Faculty Working Papers

1. A Dynamic Model of Interest and Inflation

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WICKSELL: A DYNAMIC MODEL OF INTEREST AND INFLATION

"The applications to the bank for money, then, depend on the comparison between the rate of profits that may be made by the employment of it and the rate at which they are willing to lend it."

David Ricardo (1817), Ch. XXVII.

David Hume (1752) described what would happen to prices if "all the money of Great Britain were multiplied fivefold in a night" but failed to describe how all that money would find its way into the economy. David Ricardo (1817) saw how: Banks would be willing to lend money at a rate of interest falling short of the rate of profits that could be made by employing it. Knut Wicksell (1898) restated Ricardo's idea within a rigorously defined
Figure 1. Point-input flow-of-output case; flow-of-input point-output case; and point-input point-output case.
framework employing such modern concepts as aggregate demand and aggregate supply.

The concepts survived Wicksell's specific framework. But two characteristics of that framework are worth remembering. First, in its Wicksellian form macroeconomics began as a linear descendant of capital theory—as was inevitable once the rate of profits on capital had been made part and parcel of the mechanism via which new money finds its way into the economy. Second, in its Wicksellian form macroeconomics was explicitly dynamic—a step-by-step account of how the new money finds its way into the economy. The latter characteristic was lost in the Keynesian revolution.

Of which particular capital theory was Wicksellian macroeconomics a linear descendant? Capital has something to do with time, and let us distinguish, as Adam Smith did, between two forms of capital, fixed and circulating.

In fixed capital what takes time is the utilization of durable plant and equipment. In the top part of Figure 1 let a machine be constructed by labor alone, let it be constructed instantaneously at the point of time 1, and let its useful life be two years. Ignore the labor required for its operation. Then some of the labor embodied in the machine at time 1 will mature
in the form of finished products at time 2 and the rest at time 3— as indicated by the two arrows. In other words, some labor is invested for one year, some for two years. We might call this case the point-input flow-of-output case. It moved into the mainstream of economic theory with the addition of the famous Chapter 31 to the third edition (1821) of Ricardo's *Principles*. It is the most important form of capital. Indeed, it will be assumed to be the only form of capital in all other chapters of this book.

In circulating capital what takes time is the maturing of output in slow biological growth. In the middle part of Figure 1 let a tree be planted at the point of time 1 and let it be cut two years later. The labor required for planting it is put in at time 1. Let the labor required for tending the growing tree be put in at time 2. Whether put in at time 1 or 2, all labor will mature in the form of timber at time 3— as indicated by the two arrows. As before, some labor is invested for one year, some for two years. We might call this case the flow-of-input point-output case. It goes back to Cantillon (1755) and Quesnay (1758).
A particularly simple case is shown in the bottom part of Figure 1. Here all labor is put in at time 1 and matures at time 2. We might call this case the point-input point-output case. It was the primary capital concept of the English Classicists—they called it a "wage fund," because capital advances wages. They thought of the period of production as the harvest year. They found the wage rate simply by dividing the wage fund by the labor force. Such was the concept of capital in which Wicksell's macroeconomics was anchored. We shall now set out the latter. All our references to Wicksell (1898) and (1906) will use the two translations: Wicksell (1936) and (1935), respectively.

I. NOTATION

Variables

$g_p$ proportionate rate of growth of price
L \equiv \text{ labor employed, man-years} \\
P \equiv \text{ price of consumers' goods} \\
r \equiv \text{ the money rate of interest} \\
w \equiv \text{ money wage rate, dollars per man-year} \\
X \equiv \text{ physical output of consumers' goods} \\
Z \equiv \text{ profits bill} \\
\zeta \equiv \text{ the natural rate of interest} \\

Parameters \\
F \equiv \text{ available labor force, man-years} \\
S \equiv \text{ available physical capital stock of consumers' goods} \\

II. WICKSELL'S MODEL \\

1. The Wage Fund \\

In his macroeconomics Wicksell adopted the simple and "prematurely discarded" (1936), 130, classical wage-fund doctrine. The period of production is one year "as would be the case if technical con-
ditions firmly prevented any extension or contraction" (1936), 136. Let the economy produce a single consumers' good priced \( P \). Physical capital stock \( S \) is a stock of that good, so the money value of capital stock is \( PS \). Let us follow an earlier Wicksell (1893), 94, and let the services of land be free. Then rents will not have to be advanced, only wages will. Consequently in an economy using money the money value of capital stock will equal the wage bill:

\[
(1) \quad PS = Lw
\]

Temporarily Wicksell wished to visualize an in-kind economy. Here physical capital stock \( S \) feeds labor for one year at a real wage rate \( w/P \) defined as the number of physical units of consumers' goods one man-year will buy. Wicksell's in-kind vision will emerge when we divide (1) by \( P \):

\[
(2) \quad S = Lw/P
\]

2. **Full Employment**

Full employment was not an assumption but an equilibrium result:
The equilibrating variable was the real wage rate. Paraphrasing the earlier Wicksell (1893), 102, we restate the equilibrating mechanism as follows. If the available physical capital stock could employ more man-years than are available, \( L > F \), there is positive excess demand in the labor market, and eager entrepreneurs will bid up the real wage rate until \( L = F \). Vice versa, if the available physical capital stock cannot employ all available man-years, \( L < F \), there is negative excess demand in the labor market, and eager wage earners will be willing to work at a lower real wage rate until \( L = F \). In this way the real wage rate \( w/P \) is determined by supply—available labor force \( F \)—and demand—available physical capital stock. Remember we are in the nineteenth century!

With the available labor force \( F \) fully employed in a one-year period of production, physical output \( X \) will be determined.

3. The Natural Rate of Interest

Such is the in-kind framework within which Wicksell saw his natural rate of interest. He failed to define it explicitly, but
his hints allow us to do so. The key is his description of the profits bill as "the amount by which the total product...exceeds the sum of the wages...that have been paid out," or in our notation:

(4) \[ Z = PX - Lw \]

Divide (4) by PS, insert (1) and (3), and find the rate of return

(5) \[ \zeta \equiv \frac{Z}{PS} = \frac{\frac{X}{F}}{\frac{w}{P}} - 1 \]

or in English: The rate of return equals labor productivity divided by the real wage rate minus one. Notice three things in (5). First that (5) is a pure number: Labor productivity \( \frac{X}{F} \) is number of physical units of consumers' goods one man-year will produce. The real wage rate \( \frac{w}{P} \) is number of physical units of consumers' goods one man-year will buy. Consequently numerator and denominator have the same dimension. Second, notice that (5) is greater than one: One man-year will produce more consumers' goods than it will buy—the difference is what Marx called "surplus value".
Third, notice that no absolute price enters (5), only the relative price \( w/P \) does. We may think of \( w/P \) as the ratio in which goods and man-years will exchange in an in-kind economy obeying the law of supply and demand. That was exactly the way Wicksell thought of it. The natural rate of interest would prevail, he said (1936), 103, if "the entrepreneur borrows...consumption goods from the capitalists in kind and then pays them out in kind in the shape of wages..." In short, the natural rate of interest is (1936), 102, "the rate of interest which would be determined by supply and demand if no use were made of money..."

Our (5) is an exact expression of such a rate.

4. The Money Rate of Interest

Having described his in-kind rate Wicksell introduces money. Between his entrepreneurs and his capitalists he puts a bank capable of creating money in the form of drawing rights upon itself. To bridge their one-year gap between input and output, entrepreneurs borrow in the bank. To hold their assets in an interest-bearing form, capitalists deposit in the bank. Entrepreneurs are paying and capitalists are earning the same money rate of interest, Wicksell (1936), 140.
5. Monetary Equilibrium

Wicksell now defined monetary equilibrium as the equality between the money rate and the natural rate of interest:

\( r = \zeta \)

At the beginning of each year let the entrepreneurs borrow the sum \( PS = Fw \) from the bank, immediately spend it hiring labor, and embark upon their one-year period of production. Receiving the sum \( Fw \), labor immediately spends it on the consumers' goods produced last year. Those consumers' goods are sold to labor by the capitalists who have an instant part-time job as retailers: At the end of each year they buy that year's output from the entrepreneurs, retain part of it for their own consumption, at the beginning of next year sell the rest to labor, and deposit the proceeds \( Fw \) in the bank.

At the end of each year, according to (5) the value of output \( PX \) will equal \((1 + \zeta)Fw\). The value of the capitalist-retailers' deposit and the value of the entrepreneurs' debt will both equal \((1 + r)Fw\). Since in monetary equilibrium \( r = \zeta \), the value of output, the assets of the capitalists, and the debt of the entre-
preneurs will all be equal. As a result, the capitalist-retailers will be willing and able to buy the output offered by the entrepreneurs at a value enabling the latter to pay their debt to the bank.

Having done so, entrepreneurs are left with no income of their own. This is as it should be: In a purely competitive economy with freedom of entry, anticipated net profits are zero, and in equilibrium realized net profits equal anticipated net profits.

6. Disequilibrium: First Year

Let the bank interrupt such an equilibrium by reducing the money rate by one percentage point. As a result, at the beginning of the first year it looks to the entrepreneurs as if they could make a one percentage point net profit. But anticipated positive net profits are incompatible with pure competition and freedom of entry. The anticipated positive net profits will be washed away by competitive bidding by the entrepreneurs in the labor market raising the money wage rate by approximately one per cent.

If "approximately one per cent" isn't accurate enough, by
exactly how many per cent will the money wage rate rise? In (5) replace \( \xi \) by \( r \), rearrange, and write

\[
(7) \quad (1 + r)w = \frac{PX}{F}
\]

At a money wage rate satisfying (7), entrepreneurs expect a return of no more than the money rate \( r \)——a zero net profit. On the right-hand side of (7) \( F \) is a parameter, \( X \) is determined by full employment in a one-year period of production, and no change in \( P \) is anticipated, because Wicksellian entrepreneurs always expect current prices to prevail in the future. In short: The right-hand side of (7) is a constant. Under that assumption take the derivative of (7) with respect to \( r \), rearrange, and find

\[
(8) \quad \frac{dw}{dr} = -\frac{w}{1 + r}
\]

But we need an elasticity rather than a derivative. So multiply (8) by \( r/w \) and find the elasticity
For low values of $r$, $r/(1 + r)$ is not very different from $r$. For example, if $r = 0.04$, then $r/(1 + r) = 0.0384615$, and the elasticity (9) means that if $r$ were reduced by one percentage point from 0.04 to 0.03 or by 25 per cent, $w$ would rise by $0.0384615 \times 25$ per cent or by 0.9615375 per cent—which is "approximately one per cent".

If entrepreneurs are to pay an approximately one per cent higher money wage rate, they must borrow approximately one per cent more from the bank, and the money supply must expand by approximately one per cent. As a result, labor will earn and spend approximately one per cent more. But last year's physical output minus the part retained by the capitalist-retailers for their own consumption hasn't changed. Consequently a higher aggregate consumption demand is now chasing an unchanged physical supply, and the price of consumers' goods must rise by approximately one per cent.

At the end of the first year the entrepreneurs offer that year's
physical output to the capitalist-retailers. At the old price and at the new approximately one per cent higher money wage rate satisfying (7), entrepreneurs expect a return of no more than the money rate \( r \) — a zero net profit. At the end of the first year the capitalist-retailers possess a bank deposit no more than adequate to buy the year's physical output at the price anticipated by the entrepreneurs: At the beginning of the year labor spent its approximately one per cent higher money wage bill, and the capitalist-retailers deposited the proceeds at the money rate \( r \). Nevertheless, expecting current prices to prevail in the future, the capitalist-retailers are now willing to pay a price approximately one per cent higher than anticipated by the entrepreneurs. The difference between what they need to do so and what they possess they borrow in the bank. They will need that difference only for an instant, i. e., from the end of the first year to the beginning of the second. Consequently we ignore the interest on it.

7. Disequilibrium: Second Year

Like the capitalist-retailers, entrepreneurs expect current prices to prevail in the future. Consequently, once again it looks to them as if they could make a one percentage point net profit. But
last year the reason was the reduced money rate of interest; this year the reason is the raised price expectation. Whatever the reason, anticipated positive net profits are still incompatible with pure competition and freedom of entry and will be washed away by competitive bidding by the entrepreneurs in the labor market. Once again the money wage rate will rise by approximately one per cent. If entrepreneurs are to pay approximately another one per cent higher money wage rate, they must borrow approximately another one per cent more from the bank, and the money supply must expand by approximately another one per cent. As a result, labor will earn and spend approximately another one per cent more.

But here at the beginning of the second year will not the capitalist-retailers retain less for their own consumption? At the beginning of the first year their proceeds were up by an unanticipated approximately one per cent. If they consider such an increase an instant capital gain rather than income, their income in the first year is indeed down: On an approximately one per cent larger deposit they made an exactly one percentage point lower interest rate. But this is made up for by the fact that for the first time the entrepreneurs had a realized, unanticipated, positive net profit.

As a result of all this, once again a higher aggregate consumption demand is chasing an unchanged physical supply, and once
Figure 2. Alternative rates of inflation generated by alternative money rates of interest at a four per cent natural rate of interest.
again the price of consumers' goods must rise by approximately one per cent.

8. The Cumulative Process

In this way consumers' goods prices will keep rising at the rate of approximately one per cent per year for as long as the money rate is falling short of the natural rate of interest. In similar fashion Wicksell showed that if instead of being reduced by one percentage point the money rate of interest had been raised to exceed the natural rate, consumers' goods prices would have been falling for as long as such a discrepancy were maintained.

Only a restoration of monetary equilibrium could bring such cumulative processes of inflation or deflation to an end. Once the equality between the money rate and the natural rate had been restored, price would freeze at whatever level it happened to have reached at the time of restoration.

Figure 2 summarizes graphically the simple Wicksellian relationship between the rate of inflation and the money rate of interest.
9. Wicksell’s Later Work

In his later work on money Wicksell (1906) sacrificed some of his rigor and adopted a narrative form. The natural rate of interest was now simply defined (1935), 193, as "the expected yield on the newly created capital". Circulating capital and a one-year period of production are no longer referred to. Indeed the new definition of the natural rate is much like the definition of "the marginal efficiency of capital" applied by Keynes (1936), 135, to fixed capital. A stationary economy is no longer referred to. Indeed the words "newly created capital" must refer to a growing one.

A money rate falling short of the natural rate will still encourage a demand for money which can be met only by expanding the money supply. Wicksell emphasizes this effect by defining a third rate of interest, the normal one. The normal rate of interest is defined (1935), 193, 201, as the rate which would equalize saving and investment—and presumably require no expansion of the money supply.

A money rate falling short of the natural rate will still generate inflation. Wicksell emphasizes this effect by defining a fourth rate of interest, the neutral one. The neutral rate
of interest is defined (1936), 102, as the rate which would keep prices stationary.

Is a "normal" rate of interest also a "neutral" one? Wicksell thought so. But under modern assumptions an interest rate at which saving equals investment will not guarantee absence of inflation—as we shall demonstrate in Chapter 6.

III. WHAT WAS NEW IN WICKSELL—AND WHAT WAS MISSING?

Wicksell's distinction between a natural rate and a money rate of interest enabled him to break the barrier between capital theory and monetary theory and to identify the mechanism through which a quantity theory of money must be operating. Because Wicksell wanted to deal with inflation, and because inflation is defined as the proportionate rate of growth of price, he needed a dynamic model. He carefully built one by dating his variables and emphasizing the timing of events.

Late twentieth-century readers miss four ideas in Wicksell. They miss physical output as a variable; they miss nontransaction
demand for money; they miss Fisher's distinction between a nominal rate of interest and a real one; and they miss cost-push inflation. Let us look very briefly at these four ideas.

1. Physical Output as a Variable

To Wicksell was physical output ever a variable? Arguing against the Tooke-Fullarton Banking Principle, Wicksell (1935), 195, was prepared to modify his argument "if previously there had been unemployment" but continued: "But all these are secondary considerations. As a first approximation we are entitled to assume that all production forces are already fully employed".

To the Wicksellian Bertil Ohlin (1933), (1934) physical output was a variable—as it was to be to Keynes (1936) two years later. Ohlin (1933), Sec. 6, examined the effects of a reduction of the propensity to save in an underemployed economy. As a result of increased demand for consumers' goods, physical output would rise. Ohlin saw two feedbacks, one via the income generated by the
rising physical output and another via the investment induced by it. Ohlin's feedback was not telescoped into an instant static equilibrium along a physical-output axis—as the Keynesian one was to be two years later. Involving physical output as well as prices as variables, Chlin's feedback unfolds in a cumulative process along a time axis—as did the Wicksellian one involving prices alone.

2. Nontransaction Demand for Money

Wicksellian money serves transaction purposes only. It comes into existence as a loan to entrepreneurs who immediately spend it hiring labor. Labor immediately spends it buying consumers' goods. Capitalist-retailers hold their assets in interest-bearing form. Nobody in the Wicksellian model holds money in liquid form.

Ohlin (1934), 42, 69, and 85, saw liquidity preference. Like Keynes two years later he explained it in terms of speculation in bond prices: Asset holders will hold bonds rather than money when they expect bond prices to rise. Such expectations are common when the bond yield is high. They will hold money rather than bonds when they expect bond prices to fall. Such expectations are
common when the bond yield is low.

3. Fisher's Nominal and Real Rates of Interest

Neither Wicksell, nor Ohlin, nor Keynes accepted Fisher's (1895) distinction between a nominal rate (the rate of interest in terms of gold) and a real rate (the rate of interest in terms of wheat or other goods). Wicksell (1936), 165-166, knew Fisher's work, then only two years old, and restated its "logical basis" by saying that "entrepreneurs incur their 'expense' (wages, rents, etc.) when things are cheap, and dispose of their product after prices have gone up". Strangely enough Wicksell was unimpressed: "Such a rise in prices...does not provide [the entrepreneurs] with the means of paying a higher rate of interest". Wicksell must have identified himself with his entrepreneurs who would never anticipate such a rise in prices, because they always expect current prices to prevail in the future.

4. Cost-Push Inflation

Wicksellian inflation was always of the pure demand-pull variety. Theorists had to wait for another 60 years for cost-push infla-
tion in the form of the Phillips (1958) curve. Instead of always expecting current prices to prevail in the future, labor unions and entrepreneurs learn from experience: Anticipating inflation, they are compelled to contribute to it. But their compulsion is tempered by unemployment and idle capacity, respectively.

In the following chapters of this book we shall examine such missing ideas and attempt a synthesis of them.
REFERENCES


