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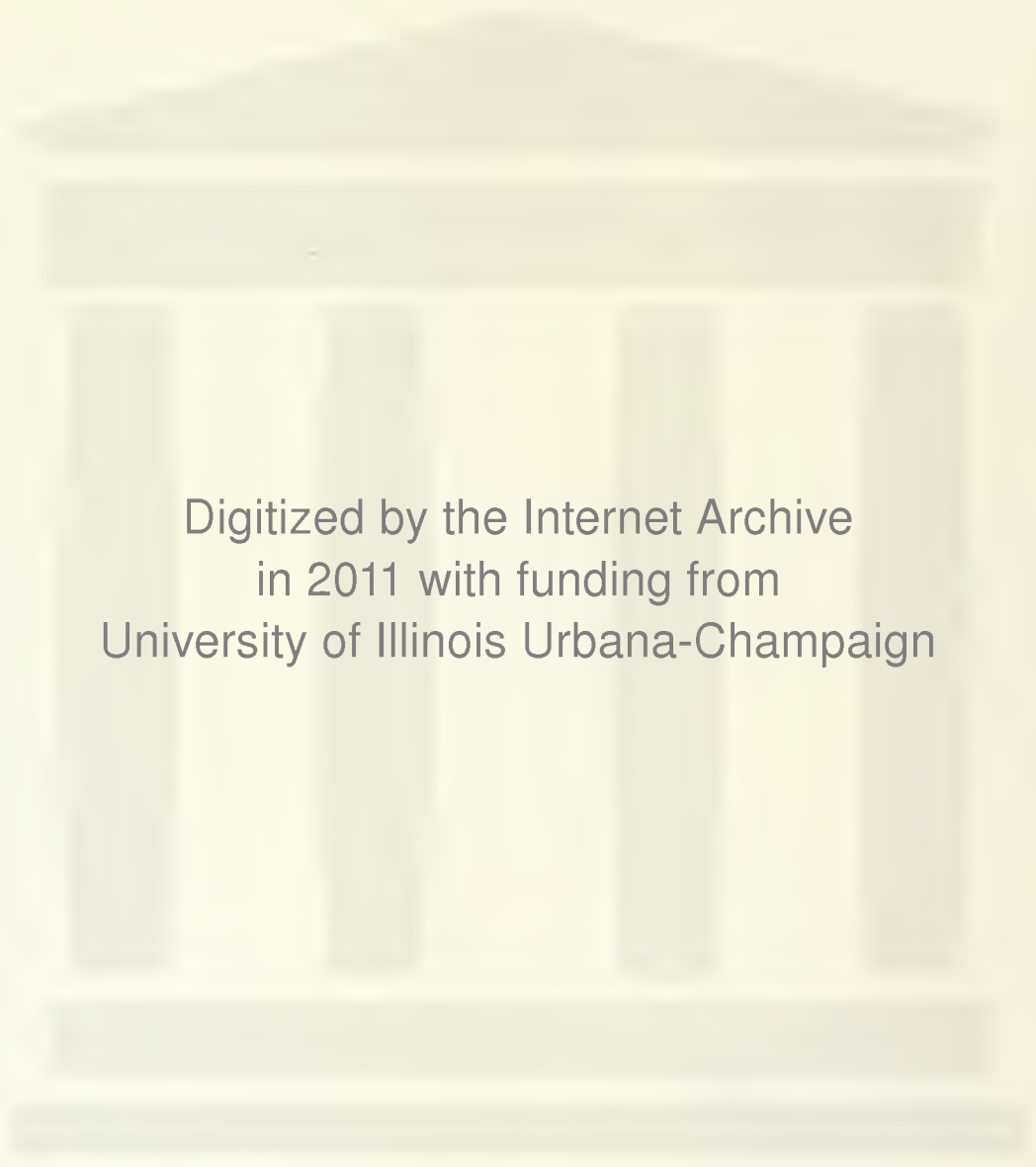
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The Effect of Learning on Cost-Volume-Profit Analysis  
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"The Effect of Learning on  
Cost-Volume-Profit Analysis"

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### Abstract

The traditional C-V-P analysis has been extended to consider the effect of learning. Learning effects have a significant impact on C-V-P analysis; they not only change the traditional breakeven point, but also affect the distribution of profits. This paper demonstrates the effect of learning on C-V-P analysis and develops a C-V-P model with consideration given to both single and probabilistic learning rate estimates. The addition of the proposed change to the traditional C-V-P analysis provides management with more accurate information for making C-V-P decisions under circumstances in which learning affects efficiency.



The accounting model of cost-volume-profit (C-V-P) analysis is widely used in profit planning and decision making. In recent years, the C-V-P model has been extended to include such factors as uncertainty, nonlinear cost and revenue functions, income tax effect, and inflationary costs and prices. The objective of this paper is to examine the effect of learning on C-V-P analysis.

When new products or processes are initiated, a learning effect phenomenon may occur. This learning phenomenon implies that unit variable cost is higher in the earlier stages of production and lower in the later stages of production. In other words, efficiency increases as the time required to complete the job decreases.

Summers and Welsch<sup>1</sup> have stated that most manufacturing operations are subject to a learning effect. Thus, the usefulness of traditional C-V-P analysis is limited in the following situations when the learning effect is ignored:

- (1) Acquisition of new equipment or a change in operational processes with which employees are unfamiliar.
- (2) Activities which are being performed by new, unexperienced employees.
- (3) Activities involving the use of raw materials which have never been used by the firm before.

The purpose of this paper is two-fold: (1) to study and demonstrate the effect of learning on C-V-P analysis, and (2) to develop a C-V-P model which includes consideration of the learning effect.

## The Traditional C-V-P Analysis - No Consideration of Learning Effect

A mathematical expression of the traditional C-V-P analysis is:

$$(1) \quad Z = (Px) - (Vx) - F$$

where:

Z = total profits

P = unit selling price

V = unit variable cost

x = sales volume in units

F = total fixed costs

The traditional C-V-P analysis is static. It is based on the assumption that unit variable cost is constant. However, through repetition of a manufacturing procedure, process, or operation, learning may result in a lower unit variable cost as efficiency increases. The decrease of unit variable cost in the later stages of the production in turn affects the contribution margin and, consequently, the results of C-V-P analysis over the learning period. If a firm is confronted with a situation where learning effects may be realized, the use of the traditional C-V-P analysis most likely produces inaccurate information. Therefore, in a case where learning effects could be realized, the incorporation of the learning effect into the C-V-P model would be helpful. In the following section, the use of a single learning rate is built into the C-V-P model.

### C-V-P Analysis with Consideration of Single Learning Rate

Mathematically, the learning effect is expressed by the following exponential function:

$$(2) \quad y = ax^b$$

where:

y = cumulative average number of labor hours per unit required to produce x units

a = number of labor hours required to produce the first unit

x = cumulative number of units produced

b = index of learning which is the log of the learning rate divided by log 2, i.e.,  $b = \log(\text{learning rate}) / \log 2$ .

The development of learning curves is based upon the assumption that learning continues at the same rate until conditions change or equilibrium is reached. The rate of learning is expressed as a percentage associated with the learning curve. To determine the rate of learning, the manager should estimate the percentage of time required to produce the first unit (or batch of goods) which yields the average expected time per unit when production doubles.

For example, assume the first unit of a new product line requires 100 labor hours to complete. If the estimated average labor time for the first two units produced is 80 hours, a learning rate of 80% (80 hours/100 hours) should be specified. Total labor required for the two units is 160 hours (100 hours for the first unit and 60 hours for the second unit).

After the learning rate has been estimated, the cumulative average labor time per unit ( $y$ ) may be computed for any level of production ( $x$ ) by using equation (2). Thus, the total number of labor hours required to produce  $x$  units may be expressed as  $yx$ .

If variable overhead cost is applied to the product at a rate based on direct labor hours, the total cost (TC) for  $x$  units of output is equal to:

$$(3) \quad TC = (yx) (\text{unit direct labor cost} + \text{variable overhead cost per direct labor hour}) + (x) (\text{unit direct material cost}) + \text{total fixed costs.}$$

By substituting equation (2) in equation (3), the learning effect is incorporated into the following computation of total cost:

$$(4) \quad (ax^{b+1}) (\text{unit direct labor cost} + \text{variable overhead cost per direct labor hour}) + (x) (\text{unit direct material cost}) + \text{total fixed cost}$$

The total revenue (TR) which would be received from these units when sold is the product of the unit selling price,  $P$ , multiplied by the number of units produced,  $x$ . By setting  $TR = TC$  and solving for  $x$ , the breakeven point may be obtained.

For purposes of illustration, an 80% learning rate is assumed in the following example:

$$a = 2 \text{ hours}$$

$$b = \log .8 / \log 2 = -.322$$

$$P = \$25$$

$$\text{unit direct labor cost} = \$2.50$$

$$\text{variable overhead cost per direct labor hour} = \$1.00$$

$$\text{unit direct material cost} = \$3.00$$

$$\text{total fixed cost} = \$100$$

Using equation (4), the total cost to produce x units in the example can be determined as follows:

$$\begin{aligned} TC &= 2x^{-.322+1} (2.5 + 1) + 3(x) + 100 \\ &= 7x^{.678} + 3(x) + 100 \end{aligned}$$

Total revenue is determined by  $TR = 25(x)$ . Therefore, the breakeven point is obtained by setting  $TR = TC$  and solving for x:

$$(5) \quad 25(x) = 7x^{.678} + 3(x) + 100$$

To solve difficult equations like equation (5), Newton's method of Tangents<sup>2</sup> provides an accurate and efficient method. A general computer program has been written using Newton's method of Tangents to solve equation (5) for the breakeven point. The resultant breakeven point is 5.56 units.

Under traditional breakeven analysis, the relationship between input and output is determined at the beginning of the learning process; thus, in most cases, the learning effect is not considered. The relationship between total cost and number of units produced is assumed to be linear. Analysis of the data from the preceding example based upon the traditional approach is as follows:

$$25(x) = (5 + 2 + 3)(x) - 100$$

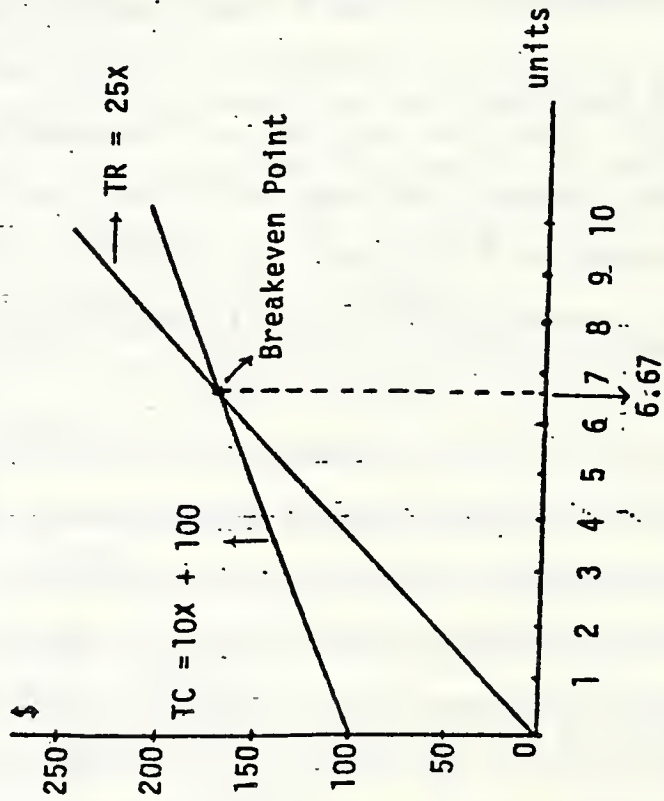
Solving for x, the breakeven point is 6.67 units. Figure 1 presents a graphical comparison of the breakeven points calculated using the traditional approach and the proposed approach.

The breakeven point under the traditional approach is higher than the breakeven point under the proposed approach. This is so because the learning effect is not considered. A profitable venture might be rejected if the learning effect is not incorporated into the manager's C-V-P analysis.

Figure 1

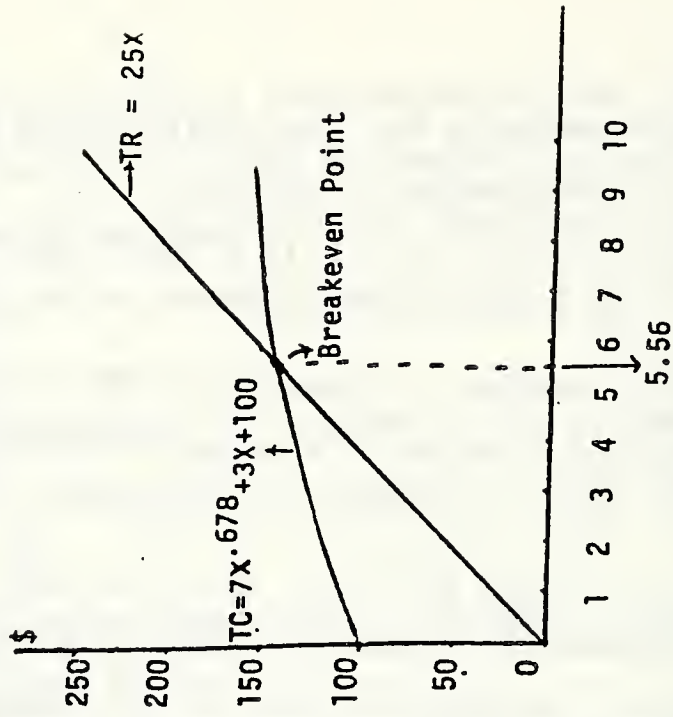
A Graphical Comparison of the Breakeven Points using the Traditional Approach and the Proposed Approach

(A) Traditional Breakeven Chart Considering No Learning Effect



ASSUMPTION: Efficiency Remains Unchanged

(B) Breakeven Chart Considering An 80% Learning Effect



ASSUMPTION: Efficiency Increases Due to Learning Effect



## C-V-P Analysis with Consideration of Probabilistic Learning Rate

One obstacle which may prevent incorporating the effects of learning in the C-V-P model is the determination of the learning rate. In the past, procedures have been developed for estimating the learning rate which include plotting historical data of similar jobs on log-log graph paper and using regression analysis. Also, previous work suggests that the learning rate may be directly related to the initial ratio of machine hours of direct labor hours. While these procedures for estimating a learning rate are useful, the validity of a single estimate of the learning rate is questionable. Estimates of a particular learning rate are by nature subject to a certain degree of uncertainty. Therefore, a probabilistic learning rate could be used to improve the analysis.

Based upon prior experience and comparative studies, a manager should be able to estimate a range of possible learning rates and assign relative probabilities to the likelihood of their occurrence. For example, the manager in the illustrated problem may estimate the possible learning rates and their relative probabilities of occurrence as shown in Table 1.

Table 1

Possible Learning Rates and Their  
Probabilities of Occurrence

<u>Learning Rate (%)</u>	<u>Probability of Occurrence</u>
75	.04
76	.06
77	.08
78	.10
79	.12
80	.20
81	.12
82	.10
83	.08
84	.06
85	.04

In this case, the breakeven point for the problem with each possible learning rate can be calculated by the method proposed in the previous section. Also, the expected breakeven point may be determined by weighting each breakeven point according to its probability of occurrence. Furthermore, the standard deviation of the expected breakeven point can be estimated from the following formula:

$$\sigma = \sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 p_i}$$

where  $x_i$  = the breakeven point for the problem with learning rate  $i$

$\bar{x}$  = the expected breakeven point

$p_i$  = the probability of occurrence assigned to learning rate  $i$

Table 2 shows the breakeven point for each possible learning rate, the calculation of the expected breakeven point, and the standard deviation of the breakeven point.

Table 2

Calculation of the Expected Breakeven Point

<u>Learning Rate (%)</u>	<u>Breakeven Point</u>	<u>Probability of Occurrence</u>	<u>Conditional Value</u>
75	5.3986	.04	.2159
76	5.4296	.06	.3258
77	5.4616	.08	.4369
78	5.4947	.10	.5495
79	5.5289	.12	.6635
80	5.5643	.20	1.1129
81	5.6010	.12	.6721
82	5.6389	.10	.5639
83	5.6782	.08	.4543
84	5.7190	.06	.3431
85	5.7613	.04	.2305
Expected breakeven point			<u>5.5684</u>
Standard deviation = 0.0919			

The expected value and standard deviation calculated in Table 2 reflect a distribution of breakeven points with a high concentration centering around 5.56. The manager can be reasonably confident that a profit will be generated beyond sales of 5.56 units. Even if the learning rate is as high as 85%, the breakeven point is only 0.2 unit more. Thus the manager can evaluate his decision considering all feasible outcomes.

It should be noted that a continuous probability distribution of the learning rate may be used. In such a case, the mean value and standard deviation of the learning rate must first be estimated. Given the mean and the standard deviation of the learning rate, random learning rates can be generated by use of a random number generator. 3

Then random breakeven points for the problem can be obtained by using the generated random learning rates in the proposed C-V-P model. Furthermore, the mean and the standard deviation of the breakeven point can be determined from the calculated random breakeven points.

### Conclusion

The traditional C-V-P analysis has been extended to consider the effect of learning. Learning effects can have a significant impact on C-V-P analysis; they not only change the traditional breakeven point but also affect the distribution of profits. This paper has demonstrated the effect of learning on C-V-P analysis and developed a C-V-P model with consideration given to both single and probabilistic learning rate estimates. The addition of the proposed change to the traditional C-V-P analysis provides management with more accurate information for making C-V-P decisions under circumstances in which learning affects efficiency.

## Footnotes

- <sup>1</sup>Summers, E. L. and Welsch, G. A., "How Learning Curve Models can be Applied to Profit Planning," Management Services (March/April, 1970), pp. 45-50.
- <sup>2</sup>See any standard quantitative methods or operations research textbook. For example, Samuel B. Richmond, Operations Research for Management Decisions (The Ronald Press Company, 1968), pp. 92-96. A copy of the computer program may be obtained by writing to the authors. By specifying values for each of the seven variables, the program can be used to calculate the breakeven point.
- <sup>3</sup>See Naylor, T. H., Balintfy, J. L., Burdick, D. S., and Chu, K., Computer Simulation Technique (Wiley, 1966), pp. 97-99.














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