

Low-Frequency Sound Propagation in Porous Media:  
Glass Spheres and Pea Gravel

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## Low-frequency sound propagation in porous media: glass spheres and pea gravel

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### Abstract

The sound propagation properties of two air-filled granular materials: large sifted pea gravel and 10 mm diameter glass spheres have been measured in an impedance tube. The experimental method was essentially the same as reported earlier [Swenson *et al.*, "Low-frequency sound wave parameter measurement in gravels," *Appl. Acoust.* **71**, 45-51 (2010)] for two other kinds of gravel: crushed limestone and undifferentiated pea gravel. Additional sampling and processing steps were applied to the microphone signals such that instead of tones, band-limited random noise was used as the input signal, and spectral domain complex pressures are now offered as input to the estimation algorithm. The estimation process extracts the best-fit attenuation coefficient, phase velocity, and characteristic impedance for the material over the signal frequencies, all with better precision than we previously obtained. Quadratic approximations for the acoustical parameters are given over the frequency range 25-160 Hz. The media are both slightly attenuating and dispersive, having attenuation coefficients within (0.13-0.34 Np/m), phase velocities smaller than those in air (180-240 m/s), and characteristic impedance approximately 3-5 times that for air. Pea gravel was more attenuating, and had slightly higher characteristic impedance, but lower phase velocities than the glass spheres.

### 1. Introduction

The study of propagation in rigid frame porous media has attracted attention of numerous investigators. The absorption and transmission of low frequency sounds is particularly significant for mitigation of sounds generated by explosions and military training. However, the spectral region from 20 to 200 Hz remains largely unexplored.

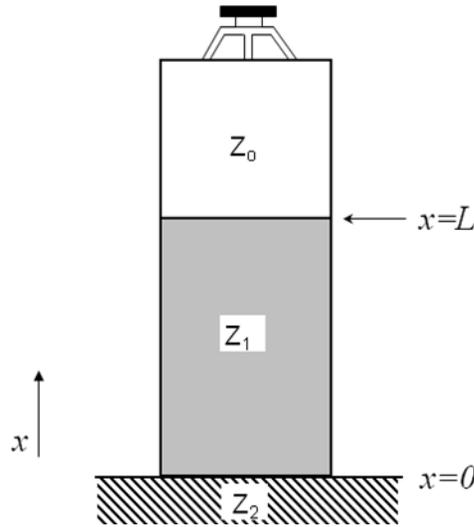
The sound propagation properties of two air-impregnated, granular materials: large sifted pea gravel and 10 mm diameter glass spheres have been measured in a large, vertical impedance tube. The experimental method was essentially the same as reported earlier (1, 2, 3) for two other kinds of gravel:

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crushed limestone and undifferentiated pea gravel. In the current experiment, the processing steps applied to the microphone signals have been improved, yielding better precision, and the ability to determine all acoustical parameters. In previous experiments with this apparatus, the phase velocity, attenuation, and input impedance of the media were estimated satisfactorily from standing wave pressure amplitudes measured in the impedance tube, but the terminating impedance of the tube and the characteristic impedance of the medium were not successfully estimated. All of these parameters can now be estimated. The essential changes in procedures were that spectral domain complex pressures are now utilized as computational input and that, instead of pure tones, band-limited random noise is used as the input signal for the measurement.

The impedance tube consists of a 7.3 m long, 76 cm diameter steel pipe standing upright (see Fig. 1). At the top end, a 12-inch direct-radiator loudspeaker is mounted in a heavy plywood baffle. The bottom end is terminated with a perforated plate for these measurements. The tube has 35 microphone ports placed along the side, approximately 20 cm apart. The tube is loaded with the material under study to depth  $L$ .



**Fig. 1.** Schematic of the impedance tube, partially filled with porous material.

In the impedance tube, sound at frequencies below 250 Hz travel as plane waves with complex pressure described by

$$\bar{P}(x, t) = \bar{P}_0 e^{\pm \bar{\gamma} x + j \omega t} \quad (1)$$

where the propagation constant  $\bar{\gamma} = \alpha + j\beta$ , for attenuation constant  $\alpha$  and phase constant  $\beta = 2\pi/\lambda$ . This equation can be reduced to phasor notation, with the  $e^{j\omega t}$  suppressed.

With the material under study placed in the impedance tube, two partially reflecting surfaces exist, one at  $x = 0$ , the terminating grate, and the other at  $x = L$ , the interface between the material under study and the air. Each surface has a complex reflection coefficient,  $\bar{\Gamma}$ , equal to the ratio

$$\bar{\Gamma}_{ab} = \frac{\bar{Z}_b - \bar{Z}_a}{\bar{Z}_b + \bar{Z}_a} \quad (2)$$

where  $\bar{Z}_b$  represents the impedance of the load and  $\bar{Z}_a$  the characteristic impedance of the incident medium. By applying the boundary conditions at the  $x = L$  interface, such that pressure and particle velocity are continuous, the full description of the pressure standing wave is given by the expression:

$$\bar{P}(x) = \begin{cases} \bar{P}_0(e^{\bar{\gamma}_1 x} + \bar{\Gamma}_{12} e^{-\bar{\gamma}_1 x}), & 0 \leq x \leq L \\ \frac{\bar{P}(L)}{1 + \bar{\Gamma}_{01}} [e^{\bar{\gamma}_0(x-L)} + \bar{\Gamma}_{01} e^{-\bar{\gamma}_0(x-L)}], & x > L \end{cases} \quad (3)$$

The characteristic impedance of the material under study can be determined from the reflection coefficients and the propagation constant of the material.

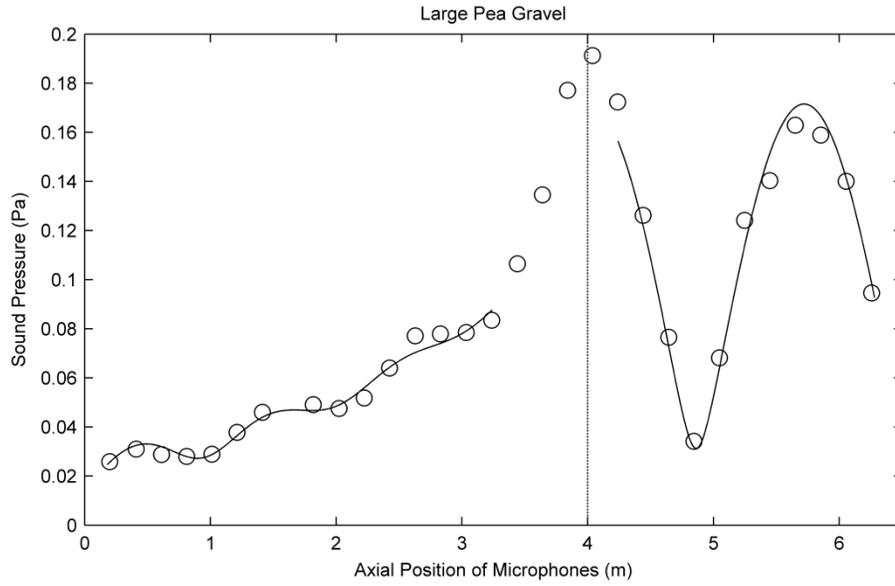
$$\frac{\bar{Z}_1}{\bar{Z}_0} = \frac{(1 + \bar{\Gamma}_{01})(1 - \bar{\Gamma}_{12} e^{-2\bar{\gamma}_1 L})}{(1 - \bar{\Gamma}_{01})(1 + \bar{\Gamma}_{12} e^{-2\bar{\gamma}_1 L})} \quad (4)$$

This model allows estimates of the attenuation, wavelength, and characteristic impedance of the material.

## 2. Data Collection and Processing

Band-limited white noise is produced by the loudspeaker. The instantaneous sound signals at the microphones are recorded by two Yokogawa DL750 digital oscilloscopes. The use of two oscilloscopes allows up to 32 microphones to be recorded simultaneously. The recordings are processed digitally by an FFT filter to give the complex pressure at each microphone for all frequency bins. An example plot of the pressure magnitudes measured in the large pea gravel at 100 Hz (bin width of 0.1 Hz) is given in Fig. 2. The circles in this plot show the magnitude of the complex sound pressures measured in the impedance tube, and the solid line shows the pressure predicted by the model with the best fit parameters. The best fit parameters for this measurement are :

$$\begin{aligned} \bar{P}_0 &= 0.0178 - 0.0124j \text{ Pa}, & \bar{\gamma}_0 &= 0 + 1.8318j, & \bar{\gamma}_1 &= 0.4350 + 2.9790j, \\ \bar{\Gamma}_{01} &= 0.6920 + 0.0126j, & \bar{\Gamma}_{12} &= -0.3208 + 0.1794j \end{aligned}$$



**Fig. 2.** Measured sound pressure amplitudes in large pea gravel at 100 Hz, with best fit function. Only 15 of the pressures in the gravel and 11 pressures in the air are fitted, owing to equipment limitations.

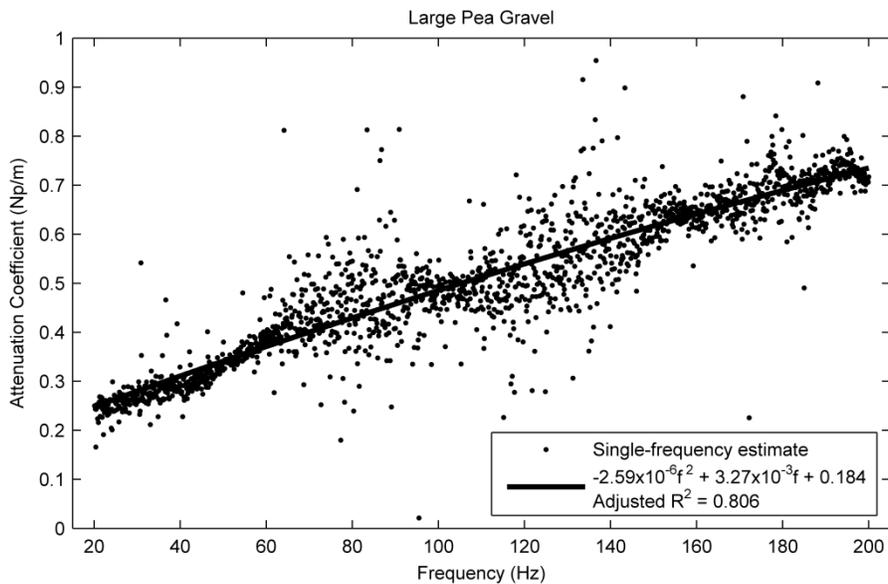
In this example, the gravel is 4 m deep. The plot to the left of the vertical line represents the pressures in the gravel and to the right that of the air above the gravel. A model for the complex pressure in the tube is then fitted to the measured complex pressures to provide the acoustic parameters of the material under study. The best fit is found by searching for the least mean squared error between the model and the measurements. The procedure adopted for this paper uses MATLAB's *nlinfit* function (an implementation of the Levenberg-Marquardt algorithm) for finding the best fit to the measured data in a least squares sense (4). Previously (1, 2, 3), pure tones were used as excitation signals. It has been found, however, that with an LMSE procedure used to estimate several unknown parameters simultaneously, one or more parameters can experience large errors fortuitously on individual frequencies. Slightly changing the frequency can yield substantially different parameter values. Repeating the LMSE routine at very closely spaced frequencies yields a band of parameter values closely clustered about a mean trend. This is accomplished with a band-limited white noise input signal, filtered by Fourier transformation to yield frequency spacing of 0.1 Hz for the pea gravel and 0.02 Hz for the glass spheres. The increase in frequency resolution is due to increasing the recording time from 10 s to 50 s. This increase in recording time will also reduce the variance of the parameter estimates. Using additional frequency bins is more computationally intensive, as the entire LMSE parameter search is performed on each frequency, but the result is more informative than is attained with widely-spaced pure tones. Examination of the statistical properties of the LMSE process is beyond the scope of this paper.

### 3. Media tested

This paper reports on two granular materials: large sifted pea gravel and 10 mm diameter glass spheres. The large sifted pea gravel, obtained by standard sifting techniques, contains grain sizes between 12.5 and 6.25 mm. The glass spheres are of uniform 10 mm diameter. Tortuosities measured by an electrical conductivity method are 1.732 for large sifted pea gravel and 1.197 for the glass spheres. Flow resistivities are  $429.93 \text{ N}\cdot\text{s}/\text{m}^4$  for the large sifted pea gravel and  $219.35 \text{ N}\cdot\text{s}/\text{m}^4$  for the glass spheres. Porosities are 0.42 for the large sifted pea gravel and 0.369 for the glass spheres. These data are from Ref. (6).

#### 3.1 Large sifted pea gravel

The sound propagation parameters of this medium are shown in Figs. 3 (attenuation), 4 (phase velocity) and 5 (characteristic impedance). Polynomial models are given for each of the parameters; these are for convenience only and are not to be extrapolated outside the given frequency ranges. In each case the coefficients of the polynomial are adjusted by a least mean squares procedure for a best fit to the (scattered) parameter estimates previously determined.



**Fig. 3.** Attenuation coefficient for large sifted pea gravel. Each dot is a separate determination of the attenuation coefficient by the LMSE process for a distinct frequency.

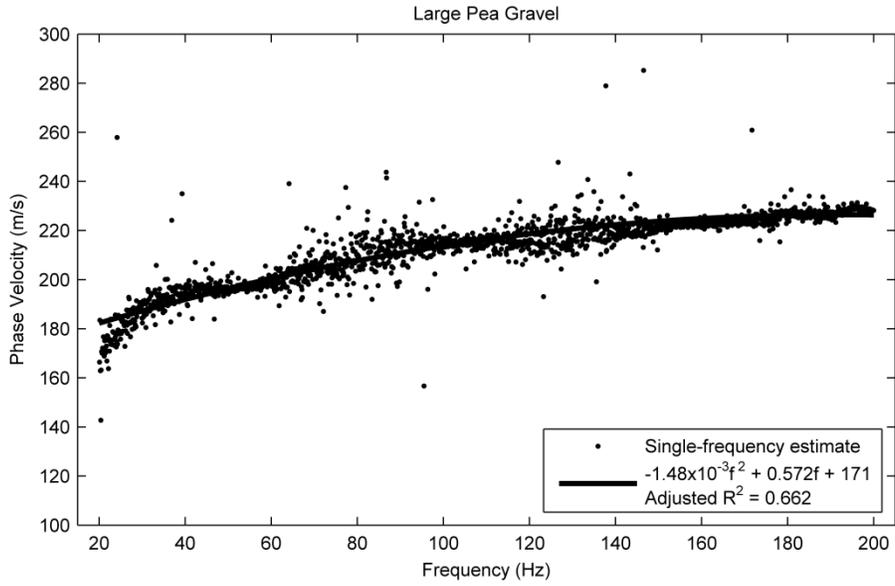


Fig. 4. Phase velocity for large sifted pea gravel.

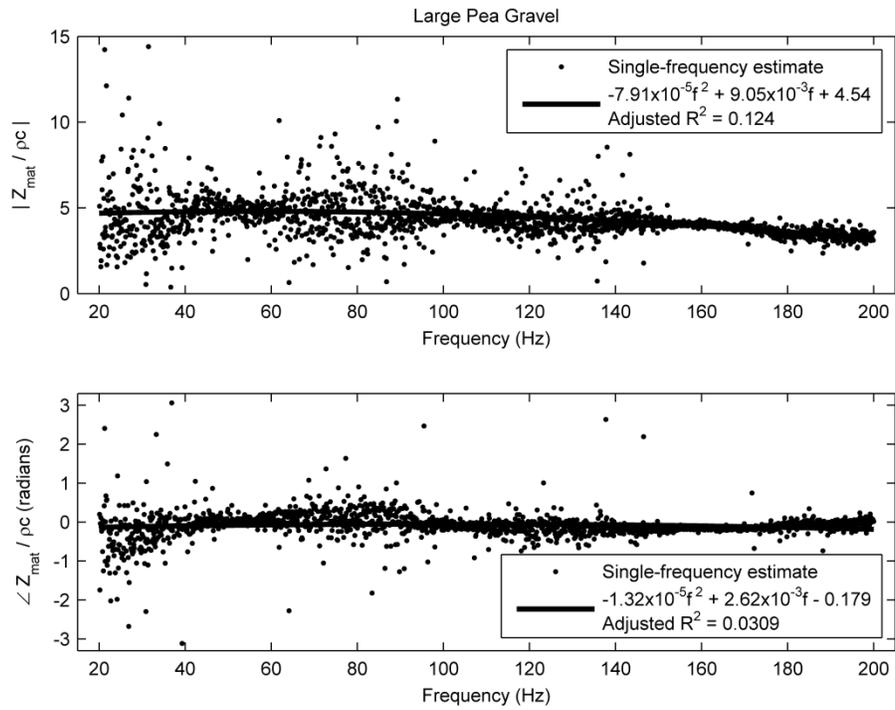
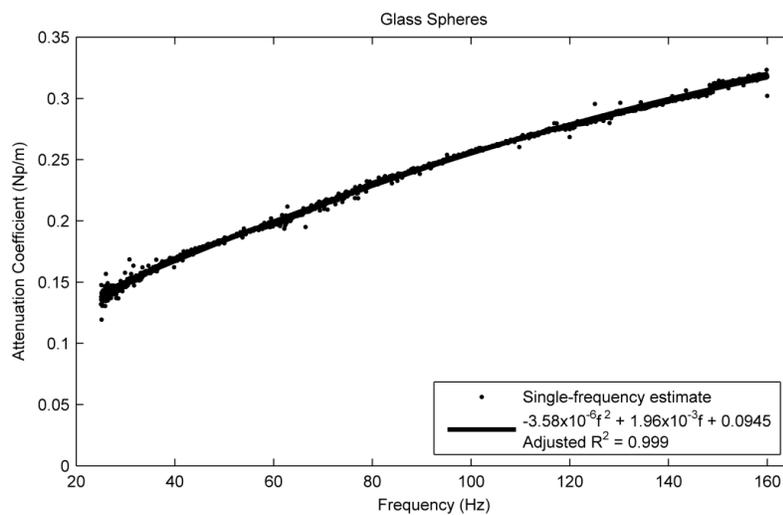


Fig. 5. Characteristic impedance for large sifted pea gravel normalized by  $\rho c$  for air.

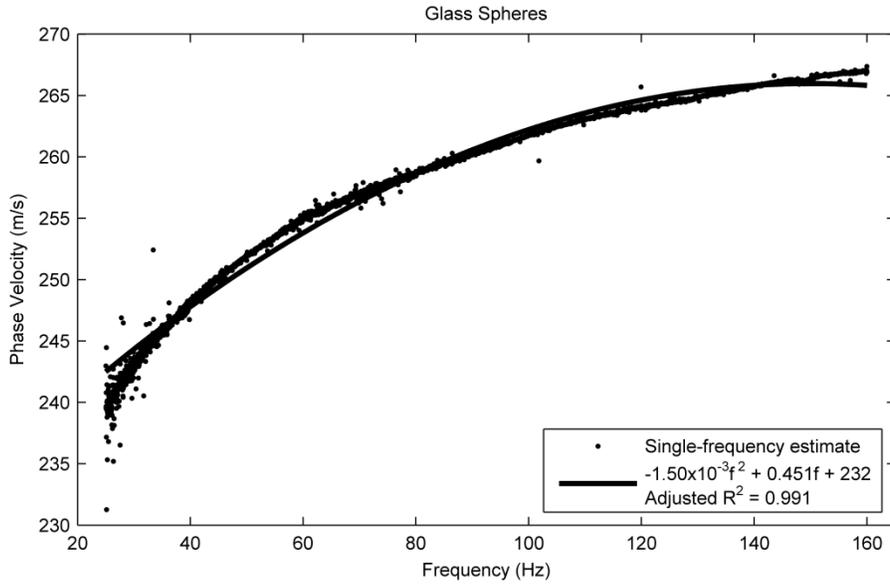
### 3.2 10 mm glass spheres

The wave propagation parameters of the 10 mm diameter glass spheres were determined from measurements of complex sound amplitudes made in the impedance tube filled to 3.8 m depth. The tube was terminated with a perforated steel plate in the expectation that a dissipative load would result in a reduced standing wave ratio. Previous experience has been that, while a high standing ratio in the medium under test expedites determination of the phase velocity, it results in excessive random anomalies in the determination of attenuation and characteristic impedance, especially when the attenuation is small and when several parameters must be determined simultaneously in a multi-parameter LMSE search. Unfortunately, the perforated plate did not result in a substantial reduction in the standing wave ratio; nonetheless, by use of the very-closely spaced frequency excitation it has been possible to estimate all the desired parameters. Only the characteristic impedance required extra processing measures, as described below.

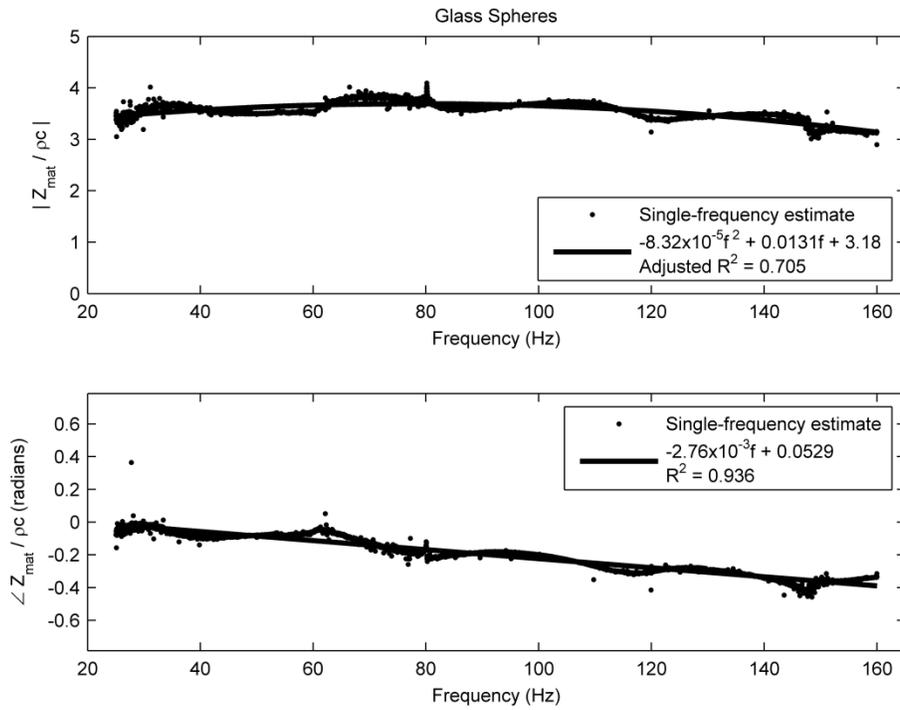
The attenuation, phase velocity, and characteristic impedance of the glass spheres are presented in Figs. 6, 7, and 8 respectively. The curves are the best second-degree polynomial fit to the estimates from the LMSE computations. In each case, the LMSE estimates are shown explicitly, with the best fit polynomial curves superimposed. In Fig. 8, the oscillations in the LMSE data are clearly not consistent with the assumed homogeneity and isotropy of the medium, but no reasonable cause has been determined. To test the validity of the polynomial approximation to the LMSE-generated characteristic impedance, Eq. 4 was solved for  $\bar{\Gamma}_{01}$  using the approximation for the characteristic impedance (Fig. 8) and the LMSE values for attenuation, phase velocity and  $\bar{\Gamma}_{12}$ . As is seen in Fig. 9, the computed values are nearly equal to the values originally calculated using the standing wave in the air above the porous medium. This is taken to indicate that a very small error in the estimate of the input impedance to the column could cause the oscillation displayed in the LMSE output of Fig. 8. A similar result was achieved when calculating the terminating impedance,  $\bar{\Gamma}_{12}$ .



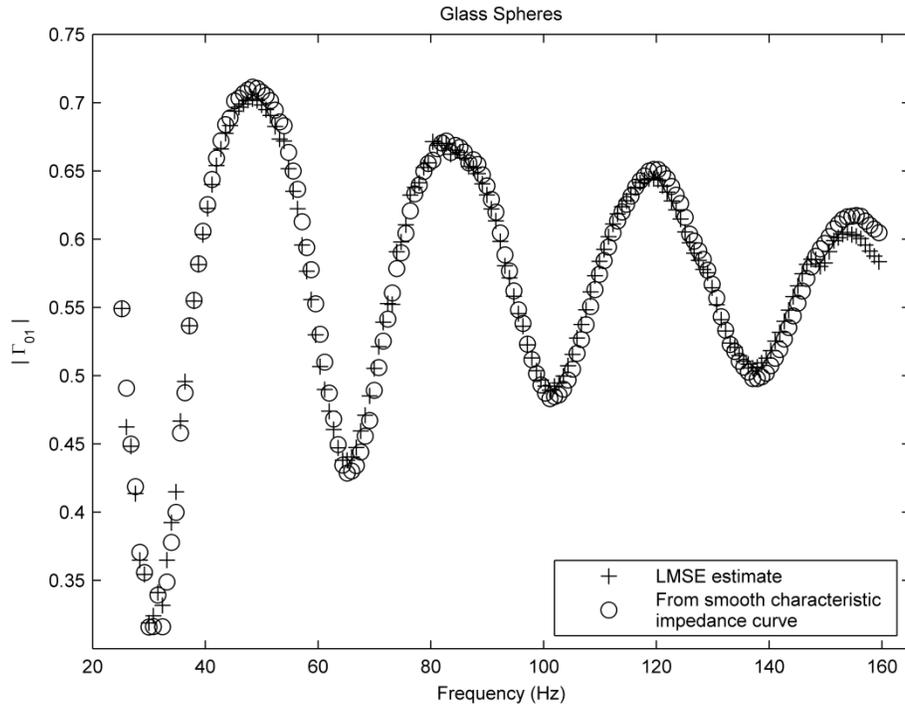
**Fig. 6.** Attenuation coefficient of 10 mm glass spheres.



**Fig. 7.** Phase velocity of 10 mm glass spheres.



**Fig. 8.** Complex characteristic impedance of 10 mm glass spheres.



**Fig. 9.** Comparison of values for  $|\bar{\Gamma}_{01}|$  for glass spheres. See text for explanation.

#### 4. Conclusion

This model and measurement procedure for complex pressures in the impedance tube has enabled us to produce estimates for attenuation, phase velocity and characteristic impedance for both the large sifted pea gravel and 10 mm glass spheres. The large sifted pea gravel's larger tortuosity conforms to the smaller phase velocity, when compared with the glass spheres. Similarly the large sifted pea gravel's higher static flow resistivity conforms to the larger characteristic impedance magnitude and attenuation coefficient as compared with the glass spheres. A detailed physical parameter model of the characteristic impedance, attenuation coefficient and phase velocity for these media is a planned topic of future investigation. Not surprisingly, the scatter of the parameter estimates decreased with increased recording time as seen when comparing the graphs of the two media.

#### 5. Acknowledgements

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