Faculty Working Papers

SPECIFICATION ERROR, RANDOM COEFFICIENT AND THE RISK-RETURN RELATIONSHIP

Cheng F. Lee, Professor, Department of Finance
Frank J. Fabozzi, Queens College, CUNY
Jack Clark Francis, II, Baruck College CUNY

#720

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

FACULTY WORKING PAPERS

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

October 24, 1980

SPECIFICATION ERROR, RANDOM COEFFICIENT AND THE RISK-RETURN RELATIONSHIP

Cheng F. Lee, Professor, Department of Finance
Frank J. Fabozzi, Queens College, CUNY
Jack Clark Francis, II, Baruck College CUNY

#720

Summary

Based upon the concepts of specification errors and random coefficients, the risk-return relationship in capital asset pricing is re-examined. It is found that the random-coefficient risk-return trade-off model is more appropriate than the fixed-coefficient risk-return trade-off model in capital asset pricing analysis.

SPECIFICATION ERROR, RANDOM COEFFICIENT AND THE RISK-RETURN RELATIONSHIP

I. INTRODUCTION

The theory of equilibrium in the capital markets developed independently by Sharpe (1964), Lintner (1965) and Treynor (1961) provides a risk-return relationship for assets and portfolios. This relationship, shown as equation (1), is called the security market line (SML) by Sharpe and the capital asset pricing model (CAPM) by others.

(1) $$E(R_j) = E(R_f) + B_j[E(R_m)-E(R_f)]$$

where $E(R_j)$ denotes the expected rate of return from the *jth* market asset, $E(R_f)$ represents the expected value of the risk-free interest rate, B_j is called the beta systematic risk coefficient, $E(R_m)$ is the expected return from the market, and

$[E(R_m) - E(R_f)]$ is the theoretical market risk premium.

Because of the pivotal role of the risk-return relationship in financial theory, researchers have tested the model empirically to determine if the estimated parameters are equal to the anticipated values observed in the marketplace. Although their findings fail to support the model, Roll (1977) has questioned the validity of the econometric tests employed. He has argued that the CAPM is testable in principle but virtually impossible to test statistically.

Despite the fact that the CAPM is a theoretical model which may not be tested statistically, an empirical market line has been employed by researchers to test the efficiency of financial markets and evaluate portfolio performance. Roll (1978), however, has explained the ambiguity of performance tests using the empirical market line.

The purpose of this paper is to present empirical evidence that the estimated risk-return relationship exhibits the characteristic of a stochastic parameter variation model. Such a finding diminishes the value of the model in empirical tests of market efficiency and portfolio performance and provides further evidence to support Roll's (1978) contention about the inappropriateness of the model for such purposes.[1]

The next section provides theoretical reasons to justify the use of a stochastic parameter regression model to describe the risk-return relationship. Section III formulates the specific stochastic parameter model employed. The data and results are described in the fourth section, followed by the conclusions in section five.

II. JUSTIFICATION FOR EMPLOYING THE STOCHASTIC PARAMETER REGRESSION MODEL

The empirical analogue of equation (1) is shown as equation (2).

$$(2) \qquad \bar{R}_j = \lambda_0 + \lambda_1 \hat{B}_j + e_j$$

where

$\bar{R}_j$ = the arithmetic mean return for security j,

$\hat{B}_j$ = the estimated systematic risk for security j, and

e_j = the stochastic error for security j.

Equation (2) is estimated using cross-sectional data with the beta coefficient estimated from a first-pass regression based on a time series of historical returns. If equation (1) is a true description of the risk-return relationshp, then λ_0 and λ_1 should equal the arithmetic mean return for the risk-free asset ($\overline{R}_f$) and arithmetic average excess of the market return ($\overline{R}_m$) over the risk-free rate (that is, the market risk premium). Moreover, the slope, λ_1, should be constant for all securities.

In a survey article about stochastic parameter regressions, Barr Rosenberg (1973, p. 381) states:

> The stochastic parameter problem arises when parameter variation includes a component which is a realization of some stochastic process in addition to whatever component is related to observable variables. Thus, stochastic parameter regression is a generalization of ordinary regression. Ideally, a model would be so well defined that no stochastic parameter variation would be present, and no generalization would be needed, but the world is less than ideal.

If the risk-return relationship is well specified, we would not observe that its slope or market risk premium in equation (1) would vary stochastically. However, there exist both theoretical economic and econometric reasons to suspect the risk-return relationship is, in fact, misspecified.

First, Arditti (1967), Kraus and Litzenberger (1976) and others have published theoretical and empirical work showing that the equilibrium return of an asset is influenced by both the second and third statistical moments of its return distribution. These findings extend the two parameter model to a third parameter, namely, skewness. Since the risk-return relationship ignores the impact of skewness, or skewness related factors, it suffers from omitted variables.[2]

Second, other studies have suggested the possibility of additional omitted variables. Sharpe (1977), for example, has given the risk-return relationship a "multi-beta" interpretation. Similarly, Ross (1976, 1977) uses an arbitrage approach to derive a multi-factor risk-return relationship. Brennan (1970) has analyzed the impact of the tax effect due to the different treatment of dividend income and capital gains. He derived a multi-index model including average excess dividend yield as an additional explanatory variable in the risk-return relationship. Bachrach and Galai (1979) have shown that the price of the stock should be included in the risk-return relationship while Lanstein and Sharpe (1978) and Joehnk and Petty (1980) have shown that duration or interest rate risk should also be considered.

Statistically, the multi-index risk-return model can be specified by equation (3).

$$(3) \qquad \bar{R}_j = \lambda_0' + \lambda_1'\hat{\beta}_j + \lambda_2\hat{X}_{2j} + \ldots + \lambda_n\hat{X}_{nj} + \tau_j$$

where $\hat{X}_2, \ldots, \hat{X}_n$ are estimates of omitted factors discussed above, $\lambda_0', \lambda_1', \lambda_2, \ldots \lambda_n$ are cross-section regression parameters, and τ_j is the stochastic error for security j. It should be noted that equation (3) is a generalized case of equation (2).

If we use the specification method specified by Theil (1971, pp. 548-549) it can be shown that

$$(4) \qquad \hat{\lambda}_1 = \hat{\lambda}_1' + b_2\hat{\lambda}_2 + \ldots + b_n\hat{\lambda}_n$$

where $b_2, b_3, \ldots b_n$ are so-called auxiliary regression coefficients.[3] In addition we also know that

$$5A) \qquad \hat{\lambda}_0 = \bar{\bar{R}}_j - \hat{\lambda}_1\overline{\hat{\beta}}_j$$

and

$$\text{5B)} \qquad \hat{\lambda}_0' = \bar{\bar{R}}_j - \hat{\lambda}_1'\bar{\hat{\beta}}_j - \hat{\lambda}_2\bar{\hat{X}}_2 - \ldots - \hat{\lambda}_n\bar{\hat{X}}_n$$

where $\bar{\bar{R}}_j = \sum_{j=1}^{n} \bar{R}_j/n$, $\bar{\hat{\beta}}_j = \sum_{j=1}^{n} \hat{\beta}_j/n$

If all auxilliary regression coefficients are zero (i.e., all the omitted variables are uncorrelated with $\hat{\beta}_j$), then $\hat{\lambda}_1$ is an unbiased estimate for $\hat{\lambda}_1'$. However, $\hat{\lambda}_0$ is no longer an unbiased estimate for $\hat{\lambda}_0'$. Therefore, $\hat{\lambda}_0$ cannot be used to test the null hypothesis that λ_0 is equal to $\bar{R}_f$ if the multi-index model is appropriate. If equation (3) does hold and all auxilliary regression coefficients are approximately equal to zero, $\hat{\lambda}_1$ will still be an unbiased estimator of $\hat{\lambda}_1$. However, $\hat{\lambda}_1$ becomes a random instead of a fixed variable.

Third, Roll (1977) and others have suggested beta estimates obtained by regressing returns from common stocks on stock market average returns are a form of partial equilibrium analysis which ignores investment in other capital assets. They suggest a general equilibrium analysis which includes other assets (such as, investments in human capital, commodities, real estate, etc.) should be used to obtain a risk-return tradeoff. If the more general equilibrium analysis suggested by Roll produces a risk-return relationship which departs significantly from the usual partial equilibrium analysis, this may explain why the estimated model exhibits the characteristics of a stochastic parameter regression model.

Fourth, Levy (1978) and Hessel (1978) have demonstrated that imperfect capital markets will modify the risk-return relationship which is predicated on the assumption of perfectly competitive markets. Levy (1978), for example, has developed a generalized risk-return relationship when (i) market participants differ in their investment strategies and do not adhere to the same risky portfolio given by their market portfolio and (ii) do not hold many risky assets in their portfolio. Levy concludes that the true risk index is somewhere between the total variance of the security and the systematic risk implied by capital market theory.

Finally, numerous studies have documented that the explanatory variable in the risk-return relationship, the beta coefficient, is subject to estimation error. The beta coefficient is estimated in the first-pass regression. However, in the first pass regression, the true market model may be a multi-index model rather than a single index model. As indicated in the discussion of the second pass regression above, the estimated beta of the market model will then exhibit the characteristics of a stochastic parameter regression model.[4] One might argue that if beta is a random coefficient as suggested by Fabozzi and Francis (1978) and Lee and Chen (1980), then the beta estimated from a random coefficient model should be employed in the second-pass regression. However, since the OLS estimate in a fixed coefficient model provides an unbiased estimate of the slope in a random coefficient model, the OLS estimate of beta will be used in estimating the risk-return relationship.

III. TEST FORMULATION

Previous research employed the classical OLS fixed-coefficient approach to estimate equation (2). The purpose here is to determine if a random coefficient relationship between return and systematic risk exists. That is, does the proportionality constant, λ_1, which represents the market risk premium in equation (1) fluctuate randomly from one security to the next?

There are several stochastic parameter regression models suggested in the literature.[5] The random coefficient model formulated by Thiel (1971) is used in this investigation. The fixed coefficient model given by equation (2) can be converted to the random coefficient model (RCM) shown by equations (6) and (7).

$$(6) \qquad \bar{R}_j = \lambda_0 + \lambda_{1j}\hat{\beta}_j + w_j$$

or

$$(7) \qquad \bar{R}_j = \lambda_0 + \bar{\lambda}_1\hat{B}_j + w_j$$

where

$$w_j = (\lambda_{1j} - \bar{\lambda}_{1j})\hat{B}_j + e_j$$

and $\bar{\lambda}_1$ is the mean of λ_{1j}. Moreover, it is assumed that the distribution of λ_{1j} is homoscedastistic and the e_j values are uncorrelated with the λ_{1j} values.

To test whether the RCM is a description of the risk-return relationship, two statistics must be estimated. First $\bar{\lambda}_1$ of equation (7), and second, the variance of the distribution of λ_{1j} around its mean $\bar{\lambda}_1$, var (λ_{1j}), must be estimated.[6] If no statistically significant variance for λ_{1j} around $\bar{\lambda}_1$ is found, then the RCM can be rejected and the traditional fixed-coefficient model accepted.

Theil (1971, p. 623) has shown that the OLS estimator of $\bar{\lambda}_1$ in equation (7) is unbiased but will result in an inefficient estimator for the variance of the estimate of $\bar{\lambda}_1$.[7] An approximate procedure suggested by Theil to estimate $\bar{\lambda}_1$ and var(λ_{1j}) is described briefly below.

First, the ordinary least squares residuals, denoted by $\hat{e}_j$, must be calculated from equation (2). Second, equation (8) must be estimated using OLS.

$$(8) \qquad \hat{e}_j^2 = m_0 P_j + m_1 Q_j + f_j$$

where[8]

$$P_j = 1 - \left(\frac{\hat{B}_j^2}{\Sigma \hat{B}_j^2} \right)$$

$$Q_j = \hat{B}_j^2 \left(1 - 2 \left\{ \frac{\hat{B}_j^2}{\Sigma \hat{B}_j^2} \right\} + \left\{ \frac{\Sigma \hat{B}_j^4}{(\Sigma \hat{B}_j^2)^2} \right\} \right)$$

and

f_j = stochastic error term.

The coefficients m_0 and m_1 are to be estimated. They represent the variance of the error term e_j in equation (7) and var (λ_{1j}), respectively. The statistical significance of m_1 (as measured by its t-statistic) then determines whether the RCM is appropriate. However, because of the heteroscedasticity in equation (8), Theil suggests that equation (7) be estimated using generalized least squares (GLS).[9] The GLS estimate for $\bar{\lambda}_1$ in equation (7) is defined in equation (9).[10]

(9) $$\hat{\bar{\lambda}}_1 = \left(\Sigma \frac{\hat{B}_j \bar{R}_j}{\hat{m}_0 + \hat{m}_1 \hat{B}_j^2}\right) \Big/ \left(\Sigma \frac{\hat{B}_j^2}{\hat{m}_0 + \hat{m}_1 \hat{B}_j^2}\right) .$$

Note that if m_1 is not statistically different from zero, equation (9) reduces to the OLS estimate for $\hat{\bar{\lambda}}_1$.

IV. EMPIRICAL RESULTS

The securities used to estimate the risk-return relationship are the common stock of 694 New York Stock Exchange companies. For each stock, $\hat{B}_j$ was estimated from the single-index market model, equation (10), using monthly non-compounded price change plus dividend returns for the 72 month period from January, 1966 to December, 1971.

(10) $$R_{jt} = \alpha_j + \beta_j R_{mt} + u_{jt}$$

where

R_{jt} = return on stock j in month t

R_{mt} = market return in month t

u_{jt} = stochastic error term in month t for stock j, and,

α_j and β_j are the parameters to be estimated.

The S&P 500 index with dividends included was used for the market index.

The time period was also partitioned into two non-overlapping 36 month periods—January, 1966 to December, 1968 and January, 1969 to December, 1971. Equation (10) was estimated for both time periods. The observed market risk premium was positive for the first sample period and negative for the second sample time period.

Estimates of the fixed coefficient OLS model equation (2) for the entire time period and the two sub-periods are presented in Table 1.

The observed market values for λ_0 and λ_1 if the CAPM is valid are also shown in Table 1.[11] For each time period, the signs of the estimated parameters were the same as the observed market values. And, each parameter was significantly different from zero at the 1% level of significance. Other researchers who estimated the risk-return relationship found that the estimated values for the parameters were significantly different from the observed market values. Table 1 suggests that for each of the entire time periods and the two sub-periods, $\hat{\lambda}_1$ was significantly different from the observed market value for the market risk premium. For the two 36 month time periods, $\hat{\lambda}_0$ was not statistically different from the observed risk-free rate, R_f.[12]

The results for $\hat{\bar{\lambda}}_1$ and $\hat{m}_1$ [= var (λ_{1j})] for the RCM are summarized in Table 1.[13] For the 72 month period, the variance of λ_{1j} was positive and significantly different from zero at the 5% level of significance.[14] This was also found for the 36 month period January, 1966 to December, 1968 in which the market risk premium was positive. Hence, for the two periods in which the market risk premium was positive, the SML was found to exhibit the property of a RCM. However, when the market risk premium was negative, namely, from January, 1968 to December, 1971, the variance of λ_{1j} was not statistically significant.

It is also interesting to note the degree of randomness of the market risk premium for the two cases in which $\hat{m}_1$ was statistically significant. The coefficient of variation, $\sqrt{\hat{m}_1}/\hat{\bar{\lambda}}_1$, for the 72 month and 36 month time periods were 1.57 and .97, respectively. This indicates considerable random movement in relation to $\hat{\bar{\lambda}}_1$. If a 95% confidence interval was constructed for the movements around $\hat{\bar{\lambda}}_1$ based on $\sqrt{\hat{m}_1}$, the interval would include the observed market value for the market risk premium.

V. CONCLUSIONS

The risk-return relationship of capital market theory is not simply a model accepted by some academicians. Regulators have used the model to estimate the appropriate return on equity for regulated firms. Corporate management has been encouraged to use the model to evaluate the performance of in-house or independent pension portfolio managers. The performance of the mutual fund industry has been questioned based on empirical results which have used the theoretical model. We must, therefore, continue to evaluate the model both theoretically and empirically.

In this paper, we disclose a disturbing empirical result of the risk-return relationship. Employing a stochastic parameter regression model, we find that the slope of the risk-return relationship varies randomly from one security to the next. Moreover, the observed randomness was substantial. General plausible explanations for such results were suggested. Even if the reader rejects any or all of these arguments, it is difficult to refute the empirical findings.

In addition to supporting Roll's (1978) theoretical argument that the estimated market line provides ambiguous results when applied to testing market efficiency and performance evaluation, the empirical results indirectly support Roll and Ross's (1979) empirical results of testing the Arbitrage pricing theory.[15] The direct relationship between the results of this study and those of Roll and Ross will be developed in future research.

FOOTNOTES

[1]It should be emphasized that this paper does not intend to test the CAPM. In light of Roll's (1977) criticism, such tests would be fruitless. Yet, readers will obviously be interested in how the observed market values and the estimated parameters compare. Hence, these valid tests are presented but should not be construed as a test of the CAPM using a different statistical model.

[2]Previous research on skewness and the risk-return relationship is summarized in footnote 2 of Kraus and Litzenberger (1976).

[3]A more precise definition can be found in Theil (1971, p. 549).

[4]This was found true for a substantial number of stocks by Fabozzi and Francis (1978) using the stochastic parameter regression model described in the next section. In such cases, the total risk can be partitioned as follows:

$$\sigma_i^2 = (B_i^2 + \sigma_{B_i}{}^2)\ \sigma_m^2 + \sigma_{e_i}^2$$

where

σ_i^2 = variance for the returns for stock i

σ_m^2 = variance for the market return

$\sigma_{e_i}^2$ = unsystematic risk for stock i

and

$\sigma_{B_i}^2$ = variance for the systematic risk of stock i

In such a case, equation (1) is then

$$\bar{R}_j = \lambda_0 + \lambda_1(\hat{B}_i^2 + \hat{\sigma}_{B_i}^2)1/2 \quad .$$

Hence, equation (2) is misspecified in that $\hat{\sigma}_{B_i}^2$ is not considered. Note also that if the traditional procedure for computing unsystematic risk is employed but the market model is a RCM, then the unsystematic risk would improperly includ $\sigma^2_{B_i} \sigma^2_m$. This might explain why some researchers have found a positive relationship between average returns and unsystematic risk.

[5]See Rosenberg (1973) for a description of various stochastic parameter variation models.

[6]It is imperative the reader understand the difference between the variance of the estimate of $\overline{\lambda}_1$ and the variance of λ_{1j} around its mean $\overline{\lambda}$. The variance of the estimate of $\overline{\lambda}_1$ is used to test whether $\overline{\lambda}$ is significantly different from zero. The variance of λ_{1j} around its mean $\overline{\lambda}_1$ is used to test if the model is a RCM. This latter variance must also be estimated and, hence, has its own variance for the estimate of var(λ_{1j}).

[7]This inefficiency results from the fact that w_j in equation (7) is heteroscedastic [see Theil (1971, pp. 623)]. This may help explain why Miller and Scholes (1972) found heteroscedasticity when they estimated equation (2).

[8]In the equations below $\hat{B}_j$ and $\overline{R}_j$ represent deviations of each variable from their respective means. The summation is over all observations.

[9]Theil (1971) has shown that to estimate the variance-covariance matrix for generalized least squares in this case, the following weights should be used:

$$Z_j = 1/2(\hat{m}_0' P_j + \hat{m}_1' Q_j)^{-2}$$

where Z_j = the weight for the <u>jth</u> observation and $\hat{m}_0$ and $\hat{m}_1$ are the ordinary least squares estimates of m_0 and m_1 for equation (8).

[10]Note again that the variables are in deviation form. (See footnote 8.)

[11]We use the terminology "observed market value if the CAPM is valid," or simply, "observed market value" in lieu of theoretical value since the market risk premium observed in the market may be negative. This would then suggest that theory implies an inverse risk-return relationship.

[12]This result was somewhat surprising in light of the analytical results derived in Section II. It was shown there that λ_0 will be a biased estimate of $\overline{R}_f$ if the multi-index model is appropriate.

[13]The GLS procedure suggested by Theil did not reduce the variance of the estimate of $\bar{\lambda}_1$ as expected. The OLS and GLS estimates were practically the same after considering rounding errors which occur in the more complex estimation procedure.

[14]A one-tail test is used since the alternative hypothesis is that the variance of λ_{1j} is positive.

[15]Roll and Ross have found that there exist at least three and probably four "priced" factors in addition to market factor in the generating process of return.

REFERENCES

Arditti, F. D. (1967), "Risk and the Required Rate of Return," Journal of Finance (March), pp. 19-36.

Bachrach B. and Galai, D. (1969), "The Risk-Return Relationship and Stock Prices," Journal of Financial and Quantitative Analysis (June), pp. 421-442.

Brennan, M. J. (1970), "Taxes, Market Valuation and Corporate Financial Policy," National Tax Journal, pp. 417-427.

Fabozzi, F. J. and Francis, J. C. (1978), "Beta as a Random Coefficient," Journal of Financial and Quantitative Analysis (March), pp. 101-115.

Hessel, C. A. (1978), "The Effects of Taxes and Imperfect Competition on an Investor's Optimum Portfolio," unpublished doctoral dissertation, New York University.

Joehnk, M. D. and Petty, J. W. (1980), "The Interest Sensitivity of Common Stock Prices," Journal of Portfolio Management (Winter), pp. 19-25.

Kraus, A. and Litzenberger, R. H. (1976), "Skewness Preference and the Valuation of Risk Assets," Journal of Finance (September), pp. 1084-1100.

Lanstein, R. and Sharpe, W. F. (1978), "Duration and Security Risk," Journal of Financial and Quantitative Analysis (November), pp. 653-668.

Lee, C. F. and Chen, S. N. (1980), "A Random Coefficient Model For Reexamining the Traditional Method of Decomposing the Total Risk," forthcoming Quarterly Review of Economics and Business.

Levy, H. (1978), "Equilibrium in an Imperfect Market: A Constraint on the Number of Securities in the Portfolio," American Economic Review (September), pp. 643-658.

Lintner, J. (1965), "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," The Review of Economics and Statistics 47 (February), pp. 13-37.

Miller, M. M. and Scholes, M. (1972), "Rates of Returns in Relation to Risk: A Re-examination of Recent Findings," published in Studies in the Theory of Capital Markets, edited by Michael Jensen Praeger, pp. 47-78.

Roll, R. (1977), "A Critique of the Asset Pricing Theory's Tests; Part I: On Past and Potential Testability of the Theory," Journal of Financial Economics, (March), pp. 129-176.

__________ (1978), "Ambiguity When Performance is Measured by the Securities Market Line," Journal of Finance (September), pp. 1051-1069.

Roll, R. and S. A. Ross (1979), "An Empirical Investigation of The Arbitrage Pricing Theory," Working paper 15-79, Graduate School of Management, UCLA.

Rosenberg, B. (1973), "A Survey of Stochastic Regression Parameters," Annals of Economic and Social Measurement, Vol. 2, pp. 381-397.

Ross, S. A. (1976), "Arbitrage Theory of Capital Asset Pricing," Journal of Economic Theory, 8, pp.

Ross, S. A. (1977), "Return, Risk and Arbitrage," In I. Friend and J. L. Bicksler, Risk and Return in Finance, Vol. 1 (edited), Ballinger Publishing Company.

Sharpe, W. F. (1964), "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," Journal of Finance, (September), pp. 425-442.

Sharpe, W. F. (1977), "The Capital Asset Pricing Model: A Multi-Beta Interpretation," in Levy, H. and M. Sarnat, eds., Financial Decision Making Under Uncertainty, (New York: Academic Press, 1977).

Theil, H. (1971), Principles of Econometrics, John Wiley & Sons.

Treynor, J. L. (1961), "Toward A Theory of Market Value of Risky Assets," unpublished paper.

TABLE I

SUMMARY OF RESULTS[1]

	Fixed Coefficient Model			Random Coefficient Model		Observed Market Values[2]	
Time Period	$\hat{\lambda}_0$	$\hat{\lambda}_1$	r^2	$\hat{\bar{\lambda}}_1$	$\hat{m}_1 = var(\hat{\lambda}_{1i})$	$\lambda_0 = \bar{R}_f$	$\lambda_1 = \bar{R}_m - \bar{R}_f$
Jan. 1966 to Dec. 1971	.0025 (2.78)	.0046 (6.27)	.054 (39.50)	.0044 (5.97)	.000048 (2.72)	.0044	.0006
Jan. 1966 to Dec. 1968	.0047 (3.99)	.0105 (11.38)	.158 (129.85)	.0100 (9.67)	.000094 (2.50)	.0040	.0026
Jan. 1969 to Dec. 1971	.0037 (2.82)	-.0036 (-3.58)	.018 (12.68)	-.0045 (-4.31)	.000041 (.93)	.0048	-.0015

[1]The t-values are shown below the estimated coefficient. The coefficient of determination is denoted as r^2. The value below r^2 represents the F-value. All estimated values are significantly different from zero at the 1% level except for the estimate of m_1 for the time period Jan. 1969 to Dec. 1971.

[2]The average monthly yields from three month Treasury Bills are used to measure the risk-free return, $\bar{R}_f$.