Equation (5) shows that if the correlation between the component equilibrium and adjustment returns is positive (negative), measured beta will be greater (smaller) than true beta. However, the direction of the discrepancy between equilibrium and measured beta does not necessarily depend on the direction of the adjustment price change. If, for example, \(\rho_{a,e}\) is always positive, both positive and negative price changes will result an increase in measured beta.

3. A Practical Example

In a recent *Bell Journal of Economics* article Pettway [18] examined whether the estimated beta of a 36 electricity utility portfolio was stable enough to provide good estimates of subsequent observed \(\beta\) values during various subperiods in the 1971-1976 period. In the middle of the test period (April 18, 1974) Consolidated Edison announced it was omitting its second quarter 1974 dividend.

Prior to the skipped dividend, 1971-1973, the utility portfolio beta was relatively low (approximately .40). For some time after the dividend omission, 1974-1975, the portfolio beta was considerably higher (approximately .70), and then it settled back to its original level in the last three quarters of 1976. Pettway argues that the skipped
dividend may have changed the systematic risk of electric utilities for the period immediately following the dividend episode.

Pettway's explanation of the data may be correct. However, this section will offer an interesting alternative interpretation of the same data based upon the adjustment phenomenon described in equation (5).

Unfortunately, a direct test may be impossible. The correlation parameter \( (\rho_{a,e}) \) in the model cannot be directly observed. However, it is possible to work backward and infer what level of correlation between \( R_e \) and \( R_a \) would create the \( \beta \) effect observed by Pettway. The reasonableness of the computed magnitude of the correlation coefficient will provide an indirect test of the adjustment beta concept.

A review of Pettway's findings [18, p. 244] shows that the beta of his 36 electric utility portfolio was about .41 before the April, 1974 dividend announcement. The portfolio beta averaged above .65 following the dividend announcement before returning to the original 1971-1973 level of .41 during the final three quarters of 1976. By substituting these values into equation (5) we can see that

\[
\beta_0 = (1 + \alpha_1)(\beta_e)
\]

\[
.65 = (1 + \alpha_1)(.41)
\]

where

\[
\alpha_1 = \frac{\rho_{a,e}}{\sigma_e} \cdot \frac{\sigma_a}{\sigma_e} = .59
\]

Parts of the Pettway study were replicated in order to estimate the variance of returns before and after the April 1974 dividend omission.
This analysis showed the standard error of returns increased over 80% after the Consolidated Edison dividend shock and settled back to its old value when the portfolio beta itself resumed its 1971-1973 value in the final three quarters of 1976. Therefore,

\[ \sigma_o = 1.80 \sigma_e, \quad \text{or} \]

\[ \sigma_e = (\sigma_e^2 + \sigma_a^2 + 2 \rho \sigma_e \sigma_a)^{1/2}. \]

Substituting (7) into (8) yields

\[ (1.80 \sigma_e)^2 = \sigma_e^2 + \sigma_a^2 + 2 \rho \sigma_e \sigma_a \]

\[ 2.24 = \frac{\sigma_a^2}{\sigma_e^2} + \frac{2 \rho \sigma_e \sigma_a}{\sigma_e} \cdot \]

Equations (6) and (9) can be solved to obtain a \( \rho_{e,a}^2 \) estimate of .178. This suggests that only about 18% of the variance of the price adjustment return series for the utility portfolio would have to be explained by the equilibrium return series for the adjustment phenomenon of equation (5) to account completely for Pettway's findings.\(^2\)

We feel a \( \rho_{e,a}^2 \) of .18 is reasonable. It suggests that the relationship between the two component return series is lower than the relationship discovered by King [11] between market and individual security returns (\( \rho^2 = 30\% \) to 60\%), but higher than the relationship between industry and security returns (\( \rho^2 = 10\% \)).

Pettway's analysis illustrates the effect on a portfolio beta from stock price adjustments to bad news. Studies of the effects of stock splits by Fama, Fisher, Jensen and Roll (1969), and Bar-Yosef and Brown (1977) provide examples of stock price adjustments to good news.
FFJR point out that prior to a stock split a security exhibits a relatively short period of intense upward price adjustments. Bar-Yosef and Brown show that the average value of beta increases substantially during this upward price adjustment period. However, as in the Pettway bad news analysis, beta later returns to its original value.\(^3\) This time pattern displayed by measured beta in both the "good news" and "bad news" studies is consistent with the adjustment return phenomenon described in equation (5). Specifically, if the price adjustment is not the result of a change in equilibrium beta, measured beta may increase during the adjustment period but it will eventually return to its original value.

Conclusions and Implications

Occasionally a security's price must adjust to reflect unanticipated new information. This paper points out that the adjustment process can create measured betas having little relation to the final, post-adjustment, equilibrium value. A model of a "\(\beta\) variability phenomenon" has been offered as a contributing source of beta non-stationarity in general, and a plausible explanation of Pettway's and Bar-Yosef and Brown's findings in particular.

Ignorance of the phenomenon can produce serious errors when utilizing \textit{ex post} \(\beta\) estimates in rate regulatory proceedings.

(1) After substantial, unanticipated new information it is natural to expect a new value for systematic risk. A researcher who is unaware of the adjustment phenomenon might assume the temporary adjustment beta is the new equilibrium beta. This is
particularly serious (as indicated by equations 1 through 5) because the adjustment beta does not necessarily bear any relation to the final equilibrium value.

(2) A researcher might blindly compute betas using historic data which includes periods of adjustment mixed in with periods of equilibrium. This approach has serious heteroscedasticity problems and may lead to serious misestimation of beta. The recommended practice [14,15] of estimating a specific utility's beta to be the beta measure of a portfolio of comparable firms does not necessarily avoid the heteroscedasticity problem posed by adjustment periods. Both individual firms and entire industries can experience significant adjustment periods as the Pettway and Bar-Yosef and Brown studies reveal.

(3) The researcher might drop adjustment periods from his database and ignore them entirely. This is also wrong. Occasional violent adjustments in security prices are an integral part of security performance. Moreover, while the price adjustment phenomenon persists, a security's adjustment beta has the same effect on portfolio performance as betas resulting from any other cause. Adjustment betas must therefore be rewarded by appropriate (market equilibrium) levels of expected return—just like any other betas.

The adjustment phenomenon is probably best handled by:

(a) Adjustment and equilibrium betas should be calculated separately to avoid heteroscedasticity.
(b) The researcher should estimate the likelihood that an adjustment beta will occur during the period for which predictions are being made. He must also assess the probable intensity and duration of such an adjustment. To make these predictions, the researcher should look at historic data over an extended period so as to get a long term feeling for the incidence, duration and intensity of these adjustment periods.\(^5\)

(c) If beta is to be used to determine appropriate rates of return for utilities, a market equilibrium expected returns should be calculated for equilibrium and adjustment betas. These different expected returns should be averaged geometrically with weightings determined by the probability assessments described above. This will produce an average return which compensates investors for adjustment and equilibrium systematic risk.\(^6\)
FOOTNOTES

1 Modern portfolio theory is more than the CAPM, and the usefulness of beta as a measure of security risk does not depend on the strict validity of the CAPM (Myers, 1978).

Incomplete lists of the application of modern portfolio theory in rate regulatory hearings can be found in Myers (1972b, 1978), Carleton (1978), Pettway (1978), and Peseau and Zepp (1978).

2 Believers in efficient markets will have trouble accepting the idea of a non-instantaneous adjustment to new information. Yet, some of the more recent studies of market efficiency allege that the market is very slow to adjust to new information. For example, Latane and Jones (1979) find that prices don't adjust to unanticipated earnings for 5 to 6 months after the end of the quarter and 3 months after the actual announcement.

If this is the adjustment period for something as simple as a change in reported earnings, what period of uncertainty (and adjustment) might result from something as ambiguous in future implication as Consolidated Edison's skipped dividend?

3 Unfortunately neither FFJR or Bar-Yosef and Brown present the change in variance accompanying the change in beta making it impossible for us to calculate the implied \( \rho_{\text{ex}} \) as in the prior example. FFJR do present the mean absolute deviation and it seems to indicate the same general level of increase in variability as the Pettway example.

4 Bar-Yosef and Brown and Pettway dealt with the heteroscedasticity problem differently, but both calculated betas which proxy to some extent the constructs of adjustment and equilibrium betas. Pettway used the occurrence of major events to segment his study period into sub-periods, while Bar-Yosef and Brown used a moving beta measure.

5 It is interesting to note the very different implications of past changes in beta due to the adjustment phenomenon and past changes in beta due to changes in equilibrium beta. A researcher has no reason to expect past equilibrium values of beta to reappear and only the most recent equilibrium betas should be used in prediction. The researcher has every reason to believe that adjustment periods will occur in the future. Therefore, such data must be used in prediction of beta.

Ideally, a prediction of future beta should include a pre-adjustment equilibrium beta and an adjustment beta and a post adjustment equilibrium beta. Unfortunately, although a researcher can be confident that the future will contain periods of adjustment, he will not normally know what the stock is adjusting to.

The methodology described here implicitly assumes the pre and post adjustment equilibrium betas are equal. If the price adjustment is not in response to a change in equilibrium beta (as seems to be the case in
Pettway's and Bar-Yosef and Brown's findings), this assumption will be correct. If equilibrium beta has changed the assumption will, hopefully, be reasonably unbiased.

The justification for the use of geometric mean is best illustrated by a simple example. Assume the capital market equilibrium expected returns for adjustment and equilibrium betas are ER\textsubscript{a} and ER\textsubscript{e} respectively and assume the security is predicted to spend one period in adjustment and one period in equilibrium. The capital market equilibrium two period return will be

\[ ER_2 = (1 + ER_a)(1 + ER_e) - 1. \]

the mean single period return will be

\[ ER_n = [(1 + ER_a)(1 + ER_e) - 1]^{1/2} - 1 \]

ER will then compensate investors for both adjustment and equilibrium betas.
References


