ON THE AUTOMATION OF THE BOX-JENKINS MODELING PROCEDURES: AN ALGORITHM WITH AN EMPIRICAL TEST

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Summary:

The study presented a univariate stochastic modeling algorithm (USA) for purposes of the Box-Jenkins modeling of one variable time series via a fully automatic process. The algorithm was programmed for the computer and tested empirically. It was found that USA forecasts were not statistically different than those generated by conventional modeling procedures.

The results indicate that there is evidence that the Box-Jenkins modeling process can be fully automated. It is felt that the use of USA can (1) increase the reproducibility of research, (2) save time, (3) be used as a "black box" by the statistically untrained, and (4) make explicit the assumptions employed in the modeling process.
In recent years there has been an increased emphasis on the use of the Box and Jenkins (1970) method of modeling univariate stochastic time series.¹ Notwithstanding this emphasis, the use of the method involves the making of sequence of highly subjective decisions.² The purpose of the present study is to present an algorithm which quantifies this subjective sequence. Like any formal modeling process the algorithm does not replace the need to make decisions, but simply quantifies them and makes them automatically.

The paper is presented in four sections. Section one discusses the potential benefits of using an automatic univariate stochastic algorithm (USA). Section two presents the USA followed by an empirical validation in section three. Section four presents a summary and conclusions.

1.0 Benefits Associated with the USA

1.1 Subjectivity and Reproducibility

As discussed by Dharan (1977) one of the major requirements or characteristics of scientific research is reproducibility. However, "while empirical research in areas other than time series analysis may contain some subjectivity, none probably relies on subjectivity as much as the Box-Jenkins procedure does" (Dharan, 1977, p. 13). The result is that different researchers are likely to fit different models to the same data. However, the use of a programmed algorithm will always produce the same model, given a particular time series, regardless of the research.
1.2 USA and Required Earnings Forecasts

There is a reasonable indication that earnings forecasts will be required in external reports in the not too distant future (see Wall Street Journal, 1977). This will probably lead to an emphasis on the disclosure of the procedures and assumptions used to generate the forecasts. The use of a USA type method makes possible the disclosure of a precise description of the procedures used.

1.3 USA and the Untrained Individual

One benefit of USA is that it can be programmed on the computer for use as a "black box". This means that it can be used by the nonstatistical auditor in the forecasting of account balances or by management in the decision making process.

1.4 USA and Time Savings

Programmed on the computer USA can model literally thousands of time series models in a very short period of time. For example, the author found that its use on the CYBER 175 required an average of .333 CPU seconds for modeling and forecasting the EPS of firms containing 50 observations.

2.0 A Proposed Algorithm

Discussion of USA can be decomposed according to the main phases of the modeling process: (1) determining the appropriate degree of differencing, (2) model identification, (3) nonlinear least squares estimation, and (4) diagnostic checks for model adequacy. The focus will be on stages (1) and (2) since they are the most difficult phases to algorithmize.
2.1 Determining the Appropriate Degree of Differencing

The purpose of differencing is to produce a transformed series which is approximately constant in mean. The criterion given by Box and Jenkins (1970) is that a consecutive difference should be taken if the autocorrelation function does not dampen out "rapidly". The determination of what is rapid is a subjective and pivotal point in the modeling process. The same criterion holds for a seasonal difference; the autocorrelation function examined at only seasonal lags must dampen out quickly.

Rule for Taking a Consecutive Difference. Based on the above criterion it was decided to make the following formal rule: If the number of initial consecutive significant autocorrelations is greater than an arbitrary but fixed constant $K$, then the series $Z_t$ is formed from taking a first difference on the original series $X_t$ (see note 3). If no first difference is taken then $Z_t = X_t$.

Rule for Taking a Seasonal Difference. If the number of initial consecutive autocorrelations of $Z_t$ (examined only at seasonal lags) is greater than $L$ (an arbitrary but fixed constant) then a series $Y_t$ is formed by taking a seasonal difference on $Z_t$ (see note 4). If no seasonal difference is taken then $Y_t = Z_t$. It is important that the two rules for consecutive and seasonal differences are applied in the order given. This is because, if a first difference is not taken (when necessary) it is possible that there will be significant the seasonal lags due to the need for a first difference. The taking of a first difference will remove these confounding significant autocorrelations.
2.2 Model Identification

The basic identification is executed by the examination of the sample autocorrelation and partial correlation functions (ACF and PCF, respectively) of the series \( Y_t \). The sample functions are compared to theoretical functions known to be associated with various ARIMA models. The next step is to select a model which has the theoretical function most closely associated with the sample pattern.

In the development of USA, ACFs and PCFs are examined for the possible existence of three basic types of patterns:

1. **Sign Alteration**: If an ACF or PCF alternates in sign for a number of initial consecutive correlations that exceeds an arbitrary but fixed constant \( m \), then that ACF or PCF is assumed to alternate in sign.\(^5\)

2. **Unidirectional Tapering**: If an ACF or PCF has a number of initial consecutive correlations that exceeds an arbitrary but fixed constant \( n \), then that ACF or PCF is assumed to taper.\(^6\)

3. **Seasonal Pattern**: If an ACF or PCF has a number of initial consecutive seasonal correlations that exceeds an arbitrary but fixed constant \( p \), then that ACF or PCF is assumed to have a seasonal pattern.\(^7\)

2.2.1 Introduction of Notation

For purposes of notation, two zero-one variables are defined to denote the existence of patterns in the ACF or PCF. These are APTRNID and PPTRNID and are set to one if at least one of the above three patterns occur in the ACF or PCF respectively (otherwise set to zero).
In addition, let $A_k$ denote the autocorrelation coefficient at lag $k$ and $\rho_k$ denotes the partial correlation coefficient at lag $k$. Also, $A_k$ and $\rho_k$ will be defined to be equal to one when significant (and zero otherwise). Finally, let the variable ACNECID (or PCNECID) equal the number of initial consecutive significant autocorrelations (partial correlations).

2.2.2 Operation of the Algorithm

The USA model identification procedure selects a model based on the values that the above defined variables take on. Table 1 presents a list of models with the associated values of the above variables required for identification. The procedure used is to sequentially proceed down the list until a model is selected. The next step is to estimate parameters for this model and then apply diagnostic checks to the residuals. If the model is accepted then forecasting is done. If the model is unacceptable then Table 1 is again used to model the residual series. The model for residual series is then added as a factor to the previous model and the process continues until a model passes the diagnostic checks.
Table 1
A Description of Procedures Employed to Identify Models by USA

<table>
<thead>
<tr>
<th>Model(s) Identified</th>
<th>Associated Correlation Structure Required for Identification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular AR of order one</td>
<td>$A_1 = 1$  $P_1 = 1$  $APTRNID = 1$  $PPTRNID = 0$</td>
</tr>
<tr>
<td>Regular AR of order two</td>
<td>$A_1 = 1$  $A_2 = 1$  $APTRNID = 1$  $PPTRNID = 0$  $PCNECID &gt; 0$</td>
</tr>
<tr>
<td>Regular MA of order one</td>
<td>$A_1 = 1$  $A_2 = 0$  $APTRNID = 0$  $PPTRNID = 1$</td>
</tr>
<tr>
<td>Regular MA of order two</td>
<td>$P_1 = 1$  $P_2 = 1$  $APTRNID = 0$  $ACNECID &gt; 0$  $PPTRNID = 1$</td>
</tr>
<tr>
<td>Mixed AR-MA first order</td>
<td>$A_1 = 1$  $P_1 = 1$  $APTRNID = 1$  $PPTRNID = 1$</td>
</tr>
<tr>
<td>Seasonal AR of order one</td>
<td>$A_4 = 1$  $A_8 = 1$  $P_4 = 1$  $P_8 = 0$</td>
</tr>
<tr>
<td>Seasonal MA of order one</td>
<td>$A_4 = 1$  $A_8 = 0$  $P_4 = 1$  $P_8 = 1$</td>
</tr>
<tr>
<td>Try Both (first order) AR and MA</td>
<td>$[A_1 = 1$  $A_2 = 0]$  $\text{or} [P_1 = 1$  $P_2 = 0]$</td>
</tr>
<tr>
<td>Try Both (second order) AR and MA</td>
<td>$[ACNECID = 1]$  $\text{or} [PCNECID = 1]$</td>
</tr>
<tr>
<td>Try Both (first order) AR and MA Seasonal</td>
<td>$A_1 = 0$  $P_1 = 0$  $A_4 = 1$  $P_4 = 1$</td>
</tr>
<tr>
<td>Try Both AR and MA with parameters at significant lags</td>
<td>This identification is used if none of the above identifications are made.</td>
</tr>
</tbody>
</table>
3.0 An Empirical Test of USA

A sample of EPS (60 quarters) was taken for 30 firms. For each firm 50 quarters were used for modeling (using both USA and conventional methods) and the remaining 10 quarters were used for comparing the USA and conventional modeling methods.

For each of the 10 periods on the forecast horizon, a computation was made as to the number of firms for which USA came closer in absolute value to the actual EPS than for the conventional method. (This computed number will henceforth be referred to as $D_k$, $k = 1, 10$.)

Under the null hypothesis of no difference between USA and the manual method, we would expect each $D_k$ to be equal to 15 (i.e., 1/2 of the 30 firms). Table 2 presents a test of this hypothesis.

Table 2

<table>
<thead>
<tr>
<th>Period on the Forecast Horizon</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Frequency</td>
<td>12</td>
<td>13</td>
<td>15</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>15</td>
<td>16</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>Expected Frequency</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>13</td>
<td>13.5</td>
<td>14</td>
</tr>
<tr>
<td>Chi Square (9 df)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.66</td>
</tr>
</tbody>
</table>

Note that the test statistic of 3.66 is substantially less than required 14.68 needed to reject the null (assuming $\alpha = .1$). The implication is that there is no difference between the USA forecasts and those generated by conventional modeling procedures.
4.0 Summary and Conclusions

The study presented a univariate stochastic modeling algorithm (USA) for purposes of the Box-Jenkins modeling of one variable time series via a fully automatic process. The algorithm was programmed for the computer and tested empirically. It was found that USA forecasts were not statistically different than those generated by conventional modeling procedures.

The results indicate that there is evidence that the Box-Jenkins modeling process can be fully automated. It is felt that the use of USA can (1) increase the reproducibility of research, (2) save time, (3) be used as a "black box" by the statistically untrained, and (4) make explicit the assumptions employed in the modeling process.
Notes

1 Some examples of the use of Box-Jenkins models are: Albrecht, Lookabill (1977), Brown and Rozeff (1977), Dopuch and Watts (1972), Foster (1977), Lorek, McDonald and Patz (1976), and Watts and Leftwich (1977).

2 For a complete discussion see Dharan (1977).

3 For purposes of the present study, a value of 3 was assumed for K.

4 For purposes of the present study a value of 1 was assumed for L.

5 For purposes of the present study, a value of 3 was assumed for m.

6 For purposes of the present study, a value of 3 was assumed for n.

7 For purposes of the present study, a value of 1 was assumed for P.

8 The reason that the model identified is added as a factor can be stated by the following:

Assume the following ARIMA model for $Z_t$

(1) $\phi(B) Z_t = \theta(B) a_t$ where $a_t$ is the nonrandom residual series which can be modeled by

(2) $\phi'(B) a_t = \theta'(B) a'_t$ where $a'_t$ is white noise.

(3) Then from (2), $a_t = \phi^{-1}(B)' \theta'(B) a'_t$ and substituting

the right hand side of (3) into (1) we get

(4) $\phi(B) Z_t = \theta(B) \phi'(B)^{-1} \theta'(B) a'_t$

or

(5) $\phi'(B) \theta(B) Z_t = \theta'(B) \theta(B) a'_t$
Note that (5) is the same as (1) but the factors of the noise model have been added. Typically we would find $\phi(B)$ to be a regular autoregressive factor, $\phi'(B)$ to be a seasonal autoregressive factor, $\theta(B)$ to be a regular moving average factor and $\theta'(B)$ to be a seasonal moving average factor.

For a discussion of multifactor models see Box and Jenkins (1970, Chapter 9).

In those cases where more than one model is tried, USA will select the one with the smaller mean square error (where the denominator is adjusted for the appropriate number of degrees of freedom).

See Appendix 1 for a list of sample firms. Also EPS was taken from Moody's Handbook and adjusted for changes in capital structure. In addition for firms 1, 2, 3, 5, 7, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, and 29, EPS were computed using information from schedule B-3 of the Civil Aeronautics Board (CAB) form 41 in conjunction with the CAB quarterly periodical Air Carrier Financial Statistics.

The models are not presented for sake of space, but will be furnished upon written request to the author.

See Appendix 2 for a description of the periods used for modeling and forecasting.

The expected frequencies for periods 7-10 have been adjusted slightly due to a small amount of missing data.
BIBLIOGRAPHY


APPENDIX 1

LIST OF SAMPLE FIRMS

1. Airlift International
2. Alaska Airlines
3. Aloha Airlines
4. American Airlines
5. Aspen Airways
6. Braniff Airways
7. Caribbean Atlantic Airlines
8. Continental Airlines
9. Delta Airlines
10. Eastern Airlines,
11. Tiger International Airlines
12. Frontier Airlines
13. Hawaiian Airlines
14. National Airlines
15. New York Airways
16. North Central Airlines
17. North West Airlines
18. Ozark Airlines
19. Pan American Airways
20. Piedmont Airlines
21. Reeve Airlines
22. SFO Airlines
23. Seaboard World Airlines
24. Southern Airways
25. Texas International Airlines
26. Trans World Airlines
27. UAL (United Airlines)
28. Western Airlines
29. Wien Airlines
30. Allegheny Airlines

Each firm will be subsequently referred to by the identifying number that precedes it.
APPENDIX 2

DESCRIPTION OF AVAILABLE DATA FOR FORECAST ERROR ANALYSIS

This appendix gives a firm by firm description of the number of quarters of data available for forecast error analysis. For each firm, the number of periods in the base period, the origin date for forecasting, and the number of absolute forecast errors is presented.

<table>
<thead>
<tr>
<th>firm number</th>
<th>number of periods in base period</th>
<th>origin date for forecasting</th>
<th>number of steps ahead forecast error was computed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>2/74</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>3/74</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>2/74</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>2/74</td>
<td>7</td>
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<tr>
<td>9</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
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<tr>
<td>10</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
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<tr>
<td>12</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
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<tr>
<td>13</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
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<tr>
<td>14</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
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<tr>
<td>15</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
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<td>16</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
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<tr>
<td>17</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
</tr>
<tr>
<td>18</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
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<tr>
<td>19</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
</tr>
<tr>
<td>21</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
</tr>
<tr>
<td>22</td>
<td>42</td>
<td>2/74</td>
<td>10</td>
</tr>
<tr>
<td>23</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
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<tr>
<td>24</td>
<td>50</td>
<td>2/74</td>
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<tr>
<td>25</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
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<tr>
<td>26</td>
<td>50</td>
<td>2/74</td>
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<td>27</td>
<td>50</td>
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<td>28</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
</tr>
<tr>
<td>29</td>
<td>30</td>
<td>2/76</td>
<td>7</td>
</tr>
<tr>
<td>30</td>
<td>50</td>
<td>2/74</td>
<td>10</td>
</tr>
</tbody>
</table>
For example in the case of firm 1, 50 quarters of data were used in transfer and univariate estimation, and actual and predicted forecasts were computed over a 10 period forecast horizon with the first forecast being for the third quarter of 1974.