Taxation, Saving, and Labor Supply:
Cross-Section Evidence

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Abstract

This study applies cross-section data to the estimation of a simultaneous model of leisure and saving behavior. Estimates of the parameters of a two-period CES utility function are used to derive compensated and uncompensated wage and interest rate elasticity estimates. The study finds that while saving and leisure are nonresponsive to changes in the interest rate, they are highly responsive to changes in the wage rate. It follows that the taxation of wage income creates large deadweight losses while the taxation of interest income has little distortionary effect on behavior. This has important implications for the taxation of wages and interest.
TAXATION, SAVING, AND LABOR SUPPLY: CROSS-SECTION EVIDENCE

In analyzing the effect of taxation on saving and labor supply, most studies neglect the interrelationship between work and saving.\(^1\) It is generally assumed that labor supply is independent of the interest rate and saving is independent of the wage rate. Neglecting the cross-substitution possibilities between work and saving can lead to erroneous conclusions regarding the effect of taxation. One purpose of the present study is to develop an analytical framework which views the work and saving decisions as part of a simultaneous decision problem.

A second objective of the present study is to apply cross-section data to the study of the work and saving decisions. It has long been common to analyze the effect of taxation on labor supply using cross-section data since several good sources of household data containing information on hours worked, wage rate, and income are available. However, household data sets typically lack information on saving and interest rates. As a consequence, studies of taxation and saving routinely rely on aggregate time series data.

The present study uses the 1986 Panel Study of Income Dynamics which applies to the 1984 interviewing year. Saving is inferred as a residual between family income and various
consumption items such as taxes, mortgage payments, rent, and food. The interest rate is imputed on the basis of state of residence. The compensated wage and interest elasticities are calculated based on utility parameter estimates derived from a generalized least squares estimation of a leisure/saving model. The model and its estimation are described in section I, the econometric results are presented in sections II and III, and their implications are explored in section IV. Conclusions are contained in section V.

I. The Model and Its Estimation

A. The Theoretical Assumptions

The model of this study is a simple two-period life cycle model. The individual is assumed to choose among consumption, saving, and leisure during the first (or working) period and to consume the savings principal plus interest during the second (or retirement) period. The tax system influences the decision in several ways: by reducing the net wage rate, by reducing the net non-wage income of the individual, and by reducing the net interest rate. The individual's problem is to choose leisure and saving during the first period so as to maximize a utility function defined over first-period consumption, \( C_1 \), first-period leisure, \( L_1 \), and second-period consumption, \( C_2 \), subject to first-and second-period budget constraints.
Assuming the utility function takes the constant elasticity of substitution (CES) form, the problem is one of maximizing:

\[ U = [a_1 C_1^{-b} + a_2 L_1^{-b} + (l-a_1-a_2)C_2^{-b}]^{-1/b} \]

(where \( a_1 \) and \( a_2 \) are positive constants less than one and \( b \) is a constant greater than minus one) subject to the following budget constraints:

\[ (2a) \quad C_1 = w(k-L_1) + A - S \]

\[ (2b) \quad C_2 = (1+r)S \]

(where the wage rate, \( w \), non-wage income, \( A \), and the interest rate, \( r \), are measured net of taxes, \( k \) is the total time available, and \( S \) is saving). When the constant \( b \) is equal to zero, the utility function becomes Cobb-Douglas.

The first order conditions to this maximization problem may be found by differentiating (1) with respect to \( L_1 \) and \( S \) subject to (2a) and (2b). After rearranging, these may be written as:

\[ (3a) \quad L_1 = M_1^{-s} w^{-s} C_1 \]

\[ (3b) \quad S = M_2^{-s} (1+r)^{s-1} C_1 \]
where $M_1 = a_1/a_2$, $M_2 = a_1/(1-a_1-a_2)$, and $s = 1/(1+b)$.

B. The Estimation Model

The theoretical model described in the previous subsection can be put in estimation form by dividing through equations (3a) and (3b) by $C_1$ and taking logs:

\[
\begin{align*}
(4a) \quad & \ln(L_1/C_1) = -s \ln M_1 - s \ln w + e_1 \\
(4b) \quad & \ln(S/C_1) = -s \ln M_2 + (s-1) \ln (1+r) + e_2
\end{align*}
\]

where $e_1$ and $e_2$ are disturbance terms introduced to capture omitted explanatory influences. Since $s$, the elasticity of substitution, is always positive, the model states that the leisure-consumption ratio is negatively related to the wage rate (i.e., people with higher wage rates have lower leisure-consumption shares). However, since $s$ may be either greater than or less than one, the relationship between the saving-consumption share and the interest rate is indeterminate.

The ordinary least squares estimation of equations (4a) and (4b) on a cross-section sample of individuals would be inappropriate for two reasons. First, the slope of equation (4a) is equal to $-s$ while the slope of equation (4b) is equal to $(s-1)$ requiring that the sum of the two slopes equals minus
one. This constraint across equations must be taken into account in the estimation. Second, possible correlation between the two error terms, e₁ and e₂, arising from the omission of variables or from heteroskedasticity must be accounted for. As described in the next section, this will be accomplished by using constrained generalized least squares as the estimation technique.

The constrained estimates of the intercept and slope coefficients of equations (4a) and (4b) can be used to derive estimates of the utility parameters b, a₁, and a₂. In particular, using the definitions of M₁, M₂, and s, yields the following formulas for the utility parameters:

\begin{align*}
(5a) \quad & b = (1-s)/s \\
(5b) \quad & a_1 = M_1M_2/(M_1 + M_2 + M_1M_2) \\
(5c) \quad & a_2 = M_2/(M_1 + M_2 + M_1M_2)
\end{align*}

which, as shown in the next section, can be used to indirectly estimate the compensated wage and interest elasticities.

C. The Uncompensated and Compensated Elasticities

The next step is to solve equations (3a) and (3b) for L₁ and S independently of each other. This is done by
substituting for $C_1$ from equation (2a) and simplifying. The result is:

(6a) $L_1 = M_1^{-S} w^{-S} (wk + A)/D$

(6b) $S = M_2^{-S} (1+r)^{1-r} (wk + A)/D$

where $D = 1 + M_1^{-S} w^{1-S} + M_2^{-S} (1+r)^{1-S}$.

The uncompensated wage and interest elasticities can be derived from equations (6a) and (6b) by differentiating the equations by $w$ and $r$, respectively, and applying the standard elasticity definitions. This gives:

(7a) $e_{Lw} = -s + (wk/(wk + A)) - ((1-s)M_1^{-S} w^{1-S}/D)$

(7b) $e_{Sr} = -(1-s)r(1 + M_1^{-S} w^{1-S})/(1+r)D$

It can be seen that the uncompensated wage and interest elasticities depend on the wage and interest rates, on non-work income, and on the parameters of the utility function.

The compensated elasticities follow directly by accounting for the income effect:

(8a) $e_{Lw}^{\text{COMP}} = e_{Lw} - (w(k-L_1)/L_1)(\partial L_1/\partial A)$

(8b) $e_{Sr}^{\text{COMP}} = e_{Sr} - (r/(1+r))(\partial S/\partial A)$
After substitution and simplification, this yields:

\[(9a) \quad e_{Lw}^{\text{COMP}} = e_{Lw} - \left(\frac{wk}{wk+A}\right) + \left(M_1^{-s}w^{1-s}/D\right)\]

\[(9b) \quad e_{Sr}^{\text{COMP}} = e_{Sr} - \left(rM_2^{-s}(1+r)s^{-1}/(1+r)D\right)\]

Both sets of wage and interest elasticities can be calculated for each household from the estimates of \(s\), \(M_1\), and \(M_2\) and from the known values of \(w\), \(A\), and \(r\). Maximum time available, \(k\), is set at 8,760 (24 hours per day times 365 days per year).

D. The Data and Estimation Technique

Data are from the 1986 Michigan Panel Study of Income Dynamics (PSID) for the 1984 interviewing year. Unmarried persons, the self-employed, families on welfare or social security, families with head of household over 60 or under 20 years of age, and families with an unemployed head of household are excluded from the estimation sample. This provides for a subsample of households whose decisions are least likely to be distorted by education or retirement decisions, households for which wage rate data are available, and households whose work/saving decisions are not biased by high implicit marginal tax rates (as with AFDC recipients, say). The estimation subsample includes 1,675 households.
Saving is defined as a residual between total family income and major consumption items. The consumption items available for exclusion are total federal income taxes, the family's annual mortgage payment, annual property tax for homeownering families, annual rent for renting families, and annual food expenditures at home and away from home. Hence, most tax, shelter, and food costs are netted out of income to arrive at a proxy for saving. Other items of consumption such as transportation, entertainment, and clothing are not available in the data for exclusion. There is no way of knowing to what extent this may bias the estimation results.

The interest rate is imputed to each family on the basis of their state of residence. The interest rate used is the median rate on time and savings deposits paid by banks in the family's state. The rate is a composite rate on total time and savings deposits calculated by dividing interest paid on deposits by average interest bearing deposits.\(^3\)

Variables are put on an after tax basis by multiplying by one minus the family's marginal tax rate. The marginal tax rate is imputed for each family in the sample based on the family's taxable income, number of exemptions, and the tax table used. Taxpayers are either assigned the standard deduction (zero-bracket amount) or an average itemized deduction for their taxable income bracket depending on whether or not the family itemizes deductions. For those families itemizing deductions, the amount of the itemized deduction is
calculated by applying an average fraction of income determined from the 1982 Statistics of Income -- Individual Tax Returns to the family's taxable income. The deduction for married couples when both work is calculated by subtracting from gross income 10% of the earned income of the lesser-earning spouse regardless of whether the couple itemizes or not.

Equations (4a) and (4b) are estimated using joint generalized least squares taking account of the restraint across equations. The maximum likelihood technique uses estimates of the covariance across residuals to increase the efficiency of estimation. The results of the estimation are described in the next section.

II. Econometric Results

A. Estimation Results

The generalized least squares estimation results for equations (4a) and (4b) are shown in Table 1. The dependent variables, the log of the leisure-consumption ratio and the log of the saving-consumption ratio, are related to the explanatory variables, the log of the wage rate and the log of one plus the interest rate, taking account of possible correlation between the two error terms and a cross-equation constraint on the slope coefficients. As seen in Table 1, the estimated slopes
# Table 1

**Generalized Least Squares Estimation Results**

For the Leisure Saving Model

(*t*-ratios in parenthesis)

<table>
<thead>
<tr>
<th></th>
<th>Equation 4a</th>
<th>Equation 4b</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variable</strong></td>
<td>ln($L_1/C_1$)</td>
<td>ln($S/C_1$)</td>
</tr>
<tr>
<td><strong>Explanatory variables:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>1.010</td>
<td>0.496</td>
</tr>
<tr>
<td></td>
<td>(22.22)</td>
<td>(36.61)</td>
</tr>
<tr>
<td>ln $w$</td>
<td>-0.821</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>(34.49)</td>
<td></td>
</tr>
<tr>
<td>ln $(1+r)$</td>
<td>--</td>
<td>-0.179</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-7.51)</td>
</tr>
<tr>
<td><strong>System Weighted R-Square:</strong></td>
<td>0.264</td>
<td></td>
</tr>
</tbody>
</table>

**Utility parameters:**

- $b$: .218
- $a_1$: .150
- $a_2$: .574
- $s$: .821
are of the expected signs and meet the constraint that they sum to minus one. All of the estimated coefficients are significantly different from zero at the .99 level of significance and the system weighted R-squared statistic suggests a reasonably good fit for cross-section data.

The implied utility function parameters are shown at the bottom of Table 1. The substitution parameter, b, whose estimated value is .218, can range between -1 and infinity. If b is equal to zero, the CES utility function reduces to the Cobb-Douglas form. The parameter is used to calculate the elasticity of substitution defined as 1/(1+b). Its value of .821 greater than zero suggests a relationship of substitutability among present consumption, leisure, and future consumption.

B. Elasticity Results

The wage and interest elasticities measure the responsiveness of leisure and saving to one percent changes in the wage and interest rates, respectively. The uncompensated elasticities reflect the operation of both the income and substitution effects while the compensated elasticities control for changes in income. The uncompensated elasticities, calculated according to equations (7a) and (7b), and the compensated elasticities calculated according to equations (9a) and (9b), are shown in Table 2.
Table 2
Wage and Interest Elasticities

Uncompensated Elasticities:

\[ e_{Lw} = -0.027 \]

\[ e_{Sr} = -0.008 \]

Compensated Elasticities:

\[ e_{Lw}^{COMP} = -0.306 \]

\[ e_{Sr}^{COMP} = -0.022 \]
The estimated uncompensated wage elasticity, $e_{LW}$, is negative and small. Theory does not tell us the expected sign of this coefficient since it is the result of a substitution effect adverse to leisure and an income effect favorable to leisure. The fact that its estimated value is small and negative tells us that the substitution effect slightly overpowers the income effect.

For policy analysis, the compensated wage elasticity is a more meaningful concept than the uncompensated elasticity since the compensated elasticity indicates what would happen to leisure with a change in the wage rate if income were changed so as to hold utility constant. Its estimated value, $-0.306$, is moderately large and negative, implying, for example, that a ten percent increase in the wage rate compensated by a decrease in income so as to hold utility constant would result in a 3.06 percent decrease in the consumption of leisure.

The estimated value of the uncompensated interest rate elasticity, $e_{r}$, is likewise small and negative. An increase in the interest rate causes a small decrease in saving. Even though this result seems counter-intuitive, in fact it is consistent with theory since an increase in the interest rate has an indeterminate effect on present consumption and on leisure. Depending on the effect of the interest rate on present consumption and leisure, an increase in the interest rate may increase or decrease saving.
Again, for policy purposes, it is the compensated interest rate elasticity that is of greatest interest. Its estimated value is negative but very small. According to the estimation results, a ten percent increase in the interest rate leads to a .22 percent decrease in saving when income is changed so as to hold utility constant. A comparison of these results with those of the literature is presented in the next section.

C. Comparison with the Results of Other Studies

C.1 Wage Elasticities

Since wage elasticities in the literature are most often reported in terms of work rather than leisure, it is first necessary to convert the leisure-wage elasticities reported in Table 2 into work-wage elasticities. Since work and leisure are related by $H_1 = k - L_1$, it follows that the uncompensated and compensated wage elasticities are related as follows:

\[(10a) \quad e_{Hw} = - e_{Lw} \frac{L_1}{H_1} \]

\[(10b) \quad e_{Hw^{\text{COMP}}} = - e_{Lw^{\text{COMP}}} \frac{L_1}{H_1} \]

With appropriate substitutions, the results of this study suggest that the uncompensated work-wage elasticity is .079 and the compensated work-wage elasticity is .910.
The early econometric studies of the labor supply of prime age males found slightly backward bending labor supply curves as would be implied by small, negative work-wage elasticities. For example, Ashenfelter and Heckman (1973) who used 1967 data on male heads of families found a work-wage elasticity of -.150 and a compensated work-wage elasticity of .120. More recent studies which are more sophisticated in their treatment of income taxation tend to find larger compensated work-wage elasticities.

Hausman (1981), using 1975 data from the Michigan Survey of Income Dynamics, an older version of the data set used in this study, found an uncompensated work-wage elasticity close to zero and a moderately large compensated work-wage elasticity of .17. Other recent studies, as summarized by Hausman (1985), report uncompensated work-wage elasticities ranging from -.13 to .09 and compensated work-wage elasticities ranging from -.08 to .20.

The uncompensated work-wage elasticity estimated in this study (.079) is within the range of these recent labor supply studies. However, the compensated work-wage elasticity estimated here (.910) is considerably larger than compensated work-wage elasticities found by other studies.

C.2 Interest Rate Elasticities

In one of the earliest studies of the saving-interest rate relationship, Wright (1967) obtained estimates of the
compensated saving-interest elasticity of 0.18 to 0.27. In contrast, a later study by Blinder (1975) found compensated saving elasticities near zero, while Boskin (1978) estimated saving elasticities in the range 0.30 to 0.60 with a preferred value of 0.40. However, it is not clear from the specification that Boskin's estimated elasticities are compensated elasticities.\

Boskin's results have generated a great deal of controversy. Since he used time series data, it was necessary to correct the market rate of return for inflation. This requires measuring the expected inflation rate, which Boskin calculated as a weighted average of past inflation rates. In a recent study, Blinder and Deaton (1985) used an alternative method for computing the expected inflation rate and found that the net real rate of interest has little impact on saving.

The results of this study also suggest that changes in the rate of interest have little effect on saving. Both the estimated uncompensated interest rate elasticity (-.008) and the estimated compensated interest rate elasticity (-.022) are very close to zero.

III. Cross-Effects

A major difference between the present study and earlier studies is that earlier studies assume away cross-effects while the present study incorporates them. Hence, the present
estimation can be used to measure the effect of changes in the interest rate on leisure and of changes in the wage rate on saving.

Following the earlier methodology for calculating the own wage and interest elasticities, differentiate equation (6a) by r and (6b) by w, and apply the standard cross-elasticity definition. This gives:

\[ e_{lr} = (1-s)rM_2^{-s}(1+r)^{S-1}/D (1+r) \]  

\[ e_{sw} = (wk/(wk+A)) - ((1-s)M_1^{-s}w^{1-s}/D) \]

The compensated cross-elasticities are computed from the uncompensated cross-elasticities by taking account of the income effect:

\[ e_{lr}^{COMP} = e_{lr} - (rM_2^{-s}(1+r)^{S-1}/(1+r)D) \]  

\[ e_{sw}^{COMP} = e_{sw} - (wk/(wk+A)) - (M_1^{-s}w^{1-s}/D) \]

The calculated values for these elasticities based on the parameter estimates from this study are shown in Table 3. The uncompensated elasticities show that leisure is nonresponsive to changes in the interest rate while saving increases rather substantially as the wage rate increases. A ten percent increase in the wage rate causes a 7.94 percent increase in
Table 3
Estimated Cross-Elasticities of Leisure and Saving

Uncompensated Cross-Elasticities:

\[ e_{Lr} = 0.002 \]
\[ e_{Sw} = 0.794 \]

Compensated Cross-Elasticities:

\[ e_{Lr}^{COMP} = -0.011 \]
\[ e_{Sw}^{COMP} = 0.515 \]
saving. The compensated elasticities likewise show a rather small cross-substitution elasticity between leisure and the interest rate but a fairly large cross-substitution elasticity between saving and the wage rate.

Hence, this study finds that not only does an increase in the wage rate tend to have a large compensated effect on leisure, it also has a large compensated cross-effect on saving. Conversely, an increase in the interest rate has a small compensated effect on saving and a relatively small compensated effect on leisure. The implications of these findings for taxation are discussed below.

IV. Implications for Taxation

A major finding of this study is the discovery of relatively large compensated own- and cross-wage elasticities and relatively small (near zero) compensated own- and cross-interest rate elasticities. This finding has important implications for the deadweight loss of the personal income tax. By reducing the disposable wage and interest rates, the personal income tax distorts economic behavior, causing a loss in efficiency. The efficiency loss associated with a distortionary tax is defined as the deadweight loss (or excess burden) of the tax.

The concept of consumer surplus is often used for calculating the deadweight loss of a tax. In general,
deadweight loss per dollar of tax revenue (sometimes called the efficiency loss ratio) is measured by:

\[(11) \quad \text{Deadweight Loss/Tax Revenue} = \frac{1}{2} t \, e^{\text{COMP}}\]

where \(t\) is the marginal tax rate.\(^5\) The deadweight loss per dollar of tax revenue varies proportionally with the marginal tax rate and with the compensated elasticity of demand.

The personal income tax reduces both the disposable wage rate and the disposable interest rate, creating two sources of deadweight loss. The estimated compensated own- and cross-wage and interest rate elasticities can be used to calculate an estimate of the deadweight loss of the income tax.\(^6\) The estimates from this study indicate that the loss per dollar of wage tax revenue is \(.177 \, (.5 \times .248 \times 1.424)\) and the loss per dollar of interest tax revenue is \(.001 \, (.5 \times .248 \times .011)\). The government could have collected \$1.18, with exactly the same effect on consumer utility, had it used a lump-sum tax. It also implies that the deadweight loss from taxing wages is much greater than the deadweight loss from taxing interest. The government could reduce the deadweight loss of the personal income tax by reducing the tax on wages and increasing the tax on interest.

The above results must be viewed with caution since they are based on the assumption that the compensated demand curves are linear (or that the tax change is small enough that linear
approximation are acceptable). Since we are calculating a deadweight loss associated with a .248 marginal tax rate, the estimated deadweight losses may be biased.

V. Conclusions

This paper presents an empirical model of labor supply and saving that allows for cross-substitution between the two activities. Estimates of the own- and cross-substitution wage and interest elasticities suggest that both work and saving behavior are much more responsive to changes in the wage rate than to changes in the interest rate. If this is correct, then wage taxation is likely to create larger efficiency costs than is interest taxation. Since these results are based on a highly simplified model, they must be viewed with caution.

In the model, saving is defined as a residual between major consumption items and income, and the interest rate is imputed on the basis of state of residence. In reality, there are many vehicles for saving, each with its own interest rate, and each taxed at a different tax rate. As several authors have pointed out, it is an oversimplification to speak of "the" interest rate elasticity of savings when, in fact, there are a wide range of elasticities.  

Another simplification of the model is its focus on the labor supply response of the head of household. Several studies have shown that female labor supply is likely to be
more sensitive to tax changes than is male labor supply. An important extension of the model would be to include the labor supply responses of secondary family workers. Likewise, the model could be extended to consider different aspects of labor supply such as job choice and human capital investment.

The motivation behind these simplifications was to make the model empirically tractable. Research of the long-run effects of taxation is still in an early stage of development. Better cross-section data sets on savings and interest rates and more realistic models of family work and saving behavior are sorely needed.
REFERENCES


1. For a comprehensive analysis of recent literature on taxation and labor supply, the reader is referred to Killingsworth (1983), and for a review of recent work in the area of savings and taxation, see Kotlikoff (1984).

2. \( e_{Lw} = \left( \frac{\partial L}{\partial w} \right)(w/L) \) and \( e_{Sr} = \left( \frac{\partial S}{\partial r} \right)(r/S) \).


6. \( e_w^{COMP} = e_{Hw}^{COMP} + e_{S_w}^{COMP} = .909 + .515 = 1.424 \) and \( e_r^{COMP} = e_{Hr}^{COMP} + e_{Sr}^{COMP} = .033 - .022 = .011 \).

