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Equilibrating Variables

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By Hans Brems

Abstract

If the demand for money were infinitely sensitive to it and in-vestment not sensitive to it at all, the rate of interest could play no role as an equilibrating variable. Leaving it out as a first approximation, the first half of the paper specifies and solves an algebraical Keynesian model of fiscal policy using physical output as its sole equilibrating variable. The last half of the paper examines a Keynesian model of fiscal policy using both physical output and the rate of interest as its equilibrating variables.

KEYNESIAN FISCAL THEORY: ONE OR TWO EQUILIBRATING VARIABLES

By Hans Brems

On fiscal policy Keynes expressed himself in nontechnical form in Keynes (1929) and Keynes (1933). But his formal model in Keynes (1936) had no fiscal policy in it. Here he tried to show why monetary policy was impotent in generating full employment. First, at low rates of interest the demand for money was so sensitive to the rate of interest that a larger money supply would have little effect upon that rate. Second, at low rates of interest investment demand was so insensitive to the rate of interest that even if that rate could have been depressed, it would have had little effect upon investment.

But if the demand for money were infinitely sensitive to it and investment not sensitive to it at all, the rate of interest could play no role as an equilibrating variable. Why not leave it out? Working out a Keynesian theory of fiscal policy Hansen (1941, 1951) did so as a first approximation and used physical output as his sole equilibrating variable. For the time being we shall follow him.

Any model of fiscal policy must deal with three magnitudes, i.e., physical government purchase of goods and services, the fiscal deficit, and the tax rate. They cannot, all three of them, be parameters at the same time, or government could decide to buy all it cared for at zero or low tax rates, yet run a fiscal surplus. A choice will have

to be made: The government can fix two of the three magnitudes as
parameters and let the economy determine the third as a variable.
Which two? There are three different ways in which two elements can
be selected from three. That gives us three alternative priority pat-
terns: Either the government fixes government demand and the tax rate
and lets the economy determine what the fiscal deficit will be. Or
the government fixes government demand and fiscal deficit and lets the
economy determine the necessary tax rate. Or, finally, the government
fixes fiscal deficit and tax rate and lets the economy determine how
much the government can afford to buy.

In chs. 6 through 8 we shall use the growth rates of the money
and bond supplies as policy instruments. As an introduction to such
analysis the third priority pattern is well suited, and we shall now
develop a Keynesian version of it using the following variables and
parameters.

I. ONE EQUILIBRATING VARIABLE

1. Variables

$C \equiv$ physical consumption

$G \equiv$ physical government purchase of goods and services

R ≡ tax revenue

X ≡ physical output

Y ≡ money national income

y ≡ money disposable income

2. Parameters

A ≡ autonomous consumption

c ≡ marginal propensity to consume real disposable income

d ≡ fiscal deficit

I ≡ physical net investment

P ≡ price of goods and services

T ≡ tax rate

The model will include no derivatives with respect to time t, hence is static.

3. The Model

Consider a one-good economy with firms, households, and government in it. Ignore capital consumption allowances, subsidies, indirect business tax, and business transfer payments. Then national income defined as the aggregate earnings arising from current production equals the money value of physical output.

(1) $Y \equiv PX$

Ignore undistributed earnings. Then all national income becomes personal income, and disposable income will equal national income minus government gross receipts plus government transfer payments to persons, subsidies, and interest paid by government. Or, ignoring what the government collects with one hand only to pay back with the other, disposable income simply equals national income minus government net receipts:

(2) $y \equiv Y - R$

Let consumption be a function of real disposable income:

(3) $C = A + cy/P$

where $0 < c < 1$.

In Western tradition, as developed from the English Magna Carta of 1215, the Swedish Magna Carta at Uppsala of 1319, and the American Revolution of 1776, taxes are collected according to statute, and statute defines tax base and tax rate. Typical modern tax bases are assets, income, final sales, or value added. As a good first approximation, let tax revenue be in proportion to money national income:

(4) $R = TY$

where $0 < T < 1$.

Ignore the government interest bill and define the fiscal deficit as the money value of government purchase of goods and services <u>minus</u> government net receipts:

(5) $d \equiv GP - R$

Goods market equilibrium requires the supply of goods to equal the demand for them:

(6) $X = C + I + G$

4. <u>Solutions</u>

To solve for our sole equilibrating variable, physical output X, insert (1) through (5) into (6) and find

(7) . $$X = \frac{A + I + d/P}{(1 - c)(1 - T)}$$

To see how physical output depends upon the fiscal deficit, take the derivative

$$(8) \qquad \frac{\partial X}{\partial d} = \frac{1}{(1 - c)(1 - T)P}$$

Under the assumptions made about the parameters c and T, (8) is always positive: A larger deficit will generate a larger physical output!

To see how physical output depends upon tax rate take the derivative:

$$(9) \qquad \frac{\partial X}{\partial T} = \frac{X}{1 - T}$$

where X stands for the output solution (7) above. Under the third priority pattern the tax rate T is a parameter assumed to lie between zero and one. Consequently, the derivative (9) must have the same sign as output X, hence if according to (7) X happens to be positive, the derivative (9) will be positive: The higher the tax rate, the higher the output. Is this surprising? Not when we remember that to keep the fiscal deficit d constant, government demand G must always increase by the same amount as tax revenue. And 215 years ago, James Steuart (1767: 272) observed that taxation amounts to taking income away from households whose marginal propensity to spend it may fall short of

unity, and transferring it to government whose marginal propensity to spend it must equal unity:

> It is no objection to this representation of the matter, that the persons from whom the money is taken, would have spent it as well as the state. The answer is, that it might be so, or not: whereas when the state gets it, it will be spent undoubtedly.

Given the fiscal deficit d and the tax rate T, how much can the government afford to buy? Insert (1) and (4) into (5) and find

(10) $G = d/P + TX$

To see how physical government purchase depends upon the fiscal deficit, take the derivative

(11) $$\frac{\partial G}{\partial d} = \frac{1 - c(1 - T)}{(1 - c)(1 - T)P}$$

Under the assumptions made about the parameters c and T, (11) is always positive: A larger deficit will permit a larger physical government purchase!

To see how physical government purchase depends upon the tax rate, take the derivative

(12) $\dfrac{\partial G}{\partial T} = \dfrac{X}{1 - T}$

where again X stands for the output solution (7) above. Like (9), and for the same reason, the derivative (12) must have the same sign as output X, hence if according to (7) X happens to be positive, the derivative (12) will be positive: The higher the tax rate, the more the government can afford to buy. Is this surprising? Not when we remember that to keep the fiscal deficit d constant, government demand G must always increase by the same amount as tax revenue, and we just say that the higher the tax rate, the higher the physical output, hence the higher the tax revenue.

5. The Balanced-Budget Multiplier

Keynesians chose to consider physical government purchase G a parameter and the tax rate T a variable. Early Keynesians like Gelting (1941), Haavelmo (1945), and Samuelson (1948) found a balanced-budget multiplier equalling unity as the derivative of their single equili-brating variable, physical output X, with respect to their parameter G--assuming T to vary and keep the budget balanced.

Our choice is the opposite one: We consider physical government purchase G a variable and the tax rate T a parameter. But we can

easily find our counterpart to the balanced-budget multiplier: Change

our tax rate T by the differential dT, write the differentials

$dX \equiv (\partial X/\partial T)dt$ and $dG \equiv (\partial G/\partial T)dT$, divide the former by the latter,

insert (9) and (12) and find

$$(13) \qquad \frac{dX}{dG} \equiv \frac{\partial X/\partial T}{\partial G/\partial T} = 1$$

So here is a multiplier equalling unity. But notice that it holds

under _any fixed_ fiscal deficit. The balanced budget is the special

case of a fixed deficit equalling zero. Nowhere did the literature

seem to notice that the balanced-budget multiplier was derived under

the unnecessarily narrow assumption of a balanced budget.

6. Conclusion

Whether balanced or nonbalanced the budget multipliers were

derived under the assumption of an autonomous physical investment.

Since our system had only one equilibrating variable, i.e., physical

output, the adjustment of saving to autonomous physical investment

had to be brought about by adjustments in physical output alone. This

is an extreme and very special case. Could there be an additional

equilibrating variable?

II. TWO EQUILIBRATING VARIABLES

## 1. The Rate of Interest As an Additional Equilibrating Variable

Realizing that the demand for money may be less sensitive and in-
vestment demand more sensitive to the rate of interest than Keynes had
imagined, Hansen (1953: 165) liked to think of the Keynesian system
as having two equilibrating variables:

> The rate of interest and the national income are
> together mutually determined by the three basic
> functions [the consumption function; the marginal
> efficiency of investment schedule; the liquidity
> preference schedule], together with the quantity
> of money.

To get a first feel for the theory of fiscal policy with physical
output and the rate of interest as equilibrating variables we follow
Hansen (1949) and ignore inflation. In that case the nominal and the
real rate of interest will coincide, and we may deal with a single rate
of interest. In that case we may invoke some simple graphical analysis,
i.e., the celebrated IS-LM diagram. Had there been inflation we would
have encountered the difficulty that the IS curve is a function of the
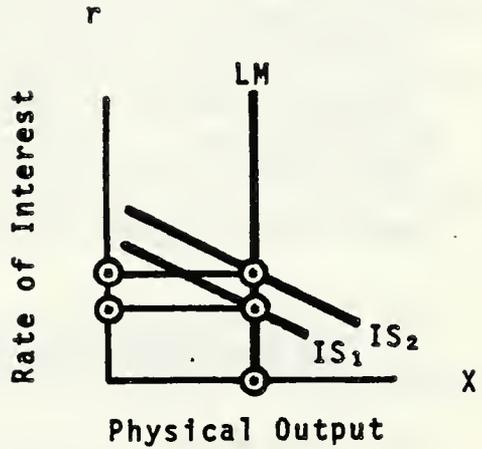real rate of interest, while the LM curve is a function of the nominal
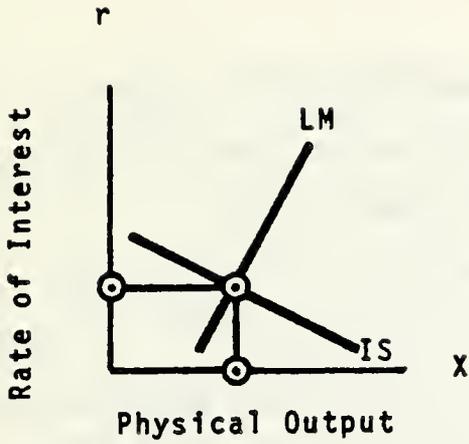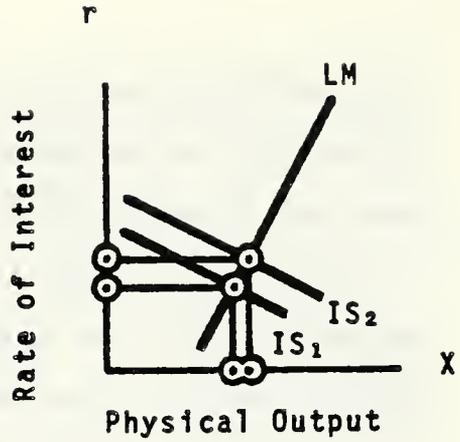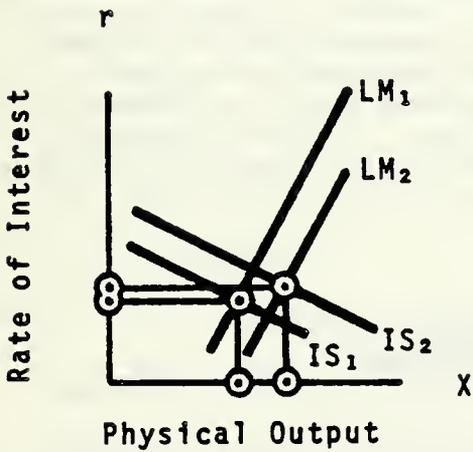
Figure 1
IS-LM Analysis of Deficit Financing

one. An IS-LM diagram has the single rate of interest r plotted on the vertical axis and physical output X on the horizontal one. The upper left corner of Figure 1 shows the IS and LM curves.

The IS curve is the locus of all combinations of interest rate and output in which the goods market is in equilibrium: X = C + I + G. Since investment I is now sensitive to the rate of interest and is the lower the higher the rate of interest, equilibrium output will be low at a high rate of interest and high at a low one. The IS curve is a negatively sloped one.

The LM curve is the locus of all combinations of interest rate and output in which the money market is in equilibrium. Call the supply of money M and the demand for it D, then M = D. Since the demand for money is now sensitive to the rate of interest and is the lower the higher the rate of interest, a given money supply can transact a larger output at a high rate of interest than at a low one: The LM curve is a positively sloped one.

2. Pure Bond Financing and Pure Money Financing

Once the rate of interest is introduced as an additional equilibrating variable, the choice between alternative methods of financing a budget deficit becomes crucial. Let us consider pure bond and pure money financing in turn.

Let government expand its demand G but fail to raise taxes ac-
cordingly. Pure bond financing of the resulting government deficit
means that the government issues interest-bearing claims upon itself
called bonds and sells them to households and firms. The money supply
is not affected, and the LM curve stays put: The economy must still
economize with the same quantity of money. But the expanded govern-
ment demand has pushed the IS curve to the right: At a given rate of
interest, the aggregate demand C + I + G is up.

An IS curve pushed to the right will intersect an unchanged LM
curve in a point whose abscissa and ordinate are both higher than
before. The upper right corner of Figure 1 shows this result. Output
is up in order to satisfy the new government demand. The rate of in-
terest is up. One effect of that is to discourage private investment--
to some extent government is being satisfied at the expense of private
investment. This is a weak case of crowding-out. Another effect is
to induce households and firms to hold less cash, so the larger output
may be transacted.

Pure money financing of the government deficit means that the
government issues noninterest-bearing claims upon itself called money.
The money supply is up, and the LM curve is now being pushed to the
right--thus modifying the increase in the rate of interest and the
resulting crowding-out, as shown in the lower left corner of Figure 1.

Alternatively we may define a strong sense of crowding-out as
zero sensitivity of physical output to a bond-financed government
deficit. For such zero sensitivity to result, what would the LM curve

have to be like? If an IS curve pushed to the right must intersect an

LM curve in a point whose abscissa remains the same as before then the

LM curve must stay put and be vertical. Such a case is shown in the

lower right corner of Figure 1. A vertical LM curve must mean that the

demand for money is insensitive to the rate of interest: no rise in

the latter will induce households and firms to hold less cash. No

rise in the rate of interest will release any money to transact a

larger output. Such complete insensitivity of the demand for money to

the rate of interest is an extreme and very special case--as extreme

and as special as the Keynesian twin assumptions of a complete sensi-

tivity of the demand for money and a complete insensitivity of the

demand for investment to the rate of interest.

There is good evidence that the demand for money is, in fact,

sensitive to the rate of interest. If the demand for money is the

lower the higher the rate of interest, then the velocity of money is

the higher the higher the rate of interest. U.S. experience 1916-1970,

shown in Figure 2 confirms this. Consequently a vertical LM curve

appears implausible. Early monetarism [Friedman (1959)] accepted it,

but later monetarism [Friedman (1966), (1972)] abandoned it and has a

different reason for crowding-out, as we shall see in ch. 4. But first

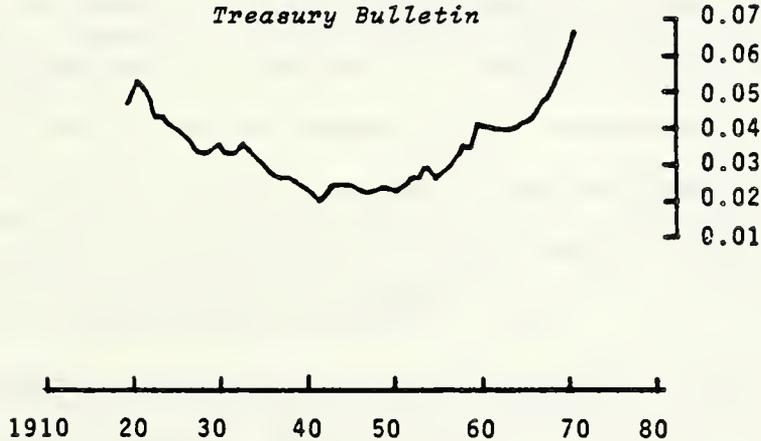we must briefly consider a different additional equilibrating variable.

3. <u>Price As an Additional Equilibrating Variable</u>

If the demand for money were infinitely sensitive to the rate of

interest and investment not sensitive to it at all, the rate of interest

Velocity of Circulation of the Money Supply
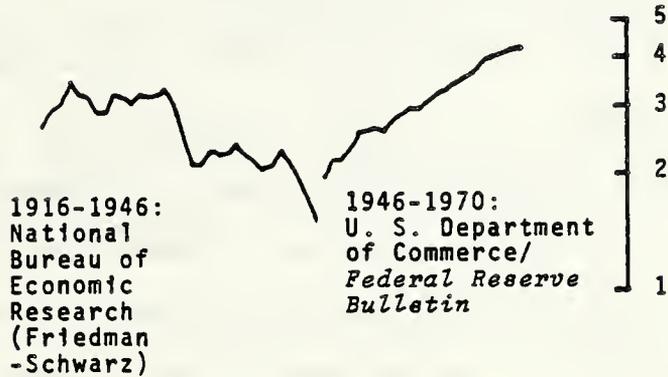
U. S. Department of Commerce, Bureau of Economic Analysis,
Long Term Economic Growth 1860-1970, Washington, D. C. 1973,
53, 61.

Figure 2
Velocity of Circulation of Money is the Higher the Higher the Rate of Interest,
U. S. Experience 1916-1970

could play no role as an equilibrating variable, and Keynesian involuntary unemployment could occur. But could price play a role? To Patinkin (1956) price rather than the rate of interest offered itself as an additional equilibrating variable.

In a static Keynesian model let price and the money wage rate be flexible and responding to excess demand or supply. Let there be excess supply in the form of unemployment. As a result, let price and the money wage rate fall in the same proportion, so the real wage rate and with it labor supply is unaffected. The price decline will continue but will raise the real value of money balances--the more so, the farther prices decline. They will keep declining until the real-balance effect has become powerful enough to stimulate demand enough to restore full employment. The stimulus is the result of adding real wealth as an argument in the consumption function and will play its role even when the rate of interest can play no role. In Haberler's (1952: 241) judgment, such a real-balance effect "removes the narrow remaining base of...static competitive underemployment equilibrium."

Generally, Keynesians have been reluctant to unfreeze price. In the cores of their fiscal-policy models neither Blinder-Solow (1974) nor Tobin-Buiter (1976) did it but did go beyond Hansen's IS-LM analysis in another respect: Like Patinkin they added real wealth as an argument in the consumption function and, for good measure, in the demand-for-money function as well.

Unfreezing price within a static framework, as Patinkin did, will not do to illuminate inflation. So let us turn to a dynamic framework.

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