THE TRANSFER FUNCTION RELATIONSHIP BETWEEN EARNINGS AND MARKET-INDUSTRY INDICES: AN EMPIRICAL STUDY

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#496

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Summary:

The study investigated the hypothesis that univariate ARIMA forecasts can be improved upon by using a more general transfer function model which consists of an ARIMA model with a market or industry index added. Statistical analysis of the data indicated that firms' forecasts have a tendency to perform either very well or very poorly under the transfer function model as compared to the ARIMA model (using an absolute value error metric).

It was demonstrated that it is possible to develop an a priori rule for the determination of when the transfer function will outperform the univariate model. In particular it was found that if a transfer function outperforms an ARIMA model for the majority of the first three periods in the forecast horizon, then there is a significant probability that it will do the same for periods four through ten.
In recent years there has been an increased emphasis on the forecasting of accounting earnings. In particular there have been a large number of studies which have utilized the Box-Jenkins method of forecasting via auto-regressive integrated moving average (ARIMA) models. Notable, however, is that these models are univariate by definition and do not provide for the statistical modeling of events which occur outside of the earnings series. The purpose of the study is to explore this limitation by employing a more general approach which incorporates market and industry index data into the forecast model.

A primary reason for exploring this more general approach is that "Financial analysts have long recognized that economy-wide and industry-wide factors affect the financial numbers of individual firms. Index models enable quantification of the effects of these factors. Such quantification can be important when assessing financial trends in a firm and forecasting financial variables" (Foster, 1978, p. 155).

Specifically, the research method involves the use of the single input transfer function method developed by Box and Jenkins (1970). This approach generalizes the ARIMA model by incorporating an additional predictor variable, in addition to past earnings, in the form of a market or industry price index. The motivation for the inclusion of these particular indices is best expressed by quoting Beaver, Clarke and Wright (1978, pp. 1-2): "Capital market equilibrium can be characterized as a mapping from states into a set of security prices. Similarly, earnings are signals from an information system which is a mapping from states into signals. In general, these could be any relationship between price and earnings depending upon the nature of the two mappings. If one assumes that prices and earnings reflect a common set of events it is
not unreasonable to assume that the two might be positively associated. In fact, the Ball and Brown study and the empirical evidence provided in cross-sectional valuation studies provide support for such a view."

The paper will consist of four sections. The first will give a brief discussion of the transfer function and the second will present the research design. In the third section the results will be presented followed by a summary and conclusions in section four.

1.0 A Generalization of the Traditional Box-Jenkins Approach

A generalization of the ARIMA model is the transfer function (TF) which has not been generally used but has recently been suggested by Foster (1977). This forecast method, which generalizes the traditional Box-Jenkins approach, avoids the univariate limitation. In particular, it generalizes the ARIMA models by allowing for the simultaneous modeling of the time series properties of more than one series of interest. The general form of the transfer function is

\[ y_t = \{f_1(y_{t-1}, y_{t-2}, \ldots), f_2(x_t^{(1)}, x_{t-1}^{(1)}, \ldots), f_3(x_t^{(2)}, x_{t-1}^{(2)}, \ldots), f_n(x_t^{(n)}, x_{t-1}^{(n)}, \ldots) + u(t) \}. \]

Note that (1) completely generalizes the ARIMA models to remove the univariate restriction. In particular, \( f_2, f_3, \ldots, f_n \) produce a generalization by allowing \( y_t \) to be modeled as a function of \( x^{(1)}, x^{(2)}, \ldots, x^{(n)} \). The net result is a very broad family of models which contain the ARIMA models as a proper subset. In summary, the transfer function, due to its generality, has the ability to utilize more data than the ARIMA models. Specifically, it can simultaneously utilize the time series and cross-correlational properties of more than one series for the purpose of forecasting EPS.
2.0 Research Design

2.1 The Sample

A sample of thirty airlines was selected. (A list of the sample firms is presented in Appendix 1.) This industry was chosen because of the availability of both an industry index and individual firm EPS for a period sufficiently long to perform the statistical analysis.

The basic requirement for a firm to be selected was the availability of EPS for 60 quarters. This provided 50 quarters recommended for model estimation and 10 quarters for forecast error computation. Since only 30 firms in the industry met the selection criteria, the sample was not random.

2.2 General Hypothesis

The General Hypothesis tested is:

\( H_0: \) ARIMA forecasts of earnings are not improved when an industry or market index is added to the basic ARIMA model.

\( H_A: H_0 \) is not true.

This general hypothesis will be operationalized by defining an error metric. In addition the null hypothesis of no interaction between firm and forecast model (ARIMA vs TF) will be examined.

2.3 Construction and Application of the Forecast Model

**Step 1** For each sample firm one univariate and a two bivariate TF models were constructed based on 50 quarters of EPS. The bivariate models were of the form

\[
(2) \ y_t = [f_1(y_{t-1}, y_{t-2}, \ldots, y_{t-n}), f_2(x_{t}, x_{t-1}, \ldots, x_{t-n}), u(t)]
\]

\((k = 1,2)\)
where $x_t^{(1)}$ corresponds to the Dow Jones Industrial Index and $x_t^{(2)}$ corresponds to the Standard and Poors' Air Transportation Industry Index. Note that (2) is a special case of (1) above where there is one $x$ variable. This restriction is made because at present there are a number of unresolved problems with using a TF model which has more than one $x$.  

Step 2

For each firm forecasts were generated from one to ten periods in the future from three models: (1) ARIMA (2) TF with the Dow index added and (3) TF with the Air transportation index added.

3.0 Empirical Results

3.1 Choice of an Error Metric and Associated Statistical Procedure for Testing the Null Hypothesis

Initially, consideration of an absolute percentage error metric was given; however, due to near zero denominators and correspondingly large denominators a large number of explosive forecast error occurred. Because of this problem it was decided to employ a nonparametric analysis that utilizes simple absolute forecast error.

One type of nonparametric analysis that has been used in the past is the performance of a series of separate nonparametric tests for each different time origin and/or period in the future. This procedure is not used here because such a method results in making a large number of nonindependent tests, and in addition it is likely that some of the tests will lead to rejection of the null by alpha error related chance. Therefore the procedure chosen was the use of a simple chi square statistic.
3.2 Test of the Null Hypothesis for Main Effects

The method used to test the null was to create a variable $\delta_{i,j,k}$ for each firm $i$, index $j$ and forecast $k$ ($i = 1,30$, $j = 1,2$, $k = 1,10$). If a TF forecast was closer in absolute value to the actual earnings number than the univariate forecast, then $\delta_{i,j,k}$ was assigned a value of 1 (and 0 otherwise). In those cases where $\delta_{i,j,k}$ equals one we shall say that the TF forecast for firm $i$, index $j$ and period $k$ dominates the univariate forecast for the same firm and period.

The result is that the number of times that a TF forecast dominates for an index $j$ and period $k$ is $\sum_{i=1}^{30} \delta_{i,j,k}$. This implies that associated with each index $j$ there is the following vector of frequencies:

$$
\begin{bmatrix}
\sum_{i=1}^{30} \delta_{i,j,1}' & \sum_{i=1}^{30} \delta_{i,j,2}' & \cdots & \sum_{i=1}^{30} \delta_{i,j,10}'
\end{bmatrix}
$$

where each element of the vector represents the number of times that the TF for index $j$ dominates the univariate forecast at time $k$.

Since there are 30 firms the null hypothesis can be stated that there is an expected frequency of $15(1/2 \times 30)$ in each cell (i.e., each vector element). The actual and expected cell frequencies are presented in Table 1 for both indices 1 and 2 (corresponding to the Dow and Air Transportation indices respectively).
TABLE 1
Actual and Expected Frequencies for the Number of Times that the TF Forecast for Index j Dominates the ARIMA forecast for Period k

<table>
<thead>
<tr>
<th>Index j</th>
<th>Frequency</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Chi Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dow j = 1 actual</td>
<td></td>
<td>13</td>
<td>17</td>
<td>21</td>
<td>17</td>
<td>16</td>
<td>19</td>
<td>15</td>
<td>19</td>
<td>13</td>
<td>11</td>
<td>5.41</td>
</tr>
<tr>
<td>Air Trans. j = 2 actual</td>
<td></td>
<td>11</td>
<td>13</td>
<td>20</td>
<td>20</td>
<td>17</td>
<td>19</td>
<td>14</td>
<td>20</td>
<td>13</td>
<td>15</td>
<td>9.79</td>
</tr>
<tr>
<td>expected</td>
<td></td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>14</td>
<td>14</td>
<td>13.5</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

For both indices the null hypothesis is not rejected at the $\alpha = .1$ level. This implies that on the average the TF forecasts are not significantly different than the ARIMA forecasts.

3.3 Test of the Null Hypothesis for Interaction Effects

A test of interaction between firms and forecast models was made to investigate the following question: Do ARIMA models tend to dominate for some firms and TF models dominate for others?

If such an interaction does exist we would expect to find $\sum_{k=1}^{10} \delta_{i,j,k}$ for a given firm $i$ and index $j$ to be close to 0 or 10 and under the hypothesis of no interaction we would expect a value of 5. Table 2 presents the results of the test.
TABLE 2
Chi Square Test of No Interaction Effect

<table>
<thead>
<tr>
<th>Index</th>
<th>Test Statistic (29 df)</th>
<th>Approximate Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOW</td>
<td>49.76</td>
<td>.025</td>
</tr>
<tr>
<td>AIR Transportation</td>
<td>37.13</td>
<td>.145</td>
</tr>
</tbody>
</table>

Note that the null is rejected (at $\alpha = .1$) for the Dow index and almost rejected at the .1 level for the air transportation index. Note that this implies that the $\delta_{i,j,k}$ (k = 1,10) are not independent since the $\delta_{i,j,k}$ for a given firm $i$ and index $j$ have a tendency to be the same for all $k$ (i.e., either 0 or 10).

3.4 A Proposed Contingency Rule for the Selection of a TF Model

The results of the interaction tests tend to indicate that in some cases the univariate modeling procedure can be improved upon by examining the performance of a given TF model for a given $i$ and $j$ over some arbitrary but fixed $L$ periods of the forecast horizon. If the TF tends to dominate the ARIMA forecasts over the $L$ periods we would expect it to tend to dominate over the remaining $10 - L$ periods of the forecast horizon.

In order to operationalize this hypothesis it was decided to select those TF models that dominated the corresponding univariate models for at least two out of the first three forecast periods. The variable $\lambda_{i,j}$ for firms $i$ and index $j$ was created and assigned a value of 1 if the TF model dominated the univariate model for the majority of the remaining seven periods (and 0 otherwise). There were 21 firms that met the selection
criterion and under the null hypothesis of equality between the TF and ARIMA methods we would expect 10.5 \((1/2 \times 21)\) firms to have a \(\lambda_1\), equal to 1. Table 3 presents a test of this hypothesis.⁹

**TABLE 3**

| Test of Equality Between TF and ARIMA Models on an A Priori Selected Subset of Firms |
|---|---|
| 1 | Number of firms for which a TF index dominated in the majority of the first 3 forecast periods | 21 |
| 2 | Number of the above 21 firms for which the TF dominated on the majority of forecast periods 4-10 | 16 |
| 3 | Expected frequencies under the null hypothesis | 10.5 |
| Chi Square Statistic with 1 df. | 2.88 |

The statistic of 2.88 is significant at the \(\alpha = .1\) level as expected.

An additional test was made by counting the total number of times that the TF dominated over periods 4-10 for the 21 firms selected. Under the null hypothesis of no difference between the TF and ARIMA methods on the restricted subpopulation, we would expect the univariate model to dominate a total of 73.5 \((21 \times 7 \times 1/2)\) times. Table 4 presents a test of this hypothesis.
TABLE 4

A Second Test of Equality Between TF and ARIMA Models on an A Priori Selected Subset of Firms

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of times which a priori selected TF dominated the corresponding ARIMA forecast</td>
<td>97</td>
</tr>
<tr>
<td>Expected frequency under the null hypothesis</td>
<td>73.5</td>
</tr>
<tr>
<td>Chi Square (1 df)</td>
<td>7.514</td>
</tr>
</tbody>
</table>

Again, as expected, the null is rejected (with $\alpha = .1$) and the data indicate that one is better off, on the average, to select the TF index model if it dominates the univariate in the majority of the first 3 forecast periods.

4.0 SUMMARY AND LIMITATIONS

4.1 Summary and Conclusions

The study investigated the hypothesis that univariate ARIMA forecasts can be improved upon by using a more general transfer function model which consists of an ARIMA model with a market or industry index added. Statistical analysis of the data indicated that firms' forecasts have a tendency to perform either very well or very poor under the transfer function model as compared to the ARIMA model (using an absolute value error metric).

It was demonstrated that it is possible to develop an a priori rule for the determination of when the transfer function will outperform the
univariate model. In particular it was found that if a transfer function outperforms an ARIMA model for the majority of the first three periods in the forecast horizon, then there is a significant probability that it will do the same for periods four through ten.

4.2 Limitations and Suggestions for Future Research

A primary limitation of the study is that it was restricted to one industry. It is suggested that the study be replicated in other industries as well as in the market as a whole.
FOOTNOTES

1 Some examples of the use of ARIMA models are: Albrecht, Lookabill and McKeown (1977), Brown and Rozeff (1977), Dopuch and Watts (1972), Foster (1977), Lorek, McDonald and Patz (1976), and Watts and Zeftwich (1977).

2 EPS was taken from Moody's Handbook and adjusted for changes in capital structure. In addition for firms 1, 2, 5, 7, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, and 29, EPS were computed using information from schedule B-3 of the Civil Aeronautics Board (CAB) and in conjunction with the CAB quarterly periodical Air Carrier Financial Statistics.

3 The modeling was done using a program first written by David Pack of the Ohio State University and modified for local use at the University of Illinois by James McKeown. In condensed form the models occupy 15 pages and are not presented in this study; however they will furnished upon written request to the author.

4 Both the Dow and industry indices were computed from averaging monthly data taken from Security Owners Stock Guide (Standard and Poor's corporation).

5 A major problem is that of modeling cases where the $X^{(k)}$ series are not independent of each other. The author is presently in the process of developing an algorithm for modeling these type of series.

6 When the absolute percentage forecast errors were computed it was found that approximately 10% of the errors were more than 3 standard deviations from the mean. In addition there were a large number of values that were a large number of values in excess of 25 standard deviations from the mean.

7 The expected cell frequencies for periods 8, 9 and 10 have been slightly adjusted for missing data. A description of data available for modeling and testing is presented in Appendix 2.

8 A maximum of one TF was selected for each firm. In the event that the two TF models were tied, the following rule was applied: (1) if one TF dominated for three periods and the other for two periods, then the one dominating for three periods was selected, (2) if both TF's dominated for two periods, the TF that dominated the other TF for the majority of the first three periods was selected. The result was that for twelve firms the Dow index was chosen and for nine firms the transportation index was chosen.

9 Of the sixteen firms in cell number two, nine were associated with the Dow index and seven were associated with the transportation index.
BIBLIOGRAPHY


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APPENDIX 1

LIST OF SAMPLE FIRMS

1. Airlift International
2. Alaska Airlines
3. Aloha Airlines
4. American Airlines
5. Aspen Airways
6. Braniff Airways
7. Caribbean Atlantic Airlines
8. Continental Airlines
9. Delta Airlines
10. Eastern Airlines
11. Tiger International Airlines
12. Frontier Airlines
13. Hawaiian Airlines
14. National Airlines
15. New York Airways
16. North Central Airlines
17. North West Airlines
18. Ozark Airlines
19. Pan American Airways
20. Piedmont Airlines
21. Reeve Airlines
22. SFO Airlines
23. Seaboard World Airlines
24. Southern Airways
25. Texas International Airlines
26. Trans World Airlines
27. UAL (United Airlines)
28. Western Airlines
29. Wien Airlines
30. Allegheny Airlines

Each firm will be subsequently referred to by the identifying number that precedes it.
APPENDIX 2

DESCRIPTION OF AVAILABLE DATA FOR FORECAST ERROR ANALYSIS

This appendix gives a firm by firm description of the number of quarters of data available for forecast error analysis. For each firm the number of periods in the base period, the origin date for forecasting, and the number of absolute forecast errors is presented.

<table>
<thead>
<tr>
<th>firm number</th>
<th>number of periods in base period</th>
<th>origin date for forecasting</th>
<th>number of steps ahead forecast error was computed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
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<tr>
<td>4</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
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<tr>
<td>5</td>
<td>30</td>
<td>2/74</td>
<td>10</td>
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<tr>
<td>6</td>
<td>30</td>
<td>3/74</td>
<td>8</td>
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<tr>
<td>7</td>
<td>40</td>
<td>2/74</td>
<td>9</td>
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<td>50</td>
<td>2/74</td>
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<tr>
<td>30</td>
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<td>2/74</td>
<td>10</td>
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</tbody>
</table>
For example in the case of firm 1, 50 quarters of data were used in transfer and univariate estimation, and actual and predicted forecasts were computed over a 10 period forecast horizon with the first forecast being for the third quarter of 1974.