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Wages, Hours, and Growth

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ABSTRACT

The paper sees wages and hours within the framework of a growth model of optimized capital stock. Its results are these. Only in the short run can labor have a higher real wage rate by accepting a lower natural rate of employment. The long-run real wage rate is invariant with the natural rate of employment. A more rapid shortening of hours will make optimized physical capital stock, physical output, and the hourly as well as the annual real wage rate grow at lower rates.

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WAGES, HOURS, AND GROWTH

By Hans Brems

Abstract

The paper sees wages and hours within the framework of a growth model of optimized capital stock. Its results are these. Only in the short run can labor have a higher real wage rate by accepting a lower natural rate of employment. The long-run real wage rate is invariant with the natural rate of employment. A more rapid shortening of hours will make optimized physical capital stock, physical output, and the hourly as well as the annual real wage rate grow at lower rates.

The real wage rate and hours worked per year per man both display an unmistakable long-run trend. According to Phelps Brown (1973: 65), over the period 1890-1960 the U.S. hourly nonagricultural real wage rate was growing by 2.08 percent per annum and the German one by 1.61 percent per annum. According to Maddison (1987: 686), over the period 1870-1984 U.S. hours of work per year per man declined from

2,964 to 1,632 or by 0.53 percent per annum. Over the same period German hours declined from 2,941 to 1,676 or by 0.48 percent per annum.

Instead of seeing changes in wages and hours as once-and-for-all events we shall take a longer view and see them within the framework of a growth model of optimized capital stock. Our longer view will pay off: the long-run scope for wage and hours policy turns out to be quite different from the short-run scope.

I. THE MODEL

1. Variables

$C \equiv$ physical consumption

$g \equiv$ proportionate rate of growth

$I \equiv$ physical investment

$k \equiv$ present gross worth of another physical unit of capital stock

$\kappa \equiv$ physical marginal productivity of capital stock

$L \equiv$ number of men employed

$n \equiv$ present net worth of another physical unit of capital stock

P $\equiv$ price of good

r $\equiv$ nominal rate of interest

ρ $\equiv$ real rate of interest

S $\equiv$ physical capital stock, number of machines

w $\equiv$ money wage rate, dollars per man-hour

X $\equiv$ physical output per year

2. Parameters

a $\equiv$ joint factor productivity of production function

α, β $\equiv$ exponents of production function

c $\equiv$ propensity to consume

F $\equiv$ available labor force

h $\equiv$ hours of work per year of men and machines alike

λ $\equiv$ natural rate of employment

All parameters are stationary except a, F, and h whose growth rates g_a, g_F, and g_h are stationary.

3. The Production Function

Let men and machines alike work h hours per year and let the aggregate production function be

$$X = a(hL)^{\alpha}(hS)^{\beta} = ahL^{\alpha}S^{\beta} \tag{1}$$

where $0 < \alpha < 1$, $0 < \beta < 1$, and $\alpha + \beta = 1$.

4. Demand for Labor

Demand for labor is a short-run commitment decided by maximization of profits. Under such short-run maximization the purely competitive firm considers its capital stock S frozen and its hourly real wage rate w/P as well as its hours h given to it by current collective agreement. The firm will be hiring man-hours until the hourly real wage rate equals the physical marginal productivity of man-hours:

$$\frac{w}{P} = \frac{\partial X}{\partial (hL)} = \frac{1}{h}\frac{\partial X}{\partial L} \tag{2}$$

Carry out the partial differentiation, raise to power $-1/\beta$, rearrange, and write short-run demand for labor

$$L = (a\alpha)^{1/\beta} \left(\frac{w}{P}\right)^{-1/\beta} S \tag{3}$$

5. Supply of Labor

Labor-market literature, e.g., Lindbeck and Snower (1986) and Blanchard and Summers (1988) distinguish between "insiders," who are employed hence decision-making, and "outsiders," who are unemployed hence disenfranchised. Let insiders accept the "natural" employment rate λ where $0 < \lambda \leq 1$. In other words, if $L > \lambda F$ insiders will insist on a higher hourly real wage rate. If

$$L = \lambda F \tag{4}$$

they will be happy with the existing hourly real wage rate. If $L < \lambda F$ they will settle for a lower hourly real wage rate.

6. Short-Run Natural Levels

The hourly real wage rate insiders will be happy with, given their natural rate λ of employment, might be called the "natural" one. Find it by inserting (4) into (3) and rearranging:

$$\frac{w}{P} = a\alpha(\lambda F)^{-\beta}S^{\beta} \tag{5}$$

At the short-run frozen capital stock S, first, labor can have a β percent higher hourly real wage rate (5) by accepting a one percent lower natural rate λ of employment. Second, (5) had no h in it; shorter hours would leave it unaffected.

The supply of goods corresponding to the natural rate λ of employment might be called the "natural" one. Find it by inserting (4) into (1):

$$X = ah(\lambda F)^{\alpha}S^{\beta} \tag{6}$$

At the short-run frozen capital stock S, then, labor would be facing an α percent lower natural supply of goods by accepting a one percent lower natural rate λ of employment.

None of these results will survive the unfreezing of capital stock.

7. Optimized Capital Stock

So far we have considered physical capital stock S frozen. We must now go beyond the short run, unfreeze it, and optimize it: optimized capital stock is a long-run commitment decided by maximization of present net worth. We begin by defining physical marginal productivity of capital stock as

$$\kappa \equiv \frac{\partial X}{\partial S} = \beta \frac{X}{S} \tag{7}$$

At time t, then, marginal value productivity of capital stock is $\kappa(t)P(t)$. Let there be a market in which money may be placed or borrowed at the stationary nominal rate of interest r. Let that rate be applied when discounting future cash flows. As seen from the present time τ, then, marginal value productivity of capital stock is $\kappa(t)P(t)e^{-r(t - \tau)}$. Let capital stock be immortal, so define present gross worth of another physical unit of it as the present worth of all such future marginal value productivities:

$$k(\tau) \equiv \int_{\tau}^{\infty} \kappa(t)P(t)e^{-r(t - \tau)}dt \tag{8}$$

Let firms expect physical marginal productivity of capital stock to be growing at the stationary rate g_κ:

$$\kappa(t) = \kappa(\tau)e^{g_\kappa(t - \tau)}$$

and price of output to be growing at the stationary rate g_P:

$$P(t) = P(\tau)e^{g_P(t - \tau)}$$

Insert these, define

$$\rho \equiv r - (g_\kappa + g_P) \tag{9}$$

take $\kappa(\tau)$ and $P(\tau)$ outside the integral sign, and find the integral (8) to be

$$k = \kappa P/\rho$$

Define present net worth of another physical unit of capital stock as its gross worth <u>minus</u> its price:

$$n \equiv k - P = (\kappa/\rho - 1)P \tag{10}$$

Optimized capital stock is the size of stock at which the present net worth of another physical unit of capital stock would be zero:

$$\kappa = \rho \tag{11}$$

Insert (7) into (11) and find optimized capital stock

$$S = \beta X/\rho \tag{12}$$

8. Optimized Investment

Optimized investment is

$$I \equiv g_S S = \beta g_S X/\rho \tag{13}$$

Optimized capital stock (12) and investment (13) are both in inverse proportion to ρ. What is ρ? As we shall see presently our model will have the solution (23) $g_X = g_S$. Consequently in (7) $g_\kappa = 0$. Historically, for good measure, κ has displayed no secular trend. But if $g_\kappa = 0$ the definition (9) of ρ collapses into the real rate of interest.

9. Consumption; Equilibrium

Let the aggregate consumption function be

$$C = cX \tag{14}$$

where $0 < c < 1$.

Aggregate equilibrium requires aggregate supply to equal aggregate demand:

$$X = C + I \tag{15}$$

II. SOLVING FOR LONG-RUN NATURAL LEVELS: SCOPE FOR WAGE POLICY

1. Long-Run Natural Levels

Now that we have unfrozen and optimized our capital stock (12), does it remain true that labor can have a higher hourly real wage rate (5) by accepting a lower natural rate λ of employment?

To see if it does, we must solve for long-run natural levels and begin with the real rate of interest. Insert (13) and (14) into (15), divide any nonzero X away, and solve for the real rate of interest

$$\rho = \frac{\beta g_S}{1 - c} \tag{16}$$

Insert (6) and (16) into (12) and solve for optimized capital stock

$$S = \left(ah \frac{1 - c}{g_S}\right)^{1/\alpha} \lambda F \tag{17}$$

Insert (17) into (6) and solve for the natural supply of goods[1]

$$X = (ah)^{1/\alpha} \left(\frac{1 - c}{g_S}\right)^{\beta/\alpha} \lambda F \tag{18}$$

Insert (17) into (5) and solve for the natural hourly real wage rate

$$\frac{w}{P} = \alpha a^{1/\alpha} \left(h \frac{1 - c}{g_S}\right)^{\beta/\alpha} \tag{19}$$

Multiply (19) by hours h per year and solve for the natural annual real wage rate

$$h \frac{w}{P} = \alpha(ah)^{1/\alpha} \left(\frac{1 - c}{g_S}\right)^{\beta/\alpha} \tag{20}$$

We may now draw our wage-policy conclusion.

2. Short-Run versus Long-Run Scope for Wage Policy

We now see a stark contrast between the short and the long run: at the short-run frozen capital stock S the natural hourly real wage rate (5) contained λ raised to the power $-\beta$. Labor could have a β percent higher hourly real wage rate by accepting a one percent lower natural rate λ of employment.

By contrast, under a long-run, unfrozen, and optimized capital stock S neither (19) nor (20) has λ in it. The long-run natural real wage rates, whether hourly or annual, are invariant with the natural rate λ of employment. In the long run labor can have a no higher real wage rate, hourly or annual, by accepting a lower natural rate λ of employment.

The clue is scale. Long-run, unfrozen, and optimized capital stock S and natural supply X were the functions (17) and (18) of λ. Both were in direct proportion to λ.

Summing up, in the long run a lower natural rate λ of employment would simply reduce the economy to a lower scale, accumulating proportionately less capital stock and producing proportionately less goods--thus impoverishing itself. Labor would not benefit. Indeed nobody would benefit.

III. SOLVING FOR LONG-RUN GROWTH RATES: SCOPE FOR HOURS POLICY

1. Long-Run Growth Rates

Now that we have unfrozen and optimized our capital stock (12), does it remain true that an hourly real wage rate (5) has no h in it, hence would remain unaffected by shorter hours?

To see if it does, we must solve for long-run growth rates and begin with convergence.

Consider the growth rates g_a of joint factor productivity, g_F of available labor force, and g_h of hours h per year per man or machine to be parameters, and let all other growth rates be variables to be solved for. Define the growth rate of a variable v as the derivative of the natural logarithm of the latter with respect to time:

$$g_v \equiv \frac{d\log_e v}{dt}$$

Unfreeze physical capital stock S in (6), consider the natural rate λ stationary, and take the growth rate of (6):

$$g_X = g_a + g_h + \alpha g_F + \beta g_S \tag{21}$$

Insert (14) and the definitional part of (13) into (15), rearrange, and write the rate of growth of physical capital stock

$$g_S = (1 - c)X/S$$

Take the growth rate of g_S, use (21), and write the rate of acceleration of physical capital stock

$$g_{gS} = g_X - g_S = \alpha[(g_a + g_h)/\alpha + g_F - g_S]$$

Here there are three possibilities. First, if $g_S > (g_a + g_h)/\alpha + g_F$, then $g_{gS} < 0$. Second if

$$g_S = (g_a + g_h)/\alpha + g_F \tag{22}$$

then $g_{gS} = 0$. Third, if $g_S < (g_a + g_h)/\alpha + g_F$, then $g_{gS} > 0$. Consequently, if greater than (22) g_S is falling; if equal to (22) g_S is stationary; and if less than (22) g_S is rising. Furthermore, g_S cannot alternate around (22), for differential equations trace continuous time paths, and as soon as a g_S-path touched (22) it would have to stay there. Finally, g_S cannot converge to anything else than (22), for if it did, by letting enough time elapse we could make the left-hand side of g_{gS} smaller than any arbitrarily assignable positive constant ε, however small, without the same being possible for the right-hand side. We conclude that g_S must either equal (22) from the outset or, if it does not, converge to that value.

Once such Solow (1956) convergence has been established we may easily find the corresponding values of other growth rates: insert (22) into (21), recall that $\alpha + \beta = 1$, and solve for the growth rate of the natural supply of goods

$$g_X = g_S \tag{23}$$

Insert (7) into (11), take the growth rate, insert (23), and solve for the growth rate of the real rate of interest

$$g_\rho = 0 \tag{24}$$

Unfreeze physical capital stock S in (5), take the growth rate of (5), insert (22), and solve for the growth rate of the natural hourly real wage rate

$$g_{w/P} = (g_a + \beta g_h)/\alpha \tag{25}$$

Unfreeze physical capital stock S in (5) once more, multiply by hours h, take the growth rate of the result, insert (22), and solve for the growth rate of the natural annual real wage rate

$$g_{hw/P} = (g_a + g_h)/\alpha \tag{26}$$

We are pleased to see that our growth-rate solutions are consistent with our long-run natural levels: our (22), (23), (24), (25), and (26) are indeed the growth rates of our (17), (18), (16), (19), and (20), respectively.

2. Sensitivities of Growth-Rate Solutions to Rate of Change of Hours

The partial derivatives of the rates of growth (22) of desired capital stock, (23) of the natural supply of goods, (24) of the real rate of interest, and (25) and (26) of the hourly and annual real wage rates with respect to the rate of change g_h are:

$$\frac{\partial g_S}{\partial g_h} = \frac{1}{\alpha} \tag{27}$$

$$\frac{\partial g_X}{\partial g_h} = \frac{1}{\alpha} \tag{28}$$

$$\frac{\partial g_\rho}{\partial g_h} = 0 \tag{29}$$

$$\frac{\partial g_{w/P}}{\partial g_h} = \frac{\beta}{\alpha} \tag{30}$$

$$\frac{\partial g_{hw/P}}{\partial g_h} = \frac{1}{\alpha} \tag{31}$$

We may now draw our hours-policy conclusion.

3. Short-Run versus Long-Run Scope for Hours Policy

We now see another stark contrast between the short and the long run: at the short-run frozen capital stock S the natural hourly real wage rate (5) had no h in it; shorter hours would leave it unaffected.

By contrast, the rate of growth g_S of long-run, unfrozen, and optimized capital stock was the function (22) of g_h. According to (27) a one percentage point lower g_h would lower g_S by $1/\alpha \cong 1.25$ percentage points and according to (30) lower $g_{w/P}$ by $\beta/\alpha \cong 0.25$ percentage points. According to (31) it would, of course, lower the rate of growth $g_{hw/P}$ of the annual real wage rate even more, i.e., by $1/\alpha \cong 1.25$ percentage points.

Summing up, a more rapid shortening of hours would slow down the accumulation of capital stock and the growth of hourly and annual real wage rates.

IV. SUMMARY AND CONCLUSIONS

We have seen wages and hours within the framework of a growth model of optimized capital stock. Our results were these. Only in the short run could labor have a higher real wage rate by accepting a lower natural rate of employment. In the long run the economy would be accumulating proportionately less capital stock, and the real wage rate would be invariant with the natural rate of employment. Only in the short run could shortening of hours leave the hourly real wage rate unaffected. In the long run a more rapid shortening of hours would slow down the accumulation of capital stock and the growth of the hourly real wage rate--and even more so of the annual one.

FOOTNOTE

[1]Our long-run natural supply of goods (18) has the elasticities $1/\alpha$ with respect to hours h and unity with respect to number of men λF. Such elasticities find some support in empirical estimates of Cobb-Douglas functions separating hours from men. From British data Feldstein (1967: 385) concluded that "the elasticity of output with respect to hours exceeds that with respect to men and is likely to exceed unity. Craine (1973: 44) found his estimate from U.S. data to paint a picture "remarkably consistent" with Feldstein's.

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