PROPERTY VALUES, LOCAL PUBLIC EXPENDITURE, AND ECONOMIC EFFICIENCY

Jan K. Brueckner

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College of Commerce and Business Administration
University of Illinois at Urbana-Champaign
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JAN K. BRUECKNER
DEPARTMENT OF ECONOMICS
UNIVERSITY OF ILLINOIS
URBANA, IL 61801

Abstract

A bid rent model of property value determination in the presence of a public good and property taxation is developed. Using the local government budget constraint, an estimating equation is derived which can indicate whether public goods are efficiently provided. The empirical results show overprovision of public goods in northeastern New Jersey.
PROPERTY VALUES, LOCAL PUBLIC EXPENDITURE, AND ECONOMIC EFFICIENCY

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Jan K. Brueckner*

Since the appearance of Oates' well-known 1969 paper, this journal has published a number of studies concerned with the Tiebout hypothesis and the effect of local public expenditures and property tax rates on property values (see Pollakowski (1973), Oates (1973), Edel and Sclar (1974), King (1977), and Rosen and Fullerton (1977)). The Tiebout hypothesis (Tiebout (1956)) states that consumers have an incentive to segregate into communities homogeneous by taste and income, where public goods are provided efficiently. Tiebout conjectured that the problem of inefficient provision of a pure public good in an economy with heterogeneous consumers, raised in Samuelson (1954), would be mitigated by this voluntary division of the population into efficient homogeneous groups. In his original paper, Oates claimed that his empirical results, which show a positive influence of local public expenditure on property values, constitute a favorable test of the Tiebout hypothesis, and many of the above investigators view their results similarly. The present paper is based on the belief that none of the previous work in this area constitutes a test of the Tiebout hypothesis, and that a proper test would bear no resemblance to previous studies. In particular, results which show a positive influence of public spending on property values only establish that consumers value the public goods they consume. Such results say nothing whatever about the efficiency of public good provision, which is the ultimate implication of the Tiebout hypothesis. Measuring changes in the degree of community homogeneity

*I am grateful to Wallace Oates for providing me with his data.
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over time would be the proper test of the hypothesis; increasing homogeneity would be evidence of the economy's movement toward an efficient Tiebout equilibrium. No data need be gathered to perform this test; casual observation shows that most communities are heterogeneous and that there exists no long run tendency for consumers to separate into homogeneous groups. These observations are consistent with recent theoretical results which have highlighted many difficulties in the Tiebout model (see, for example, Stiglitz (1974), Wheaton (1975), McQuire (1974), Westhoff (1977)).

This paper will attempt to eliminate the confusion of previous studies by abandoning the Tiebout hypothesis and focusing on the efficiency question in a world which is not in Tiebout equilibrium. A theoretical model of the determination of property values is developed which yields an estimating equation that can indicate whether or not public goods are provided efficiently, even in heterogeneous communities. Regression results based on Oates' original sample are used to evaluate the efficiency of public good provision in northeastern New Jersey.

Edel and Sclar have presented an informal analysis with results somewhat similar to ours, but their approach is marred by imprecision as well as the authors' confused notion that they were testing the Tiebout hypothesis. The following analysis will clearly and rigorously demonstrate for the first time what can be inferred from empirical results relating property values to local fiscal variables.
In this section, we develop a model of house value in the presence of a public good and property taxation based on the bid-rent model of a housing market. For a recent exposition of this model, see Wheaton (1977).

The consumer utility function depends on $x$, $q$, and $z$, the consumption levels of a numéraire non-housing private good, housing services, and a public good, respectively, and each consumer has the same utility function. The consumption level of the public good will be the same for all households in the community which provides the good. However, the good need not be purely public; increasing the community population while holding the public good output fixed may result in a lower per capita consumption level through congestion. We also assume that there are no externalities between communities associated with the public good. A crucial assumption is that all consumers with the same income level reach the same level of utility regardless of where they live. Since our sample for the empirical work is a group of bedroom communities whose residents commute, for the most part, to New York City, this assumption seems natural. If utility levels differed across communities, residences would change until the disparity was eliminated. Formally, we assume $u = h(y)$, where $u$ is the utility level, $y$ is income, and $h' > 0$. The relationship between $u$ and $y$ is not explained within the model; it depends on the general equilibrium solution for the entire economy.

The condition $u(x, q, z) = h(y)$ is equivalent to the condition $x = x(q, z, y)$; fixing $y$ determines the indifference curve on which the consumption bundle lies, and given values of $q$ and $z$ then determine $x$. It
follows that $\partial x/\partial q = -u_2/u_1 < 0$, $\partial x/\partial z = -u_3/u_1 < 0$, and $\partial x/\partial y = h'/u_1 > 0$. The budget constraint for an individual is $R + x + t = y$, where $R$ is rent and $t$ is commuting cost. Combining the budget constraint with the fixed-utility requirement yields $R = y - t - x(q, z, y)$.

The owner of the rental dwelling providing the housing service level $q$ must pay property taxes on the value of the dwelling. The value $v$ of the unit will be the capitalized flow of returns per period to the owner, $r$. Value will be proportional to $r$, and we assume $\omega$, the constant of proportionality, is the same for all units, which implies that the discount rate and the lifespan of units is the same throughout the economy. Thus, since $R$ equals $r$ plus property taxes, we have $R = r + \omega r$, where $\tau$ is the effective property tax rate. Consequently, $r = (y - t - x(q, z, y))/(1 + \omega \tau)$ and value is given by

$$
    v = \frac{y - t - x(q, z, y)}{\theta + \tau} \equiv f(q, z, \tau, y, t),
$$

where $\theta = 1/\omega$. Also, owner-occupiers must be indifferent between owning a unit and renting an identical unit. That is, the capitalized value of rent payments, $\omega R$, must equal the present value of the cost of owning a unit, $v + \omega tv$, again yielding (1). We have

$$
    f_1 = \frac{u_2}{(\theta + \tau)u_1} > 0 
$$

$$
    f_2 = \frac{u_3}{(\theta + \tau)u_1} > 0 
$$

$$
    f_3 = -\frac{v}{\theta + \tau} < 0 
$$
The sign of $f_1$ is positive because when the level of the housing services from a unit increases, other things being equal, the amount of $x$ the resident consumes must decrease to keep him on the fixed indifference curve. The rent (and the value) of the housing unit must increase to yield the required reduction in $x$. The same argument explains the sign of $f_2$. When the property tax rate increases, less of the fixed rent $R$ of the unit is available as a return to the owner, and value falls. When commuting cost rises, less income is left for expenditure on $x$ and $q$, and $R$ (and $v$) must fall to keep the consumer on the given indifference curve. The ambiguous sign of $f_4$ is due to two opposing effects. Holding utility fixed, an increase in $y$ must be accompanied by an increase in $R$ (and $v$) to keep the consumer's consumption of $x$ constant. However, utility increases with $y$, and this means that for fixed $q$ and $z$, more $x$ may be consumed. Hence, $R$ need not increase and may decrease with an increase in $y$.

The bid-rent model is different from standard consumer theory, where consumers choose a consumption bundle to maximize utility subject to a budget constraint. In standard theory, prices are exogenous and the utility level endogenous, but in the bid-rent model, the utility level is exogenous and prices (values) are endogenous. The nature of the housing stock is the reason for this shift in emphasis. Housing is a durable commodity, and a spectrum of different levels of housing services is available in the market, due to, among other things, the different ages of units. If the price per unit of
housing services were the same for all housing units, those units which did not provide some consumer's utility-maximizing level of services would not be inhabited. All units in the housing stock must be inhabited by someone, and this is assured by rents adjusting so that the utility level of consumers is the same in all units.

The bid-rent model leads to unconventional results on property tax capitalization. Holding the unit size \( q \) fixed, an increase in the tax rate \( \tau \) is fully capitalized in property value, a result which is analogous to the conventional short-run capitalization effect. Since we will be concerned with cross-section estimation, an alternative statement of this result is that the difference in value of identical units in communities with different property tax rates will reflect full capitalization of the property tax difference. Conventional long-run analysis (see Mieszkowski (1972)) predicts above-average housing prices in communities with above-average property tax rates, other things being equal. This outcome is inconsistent with our model because it violates the equal-utility requirement; consumers must pay the same rent for a unit of a given size regardless of the amount of property taxes levied on the unit. While conventional analysis assumes that capital is mobile and that consumers do not move to eliminate rent differentials, consumer mobility is fundamental to our analysis.

The durability of the housing stock means that changes in the stock in a community arise only through the decisions of producers supplying new housing units (we implicitly ignore the effect of maintenance expenditures in changing service levels from existing units). In the long run,
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changes in fiscal variables will effect the supply decisions of producers; the housing stocks in communities with different values for fiscal variables may have different age distributions and average sizes of units. However, even though the stocks may differ, the model says that values of identical units in communities with different property tax rates will, ceteris paribus, reflect full capitalization of the tax rate differences.

This paper is not concerned with modeling the supply decisions of producers of durable housing units; an attractive model of this complex problem may be found in Muth(1973). In equilibrium, however, the profits of housing producers must be the same in all communities where new construction occurs. Presumably, in a complete model, this condition would imply a variation in land prices which would yield constant profits across communities.

Equation (1) is not a complete model of property value determination because it ignores the budget constraint of the local government. In our sample, nearly all local revenue comes from the taxation of business and residential property. Consider a community where the n residents all have the same income y (an assumption which is relaxed later), where the n housing units provide service levels q_i, i=1, 2, ..., n, and have associated commuting costs t_i, i=1, 2, ..., n, and where business and commercial property is valued at B. Let the cost of providing the public good be C(z, n). Then the budget constraint of the local government is

$$\tau \sum f(q_i, z, \tau, y, t_i) + \tau B = C(z, n),$$  \hspace{1cm} (7)$$

where the presence of n in the cost function for the public good reflects congestion effects. If C_2 \equiv 0, z is a pure public good, while C_2 > 0 implies the presence of congestion.
The value of business property may be determined according to a bid-rent model. Suppose \( \pi_j \) is the before-rent profit level of the \( j^{th} \) business. Rent payments must reduce profits to zero: \( R_j = \pi_j \). Since the value of business property is related to rent in the same way as is the value of residential property, the value of the property housing the \( j^{th} \) business is \( \pi_j/(\theta + \tau) \). Aggregate business property value \( B \) equals \( \Sigma \pi_j/(\theta + \tau) \equiv \pi/(\theta + \tau) \), where \( \pi \) is aggregate profit before rent. After substituting for \( B \), (7) yields an implicit relationship between the property tax rate and the other variables:

\[
\tau = \tau(Q, z, y, T, n, \pi),
\]

where \( Q \) and \( T \) are the vectors of \( q_i \)'s and \( t_i \)'s respectively. Equation (8) gives the tax rate required to support a public good level \( z \) for given values of \( Q, y, T, n, \) and \( \pi \). The derivative of the LHS of (7) with respect to \( \tau \) is \( (B + \Sigma v_i) - \tau (B + \Sigma v_i)/(\theta + \tau) \) using (4) and noting the dependence of \( B \) on \( \tau \). Letting \( P = \Sigma v_i \), this reduces to \( \theta(P + B)/(\theta + \tau) \). This expression is positive, which means that property tax revenue is increasing in \( \tau \).

Totally differentiating (7) we have

\[
\frac{\partial \tau}{\partial q_i} = \frac{-\tau \theta + \tau f(q_i, t_i)}{\theta(P + B)} < 0
\]

(9)

The arguments of \( f \) which do not depend on \( i \) have been suppressed in (9).

Similarly

\[
\frac{\partial \tau}{\partial t_i} = \frac{-\tau \theta + \tau f(q_i, t_i)}{\theta(P + B)} = \frac{\tau}{\theta(P + B)} > 0.
\]

(10)

Also

\[
\frac{\partial \tau}{\partial z} = \frac{(\theta + \tau)(C_1 - \tau E f(q_i, t_i))}{\theta(P + B)} \geq 0,
\]

(11)
where the ambiguity is due to $C_1 > 0$ and $f_2 > 0$. In addition
\[ \frac{\partial \tau}{\partial y} = \frac{-\tau (\theta + \tau) \sum f_4(q_j, \tau)}{\theta (P + B)} > 0 \]  \hspace{1cm} (12)

\[ \frac{\partial \tau}{\partial \pi} = \frac{-\tau}{\theta (P + B)} < 0 . \]  \hspace{1cm} (13)

Changing $n$ requires adding a unit to the housing stock. Suppose a unit with value $v$ is added. If $n$ is large, we can approximate the change in $\tau$ resulting from a unit increase in $n$ by
\[ d\tau = \frac{(\theta + \tau)(C_2 - \tau v)}{\theta (P + B)} , \]  \hspace{1cm} (14)

which is ambiguous in sign when $C_2 > 0$ but is negative when $C_2 = 0$.

The intuition behind these results is straightforward. An increase in $q_j$ increases the value of the housing stock for fixed values of the values of the other variables, raising revenue above expenditures. Since property tax revenue is increasing in $\tau$, decreasing $\tau$ reduces revenue and again balances the budget. Similarly, increasing the commuting time from the $j$th unit (moving the unit farther away from the employment center) reduces aggregate property value and requires an increase in $\tau$ to offset the decline in revenue. Since the effect of changes in $y$ on property value is ambiguous, $\partial \tau / \partial y$ is also ambiguous in sign. An increase in $\pi$ increases revenue and requires a decrease in $\tau$ to maintain budget balance. An increase in $z$ has two effects: first, it increases expenditures, and second, it increases property value. After the change, it is unclear whether revenues are greater than or less than expenditures, and hence the required direction of the property tax change that
balances the budget is uncertain. Similarly, when \( n \) increases by one and
a unit is added to the housing stock, the increase in revenue may or may
not exceed the increase in cost when \( C_2 > 0 \), and the required change
in \( \tau \) is uncertain. When \( C_2 = 0 \), no increase in cost results from
increasing \( n \), and since revenues rise, \( \tau \) must fall to maintain budget balance.

Substituting (8) in (7), we have

\[
v_i = f(q_i, z, \tau(Q, z, y, T, n, \pi), y, t_i)
\]  (15)

Using (15), we can deduce how a change in any variable effects the value of
the \( i \)th unit when the property tax rate adjusts to maintain budget balance
for the local government. Since \( f_3 < 0 \), we have, using (9), (10), and (13),

\[
\frac{\partial v_i}{\partial \tau} > 0, \quad \frac{\partial v_i}{\partial q_j} > 0, \quad \frac{\partial v_i}{\partial t_j} < 0, \quad i \neq j.
\]  (16)

The change in \( v_i \) from increasing \( n \) is, from (14), ambiguous when \( C_2 > 0 \) and
positive when \( C_2 = 0 \). In addition

\[
\frac{\partial v_i}{\partial q_i} = f_1(q_i, t_i) + f_3(q_i, t_i) \frac{\partial \tau}{\partial q_i}
\]

\[
= \left( 1 + \frac{\tau v_i}{\theta(P + B)} \right) f_1(q_i, t_i) > 0.
\]  (17)

Similar calculations yield

\[
\frac{\partial v_i}{\partial t_i} = - \frac{1}{\theta + \tau} \left( 1 + \frac{\tau v_i}{P + B} \right) < 0
\]  (18)
\[
\frac{\partial v_i}{\partial y} = f_4(q_i, t_i) + \frac{\tau v_i}{\theta (P + B)} \sum f_4(q_j, t_j) \geq 0 \tag{19}
\]

\[
\frac{\partial v_i}{\partial z} = f_2(q_i, t_i) + \frac{v_i}{\theta (P + B)} \left( \tau \sum f_2(q_j, t_j) - C_1 \right) \geq 0. \tag{20}
\]

The intuition behind (16) is that any change which effects the property tax rate alone changes value in the opposite direction. From (17), increasing \( q_i \) has two effects: it directly increases \( v_i \) and it indirectly increases \( v_i \) by allowing a reduction in the property tax rate. An equivalent argument holds for (18) and increases in \( t_i \). Since the direct and indirect effects of changes in \( y \) are ambiguous, the sign of (19) is ambiguous. Although the direct effect on property value of an increase in \( z \) is positive, the tax rate may either rise or fall with an increase in \( z \), and hence the total effect and the sign of (20) are uncertain.

Previous investigators have estimated an equation resembling (15) with the property tax rate included among the explanatory variables. This renders the estimated equation useless as a predictive tool because it does not incorporate the government's budget constraint. For instance, it makes no sense to vary \( \tau \) and \( z \) independently, as the estimated equation allows, and deduce changes in property values, because the government budget constraint may no longer be satisfied after such variation. However, if (15) is estimated in cross-section, then the only fiscal variable included in the equation is \( z \), and variations in \( z \) holding the other variables fixed are implicitly accompanied by variations in \( \tau \) which maintain budget balance for the government.
In the next section we show that the coefficient on \( z \) has a special significance because its magnitude provides information about the efficiency of public good provision.

II.

We noted above that the \( q_i \) may be effected in the long run by the fiscal variables in the community. We could also argue that business profits before rent might respond in the long-run to changes in the fiscal variables. Our efficiency notion will ignore both these possible long-run effects: the efficient values of the fiscal variables will be those which maximize aggregate property value holding \( \pi \) and the \( q_i \) fixed. Aggregate property value equals \( Ev_1 + \pi/(\theta + \tau) \), and maximizing this expression subject to the government budget constraint (7) requires

\[
\Sigma (f_2(q_j, t_j) + f_3(q_j, t_j) \partial \tau/\partial z) - \frac{B}{\theta + \tau} \partial \tau/\partial z = 0, \tag{21}
\]

which reduces to

\[
\Sigma \frac{u_3(q_j)}{u_1(q_j)} = C_1, \tag{22}
\]

using (3), (4), and (11). Equation (22) is the well-known Samuelson condition which states that the sum of the marginal rates of substitution between the public good and the numeraire equals the marginal cost of the public good. This condition also emerges if we require that the utility level \( u = h(y) \) be the maximum level attainable for any values of the fiscal variables which satisfy the government budget constraint, given suitable assumptions. Under these assumptions, if (22) fails to hold, utility can be increased above \( h(y) \) by changing the values of \( z \) and \( \tau \).
We now investigate the properties of $\partial v_i/\partial z$ from (20) when the public good is provided efficiently. First, we perform the analysis for the case $B = 0$, which corresponds to a completely residential community, treating the $B > 0$ case later. Our first result is that if housing units are identical, or $q_1 = q$ and $t_i = t$ for all $i$, then $\partial v_i/\partial z = 0$ for all $i$ when (22) holds. This follows because when units are identical and $B = 0$, (20) becomes, using (3),

$$f_2(q,t) + \frac{\tau}{\theta}f_2(q,t) - \frac{C_1}{\theta n} = \frac{1}{\theta n} \left\{ n \frac{u_3}{u_1} - C_1 \right\},$$

which is zero when (22) holds with $q_1 = q$. Therefore, when units are identical and the public good is provided efficiently, $\partial v_i/\partial z = 0$ for all $i$; an increase in $z$ accompanied by a change in $\tau$ which maintains budget balance has no effect on property values. This follows intuitively from the efficiency condition; if aggregate property value (and hence individual unit values in the identical dwelling case) have been maximized by choice of $z$ and $\tau$, a change in $z$ and a corresponding change in $\tau$ will have no effect on unit values since they are already stationary.

When housing units are different, (20) can be written

$$\frac{1}{\theta + \tau} \frac{u_3(q_1)}{u_1(q_1)} + \frac{v_i}{\theta P} \left\{ \frac{\tau}{\tau + \theta} \sum \frac{u_3(q_j)}{u_1(q_j)} - C_1 \right\},$$

which equals

$$\frac{1}{\theta + \tau} \left\{ \frac{u_3(q_1)}{u_1(q_1)} - \frac{v_i}{P} \sum \frac{u_3(q_j)}{u_1(q_j)} \right\},$$

when (22) holds. If $v_i$ is close to $P/n$ we might expect $u_3(q_1)/u_1(q_1)$ to be approximately $(\sum u_3/u_1)/n$, making (25) equal to zero. To make this notion
more precise, suppose the utility function is Cobb-Douglas: \( u = x^\alpha q^\beta z^{\gamma} \).

Then \( \frac{u_2}{u_1} = \frac{\gamma x_i}{az} \) and (25) can be written

\[
\frac{1}{\theta + \tau} \left[ \frac{\gamma x_i}{az} - \frac{y - t_i - x_i}{n(y - t - x)} \sum \frac{\gamma x_i}{az} \right] = \frac{[\gamma y(x_i - \bar{x}) + \gamma(t_i \bar{x} - t\bar{x}_i)]}{az(\theta + \tau)(y - t - \bar{x})},
\]

where \( \bar{t} \) and \( \bar{x} \) are mean values. Suppose all the units in a community are roughly equidistant from the employment center, as would be the case in a suburban residential community. Then we may set \( t_i = t \) for all \( i \), which gives (26) the same sign as

\[
\gamma(y - t)(x_i - \bar{x}) .
\]

For the unit whose associated \( x \) level equals \( \bar{x} \), (27) is zero and hence \( \partial v/\partial z = 0 \); the unit's value is unchanged when \( z \) increases. Those units with high associated \( x \) levels have low \( q \) levels from the uniform utility requirement and therefore have low values, because value is increasing in \( q \) from (2). Similarly, units with low associated \( x \) levels have high values. Therefore, (27) says that \( \partial v/\partial z \) is positive for low-valued units and negative for high-valued units.

The median-valued unit will be associated with the median \( x \) level, \( \bar{x} \). The relationship of \( \bar{x} \) to \( \bar{x} \) depends both on the distribution of the \( q_i \) and on the shape of the indifference curve, and nothing can be said in general about it. However, \( \bar{x} \) will be close to \( \bar{x} \), and from (27), the change in the value of the median-valued unit when \( z \) increases and the public good is efficiently provided will be approximately zero. A similar argument can be made for a general utility function.\(^2\)

Results similar to some of our conclusions were noted by Hamilton (1976), whose model is much less general than the one analyzed in this paper.
intuition behind our results is that while efficiency leads to the maximization of aggregate property value, it does not maximize the value of each unit. Those units which contribute relatively small amounts of property taxes to government revenue enjoy a positive fiscal surplus and their values increase when \( z \) increases. Those units with a negative fiscal surplus, large units, suffer a decline in value when \( z \) expands.

What does inefficient provision of the public good imply for the variation of property values as \( z \) changes? An assumption that appears to be empirically supported is that the public good is characterized by constant returns to scale: \( C_{11} = 0 \).\(^3\) In the Cobb-Douglas example, when \( u \) and \( q_1 \) are fixed, \( \gamma x_1 / \partial z \) is decreasing in \( z \). Together, these properties imply that if \( z \) is above the efficient level, \( \Sigma \gamma x_1 / \partial z - C_1 < 0 \), and if \( z \) is below the efficient level, \( \Sigma \gamma x_1 / \partial z - C_1 > 0 \). It is easily seen that this implies that (24) is greater than (less than) (26) when \( z \) is less than (greater than) the efficient level. When \( t = t \), this means that

\[
\frac{3v_1}{3z} > a(x_1 - \bar{x}) \quad \text{as} \quad z \leq z^*,
\]

where \( z^* \) is the efficient level of \( z \) and \( a = \gamma(y - t) / \partial z(\theta + \tau)(y - t - \bar{x}) \).

For the housing unit whose associated \( x \) value equals \( \bar{x} \), the RHS of (28) is zero, and \( 3v/3z < 0 \) as \( z \leq z^* \). Since \( \bar{x} \) is close to \( \bar{x} \), we can make the following claim: approximately, the value of the median-valued unit is increasing (decreasing) in \( z \) when \( z \) is less than (greater than) \( z^* \). A similar result holds for a general utility function.\(^4\)

When \( B > 0 \), the results are slightly different. When the efficiency condition (22) holds, \( 3v_1/3z \) becomes
\[
\frac{1}{\theta + \tau} \left\{ \frac{u_3(q_i)}{u_1(q_i)} - \frac{v_1}{P + B} \sum \frac{u_3(q_j)}{u_1(q_j)} \right\}.
\]  

(29)

In the Cobb-Douglas case when \( t_i = t \) for all \( i \), this reduces to

\[
\frac{\gamma(y - t)(x_i - \overline{x}) + \gamma x_i(\theta + \tau) B/n}{(\theta + \tau) az (y - t - \overline{x} + (\theta + \tau) B/n)} \equiv \Omega_i,
\]  

(30)

which says that \( \partial v / \partial z > 0 \) for the unit whose associated \( x \) value is \( \overline{x} \). As in (28), we have \( \partial v_i / \partial z \geq \Omega_i \) as \( z \leq z^* \). Since \( \Omega_i > 0 \) when \( x_i = \overline{x} \), this says that \( \partial v / \partial z > 0 \) for the unit with \( x_i = \overline{x} \) when \( z < z^* \), but that \( \partial v / \partial z \) may be either positive or negative for this unit when \( z > z^* \). These statements hold approximately for the median-valued unit: when \( B > 0 \), the value of the median-valued unit is increasing in \( z \) when (22) holds; when \( z < z^* \), the value of the median-valued unit is increasing in \( z \), but its value may either increase or decrease with \( z \) when \( z > z^* \).

When more than two public goods are provided, analysis similar to that presented above may be performed. With efficiency, \( \partial v / \partial z_i \) is approximately zero for the median-valued unit for all public goods \( z_i \) when \( B = 0 \), and is positive for that unit for all \( z_i \) when \( B > 0 \). To characterize \( \partial v / \partial z_i \) when the public goods are not provided efficiently, we require separable utility and cost functions so that marginal rate of substitution between each public good and the numeraire does not depend on the levels of other public goods and so that the marginal cost of each public good is independent of the levels of the other public goods. The Cobb-Douglas utility function satisfies the separability requirement. With separability, the following statements are approximately
true for all public goods \( z_i \): \( \partial v / \partial z_i < 0 \) for the median-valued unit as \( z_i < z^*_i \) when \( B = 0 \); if \( B > 0 \), \( \partial v / \partial z_i \) is positive for the median-valued unit when \( z_i > z^*_i \) but may be of either sign when \( z_i > z^*_i \).

In reality, income is not uniform in communities, and our model must be modified to take account of this fact. If \( y_i \) is the income of the occupant of the \( i \text{th} \) unit, then \( v_i = f(q_i, z, \tau, y_i, t_i) \). The equation corresponding to (8) derived from the government budget constraint is \( \tau = \tau(Q, z, Y, T, n, \pi) \), where \( Y \) is the vector of the \( y_i \)'s. As before, the signs of \( \partial v_i / \partial y_i \) and \( \partial v_i / \partial y_j \), \( j \neq i \), are ambiguous. Also, the partial derivatives of \( v_i \) with respect \( q_i, t_i, q_j, t_j, j \neq i, n, \) and \( \pi \) all have the same signs as in the uniform-income case.

Efficiency still requires that equation (22) be satisfied, and \( \partial v_i / \partial z \) still is given by (29) when \( B > 0 \). Using the Cobb-Douglas example, we can see how the above analysis changes when income is not uniform. Assuming that the public good is efficiently provided and that \( t_i = t \) for all \( i \), \( \partial v_i / \partial z \) becomes

\[
\gamma \left[ (x_i y_i - x_i) - t(x_i - x) + x_i (\theta + \tau) B/n \right] / az(\theta + \tau) (y - t - x) + (\theta + \tau) B/n
\]

We are interested in the sign of (31) for the median-valued unit, but when incomes vary, the median-valued unit and the unit which provides the median level of housing services are no longer identical because unit value depends both on housing services provided and on the income of the occupant. However, given two reasonable assumptions which seem to reflect reality, the sign of (31) will be close to zero when \( B = 0 \) and positive when \( B > 0 \) for the median-valued unit. The assumptions are that as a consumer's income rises,
his x consumption increases and the value of the dwelling he chooses increases as well. The latter requirement is not the same as assuming $f_q > 0$ in (5); that derivative requires a fixed dwelling size when income increases, while our assumption permits changes in $q$ when income rises. These assumptions are quite innocuous and seem to be borne out in the real world. Since unit value increases with the income of the occupant, the median-valued unit is occupied by the individual with the median income, and this individual also consumes the median amount of $x$ in the community. Thus for the $i$ corresponding to the median-valued dwelling, the numerator of (31) is 

$$\gamma [(\tilde{x}y - \tilde{y}x) - t(\tilde{x} - \bar{x}) + \tilde{x}(\theta + \tau) B/n],$$

where the tildes refer to median values. The first part of this expression will be near zero since the means will be close to the medians. Hence, given our assumptions, $\partial v/\partial z$ will be approximately zero for the median-valued dwelling when the public good is provided efficiently and $B = 0$.

As before, when $B = 0$, $\partial v/\partial z$ for the median-valued dwelling will be (approximately) greater than (less than) zero when $z$ is less than (greater than) the efficient level. The results for the $B > 0$ case are also the same as before.

We have shown that under our assumptions, the modified model has the same qualitative properties as the constant-income model.

One further assumption, however, is required to make the modified model operational empirically. The difficulty is that the analysis yields no prediction about the $q$ level of the median-valued unit. The assumption we require is that as income increases, people live in units with higher $q$ levels. Then the median-valued unit will be the unit providing the median level of services. This last assumption, like the others, appears to hold in reality.
III.

In this section we present linear cross-section regression equations for the year 1960 based on the relation \( v_i = f(q_i, z, \tau(Q, z, Y, T, n, \pi), y_i, t_i) \), which can be written

\[
v_i = h(q_i, y_i, z, t, n, \pi, Q^i, Y^i), \tag{32}
\]

assuming \( t_i = t \) for all \( i \) and defining \( Q^i \) and \( Y^i \) to be the vectors of the \( q_j \)'s and \( y_j \)'s with \( q_i \) and \( y_i \) deleted.

We are principally interested in the sign of the coefficient on \( z \) when \( i \) corresponds to the median-valued unit. It should be noted that the linear form of the regression equation implies that the coefficient on \( z \) is the same in all communities, which means that the public good is overprovided, underprovided, or efficiently provided in all communities. This implicit assumption is restrictive, but it simplifies the estimation problem and may be justified by the notion that a bias for over- or underprovision of public goods may be present in all communities.

The theory allows us to interpret the sign of the coefficient on \( z \) in (32) when \( q_i = \hat{q}, y_i = \hat{y}, \) and \( v_i = \hat{v}, \) the median property value. However, the only available value data give the median value of owner-occupied units for the communities in the sample. The median-valued owner-occupied unit will not be the same as the median-valued unit among all units in the community because owner-occupied units tend to be larger than renter-occupied units. Thus, no unit value data is available which corresponds exactly to the requirements of the theory. This problem was circumvented in the following way.
First, we estimated (32) with the available data on the median value of owner-occupied units, using the median number of rooms of owner-occupied units as a measure of the q value of the median-valued owner-occupied unit. Then, using data on the median number of rooms in all units, both renter-and owner-occupied, we computed the median value for all units using the expression \[ \hat{V}^A = \hat{V}^0 + b(\hat{q}^A - \hat{q}^0), \] where \( \hat{V}^A \) and \( \hat{V}^0 \) are the median values for all units and owner-occupied units respectively, \( \hat{q}^A \) and \( \hat{q}^0 \) are the median number of rooms in all units and owner-occupied units respectively, and \( b \) is the estimated coefficient relating \( \hat{V}^0 \) to \( \hat{q}^0 \). Since the estimated coefficient \( b \) is positive and \( \hat{q}^A < \hat{q}^0 \), \( \hat{V}^A < \hat{V}^0 \). With \( \hat{V}^A \) as the dependent variable, we then estimated (32) with \( q_1 = \hat{q}^A \) and \( y_1 = \hat{y} \). It is the estimated coefficients from this equation that bear on the efficiency question. It should be noted that in the regression relating \( \hat{V}^0 \) to \( \hat{q}^0 \), \( \hat{y} \) was also used as the measure of the income of the occupant of the median-valued owner-occupied unit, a procedure which is not strictly correct given the discussion at the end of section II.

In the regressions, the percent of structures in the community built before 1950 was used as a proxy for \( Q^1 \), and the percent of families with incomes below \$3000 \) was used as a proxy for \( Y^1 \). These variables were labeled OLD and POOR, while median income was denoted YM. Population was 1960 census population, N. As a measure of \( t \), we followed Oates and used linear distance in miles to downtown Manhattan, MI. The median number of rooms in all units and owner-occupied units were denoted RMSA and RMSO respectively. As a proxy for \( \pi \), business profits before rent, we used total 1963 sales of retail, wholesale, and service establishments, denoted SLS.

The level of expenditures on public goods was employed as a proxy for public good output. Letting \( C \) denote expenditures, the expression \( C = C(z, n) \)
may be inverted to yield \( z = z(C, n) \), with \( \frac{\partial z}{\partial C} = \frac{1}{C_1} \) and \( \frac{\partial z}{\partial n} = -\frac{C_2}{C_1} \). Substituting \( z(C, n) \) in (32), we have \( \frac{\partial y}{\partial C} = (\frac{\partial h}{\partial z})\frac{C_2}{C_1} + \frac{\partial h}{\partial n} \). Since \( C_1 > 0 \), the sign of the coefficient relating value to \( C \) has exactly the same efficiency interpretation as the sign of the coefficient relating value to \( z \). Since \( n \) appears twice in (32) when \( z(C, n) \) is substituted for \( z \), \( \frac{\partial y}{\partial n} = -\frac{\partial h}{\partial z}(\frac{C_2}{C_1}) + \frac{\partial h}{\partial n} \). When \( C_2 = 0 \), \( \frac{\partial y}{\partial n} = \frac{\partial h}{\partial n} \), which is positive from above when \( C_2 = 0 \). When \( C_2 > 0 \), the sign of \( \frac{\partial h}{\partial n} \) is ambiguous, and since \( \frac{\partial h}{\partial z} \) may have either sign, \( \frac{\partial y}{\partial n} \) is of indeterminate sign. It should be noted that a zero regression coefficient on \( n \) need not imply \( C_2 = 0 \). When two public goods have the separable cost function \( C^1(z_1, n) + C^2(z_2, n) \), we can replace \( z_1 \) by \( C^1 \) and \( z_2 \) by \( C^2 \) in an equation such as (32), and the coefficients on \( C^1 \) and \( C^2 \) will have the same efficiency interpretations as those on \( z_1 \) and \( z_2 \).

We computed two sets of regressions reflecting different assumptions about the number of public goods provided in the communities. In the first set, the communities were viewed as producing a composite public good, and total community expenditures, \( TEXP \), was the expenditure variable. In the second set of regressions, we assumed communities produce two public goods, education and other municipal services, and the expenditure variables were \( EEXP \), educational expenditure, and \( MEXP \), other municipal expenditure.

The estimating equations using the owner-occupied value data were

\[
\begin{align*}
\hat{v}_0 &= a_0 + a_1 \text{RMSO} + a_2 \text{YM} + a_3 \text{TEXP} + a_4 \text{MI} + \\
&+ a_5 \text{N} + a_6 \text{SLS} + a_7 \text{OLD} + a_8 \text{POOR},
\end{align*}
\]

and another linear equation with \( EEXP \) and \( MEXP \) in place of \( TEXP \). Results from log-linear equations were inferior to the linear results, and are not
reported. The discussion in sections I and II suggested that the housing stock and business profits may be endogenous variables which are affected by the fiscal variables as well as by other variables. In addition, public good expenditures are themselves endogenous, representing the outcome of a political process. Although it could argued that the income distribution and the population of the community are also endogenous variables, we have decided to treat them as exogenous. Given these considerations, we computed two-stage least squares regressions treating RMSO, SLS, OLD, and the expenditure variables as endogenous, and also computed ordinary least squares estimates. In addition to YM, MI, N, and POOR, other exogenous variables were EDUC, median number of years of school completed by males over twenty-five years of age, a dummy variable which assumed the value of unity for communities in Hudson County and zero otherwise, PRPUB, the percent of the population enrolled in public schools, DEN, population density, WC, percent of employed persons in white collar professions, PROWN, percent of housing units owner-occupied, DELTN, percent change in population between 1950 and 1960, and MIGR, the percentage of residents over five years of age who were living in a different community in 1955.

The OLS and TSLS estimates with \( \hat{v}_0 \) as the dependent variable are presented in Table I. Recall that the theory is ambiguous about the signs of the coefficients of YM, POOR, N, and the expenditure variables. It predicts positive coefficients for RMSO and SLS and negative coefficients for MI and OLD. The model is not contradicted by the OLS results; RMSO, SLS, MI, and OLD have significant coefficients with the correct signs, while the coefficient of N is insignificant and YM and POOR have significantly positive coefficients. POOR's positive coefficient is somewhat mystifying; it says that holding YM
and the other variables fixed, a community with a large poor population has a higher median value for owner-occupied units than one with a small poor population. The estimated coefficients for the expenditure variables are not significantly different from zero. Inspection of (30), however, shows that when \( x_i < \bar{x} \), an inequality which is probably satisfied for the median-valued owner-occupied unit (which has \( q > \bar{q} \) and \( x < \bar{x} \)), the sign of \( \partial v/\partial z_i \) may be either positive or negative when public goods are provided efficiently. Thus, the signs of the estimated coefficients have no implications for efficiency.

The TSLS estimates are similar to the OLS estimates, the principal differences being the insignificance of the POOR coefficient in the EEXP-MEXP equation and the significant negativity of the TEXP and EEXP coefficients in their respective equations. As before, no efficiency statement can be made on the basis of these results.

Table II presents TSLS estimates based on the computed \( v^A \) variable, which permit us to discuss efficiency. The estimated coefficients of RMSO from the two TSLS equations with \( v^O \) as the dependent variable were used as described above to derive two vectors of \( v^A \) values, one corresponding to each equation. Then the TSLS equations were reestimated using the appropriate \( v^A \) vector and RMSA in place of RMSO. The mean of the \( v^O \) variable in the sample is $19,154 while the means of the computed \( v^A \) variables from the TEXP and EEXP-MEXP equations are $17,028 and $16,845 respectively.

Aside from the expenditure variables, the principal changes in the estimates are the insignificance of the SLS coefficients and the insignificance of the POOR coefficient in the TEXP equation. The former is somewhat worrying, although comfort may be taken from the fact that the t-ratios for the (positive)
SLS estimates are relatively large. All the expenditure coefficient estimates in the equations are significantly negative, and efficiency requires that they be positive. Recall from section II that where \( B > 0, \partial v/\partial z > 0 \) when \( z < z^* \) and that \( \partial v/\partial z \) may be either positive or negative when \( z > z^* \). Since our estimated coefficients are significantly negative, the theory implies that public goods are overprovided in our sample, regardless of whether we view the public good output as one or two goods.

IV.

Our efficiency result is somewhat surprising; economists have long been concerned with possible underprovision of public goods as a result of free-rider behavior. The result may, of course, be peculiar to our sample, which contains relatively affluent bedroom communities with high percentages of white-collar workers. Perhaps the altruism of well-educated people such as those in our sample imparts an over-production bias to the public sector in the communities where they reside. This conjecture could be tested by repeating the empirical work using a sample of less affluent communities.

It is important to know how much credence may be placed in our conclusion that public goods are overprovided. Recall that the analysis relied heavily on approximate statements which arose because of uncertainty about the relative magnitudes of \( \bar{x} \) and \( \tilde{x} \) and \( \bar{y} \) and \( \tilde{y} \). Non-zero coefficients on the expenditure variables might not be inconsistent with efficiency if there is a great difference between \( \tilde{x} \) and \( \bar{x} \) or between \( \tilde{y} \) and \( \bar{y} \). It seems unlikely, however, that statistically significant negative coefficients could be generated by the divergence between medians and means when the public goods are provided efficiently. Another difficulty with the argument is the rough fashion in which the \( \tilde{v}^A \) variable
was constructed. If we have badly misestimated the median value, then our interpretation of the results will not be correct. However, improving on our technique for estimating  \( \gamma_A \) does not seem possible. Another problem is the lack of data on house quality, a determinant of \( q \). Almost all units in the sample qualify as sound housing, so data on percent of units dilapidated or with substandard plumbing are not useful for measuring quality, as they might be in poorer communities. It is possible that the strong positive significance of median income in the regressions captures the effect of income on variables such as the size of yards and the quality of construction which are not represented by the median number of rooms variable. It is uncertain, however, whether the mis-specification introduced by the omission of quality variables seriously biases the results.

In conclusion, it must be stressed that the importance of the empirical results in this paper is due more to their very existence than to the particular implication that public goods are overprovided in northeastern New Jersey. We have constructed a model which permits evaluation of the efficiency of the provision of public goods using readily available data. While refinement of empirical technique and better data from other samples might lead eventually to more reliable verdicts on efficiency, this paper has provided the first complete framework for investigating this important issue.
<table>
<thead>
<tr>
<th></th>
<th>CONSTANT</th>
<th>RMSO</th>
<th>YM</th>
<th>TEXP</th>
<th>EEXP</th>
<th>MEXP</th>
<th>MI</th>
<th>N</th>
<th>SLS</th>
<th>OLD</th>
<th>POOR</th>
<th>R²</th>
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<td>.1674a</td>
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<td>------</td>
<td>-.1078a</td>
<td>-.0330</td>
<td>.0080b</td>
<td>-.0585a</td>
<td>.2610a</td>
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<td>(.095)</td>
<td>(2.735)</td>
<td>(7.868)</td>
<td>(-.155)</td>
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<td>------</td>
<td>-.1445a</td>
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<td>-.0585a</td>
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<td>(7.712)</td>
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<td>(.486)</td>
<td>(-3.920)</td>
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<td>(1.135)</td>
<td>(1.922)</td>
<td>( -1.951)</td>
<td>(.625)</td>
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(t-ratios are in parentheses below estimates.)

a - significant at one-tailed 5% level

b - significant at one-tailed 10% level
## TABLE II - DEPENDENT VARIABLE $\gamma^A$, TSLS

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<thead>
<tr>
<th>CONSTANT</th>
<th>RMSA</th>
<th>YM</th>
<th>TEXP</th>
<th>EEXP</th>
<th>MEXP</th>
<th>MI</th>
<th>N</th>
<th>SLS</th>
<th>OLD</th>
<th>POOR</th>
<th>$R^2$</th>
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<td>.2167$^a$</td>
<td>-.1858$^a$</td>
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<td></td>
<td></td>
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<td>(1.199)</td>
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</tbody>
</table>

| -6.3405 | .1901$^b$ | .2170$^a$ | ----- | -.2675$^b$ | -.1383$^b$ | -.0832$^a$ | .1703$^b$ | .0307 | -.0544$^a$ | .0898 | (.8896) |
| (-1.209) | (1.462) | (5.686) | (-1.391) | (-1.364) | (-1.780) | (1.313) | (1.137) | (-1.992) | (.587) |       |

(t-ratios are in parenthesis below estimates)

a - significant at one-tailed 5% level

b - significant at one-tailed 10% level
Data Sources

The sources for the variables $\psi$, RMSO, YM, MI, N, OLD, POOR, EDUC, PRPUB, DEN, PROWN, and DELTN are listed in Oates (1969). RMSA was taken from the 1960 Census of Housing; SLS was taken from the 1963 Census of Business; WC and MIGR were taken from the Municipal Yearbook, 1963 (see Oates); and TEXP, EEXP, and MEXP were taken from the Twenty-Third Annual Report of the Division of Local Government, State of New Jersey, 1960 (see Oates).
Footnotes

1 So far we have ignored the possibility that rental income may be a component of income for some consumers. The model becomes intractable when this feature is added; it turns out that the values of units must be simultaneously determined when rental income is included. Since only a small fraction of the population receives rental income, its omission from the model is defensible. However, we will demonstrate that (22) holds at a utility maximum when rental income is added to the model in a certain way, but we must first introduce some convenient simplifications to avoid dealing with issues related to the production of structures. In particular, imagine that "housing" and business "property" are non-produced goods similar to free land, being owned by individuals with no initial outlay required for acquisition. Suppose that the community has decided to divide total rental income equally among the residents. Net rental income from the \(i^{th}\) "housing" unit is \(\theta v_i\) since there are no costs of ownership. Each consumer's share of total rental income is thus \((\Sigma \theta v_i + \theta \pi/(\theta + \tau))/n\). Letting \(w\) be wage income, the value of the \(i^{th}\) unit is

\[
V_i = \frac{w + (\Sigma \theta v_i + \theta \pi/(\theta + \tau))/n - t_i - x_i}{\theta + \tau} \tag{1'}
\]

Consider the problem of maximizing \(u(x_k, q_k, z)\) for an arbitrary index \(k\) subject to the \(n\) constraints corresponding to (1"), to the \(n - 1\) constraints \(u(x_k, q_k, z) = u(x_{i}, q_{i}, z) \ i \neq k\), and to the government budget constraint \(\tau \Sigma v_i + \tau \pi/(\theta + \tau) = C(z,n)\). The choice variables are \(z, \tau, x_i, v_i, i = 1, 2, \ldots, n\). The solution to this problem yields the maximum uniform
utility level consistent with the consumer and government budget constraints, and it is characterized in part by (22). If (22) does not hold utility may be increased by changing \( z \) and \( \tau \).

A rigorous characterization of efficiency requires a complete general equilibrium model. The model sketched above is incomplete because the determination of \( w \) and \( \pi \) is not specified and the industries producing \( x \) and residential and business structures are not described. Construction of a true general equilibrium model is beyond the scope of this paper; our incomplete model is designed to show how (22) emerges when rental income is taken into account. Condition (22) would undoubtedly emerge in a complete general equilibrium model as well.

For the Cobb-Douglas utility function, \( \frac{u_3}{u_1} \) increases when \( u \) and \( z \) are fixed and \( x \) increases, a property which is shared by the CES utility function. For any utility function which has this property, an argument qualitatively similar to the one presented for the Cobb-Douglas function holds. In particular, suppose (25) equals zero for \( q_1 = q^* \). A unit with \( q_1 > q^* \) has \( v_1 > v^* \) and \( x_1 < x^* \), which means (25) for that unit is negative. Similarly (25) is positive when \( q_1 < q^* \). Hence \( \frac{\partial v_1}{\partial z} \geq 0 \) as \( q_1 \leq q^* \) when the public good is provided efficiently. The relationship of \( q^* \) to \( \hat{q} \), the median \( q \) level is uncertain in general, but \( q^* \) will probably be close to \( \hat{q} \).

Hirsch (1968) cites evidence which shows constant average costs for primary and secondary education, police services, and refuse collection, with scale economies present only for five services. It appears reasonable to assume \( C_{11} = 0 \) in light of the evidence.
When $q$ and $u$ are fixed, $u_j/u_1$ is decreasing in $z$ when the utility function is quasi-concave, and it follows that for the housing unit with $q = q^*$ (see footnote (2)), $\partial v/\partial z$ will be positive (negative) when $z$ is less than (greater than) the efficient level as long as $c_{11} = 0$. 
References


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