POWER TRANSFORMATIONS IN TIME SERIES MODELS OF QUARTERLY EARNINGS PER SHARE

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Fligstein, N. and Wolf, W. (1978), "Sex Similarities in Occupational Status Attainment: Are the Results Due to the Restriction of the Sample to Employed Women?" Social Science Research, 7.


Summary

For many quarterly time series of corporate earnings per share, the data strongly indicates the desirability of incorporating a power transformation into the time series model. Our empirical results strongly suggest that, for such series, this will generally lead to forecasts of improved quality. Nevertheless, the resulting forecasts, on the average, remain inferior to those produced by financial analysts.
Predicted earnings have recently become accepted as important variables in the investment decision making process, as evidenced, for example, by the fact that the Securities and Exchange Commission has considered making earnings forecasts a required item for public reporting (Wall Street Journal, 1978). Moreover, in their conceptual framework project, the Financial Accounting Standards Board (FASB) has recognized the importance of future earnings.\(^1\)

In response to this interest, several recent studies\(^2\) have examined the possibility of generating forecasts through the time series model building methodology of Box and Jenkins (1970), fitting autoregressive integrated moving average (ARIMA) models to observed earnings series. Brown and Rozef (1978) have compared such time series forecasts with predictions made by financial analysts, finding on the whole the latter to be more accurate.

Our graphical examination of 50 individual corporate earnings series often suggested greater variability in the data at higher earnings levels, thus violating one of the assumptions of the ARIMA model formulation. In these circumstances it is common statistical practice to contemplate an extension of the model in which it is assumed that some transformation \(X_t^{(\lambda)}\) of the observed series \(X_t\) obeys an ARIMA model. In studying the analysis of variance and other linear regression models, Box and Cox (1964) introduced the class of power transformation

\[
x_t^{(\lambda)} = (X_t - 1)^{\lambda / \lambda} \quad (\lambda \neq 0)
\]

\[
\log X_t \quad (\lambda = 0)
\]
The use of these transformations has, in the last few years, been incorporated into time series model building methodology, the parameter \( \lambda \) being regarded as an additional coefficient to be estimated together with the autoregressive and moving average parameters of the model.

In this paper we present empirical evidence on two questions. Does the incorporation of power transformations into the ARIMA model result in improved forecasts? and how do the resulting forecasts compare with those of financial analysts?

POWER TRANSFORMATIONS IN ARIMA MODELS

Although the possibility of employing power transformations was very briefly noted by Box and Jenkins (1970), widespread interest in their use in ARIMA model building was perhaps first stimulated by results of Chatfield and Prothero (1973). These authors produced models yielding very unsatisfactory sales forecasts, and it emerged from the discussion of the paper, particularly Box and Jenkins (1973), that this was almost certainly due to an inappropriate use of the logarithmic transformation. Substantially better forecasts resulted when the model was broadened to include the general class of power transformations. Jenkins (1979) now advocates considering the use of power transformations and discusses some of the relevant methodology.

The transformation parameter \( \lambda \) can be estimated jointly with the coefficients of the ARIMA model by full maximum likelihood. Granger and Newbold (1976) show how forecasts can be calculated from the fitted model, while Nelson and Granger (1979) summarize some empirical evidence of the use of power transformation in the analysis of macroeconomic time series.
EMPIRICAL RESULTS

A sample of 50 firms was randomly selected from calendar year-end companies whose primary reported earnings per share (before extraordinary items and adjusted for capital changes) were available for 96 quarters beginning in the first quarter of 1951. These observations were obtained from the Value Line Investment Survey and the Compustat file. The first 84 values in each series were used to fit the time series models and the remaining observations to evaluate forecast performance.

We fitted models to each series, using a full Box-Jenkins analysis, in which the data is used to select a specific model from the general ARIMA class. We also considered the possibility of requiring the same model type for every series. (Jenkins (1979) recommends this procedure for sales data.) Three such "premier" models have been employed in the accounting literature.

(i) Foster (1977) proposes a model in which the series $X_t$ is seasonally differenced and a regular first order autoregressive model is fitted to the differenced data: that is, in the notation of Box and Jenkins (1970), the model

$$(1-\delta B)(1-B^4)X_t = a_t$$

where $B$ is a back-shift operator on the index of the time series, so that $B^4X_t = X_{t-4}$, and $a_t$ is a purely random process.

(ii) Griffin (1977) and Watts (1975) employ a model in which both seasonal and non-seasonal differencing is applied, and a multiplicative first order moving average model is fitted, that is

$$(1-B)(1-B^4)X_t = (1-\delta B)(1-\delta B^4)a_t$$
(iii) Brown and Rozeff (1979) propose a model involving seasonal differencing, a first order regular autoregressive term and a first order seasonal moving average term, i.e.

\[(1-\phi B)(1-B^4)X_t = (1-\theta B^4)a_t\]

Thus the Foster model is a special case of this with \(\theta = 0\).

We considered four possible transformation strategies:

(a) Use no transformation

(b) Use the logarithmic transformation

(c) Use a power transformation with \(\lambda\) estimated by the maximum likelihood estimate \(\hat{\lambda}\)

(d) Use a decision rule based on the 95\% confidence interval for \(\lambda\). If this interval contains \(\lambda=1\) but not \(\lambda=0\), use no transformation. If the interval contains \(\lambda=0\) but not \(\lambda=1\) use the logarithmic transformation. If the interval contains neither \(\lambda=0\) nor \(\lambda=1\), use \(\hat{\lambda}\). If the interval contains both \(\lambda=0\) and \(\lambda=1\), use whichever is closer to \(\hat{\lambda}\).

Strategy (d) is motivated by a point stressed by Box and Cox (1964), but frequently ignored in subsequent studies. If possible the transformation used should make physical sense. For earnings data this suggests a pre-disposition to favor no transformation (\(\lambda=1\)) or the logarithmic (\(\lambda=0\)) transformation (which can be justified in terms of percentage changes).

The forecasts were compared in terms of mean absolute proportionate errors, that is the average of \(|(A-P)/A|\) where \(A\) and \(P\) denote actual and predicted values. The results for the 50 series in our
TABLE 1

Mean Absolute Proportionate Errors of Forecasts over 50 Earnings Series for Financial Analysts and (i) Foster model, (ii) Griffin-Watts model, (iii) Brown and Rozef model, (iv) full Box-Jenkins analysis each with (a) no transformation, (b) logarithmic transformation, (c) power transformation estimated by maximum likelihood, (d) power transformation determined by decision rule.

<table>
<thead>
<tr>
<th>Model and Transformation</th>
<th>Forecast Horizon (in Quarters)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>(i) (a)</td>
<td>.349</td>
</tr>
<tr>
<td>(b)</td>
<td>.429</td>
</tr>
<tr>
<td>(c)</td>
<td>.350</td>
</tr>
<tr>
<td>(d)</td>
<td>.332</td>
</tr>
<tr>
<td>(ii) (a)</td>
<td>.286</td>
</tr>
<tr>
<td>(b)</td>
<td>.429</td>
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<td>(c)</td>
<td>.271</td>
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<td>(d)</td>
<td>.258</td>
</tr>
<tr>
<td>(iii) (a)</td>
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<td>.492</td>
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<tr>
<td>(c)</td>
<td>.408</td>
</tr>
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<td>(d)</td>
<td>.399</td>
</tr>
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<td>(iv) (a)</td>
<td>.292</td>
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<tr>
<td>(b)</td>
<td>.456</td>
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<td>(c)</td>
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<tr>
<td>(d)</td>
<td>.285</td>
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<tr>
<td>Analysts</td>
<td>.198</td>
</tr>
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</table>

1 Not available
sample for forecast horizons of 1, 2, 3, 4 and 12 quarters are given in table 1. For comparison, the forecast performance of financial analysts is also included. The following conclusions emerge from the table:

(i) At least for short forecast horizons the Griffin-Watts model performs a good deal better than the other two premier models, and is very slightly more accurate on the average than a full Box-Jenkins analysis.

(ii) Taking logarithms of every series is clearly a poor strategy. The resulting forecasts are on average far less accurate than when no transformation at all is used.

(iii) On the average, there is a small improvement in forecast performance when power transformations are used, with either a maximum likelihood estimate of \( \lambda \) or the decision rule. However, the resulting forecasts remain inferior to those of financial analysts.

The aggregate results in table 1 do not bring out sufficiently clearly the potential gains from the use of power transformations in modelling corporate earnings series. This is because, for a large proportion of the series, the data fails to indicate the desirability of using any transformation. When the remaining series (i.e., those where \( \hat{\lambda} \) was found to be significantly different from 1) are viewed in isolation in table 2, the desirability of entertaining the possibility of a transformation becomes clearer. The decision rule picks some transformation (other than \( \lambda = 1 \)) for about half of our series. For these particular series the use of a transformation results generally in a substantial improvement in mean absolute proportionate forecast
TABLE 2

Mean Absolute Proportionate Errors of Forecasts for those series in which the Decision Rule selected some transformation other than $\lambda = 1$.

<table>
<thead>
<tr>
<th>Model and Transformation $^1$</th>
<th>Number of Series $^2$</th>
<th>Forecast Horizon (in Quarters)</th>
</tr>
</thead>
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<td>(i) (a)</td>
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<td>.271</td>
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<td>(d)</td>
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<td>.242</td>
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<td>(iii) (a)</td>
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<td>.343</td>
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<tr>
<td>(d)</td>
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<td>.313</td>
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<tr>
<td>(iv) (a)</td>
<td>23</td>
<td>.301</td>
</tr>
<tr>
<td>(d)</td>
<td></td>
<td>.287</td>
</tr>
</tbody>
</table>

$^1$The notation is the same as in Table 1.

$^2$Number of series, from a total of 50, for which the decision rule selected a transformation other than $\lambda = 1$. 

error over short horizons. For example, for prediction one quarter ahead from the Griffin-Watts model the reduction in mean absolute proportionate error is a little over 19%.

CONCLUSIONS

Our empirical results on 50 time series of corporate quarterly earnings per share show that for about half of these series the data clearly indicates the desirability of incorporating a power transformation into the time series model. When this is done, a fairly substantial improvement in forecast accuracy generally results for those series. Nevertheless, this improvement is not sufficiently substantial to contradict the conclusion of Brown and Rozeff (1978) that, on the average, forecasts produced by financial analysts are superior to those derived from single series time series models.
REFERENCES


FOOTNOTES

1 In the Statement of Financial Accounting Concepts No. 1 the FASB (1978b, Page ix) stated, "The primary focus of financial reporting is information about earnings and its components." The reasoning behind this statement can be traced to the Tentative Conclusions on Objectives of Financial Statements of Business Enterprises (FASB, 1978a) where the FASB relied upon four propositions:

(1) The primary interest of the investor is in a return on his investment in the form of cash flows (p. 45).
(2) Earnings as measured by accrual accounting are generally thought to be the most relevant indicator of an enterprise's cash earning ability (p. 45).
(3) Fundamental financial analysis focuses on the earning power of an enterprise in estimating the intrinsic value of the stock (p. 57).
(4) The most important single factor in determining a stock's value is now held to be the indicated average future earning power (p. 57).

Also, forecasted earnings is an important variable in research involving cost of capital, dividend policy, and capital markets and information content. For a more complete discussion see Foster (1977, p. 2).


3 The likelihood function can be computed by incorporating the algorithm of Ansley, Spivey and Wrobleski (1977) into that of Ansley (1979). Simulation results of Ansley and Newbold (1980) indicate that use of full maximum likelihood rather than either of the least squares procedures described by Box and Jenkins (1970) generally produces superior parameter estimates and forecasts, particularly for relatively short seasonal time series.

4 The methodology is described in Box and Jenkins (1970), Nelson (1973) and Granger and Newbold (1977).