Faculty Working Papers

DISCRETIONARY PRICING AND TAX SHIFTING

Marvin Frankel

#98

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign
FACULTY WORKING PAPERS
College of Commerce and Business Administration
University of Illinois at Urbana-Champaign
March 16, 1973

DISCRETIONARY PRICING AND TAX SHIFTING

Marvin Frankel

#98
The state of belief concerning the shifting of the corporate income tax, and to a lesser extent, excise and sales taxes, might fairly be described as unsettled. (See e.g., Mieszkowski.) Although traditional price theory affords explicit judgments about such shifting (Herber, pp. 405-419), it is recognized that these results flow from quite restrictive assumptions about economic behavior and the organization of the marketplace. Moreover, empirical work of the past several years, particularly on the income tax (Oakland), has not much reduced the areas of doubt or laid a basis for viable generalizations.

The dissent from traditional theory rests in part on the intuitively plausible premise that the firm occupies an essentially discretionary position in its pricing actions. It desires continuously to maximize profits. At the same time it can, of its own initiative, in response to a tax or other event, adjust its price in order to preserve or augment profits without somehow being constrained in the manner of the stereotyped monopolistic or competitive firm. Yet discussions that invoke this type of firm behavior, with its implication that firms are not strictly maximizing profits prior to the levy of a tax, usually offer no clearly structured theory to support it. Rather such behavior is seen to arise out of the influence on pricing decisions of one or more special circumstances, such as fear of new entry, or government intervention.¹

Utilizing the not unreasonable assumption that enterprises do not know their demand functions in any objective way, the present paper develops a discretionary pricing model of the firm. With the aid of this model it then considers, in a
partial equilibrium short run micro setting of imperfect competition, the reactions of the firm to the imposition of sales and income taxes. It also considers the outcomes of certain formula pricing procedures when used in conjunction with a tax levy. The model retains simple, well-defined links with traditional theory and is sufficiently broad to encompass the standard monopolistic and competitive situations as special cases.  

Our approach does not offer conclusive answers to unresolved questions on tax shifting. It does provide a formal rationale for the firm's discretionary position and a framework in which to examine its response to a tax. One finding of the analysis is that a price response by the firm to an income tax is a plausible expectation and perhaps but modestly less likely than with a sales tax. A second finding is that reliance on a pricing formula, whether in conjunction with a tax or otherwise, is an expected by-product of situations in which demand is unknown, and while a formula cannot serve the firm's profit goals with precision, it may nonetheless serve as well as any alternative procedure.

Section I that follows, briefly sketches the framework to be used in assessing the firm's pricing decisions. Sections II and III employ this framework to examine the effects of sales and income taxes, while Section IV extends the analysis to a consideration of formula procedures. A concluding section comments on the allocation implications of the model.

I. THE MARKET SETTING

A. Demand

Consider a firm that is selling a product in an imperfectly competitive market. The setting might be monopolistic, or oligopolistic or Chamberlinian, the precise nature of the imperfections not being critical for the discussion.
(The case of perfect competition, neglected here, would be a straightforward extension of the analysis.) How might the firm perceive the demand for its product? It, of course, knows its current price and sales rate. We assume, not implausibly, that it lacks hard objective information on what quantities the market would absorb at other prices. In keeping with common practice our firm quotes a price, which it may be moved to change from time to time, and sells all that the market will absorb at that price. How might such a firm perceive the demand for its product? For convenience, let us suppose that at the currently quoted price of $13, the sales rate, while varying, has held for some interval within 10 to 20% of the current rate of 3,750 units. The firm's position is then indicated in Figure 1 by the point labeled CSP, or current sales point. The firm might infer that, in the absence of a price change, sales will continue in the general neighborhood of the current rate for some future, decision-relevant interval. More specifically it might attach the subjective probability of 1.00 to the prospect that sales will equal or exceed 3,000 units (point A) and the subjective probability of 0.00 to the prospect that they will exceed 4,250 units (point B). It might further believe that the chances for sales to equal or exceed the current rate of 3,750 are .45. These prospects are described by the short horizontal line segment in the figure.

What sales would be realized at other prices, higher or lower, than the current one? Such information as management has, including that which market research might provide, will generally be of a subjective, conjectural sort. (Baumol, p. 106; Fellner, p. 147.) "Knowledge" will typically take the form of a judgment, whether more or less informed, of the chances of selling varying quantities at alternative prices. The content of this knowledge might be developed and set out by asking the firm, for each of a succession of prices, "What do you think are the chances you will sell at least twenty units per
Figure 1

Price-Sales Expectations Under
Unknown Demand

![Graph showing price-sales expectations under unknown demand. The graph plots price (in dollars) on the y-axis and quantity (in hundreds) on the x-axis. The graph includes points A, CSP, and B, indicating various price and quantity combinations.](image-url)
period? Thirty? Forty?" Such a procedure would generate curves - let us call them subject sales curves (SSC's) - of the sort shown in Figure 1. The succession of points on any such curve tells us, for alternative prices, the corresponding sales that will, with the designated (subjective) probability, be equalled or exceeded. The probability that sales will fall in the zone bounded by any two SSC's is, of course, equal to the difference in their respective designated probabilities.

The SSC's must converge from above and below to the horizontal line segment (which we might term a subjective sales segment or SSS), with the 1.00 SSC and the 0.00 SSC hinged at its respective extremities. These SSC's must further drift from the northwest to the southeast, for much the same reasons that a conventional demand curve is negatively sloped. Correlatively, we can say that the firm is wholly confident that at prices below the current price of $13 sales will equal or exceed 3,000 units, and equally confident that at prices above $13 sales will not exceed 4,250 units.

It should be emphasized that the SSC's reflect the characteristics not of the marketplace but of the firm's view of the marketplace. Hence the description they provide will be no more complete or logical than is this view. The curves may be discontinuous in places or segmented, and they may be linear or convex or concave to the origin. It should also be noted that unlike a conventional demand curve, the SSC's allow for the effects of any anticipated reactions by competitors. In contemplating its decisions, the firm necessarily is concerned with the probable responses of both its customers and its rivals. It must reckon with the reactions of both groups and, in particular, with their net effect.
B. Cost

We can complete our framework by specifying a simple cost function of the form

\[ C = k + bQ \]

where \( C \) = Annual total dollar cost;

\( Q \) = annual rate of output in number of units;

\( k \) = a parameter denoting annual dollar fixed outlays for salaried staff, rent, interest on borrowed funds, depreciation allowance, etc.;

\( b \) = a parameter determining incremental cost.

The constant marginal cost characteristic of this function contributes to its simplicity, yet does not limit the analysis in any critical way.

The profit to be earned by the firm can now be expressed as

\[ \Pi = PQ - C = PQ - (k + bQ) \]

where \( \Pi \) denotes annual total profit and \( P \) price per unit, so that \( PQ \) expresses total annual revenue. This expression may in turn be used to generate a family of iso-profit curves (IPC's). The IPC's are shown in Figure 2, which extends Figure 1. The $3,000 curve, for example is the locus of price-quantity pairs which, if realized, will yield that amount of profit. The curves thus show what must be accomplished in the marketplace to attain any profit goal.

C. The Pricing Decision

The IPC's and the SSC's of Figure 2 jointly display some essentials of the firm's price-quantity decision problem. The location of the CSP on the $6,000 IPC indicates the firm's current earnings at the existing price of $13 and sales rate of 3,750 units. One option for the firm is to retain the current price for the future interval in question. It could do this with a
The IPC's of $0$, $3000$, $6000$ and $9000$ are generated by (2) in the text, with parameter values of $b = 59$ and $k = 9000$. Thus at the current price of $13$ and sales rate of 3750, unit variable costs are about 70% of price, unit fixed costs about 18% and unit profit about 12%. Moving the CSP to a different PQ position would, of course, change these figures.
.45 (subjective) confidence that sales and profits would equal or exceed their current amounts with possible respective maximums of 4,250 and $8,000. Correlatively, it would recognize a .55 chance that sales and profits will fall short of current levels to possible minima of 3,000 units and $3,000. Other options entail upward or downward price changes of varying amounts, with the possible consequences as indicated by the SSC's and IPC's.5

The figure as drawn shows a maximin at the current price. This characteristic - not a necessary outcome - is related to the location of the 1.00 SSC, which lies to the left of the $3,000 IPC at prices above and below $13. All prices but the current one, whatever the chances they may offer for higher profit, will carry the risk of a lower one.

The firm's situation at the CSP clearly is a discretionary one. It quotes a price, as firms in imperfectly competitive market situations commonly do, and sells what the market will absorb at that price.6 It may, as our framework suggests, make one or more price adjustments over a sequence of decision periods, depending upon its evaluation of market circumstances and its willingness to accept risk. At the same time, there are likely to be practical limits to the frequency and size of price changes that can be effected in the short run. Note that the discretionary nature of the firm's position is a natural byproduct of market uncertainty. This uncertainty gives rise to a number of pricing options from which the firm must choose. In contrast, the maximizing firm facing a known demand function need make no choice, for there is but one price whose foreknown outcome meets its objective. Note also that with an unknown demand function, the marginal revenue function is undefined and can play no role in the firm's decision. The price-quantity position to which the firm's decision leads will not, except by accident, equal the \( MC = MR \).
solution or be in any systematic relation to the latter. Indeed, the \( MC = MR \) solution has no operational meaning in our decision context. Different firms, in identical 'objective" circumstances but with different subjective assessments of market possibilities and different risk-utility functions, would end up in different price-quantity positions. Some of these positions would yield greater profit than others, but in terms of the underlying ex ante assessments, none could be said to be preferred from a resource allocation point of view. This last point is important when considering the allocation implications of a price adjustment by the firm, whether in response to a tax or some other factor.

That this portrayal of the firm's situation is but a simple extension of traditional theory may be seen as follows: Were the firm somehow to acquire a full knowledge of its demand function, the family of SSC's would collapse into a single curve. The point of tangency of this curve with an IPC curve would represent the profit-maximizing position and would be the equivalent of the marginal cost equals marginal revenue solution.

II. A SALES TAX

Consider the firm to be situated as in Figure 2, satisfied for the moment with its position at the CSP. Suppose now that a general sales tax, specified as a flat percentage of the sales price, is enacted. Alternatively, the tax might be an excise levied on the principal product of an industry. What are the consequences?

The tax need not, though it may, affect the firm's assessment of its market opportunities. Let us defer until later a consideration of such possible effects and assume provisionally that the SSC's remain unchanged. In contrast, the tax will perforce affect the IPC's. From (2) we have

\[
(3) \quad \pi_s = PQ - (k + bQ) - \frac{sPQ}{1+s}
\]

\[
(3a) \quad \pi_s = \frac{PQ}{1+s} - k - bQ
\]
Figure 3

The Effect of a 5% Sales Tax
where $s$ denotes the rate of sales tax, $\Pi_s$ total after-tax profit and $P$ the price gross of tax. The third term on the right, which represents the firm's tax liability, treats the tax as a levy on the price net of tax. Expression (3a) can be used to generate a new set of IPC's, as shown in Figure 3. The IPC's have been displaced to the northeast and have rotated in a counterclockwise direction. The rotation effect is indicated by the difference between the dash-dot curve, which shows the pretax $6,000$ IPC, and the $3,680$ IPC. Profits have fallen from $6,000$ to $3,680$ at the CSP, and have declined as well at all other PQ positions. Similarly, expected profit at each price has declined, and by a larger percentage for lower profit levels. Of particular interest is the relative rise in the expected gain from price increases as compared with price reductions - a result of rotation. The tax also causes the dispersion of possible outcomes at each price to decline, and the post-tax, risk-reward ratio tends to improve for price increases relative to price decreases than was the case before the tax.

The essentials of this type of situation are contained in Figure 4, which is schematic and not based upon the particulars of Figure 3. The before-tax profit-price profile shows the expected profit at each price alternative, including the current price (containing the CSP). The shape of the curve as drawn is arbitrary. It will in practice depend upon the perceived SSC's in relation to the PRC's and may be quite irregular. The tax causes the profile to shift downward and to rotate counterclockwise around the current price. The dispersion associated with selected points on the profiles (and their corresponding expected profits) is shown in the left half of the figure, which thus displays the risk-profit combination implied by each price. With the tax, the risk-profit points shift downward and to the right from $A$ to $A_1$, $B$ to $B_2$, etc. In the example here, we see that because of the tax, a higher price ($B_1$) is now
Figure 4

Changes in Expected Profit and Risk, Given a Sales Tax

Risk-Profit Indifference Curves

Expected Profit, $E(r)$

Profit-Price Profiles

Before Tax

After Tax

Risk, $\sigma$

Price, $P$
preferred to the current one \((\text{CSP}_1)\). The precise outcome is obviously dependent on the shape of the indifference curves, and it may also be affected by the measure chosen to represent risk.

The maximin, it may be noted, responds to the tax in the same way as expected profit, rising relatively for prices higher than the current one, as compared to lower prices. Use of this criterion would, accordingly, also militate toward a price increase. The shape of the 1.00 SSC, whose intersection with the SSC's determines the maximin value for each price, is critical for the outcome.

The foregoing remarks point only to tendencies. They do not preclude a continuation of the current price, nor do they rule out the possibility of a price reduction. However, other considerations suggest rather strongly that a price adjustment will be made. First, a tax at even a nominal rate on sales will, in the absence of a price adjustment, typically cut heavily with profits. In the situation of Figure 3 with the before-tax profit margin on sales equal to about 12%, the 5% tax has reduced profits by over 40%. But even with a 20% margin, which is well above that for most industries\(^ {11}\), a 5% tax would reduce profits by 25%. Confronted with the prospect of such a profit erosion, it seems likely that the firm will be drawn to a serious reassessment of its price-quantity position and to the contemplation of retrieving moves. The pre-tax profit that it has enjoyed may now come to be viewed as a minimum acceptable or target rate. Though the firm may insist on quite favorable odds before venturing after a higher rate, it may accept significantly greater risks to protect an established position.

Second, with the imposition of a tax, the SSC's are subject to reappraisal. The tax bears not only upon our firm, but also upon its competitors, and the SSC's reflect the firm's assessment of their actions as well as the actions of
households. Aware that its competitors are also concerned to avoid the burden of the tax, it may quite plausibly believe that opportunities for a price adjustment are now favorable. Such a belief would be reinforced by sales tax laws which, as in many states of the U.S., specify that the tax and the sales price must be separately quoted to buyers (Due and Friedlander, p. 367). This requirement would contribute to a presumption in the mind of each seller that his competitors will add the tax to prevailing prices. The resulting revised SSC's could well prompt an upward price adjustment.

Third, with the demand function unknown, formula pricing conventions in the form, for example, of some type of markup on prime costs, are likely to hold strong appeal and be frequently employed. Indeed, such conventions can be understood as a means of coping with uncertain demand, and their use would, in its absence, make little sense. Moreover, the industry-wide use of formula procedures gives to every firm a measure of safety in adjusting its price to compensate for the impact of a tax. Accordingly, so long as the firm employs a formula and treats the tax as a cost, an upward price adjustment is sure to follow.

These considerations, which apply regardless of what the details of Figures 3 and 4 may be, suggest that a price adjustment is highly likely, and the last two paragraphs in particular, along with the rotation effect on the SSC's, point to a price increase rather than a decrease. For a price decrease to be seriously considered, firms would have to hold expectations of a higher demand elasticity than is often likely to prevail.

The consequences of a price increase and of the use of formula procedures for the firm's ability to escape the burdens of the tax are discussed below in section IV.
III. AN INCOME TAX

In terms of our framework, the impact of a proportional income tax involves a more restricted set of considerations than does that of the sales tax. Given the levy of the tax, after tax profits, $\Pi_1$, can be expressed as a modification of (2) above,

\begin{equation}
\Pi_1 = PQ - (k + bQ) - i\Pi
\end{equation}

\begin{equation}
(4a) \quad \Pi_1 = [PQ - (k + bQ)](1 - i)
\end{equation}

where $i$ denotes the rate at which profits are taxed. Expression (4a) will in turn generate a new set of IPC's. As compared with the original IPC's, the new IPC's are displaced to the northeast. There is no rotation effect. The sole result of the tax is to cause a remembering of the original curves, so that profits at each PQ position are reduced by a percentage equal to the tax rate $i$. Thus, for example, with a 50% profits tax each of the IPC's in Figure 2 would fall to half its original value. Similarly, the expected profit associated with each price option would fall by 50 percent, and the dispersion corresponding to each expected profit would decline by an equal percentage. In terms of Figure 4, the pre-tax profit-price profile would shift downward, but without rotation. While the bias toward a price increase that existed with the sales tax will be absent, the downward and rightward shift of the expected profit-dispersion points leaves open the possibility of a price adjustment. As before, the shape of the profit-price profile, the shapes of the indifference curves, and the measure used of risk may importantly affect the outcome. However, for firms employing a single-valued criterion like the maximin or expected profit, there would be no disposition to change from the current price.

This conclusion and the rationale from which it derives are, it may be suggested, of quite secondary importance in assessing the firm's response to an income tax. Far more relevant are certain of the considerations noted
previously in connection with the sales tax. First, with the prospect of much reduced profits, it is probable that the firm — whether initially risk averting, seeking or neutral — will undertake a review of its position and of the price alternatives open to it. To protect its pre-tax position, it may now be amenable to price moves that it would not earlier consider. The CSP that heretofore has satisfied it may fail now to do so. Second, the firm will reassess its SSC's in light of its beliefs about how its near and distant competitors will react to the tax and its customers to a price change. It can perhaps less readily presume than in the case of the sales tax, that other firms will raise price. This is so because the tax is not levied on product prices, is not viewed by customers as an appropriate addition to such prices, and is not, unlike the sales tax, susceptible of mechanical treatment as an element of cost. In addition, a profits tax will bear differentially upon firms, depending upon their profit position, so that a uniform response by all firms can hardly be presumed. But the firm must necessarily be aware that other firms share with it a common concern to avoid the burdens of the tax and to discover a means to this end. Moreover, there is nothing about the tax that makes it immune to recognition by individual firms of formula or target pricing methods. Given a disposition to do so, it can proceed, much as with the sales tax though with a necessarily greater sensitivity for the actions of differentially situated competitors, to seek a restoration of pre-tax profits.

Will a price adjustment, if made, tend to be upward or downward? Our discretionary model offers no definite answer. The outcome depends upon the relationship of the SSC's to the IPC's, and a downward move cannot be ruled out. However, a few circumstances suggest an upward move as more likely. First, if the firm employs some form of target pricing, it will raise price.
Second, as discussed in the next section, the increases in sales that must, under a range of conditions, be realized to make a price decrease profitable imply higher sales elasticities than frequently in fact will prevail. If the firm's SSC's stand in proximate relation to what is realizable - not a requirement of the model - it will not often find the prospects of a price decrease encouraging. Third, a decrease in price tends toward an increase in costs and financial commitments which may not be quickly reversible, and this prospect, together with uncertainty about revenues, may discourage such a move. Fourth, where there are rivals to be reckoned with and a high potential for price instability, the firm's SSC's will be of a form unfavorable to a price decrease.

It perhaps deserves emphasis that the firm's position remains essentially discretionary, whether the tax it faces be a sales tax or an income tax. In both cases, it can initiate a price adjustment and will do so if only it believes there is sufficient chance that the adjustment will make a positive contribution to profit restoration. In this regard, and details aside, both types of tax are not basically different from other impositions that may, in the absence of responsive strategies, cut significantly into profits. Increases in raw material or labor costs, for example, also adversely affect the IPC's and encourage a review by the firm of its market position. A drop in sales at the current price, driving the firm leftward to a lower IPC, will similarly prompt such a review. The review need not, of course, be limited to price policy. It may extend also to marketing questions, including advertising and differentiation, and to questions of organization, supply sources and factor proportions. But these issues are beyond the scope of the present paper.

We cannot conclude from this brief analysis that the firm will respond to an income tax by raising its price or, if it does so, by what amount. But the circumstances cited indicate that certain of the consequences of the
tax resemble those of other impositions that are typically held to induce price increases. They indicate also that contrary to the findings of traditional theory, the firm does have "someplace to go" in response to the tax—almost as much so as in the case of a sales tax. A price adjustment, while not a necessary outcome, would appear to be a plausible expectation.16

IV. FORMULA PRICING

Formula pricing procedures, tempered occasionally as special circumstances may require, offer to the firm facing unknown market demand, and occupying thereby an essentially discretionary position, a convenient means for coping with shifting market conditions. Such procedures may be especially attractive to firms that must deal with changes in the cost of labor, materials or other components or which produce items that share common overheads. They may be attractive also to firms lacking well-defined expectations about product demand or fully determined risk-utility functions. In such circumstances, a formula might serve, partially or substantially, as an economic substitute for a succession of difficult and dubiously useful reviews of market conditions and price alternatives.17 The following few paragraphs discuss some implications of price formula usage in connection with the levy of a sales tax.

There are various formulae that firms might employ, and examples of two are briefly discussed below. Whatever the specific version used, we might suppose that, following the levy of a sales tax, it will lead the firm to implement some percentage increase in price. Sales will in turn be affected in accord with the objective possibilities of the marketplace. This set of possibilities might be represented by a realized sales curve (RSC) which, unlike the usual demand function, allows for the effects on the firm's sales of any concurrent price changes by other firms. (A point like the CSP in Figure 3 represents one point on an RSC.)
Let our RSC be of the constant elasticity form

\[ Q = aP^n \]

where \( n \) is the elasticity parameter \( (n \leq 0) \), \( a \) is a constant \( (a > 0) \) and \( Q \) and \( P \) have the meanings previously assigned. Then the realized revenue function is

\[ (5a) \quad PQ = aP^{n+1} \]

Using (1) as our cost function, the realized before tax profit function is

\[ (6) \quad \Pi_o = aP_o^{n+1} - (k + bQ_o) \]

where the subscripts denote the pre-tax price and quantity, i.e., the firm's pre-tax position at the CSP in Figure 2. With the levy of a sales tax, the firm increases its price by some fraction \( t \), which may or may not equal the rate of tax, and pays the government its due. Then after-tax profit may be expressed as

\[ (7) \quad \Pi_1 = aP_1^{n+1} - (k + bQ_1) - \frac{SP_1Q_1}{1+s} \]

where the subscripts indicate post-tax prices and quantities, and the last term on the right expresses the firm's tax obligation.

A few substitutions will provide a basis for comparing pre-tax profits with post-tax, post-price-adjustment profits.

\[ (8) \quad P_1 = P_o (1+t) \]

\[ (8a) \quad b = zP_o \]

\[ (8b) \quad k = mQ_o P_o \]

In (8) \( t \), in decimal form, specifies the markup addition by the firm on the pre-tax price. In (8a) \( z \) expresses the ratio of unit variable costs to the pre-tax price. In (8b) \( m \) is the ratio, at the pre-tax rate of output, of unit fixed costs to the pre-tax price. Substituting the relations into (7) and (8) and dividing the latter equation by the former yields the ratio of post-tax to pre-tax profits:
Equation (9) serves as the basis for Figure 5 below, which sets out for the firm, for selected sets of cost conditions, markup percentages and RSC elasticities, the relation of post-tax, post-price adjustment profits to pre-tax profits. Implicitly, the rate of profit on sales is also a determining factor, since this rate is given by $1 - z - m$. The rate of tax, $s$, is assumed in all instances to be .05.

Let us note first, in passing, that if the firm makes no price change in response to the tax -- $t=0$ -- it must fully absorb the imposition, with consequences for profits that depend on the rate of profit on sales. E.g., if the rate is 10%, a 5% sales tax will cut profits by half. More generally, profits will fall by the dollar amount of the tax. A more interesting case is that where $t=s$ -- a percentage increase in price exactly equal to the 5% tax. Such a procedure would appear as a plausible reaction by the firm to the tax, and might result from its use of a simple formula in which price is determined by summing the major components of unit cost -- in the present case unit prime cost, unit fixed costs and, where a tax is levied, the unit amount of the tax -- and adding a target unit profit. 18

Results from this procedure, along with others covering other values for $t$, are shown in Figure 5. All cases shown in the figure involve a rate of profit on (pre-tax) sales of .05 -- the approximate median ratio across some 22 industries (Fortune, p. 201). The curves labeled C and C' cover the case of $t=.05$. We see that elasticities greater than zero (disregarding the negative signs) produce profit declines as indicated, with larger declines for the $C'$
Figure 5
Profit Outcomes Using a Markup Formula, Given a 5% Sales Tax*

*With certain combinations of parameters, the profit ratio may turn negative, as in the lower right corner of the figure. These are cases in which, because of the tax, positive pre-tax profits have not only fallen but have turned into losses.
curve which represents a lower ratio of variable cost to price. Thus with 
z=.80 and \( \eta = -0.8 \), post-tax profits fall to 85 percent of the pre-tax level, 
whereas if \( z = .40 \) they fall to about 55 percent of the original amount. 19

The requirement with \( t = .05 \), of zero elasticity to preserve profits intact 
is severe. Yet this condition may be approximately met in the case of a 
general sales tax, which leaves relative prices unchanged, if households sus-
tain their real consumption of all items. Studies of consumer behavior offer 
evidence to support the idea of such consumer resistance to a cut in their 
consumption (Ferber, pp. 23-24).

The zero elasticity condition might also be met if households continued 
 to spend the same dollar amount after the tax and price increase as before 
them, while the government raised its outlays by the full amount of the sales 
tax revenue. In that event, aggregate expenditures would rise by the amount 
of the tax, with reduced household purchases offset by increased government 
purchases. But if at the other extreme the government instead withheld the 
sales tax revenue while households spent just the pre-tax amount, the outcome 
would be as indicated for \( \eta = -1.0 \). Profits would decline, and decline more the 
lower the values for \( z \) and the profit-sales ratio. In this case, an effective 
formula would require a markup on price in excess of the amount of the tax.

Firms in industries experiencing selective excises are more vulnerable, 
since in most instances, beyond the short run, they face elasticities greater 
than zero. How much greater is unclear. Data for a wide range of industries 
suggest considerable variation, but with elasticities ordinarily below unity 
and often significantly so (Houthakker and Taylor, ch. 4, and Watson, chs. 1-8). 
With selective excises, therefore, a markup over price greater than the rate 
of tax ordinarily will be required. An idea of the outcomes for markups of 
.075 and .10, given alternative RSC elasticities, can be gleaned from curves 
B and \( B' \) and \( A \) and \( A' \) in Figure 5.
A formula employing a markup on prime cost typically will yield an increase in price in excess of the rate of tax. That is, \( t > s \) will prevail. To avoid ambiguity, let us define the markup on prime cost as \( \frac{P}{b} = \frac{1}{z} \). Let us term this the gross markup and distinguish it from \( \frac{P}{P_o} = (1+t) \), which we shall call the net markup. Using this terminology, a value for \( z \) of .80 implies a gross markup of 1.25. If a tax at the rate of .05 is treated by the firm as a component of prime cost, \( z \) then rises to .85, and applying the gross markup gives \( .85 \times 1.25 = 1.0625 \). The last figure is our net markup, \( 1+t \), and the new price becomes \( 1.0625 \times P_o = P_1 \). Outcomes for \( t = .0625 \) are not shown in Figure 5 but can be quite roughly interpolated. They are clearly better for the firm than in the earlier case where \( t = .05 \). In similar fashion, a \( z \) of .40 instead of .80 gives a gross markup of 2.50 and a net markup of 1.125. Figure 5 does not give results for \( t = .125 \), but their general magnitude can be guessed from the figures for \( t = .10 \). Observe that the markup on prime cost formula tends to compensate for the handicap of low prime costs (in that costs do not decline much as output falls) by insuring higher \( t \) values as \( z \) declines.

Another way of examining markup implications is to ask, For any given value for \( t \), what elasticity is required to maintain post-tax profits exactly equal to pre-tax profits. In terms of (9) above, the equality of pre-tax and post-tax profits means that \( \frac{\Pi_1}{\Pi_o} = 1 \). Solving for \( \eta \) on this basis yields

\[
\log \left[ \frac{1 - z}{1 + t - z - \frac{s(1+t)}{1+s}} \right] = \log (1+t)
\]

(10) \( \eta = \frac{\log \left[ \frac{1 - z}{1 + t - z - \frac{s(1+t)}{1+s}} \right]}{\log (1+t)} \)

The fixed cost parameter, \( m \), and with it, implicitly, the profit-sales ratio, are absent from the expression. The equation stipulates, in effect, that \( \eta \) be such as to preserve intact the product of pre-tax volume multiplied by the absolute amount of pre-tax gross margin – unit fixed cost plus unit profit – and
it makes no difference to the outcome how this margin is divided between its two components. Thus, with the rate of tax given at .05, the requisite \( \eta \) is seen to depend entirely on the net markup, \( 1+t \), and on the unit variable cost parameter, \( z \). Values for \( \eta \) under alternative conditions are presented in Table 1.

**TABLE 1**

Revenue Elasticities Required for Equality of Post- with Pre-Tax Profits

<table>
<thead>
<tr>
<th>( z )</th>
<th>( t )</th>
<th>.05</th>
<th>.06</th>
<th>.075</th>
<th>.10</th>
<th>.125</th>
</tr>
</thead>
<tbody>
<tr>
<td>.20</td>
<td>.00</td>
<td>-.20</td>
<td>-.41</td>
<td>-.61</td>
<td>-.73</td>
<td></td>
</tr>
<tr>
<td>.40</td>
<td>.00</td>
<td>-.27</td>
<td>-.54</td>
<td>-.80</td>
<td>-.95</td>
<td></td>
</tr>
<tr>
<td>.60</td>
<td>.00</td>
<td>-.40</td>
<td>-.80</td>
<td>-1.18</td>
<td>-1.40</td>
<td></td>
</tr>
<tr>
<td>.80</td>
<td>.00</td>
<td>-.80</td>
<td>-1.56</td>
<td>-2.24</td>
<td>-2.59</td>
<td></td>
</tr>
</tbody>
</table>

Three points deserve note. First, except where \( z \) is relatively low, formula procedures that yield markups modestly in excess of the tax rate call for elasticities no lower than those prevailing in many industries. Thus, with \( t = .06 \) and \( z = .60 \), \( \eta \) can be as great (neglecting the sign) as -.40 without adversely affecting profits; with \( t = .075 \), \( \eta \) could be as high as -.80. Second, procedures involving markups on prime cost must often more than compensate for the tax. For example, as previously described, a \( z \) of .40 implies a gross markup of 2.50. If then a tax of .05 is incorporated into prime cost, the relevant \( t \) will be .125, and the associated "break-even" \( \eta \) will be a high -.95. For many firms which find themselves in this kind of situation, the elasticities actually confronted must be much lower. Third, observe that while for a given \( t \), say .06, a low \( z \) calls for a comparatively low elasticity, low \( z \)'s imply
high gross and net markups, and hence high t's. Using a prime cost formula therefore, the firm with a low z and hence minimal benefit from reduced costs as price rises and output contracts, may be no less-well situated than the firm with a high z.

A general conclusion from Figure 5 and Table 1 is that no formula is likely to be right in the sense of insuring, independently of the realized sales curve elasticity, the equality of post- with pre-tax profits. Yet a formula, to be useful to the firm, need not pass so severe a test. It need merely do as well as the firm might do if, with limited and uncertain information, it sought to make its decisions by some other means. In a dynamic world in which critical parameters change, the use of a formula tempered by judgment as circumstances permit, may serve as an economizing approach to otherwise difficult decisions. The potential attractiveness of a formula is enhanced if the firm treats it not as an instrument for providing definitive answers but as a convenient means for attaining acceptable first approximations. The initial outcome need not be a terminal one. It may take three or six months or longer for the consequences of a tax and a reactive price change to be perceived. When the information is in, the firm can take stock. If post-tax profits equal or exceed pre-tax profits, it may decide upon no further price change. If profits are lower, it can opt for a further adjustment and/or a formula revision. Insofar as firms behave in this way we would expect that, over time, prices would tend to rise by as much and sometimes more than the tax.

V. ALLOCATION IMPLICATIONS

In concluding, let us briefly compare the allocation implications of our discretionary pricing (DP) model with those of the standard monopoly (SH) model.
One way of doing this is to consider two firms that are identically situated, except that the SM firm knows its demand function in the usual objective sense while the DP firm does not. We continue to presume the case of constant marginal costs.

With the levy of a sales tax, the DP firm will, we assume, through formula usage or otherwise, raise its price by more than the SM firm and, other things equal, will experience a larger contraction in sales. However, the DP firm may occupy a different initial position, having either a higher or lower price, than the SM firm. Hence its post-tax price may be higher or lower, and its output lower or higher, than the latter's. Generally, therefore, the DP firm will earn lower profits, both in its pre- and post-tax positions than the objectively maximizing SM firm. With constant marginal costs, one cannot invoke "price equals marginal cost" as an allocation criterion - not if one accepts that total costs must be covered. Instead, one might use the criterion of "price equals average total costs" (including necessary profit). It is interesting that by this yardstick the DP firm would, except where its position chances to coincide with the SM firm, render a better showing - that is, it would exhibit smaller excess profits.

The levy of an income tax does not affect the optimizing position of the SM firm, whose post-tax price and quantity are equal to its pre-tax price and quantity. However, for reasons previously indicated, the levy of such a tax may well prompt a price rise by the DP firm. Again, the initial position of the DP firm may be higher or lower than that of the SM firm. But now, with no price adjustment by the SM firm, there is a greater likelihood that the latter's price will be equalled or exceeded by the post-tax price of the DP firm. Correlatively, the output of the DP firm will tend to be lower. Again, profits will be lower with the DP firm, as one would expect with any departure from an objective maximum, and this firm will appear - deceptively -
to better meet the "price equals average total cost" criterion.

The foregoing comparison necessarily presumes, (1) that the SM and DP firms are insulated from the competitive pressures of other firms - otherwise the demand function of the SM firm cannot be defined in the usual way, and (2) that the SM firm always, and the DP firm when its price is equal to or less than that of the SM firm, operate over an elastic range of the demand curve - otherwise there can be no equilibrium at positive output for the SM firm. However, these characteristics are not essential attributes of the DP model, and the responses of a DP firm to a tax, as described in this essay, do not depend on them.
Footnotes

1 Other circumstances accounting for non-maximization of pre-tax profits include a fear of prompting union wage demands, public utility regulation, fear of an unfavorable public image, and imperfect knowledge of costs or of the market. (See Herber, pp. 411-414.) A more formal hypothesis, with greater potential for generalization, that might be used to explain such behavior is that of sales maximization subject to a profit constraint (Baumol, pp. 53-55). Search and satisficing models (Simon) and managerial discretion models (Williamson) might be adapted to the same end.

2 In contrast to many analyses of pricing under uncertainty, our discussion does not make restrictive assumptions as to the distribution of the firm's subjective expectations or their relation to objective market possibilities. Neither does it make particular assumptions about the nature of the firm's utility function. It stresses instead, (1) the essential subjectivity of the firm's perception of its market opportunities, (2) the possible importance of the use by firms of price formulae or other pricing conventions, (3) the effect a tax may have in causing firms to reassess their market opportunities, and (4) the potential for a significant shift in the firm's attitude toward risk, following the imposition of a tax or other burden.

3 "...it is our view that the model of the price-quoter provides the most accurate and comprehensive portrayal... The firm of the real world quotes a price, albeit perhaps the 'market,' and to all intents and purposes waits to observe the buyers' reactions to that price..." (Horowitz, p. 412).

4 At the limit, this line segment could shrink to a point at the CSP, denoting full confidence by the firm that the exact current rate of sales would continue. Alternatively, the distance between the 1.00 and 0.00 end points might increase, indicating greater uncertainty about sales than Figure 1 shows. One would expect that the longer the decision or planning period, the longer would the line segment tend to be and the lower the firm's confidence in a continuation of sales at approximately the current rate.

5 "The decision-maker cannot know what state of nature will prevail... consequently, each strategy must be associated in his mind with a variety of outcomes to which he would attach probabilities. Thus we visualize a price-setter selecting from several sets of potential outcomes..." (Oxenfeldt, pp. 221-2).

6 Our framework would serve as well for the case in which the firm fixes its rate of output and allows price to adjust. But given the realities of most markets, this is a less interesting case.

7 Theories stressing the organization, goals and decision processes of the firm (e.g., Cyert and George) or its internal efficiency (Leibenstein) lead to a similar conclusion.
For the monopolistic firm, this would be a ceteris paribus curve. For the oligopolist, it would be a function that incorporated the reactions of rivals.

Let \( P^* \) be the price net of tax. Then the total tax take is \( sP^*Q \). Since

\[
P = (1+s)P^*\]

the tax take is \( sPQ \). Since

\[
\frac{\partial P}{\partial Q} = \frac{b(1+s) - P}{Q}
\]

which differs from the pre-tax derivative by the presence of \( 1+s \) in the numerator. With \( b \), or marginal cost, less than price, the slope of the IPC will be negative, and a positive \( s \) will result in lower negative values, i.e., counterclockwise rotation. Ours is the case of constant marginal cost, but the rotation effect is equally present with declining or rising marginal costs.

For the 500 largest industrial companies, the median return on sales (after taxes) was 4.8% in 1968 and 4.6% in 1969. For the mining industry, which had the highest return, the median figures were 18.2% in 1968 and 11.9% in 1969. (Fortune, p. 201).

An occasional firm, anticipating price rises by competitors in response to a sales tax, might expect gains for itself in holding price constant or lowering it. Such a belief would simply affirm that the predominant tendency of firms is toward a price increase. A price increase seems generally to be assumed by economists. Due and Friedlander, for example, remark (p. 369), "...casual observations suggest that retailers almost universally add the tax to the price, except in some instances on large ticket items..." The empirical evidence they cite (pp. 368-9) supports forward shifting.

If a sales tax is levied at the rate of, say, 5%, if unit prime costs are one-half the pre-tax price, and if a 5% price cut is to suffice to restore pre-tax profits, then the elasticity of demand over the relevant range must, at the least (neglecting the sign), be about \(-4\). For a 10% price cut to suffice an elasticity of at least \(-3\) would be needed. If unit prime units are but one-quarter the pre-tax price, the respective elasticity figures become \(-2.7\) and \(-2\).

Full-loss-offsets are implied in this formulation. This assumption, though it may be important to the results of particular cases, does not affect the discussion here.

The notions of \( X \)-inefficiency and organizational slack are germane here. (Leibenstein, Cohen and Cyert, PP. 333-335). Pressure on profits is apt to lead to a review of practices previously regarded as satisfactory or even optimal. The discretionary nature of the firm's position on the supply side arises in part from its lack of full information about costs and of the cost consequences of a departure from current output levels. Although this paper assumes costs to be fully known, the framework employed could readily be extended to deal with cost uncertainty.
Whether and to what extent the corporate income tax is in fact shifted continues to be a controversial matter, with empirical studies producing conflicting results. (E.g., Kryzaniak and Musgrave, Gordon, Oakland.)

The rather common use of formula-like procedures as an ingredient - though not necessarily the only ingredient - in the pricing process is suggested by the findings of a number of studies. (E.g., Barback; Kaplan, Dirlam and Lanzillotti; Hall and Hitch; U.S. Senate Committee on the Judiciary.)

More specifically, suppose the firm determined price by the following formula:

\[ (8c) \; P_0 = b + \frac{k}{Q} + \frac{\Pi^*}{Q} \]

where \(\frac{k}{Q}\) represents unit fixed cost and \(\frac{\Pi^*}{Q}\) represents desired unit profit, both calculated on the basis of some standard volume \(Q\). If a sales tax is levied in the unit amount of \(sP\), this sum might become another cost component in the price formula, being treated in effect like another element of prime cost, so that

\[ (8d) \; P_1 = sP_0 + b + \frac{k+\Pi^*}{Q} \]

and substituting (8c) into (8d) gives

\[ (8e) \; P_1 = P_0 (1+s). \]

When \(s=t\), this is of course the equivalent of (8) in the text.

Were the profit-sales ratio .025 instead of .05, the three sets of curves would fall steeply and, in the instant example, profits would decline to 69 percent of this pre-tax level. However, the \(B + B'\) curves would commence on the ordinate from a higher point - 2.00 instead of 1.5. For the \(A + A'\) curves, the figure would be 3.00 instead of 2.00.

If profits unexpectedly rise, it may even be content to sustain the new price, avoiding the risks associated with a change, in face of some subsequent circumstance, like a cost increase, that adversely affects profits.
References


Fellner, H.J., Competition Among the Law, New York, ... Knopf, 1949.


