A Sequential Equilibrium Model of Cost Overruns in Long Term Projects

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WORKING PAPER SERIES ON THE POLITICAL ECONOMY OF INSTITUTIONS
NO. 21

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign
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Abstract

We consider a repeated contract game between a sponsor and a contractor concerning a large scale project where the project requires a number of tasks to be completed before the benefit from the project can be realized. There is cost uncertainty and the contractor has private, task specific information which is relevant in cost determination. Moreover the contractor supplies effort which affects the cost of completing a task. Thus the sponsor must resolve both moral hazard and adverse selection problems in designing the remuneration scheme offered to the contractor. We focus on the case where the sponsor cannot precommit to compensation per task and where the contractor is not bound to complete the project. We demonstrate that the equilibrium compensation path rises as the project nears completion giving the appearance of cost overruns towards the tail end of the project. We also consider several extensions of the basic model which amplify on these results.

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Introduction

Anyone with a rough familiarity of large scale contracts sponsored by government agencies is well aware that such projects often involve cost overruns. The procurement of new weaponry and the development of new nuclear power plants offers just two of the more prominent examples where cost overruns appear common. The 'horror stories' abound and frequently receive extensive media coverage. From the vantage of professional economists this phenomenon offers two distinct sources of interest. First, is there a satisfactory explanation? Such an explanation would obviously have to account for whether it matters that the sponsorship of such projects is typically by a government agency rather than by private enterprise and would also have to account for whether it matters that the scale of the project is usually very large rather than not so large. Second, is this something that economists should be worrying about? That is, do cost overruns constitute prima facie evidence of gross inefficiency in government procurement? In more popular jargon, is the public being bilked over and over again and, if so, are there any recommendations that we as economists can make as partial remedy?

In this paper we provide a positive model which predicts cost overruns as the sequential equilibrium outcome of a dynamic contract game concerning a large scale project between a single sponsor and contractor.

1 A sampling of one month of the Wall Street Journal has provided two examples of such horror stories: The stealth bomber program (February 25, 1988 p.3) and the Comanche Peak nuclear power plant (February 16, 1988, p.7)
We follow the optimal regulation literature, Baron and Myerson (1982), Baron and Besanko (1984) and (1987), Laffont and Tirole (1986), et. al., in assuming that this contracting problem contains elements of both adverse selection and moral hazard. That is, we assume the contractor has some private information as to project cost which is unobservable by the sponsor and the contractor makes some decisions over the inputs it supplies in completing the project based on this private information. We also follow the optimal regulation literature in assuming that the sponsor's benefit from completion of the project is publicly known. Other recent papers dealing specifically with procurement, e.g., Riordan (1986) and Tirole (1986) have focused on the contracting problem under bilateral asymmetric information, where the sponsor's valuation as well as the contractor's cost is uncertain. Because we are interested in the underlying dynamics rather than in mechanism design issues per se, we take a simpler route in the development of the model by focusing exclusively on cost uncertainty.

The formulations in the above mentioned optimal regulation papers are static and have the feature that both benefits and costs are realized simultaneously. We depart from the optimal regulation literature in our assumption that the project requires a multiplicity of tasks. In our model there is a cost realization after each task has been completed but the benefit does not accrue to the sponsor until the entire project is finished. The moral hazard in our paper arises because upon completion of each task

\[ \text{2 See Baron (1988) for a dynamic version of the optimal regulation model. The only paper that we are aware of which specifically deals with procurement in a dynamic context is Lewis (1986). Lewis considers the Bayes equilibrium of a repeated contract game where the sponsor learns over time about the contractor's private information and the sponsor may cancel the project at any time if the sponsor's beliefs about this private information are sufficiently pessimistic. Lewis' model does predict cost overruns as equilibrium outcomes but he takes the sharing rule by which the sponsor is compensated to be parametric. See note 12.} \]
the contractor sends a signal to the sponsor as to the cost involved. This signalled cost accurately reflects the cost reducing effort provided by the contractor. However, signalled cost is but one component of the contractor's actual cost in completing the task. The contractor faces a cost from taking the effort which affects the signalled cost. This effort cost component is not observed by the sponsor since it depends on the task specific information. Moreover, the sum of the effort and signalled cost may or may not be minimized by the contractor taking cost reducing effort, depending on the realization of the task specific information. Since we model the sponsor as the residual claimant, the sponsor has incentive to design the contract so as to minimize the sum of the effort and signalled cost. But since the remuneration the contractor receives is a function of the signalled cost only, the sponsor must design the contract to induce self-selection on the part of the contractor.

In the absence of any nonlinearity in the contractor's utility function a fixed payment, independent of the signalled cost, would indeed induce self-selection. However, we assume that the contractor is not bound to complete the project and can drop out at any time prior to project completion. In designing the contract, the sponsor is well aware that the contractor has the option to separate. Hence, fixed payment schemes will not be optimal because such schemes will induce excessive separation. Holmstrom (1983) and Harris and Holmstrom (1982) obtain a similar result in a labor contract model where the reservation wage is random. The conclusion these authors obtain is that the contract wage is downward rigid, but adjusts in an upward direction when the realization of the reservation wage is particularly high. The effect is to create a "tilting" in
the employee compensation scheme, i.e., the wage-earnings profile is steeper than the reservation wage profile, whereby rent accruing to the employee is increasing in seniority. Lazear (1979) and (1981) obtains a similar result when the problem is the employee effort rather than the employee quit moral hazard. In Lazear's model the tilting arises because the firm wishes to have its employees post implicit bonds which are forfeited in the event that workers are caught shirking.3

In these dynamic labor contract models it is assumed that the firm can credibly commit to the entire lifetime compensation scheme of its employees. In fact most of the contracting literature has adopted this asymmetric approach where the firm has total precommitment power while workers have no precommitment power.4 In our model we assume that neither the sponsor nor the contractor has a precommitment capability. That is, the possibility that the contractor drops out is accompanied by the chance that the sponsor will terminate the project and by the fact that the sponsor is free to alter the contractor's remuneration function after each task has been completed.5 We show that cost overruns can be viewed as the equilibrium outcome of such a contract model.6

3 Other authors have explained tilting in the employee compensation scheme as a consequence of the firm's desire to screen workers for specific attributes. For example see Salop and Salop (1976).
4 However, see Macleod and Malcomson (1987) for a noteworthy exception.
5 Since the sponsor can alter the remuneration function after the completion of each task, the sponsor can induce the contractor to drop out by making the remuneration sufficiently unattractive. This is the method by which the sponsor cancels the project in our model.
6 This result is somewhat surprising in that Arvan (1988) has shown, in a Lazear-style shirking model, that when the firm does not have a precommitment capability with regard to the wage it offers the equilibrium wage-earnings profile tends to be flat.
The intuition behind the tilting which occurs in our model is that the sponsor's attitude towards project completion changes with the number of tasks remaining. Towards the end of the project the sponsor is 'locked in,' the costs associated with contractor remuneration for already completed tasks are sunk and the sponsor will fund the project henceforth, regardless of the remaining costs involved. During lock-in the sponsor's prevalent concern is the threat of contractor drop-out. As a result the sponsor is willing to make high payments to the contractor during lock-in. The contractor can expect to earn some quasi-rents during lock-in since the remuneration scheme satisfies self-selection. Early on in the contract the sponsor is less concerned about project completion and will cancel the project unless tasks come in at low cost. In the early stages the equilibrium remuneration scheme is cost plus, i.e., the contractor is paid the minimum to satisfy the drop-out moral hazard constraint. During this period there are no rents earned by the contractor. The overall effect is to make payments rise over time as tasks are completed, giving the appearance of cost overruns toward the end of the project.

The remainder of the paper is organized as follows. In the next section we develop the basic model. Section 3 is devoted to an analysis of sequential equilibrium of the dynamic contract game. In section 4 we provide three extensions of the basic model to separately consider the effects of contractor risk aversion, the effects of noise in the cost signalling process, and the effects of contractor impatience. Finally, we offer a brief conclusion in section 5 where we take up the policy implications of our analysis.
We consider the relationship between a sponsor and a contractor over the construction of a large scale project. Assume that the project requires a prespecified number of tasks, $N$, for completion. For simplicity assume that each task lasts 1 period. Let $i$ be an index which refers both to the number of tasks and the number of periods remaining till completion of the project; $i = 1,...,N$. Let $B$ denote the benefit accruing to the sponsor in the event the project is completed. It is assumed that the benefit to the sponsor is zero in the event that the project is not completed.

At the start of period $i$ nature makes a move. Let $\tilde{t}_i$ denote this random variable and $t_i$ denote the realization of this random variable. $t_i \in \{E,H\}$, where $t_i = E$ means that the $i$th task is easy while $t_i = H$ means that the $i$th task is hard. We assume that $\tilde{t}_h$ and $\tilde{t}_i$ are statistically independent for all $h,i = 1,...,N$, $h \neq i$. Let $Pr\{\tilde{t}_i = H\} = \theta$. This probability is taken to represent the subjective beliefs of the sponsor concerning the difficulty of task $i$. Assume $0 < \theta < 1$. $\tilde{t}_i$ is observed by the contractor but not by the sponsor. However, $\theta$ is taken to be common knowledge.

After nature's move the sponsor chooses the compensation function, $p_i$. We elaborate on $p_i$ below. The contractor then chooses an indicator variable, $Q_i$, $Q_i \in \{0,1\}$, such that if $Q_i = 1$ the contractor has decided to complete task $i$ while if $Q_i = 0$ the contractor has decided to quit and drop out of the project in period $i$. The choice of $Q_i$ is based on $\pi$, $\pi \geq 0$, the per period opportunity profit level the contractor can experience elsewhere, and the rents the contractor expects to earn from continuing to participate in the project. The game ends if the contractor drops out. If the contractor has decided to complete task $i$ the contractor then chooses the level of cost
reducing effort, \( e_i, e_i \in \{0, 1\} \), where \( e_i = 1 \) denotes that cost reducing effort has been taken and \( e_i = 0 \) denotes that no cost reducing effort has been taken.

The sponsor receives a perfectly informative signal of the cost reducing effort, \( s_i, s_i = e_i w^L + (1-e_i) w^H \), where \( w^L < w^H \). We refer to \( s_i \) as the signalled cost. When \( s_i = w^L \) the contractor indicates to the sponsor that the task is low cost while when \( s_i = w^H \) the contractor indicates the task is high cost. Signalled cost is but one component of the actual cost involved in completing task \( i \), \( c^a(t_i, s_i) \). \( c^a(t_i, s_i) = s_i + c^e(t_i, s_i) + c^{ad}(t_i) \), where \( c^e(t_i, s_i) \) denotes the cost to the contractor from taking cost reducing effort and \( c^{ad}(t_i) \) denotes the cost to the contractor due to the task advantage, i.e., whether the task is easy or hard. The effort cost component is given by

\[
(1) \quad c^e(t_i, w^H) = 0 \quad \text{for } t_i = E, H, \\
\quad c^e(E, w^L) = w^H - w^L - a, \text{ and } c^e(H, w^L) > w^H - w^L,
\]

where \( a \) is a constant, \( 0 < a < w^H - w^L \). Thus cost reducing effort requires the expenditure of real resources and the sponsor will need to provide incentives to the contractor to induce such effort. The task advantage term is given by

\[
(2) \quad c^{ad}(E) = -(w^H - w^L - a) \text{ and } c^{ad}(H) = 0.
\]

Thus the actual cost function is given by
\( c^a(E, w^L) = w^L, \quad c^a(E, w^H) = w^L + a. \)
\( c^a(H, w^L) > w^H, \) and
\( c^a(H, w^H) = w^H. \)

Note that cost reducing effort is efficient, in the sense of minimizing actual costs in period \( i \), only if task \( i \) is easy. Also note that we have designed the actual cost function so that actual cost coincides with signalled cost when the task is easy, if the contractor takes cost reducing effort, and again when the task is hard, if the contractor does not take cost reducing effort. We shall refer to this case where actual and signalled cost coincide as truthful signalling on the part of the contractor.

Neither the effort cost nor the task advantage component are observed by the sponsor. Hence, the compensation function depends on signalled costs only, not on actual cost. That is \( p_i : (w^L, w^H) \rightarrow \mathbb{R} \), where \( p_i(w^L) \) is the payment the sponsor makes to the contractor upon completion of task \( i \) when the signalled cost is \( w^L \) and \( p_i(w^H) \) is defined similarly when the signalled cost is \( w^H \).

III. Equilibrium

We wish to analyze sequential equilibrium of this contract game.\(^8\) To get the reader familiar with the underlying issues, we shall first consider

\(^7\) Note that in its most general form we can imagine the compensation function to depend on the history of signalled costs through period \( i \), \( H_i = (t_{i,1}, t_{i,2}, \ldots, t_{i,i}) \). In this paper we restrict attention to history independent compensation functions. Since the contractor only observes \( t_{i,1} \) at the start of period \( i \), there does not appear to be any loss in making this restriction.

\(^8\) Note that we don't have well defined subgames in our model because when the sponsor chooses the compensation function the sponsor's information set is not a singleton. We trust that this will not create any confusion for the reader in the subsequent analysis.
the special case where $N = 1$. The main ideas are transparent in this case and can then be readily extended to the more general case.

We proceed via backward induction. First, consider the cost reducing effort decision of the contractor given the prespecified compensation scheme, $p_1$, and given that the contractor has chosen to complete the project. Then, $e_1 = 1$ only if

$$p_1(w^L) - c^a(t_1,w^L) > p_1(w^H) - c^a(t_1,w^H).$$

Substituting (3) into (4) when $t_1 = E$ yields

$$p_1(w^L) > p_1(w^H) - a.$$ When (5) is violated the contractor has no incentive to take cost reducing effort. In the sequel we will restrict attention to the case where $p_1(w^L) \leq p_1(w^H)$. Then (3) and (4) imply that the contractor will not take cost reducing effort when $t_1 = H$. Moreover, we will assume that when task 1 is easy the contractor does indeed take cost reducing effort as long as (5) is satisfied. Given these additional assumptions we shall refer to (5) as a self-selection constraint in that (5) induces truthful signalling.

Second, consider the contractor participation decision. Recall that if the contractor chooses to drop out the contractor earns $\pi$. Thus, $Q_1 = 1$ only if

$$\max_{s_1} p_1(s_1) - c^a(t_1,s_1) \geq \pi$$

Note that when $t_1 = H$, (6) reduces to
Substituting (7) into (5) yields

\( p_L(w^L) \geq w^H - a + \pi. \)

Evidently, if (8) holds then (6) does not bind when \( s_1 = E \). In other words, if the sponsor induces participation by the contractor with a hard task and induces cost reducing effort by the contractor with an easy task then the contractor with and easy task earns a rent of at least \( w^H - w^L - a \). The sponsor can extract this rent from the contractor with an easy task but only by inducing the contractor with a hard task to drop out. This is the tradeoff the sponsor contemplates in choosing the compensation scheme.

Finally, we consider the sponsor's choice of the compensation function, \( p_I \). We assume that the sponsor's goal is to maximize the expected net benefit, i.e., the product of the conditional expectation of the benefit minus the construction costs associated with completion of the project and the probability that the project is completed. The sponsor's problem is given by

\[
\begin{align*}
\text{maximize} \quad & (1 - \theta)Q_1(w^L)[B - p_I(w^L)] + \theta Q_1(w^H)[B - p_I(w^H)] \\
\text{Subject to:} \quad & 0 \leq Q_1(s_1) \leq 1, Q_1(s_1)[p_I(s_1) - s_1 - \pi] \geq 0, \text{ and} \\
& (1 - Q_1(s_1))[\pi - p_I(s_1) + s_1] \geq 0 \text{ for } s_1 = w^L, w^H; \text{ as well as (5).}
\end{align*}
\]

\[9\] In (9) we invoke the revelation principal, see Myerson (1979) or Harris and Townsend (1981), by asserting that there is an optimal compensation function which induces truthful signalling on the part of the contractor. This is why (5) is taken as a constraint and why we have omitted \( t_1 \) as an argument of \( Q_1 \).
Note that in (9) we treat $Q_j(s_j)$ as a probability rather than as an indicator variable. If at the optimum $0 < Q_j(s_j) < 1$, then it must be that $p_j(s_j) - s_j - \pi = 0$ and $B - p_j(s_j) = 0$, in which case $Q_j(s_j) = 0$ or 1 is also optimal. The solution to (9) is easy to characterize and is given in the following proposition.

**Proposition 1**: Let $(p_j^*, Q_j^*)$ denote a solution to (9). Then

(i) $p_j^*(w^L) < w^L + \pi$, $p_j^*(w^H) < w^L + a + \pi$, and $Q_j^* = 0$

if $B \leq w^L + \pi$;

(ii) $p_j^*(w^L) = w^L + \pi$, $p_j^*(w^H) < w^L + a + \pi$, and $Q_j^*(w^L) = 1$, and $Q_j^*(w^H) = 0$ if $w^L + \pi \leq B \leq w^H + \pi$ or if $B \geq w^H + \pi$ and $\theta \leq \frac{w^H - w^L - a}{B - w^L - a - \pi}$;

(iii) $p_j^*(w^L) = w^H - a + \pi$, $p_j^*(w^H) = w^H + \pi$, and $Q_j^* = 1$

if $B \geq w^H + \pi$ and $\theta \geq \frac{w^H - w^L - a}{B - w^L - a - \pi}$.

**Proof**: First note that if if $B \leq s_j + \pi$ then it is optimal for the sponsor to cancel the project when the actual cost is $s_j$. In our formulation, the sponsor cancels the project by inducing the sponsor to drop out. This is accomplished by violating the voluntary participation constraint (6), in which case $Q_j(s_j) = 0$. If $Q_j(s_j) > 0$ at the optimum then the objective function is decreasing in $p_j(s_j)$. Hence, either the voluntary participation or the self-selection constraint must bind. If $B > w^L + \pi$ then it is always optimal to induce the contractor with an easy task to complete the project, i.e., $Q_j(w^L) = 1$ is optimal, since inducing the low cost contractor to quit implies that the project will never be built. Finally, suppose $B > w^H + \pi$. If $Q_j(w^L) = 1$ and $Q_j(w^H) = 0$, then the maximum value the objective takes on is
$(1 - \theta)(B - w^L - \pi)$. If $Q_1 \equiv 1$, then the maximum value the objective takes on is $B - w^H - \pi + (1 - \theta)a$. The former exceeds the latter if and only if

$$\theta \leq \frac{w^H - w^L - a}{B - w^L - a - \pi}.$$ 

Suppose the sponsor would want the project completed if the sponsor knew that the task was hard. When the sponsor doesn't know whether the task is easy or hard proposition 1 demonstrates that the tradeoff between rent extraction from the contractor with an easy task and completion of the project from the contractor with a hard task is resolved on the basis of the sponsor's beliefs. If the sponsor views it likely that the task will be easy then the sponsor will opt for the rent extraction compensation function and be willing to forego the loss of surplus in the event that the task proves to be hard. On the other hand, if the sponsor deems it likely that the task will be hard then the risk of contractor dropout becomes too great, in which case the compensation function rises to ensure contractor participation.

We now turn to the case where $N > 1$. Since we restrict attention to history independent compensation schemes the condition that governs the choice of cost reducing effort is the same as in the one period game. The substantive difference between this multiperiod game and the single period game lies in the participation decision for the contractor and the compensation function decision for the sponsor. Both of these decisions are influenced by the equilibrium play in periods closer to completion of the project. That is, the expected rents to be earned influence these decisions.
In this section we assume that the contractor is risk neutral and, hence, acts as an expected profit maximizer. Let the expected rent earned by the contractor in the equilibrium of the one period game, $R_1$, be defined by

$$R_1 = (1 - \theta)Q_1(w^L)[p_1(w^L) - w^L - \pi] + \theta Q_1(w^H)[p_1(w^H) - w^H - \pi].$$

Then, recursively define the expected rent earned by the contractor from period $i$ till completion of the game, $R_i$, by

$$R_i = (1 - \theta)Q_i(w^L)[R_{i-1} + p_i(w^L) - w^L - \pi] + \theta Q_i(w^H)[R_{i-1} + p_i(w^H) - w^H - \pi],$$

for $i = 2, \ldots, N$. It follows that the contractor will continue to participate in the project in period $i$ only if

$$R_{i-1} + p_i(s_i) - s_i - \pi \geq 0.10$$

In a similar fashion let the expected net benefit earned by the sponsor in the equilibrium of the one period game, $V_1$, be defined by

$$V_1 = (1 - \theta)Q_1(w^L)[B - p_1(w^L)] + \theta Q_1(w^H)[B - p_1(w^H)].$$

Then, recursively define the expected net benefit earned by the sponsor from period $i$ till completion of the game, $V_i$, by

$$V_i = (1 - \theta)Q_i(w^L)[V_{i-1} - p_i(w^L)] + \theta Q_i(w^H)[V_{i-1} - p_i(w^H)],$$

---

10 (12) is written under the assumption that truthful signalling is optimal for the contractor. We will continue to make this assumption in the remainder of the paper for ease in notation.
for $i = 2, \ldots, N$. Evidently, the same reasoning as is given to prove proposition 1 determines the optimal compensation scheme in period $i$. We summarize this result in the following corollary.

**Corollary 1:** Let $(p^*_i, Q^*_i)$ denote the equilibrium compensation scheme and contractor quit-stay function in period $i$ for $i = 2, \ldots, N$. Then

(i) $p^*_i(w^L) < -R_{i-1} + w^L + \pi$, $p^*_i(w^H) < -R_{i-1} + w^L + a + \pi$, and

$Q^*_i \equiv 0$ if $V_{i-1} \leq w^L + \pi$;

(ii) $p^*_i(w^L) = -R_{i-1} + w^L + \pi$, $p^*_i(w^H) < -R_{i-1} + w^L + a + \pi$, $Q^*_i(w^L) = 1$, and $Q^*_i(w^H) = 0$ if $w^L + \pi \leq V_{i-1} \leq w^H + \pi$ or if $V_{i-1} \geq w^H + \pi$ and

$\theta \leq \frac{w^H - w^L - a}{V_{i-1} - w^L - a - \pi + R_{i-1}}$, and

(iii) $p^*_i(w^L) = -R_{i-1} + w^H - a + \pi$, $p^*_i(w^H) = -R_{i-1} + w^H + \pi$, and

$Q^*_i \equiv 1$ if $V_{i-1} \geq w^H + \pi$ and $\theta \geq \frac{w^H - w^L - a}{V_{i-1} - w^L - a - \pi + R_{i-1}}$.

By substituting the results from corollary 1 into (11) it follows that $R_i = 0$ if either case (i) or case (ii) holds and that $R_i = (1 - \theta)[w^H - w^L - a]$ if case (iii) holds. This limits the possible compensation schemes over time to a manageable number. Since the sponsor can always induce contractor drop-out by making the compensation function sufficiently low, it follows that $V_i \geq 0$. Then (14) implies that $V_i$ is nonincreasing in $i$ and is decreasing in $i$ as long as $V_i > 0$. It follows that if case (iii) holds in period $i$ then it necessarily holds in period $i - 1$. In other words, the equilibrium contract may entail an interval near completion where the sponsor is "locked in," i.e.,
the project is sufficiently close to completion that the sponsor funds the project regardless of the costs. Prior to this lock-in phase the sponsor funds the project only if the task proves to be easy and cancels if the task proves to be hard. With the exception of the period just before lock-in, there are no future rents to be earned by the contractor in this initial phase and, consequently, the payment made to the contractor in this initial phase equals project cost plus opportunity cost. Of course if the initial phase is sufficiently long that the contractor believes the chance of dropout before lock-in is sufficiently great, then the sponsor cancels the project initially rather than endure the risk of incurring substantial costs without completing the project. We characterize the contractual equilibrium in the following corollary.

Corollary 2: There exists two nonnegative integers, $l$ and $\sigma$, $\sigma \geq l + 1$, such that $l \neq 1$ and if $l \geq 2$ then

(i) $\pi'(w^L) = w^H - a + \pi$, $\pi'_i(w^H) = w^H + \pi$, and $Q_i^* = 1$;

(ii) $\pi'_i(w^L) = (1 - \theta)w^L + \theta(w^H - a) + \pi$,

(iii) $\pi'_i(w^H) = (1 - \theta)(w^L + a) + \theta w^H + \pi$, and $Q_i^* = 1$ for $i = 2, ..., l - 1$;

(iv) $p^*_i(w^L) = w^L - (1 - \theta)(w^H - w^L - a) + \pi$, $Q^*_i(w^L) = 1, Q^*_i(w^H) = 0$;

while for all values of $l$

(iv) $p^*_i(w^L) = w^L + \pi$, $p^*_i(w^H) < w^L + a + \pi$, $Q^*_i(w^L) = 1$, and $Q^*_i(w^H) = 0$ for $i = l + 1, ..., \sigma - 1$; and

(v) $p^*_i(w^L) < w^L + \pi$, $p^*_i(w^H) < w^L + a + \pi$, and $Q_i^* = 0$ for $i \geq \sigma$.
In figure 1 we graph the time path of the equilibrium compensation scheme under the assumption that \( \ell = 0 \) and \( N < \sigma \). The payment scheme is flat in this case. That is, there will not be any cost overruns. In figure 2 we graph the time path of the compensation scheme under the assumption that \( \ell \geq 2 \) and \( N < \sigma \). In this case there is a tilting of the compensation scheme. Payments are higher during lock-in than they are during the initial phase of the project. We feel that since the initial phase is characterized by cost plus compensation the lock-in phase can be reasonably interpreted as the occurrence of cost overruns.
This completes the description of the basic model. In the next section we present some extensions which embellish our basic results.
III. Extensions

Contractor Risk Aversion

In the basic model payments during the lock-in phase are governed by the idea that the contractor can earn rents with expected value equal to $(1 - \theta)(w^H - w^L - a)$ from continuing to participate in the project. This expected rent is extracted from the contractor by reducing the current period payment by an amount equal to the expected rent, though an additional rent may be created if the self-selection constraint binds in the current period. For a variety of reasons it may not be possible for the sponsor to fully extract these rents from the contractor. For example, when $\pi < (1 - \theta)(w^H - w^L - a)$ the contractor's out of pocket expense on the project exceeds the payment received from the sponsor during any period in the lock-in phase when the contractor has a hard task. If the contractor does not have sizeable receipts from other activities in which the contractor is simultaneously engaged and if the contractor has limited collateral for accessing credit markets, liquidity constraints may make such negative cash flow scenarios infeasible. In this section we present an alternate reason for limited rent extraction based on contractor risk aversion.

The intuition we have in mind is quite simple. The rent that the contractor anticipates from participation in the project is risky. The contractor would like to insure against this risk but the sponsor refuses to provide insurance in the compensation scheme because such insurance mitigates against the incentives for taking cost reducing effort. Thus, while the sponsor can extract the expected rent from the contractor by
reducing the current period payment, the sponsor must pay the contractor a risk premium. This risk premium has the effect of increasing the contractor's compensation as compared to the risk neutral case.

Let \( u \) denote the contractor's von Neumann-Morgenstern utility function defined over current period profit. Assume that \( u' > 0 \) and \( u'' < 0 \). During the lock-in phase, which we focus on below, the participation constraint for a contractor with a hard task in period 1 determines \( p_i(w^H) \) and then the self-selection constraint yields \( p_i(w^L) = p_i(w^H) - a \). In period 1 the participation constraint when \( t_1 = H \) becomes

\[
(15) \quad u(p_i(w^H) - w^H) \geq u(\pi).
\]

Since \( u \) is increasing this is identical to (7). Thus, there is no effect on the period 1 payments, regardless of contractor risk aversion. For \( i \geq 2 \) the participation constraint in period 1 when \( t_i = H \) is

\[
(16) \quad u(p_i(w^H) - w^H) + \sum_{h=1}^{i-1} [(1-\theta)u(p_h(w^L) - w^L) + \theta u(p_h(w^H) - w^H)] \geq i[u(\pi)].
\]

Since both the participation constraint when \( t_{i-1} = H \) and the self-selection constraint when \( t_{i-1} = E \) bind in period 1 - 1, (16) reduces to

\[
(17) \quad u(p_i(w^H) - w^H) \geq u(\pi) - (1 - \theta)[u(p_{i-1}(w^H) - w^L - a) - u(p_{i-1}(w^H) - w^H)].
\]

This inequality binds for all the periods in which lock-in occurs. Note that the right hand side of (17) is increasing in \( p_{i-1}(w^H) \) since \( u \) is strictly
concave. Moreover, \( \pi = p_1(w^H) - w^H > p_2(w^H) - w^H \). Thus, it follows from a straightforward induction argument that \( p_i(w^H) \) is decreasing in \( i \), i.e., compensation rises during lock-in as the project nears completion. That is, the compensation is both higher and more sharply sloped when the contractor is risk averse than when the contractor is risk neutral.\(^{11}\)

**Noisy Signalling**

In the basic model we assumed that the sponsor observed a perfectly informative signal of the contractor's cost reducing effort. It may be more reasonable to assume that there are factors which influence these signalled costs outside the control of the contractor, in which case the signalled cost should be taken as a random variable. But then upon observing a high signalled cost the sponsor will be unsure whether this is the result of a lack of effort on the part of the contractor or whether this is simply the consequence of bad luck. This makes the model more like the standard principal-agent model.\(^{12}\) It is well known that in the standard principal-agent model it is optimal to make the agent a residual claimant when the agent is risk neutral. When the agent bears all the risk the agent faces the right incentives for taking effort. However, in our model making the contractor the residual claimant is not optimal during the lock-in phase, because excessive rents will be earned by the contractor when the task is easy. But, we show that there is an optimal linear compensation

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11 The effect that contractor risk aversion has on the shape of the compensation function lowers the net expected benefit accruing to the sponsor and may consequently shorten the length of the lock-in phase.

12 For a good exposition of the principal-agent model see Holmstrom (1979).
schedule for which the results under perfectly informative signalling can be duplicated under noisy signalling, in an expected value sense.

Let \( \tilde{c}_i \) be the random variable denoting signalled costs in period \( i \). \( \tilde{c}_i \) is taken to have support over the interval \( [c, \tilde{c}] \) and have distribution function \( F(e_i) \), where \( F(1) \) dominates \( F(0) \) in the sense of first order stochastic dominance. Moreover, assume \( \int_{\tilde{c}} cf(c,0)dc = w^H \) and \( \int_{\tilde{c}} cf(c,1)dc = w^L \). In other words, \( w^H \) is the expected signalled cost when no cost reducing effort is taken and \( w^L \) is the expected signalled cost when cost reducing effort is taken. The remainder of the setup of the basic model is unchanged.

Suppose the compensation function is a linear function of the observed signalled cost, i.e., \( p_i(c_i) = p_{i0} + p_{i1}c_i \). Then it is straightforward to show that a contractor who has an easy task in period \( i \) and who has agreed to complete task \( i \) will take cost reducing effort only if

\[
(18) \quad p_{i1} \leq \frac{a}{w^H - w^L}
\]

Similarly, it is straightforward to show that a contractor who has a hard task in period \( i \) and who has agreed to complete task \( i \) will not take cost reducing effort only if

\[
(19) \quad p_{i1} \geq \frac{w^H - w^L - c\epsilon(H,w)L}{w^H - w^L}.
\]
We will take $p_{i1}$ to be delimited by the closed interval determined by (18) and (19). For a given value of $p_{i1}$ in this interval, $p_{i0}$ takes on the lowest possible value consistent with satisfaction of the participation constraint for the contractor with a hard task in period $i$. This yields

$$p_{i0} = w^H(1 - p_{i1}) + \pi - R_{i-1}.$$  

Any pair, $(p_{i0}, p_{i1})$, which satisfies (20) yields the same expected payments to a contractor with a hard task in period $i$. However, for such pairs the payment to a contractor with a low cost in period $i$ as a function of $p_{i1}$ is

$$E[p_{i} | \tilde{e}_i = E] = w^H + p_{i1}(w^L - w^H) + \pi - R_{i-1}.$$  

The sponsor would like to minimize the right hand side of (21). Hence in the optimal linear compensation scheme, (18) will bind.

The reader can readily verify that when

$$p_{i}(c_1) = \left[ \frac{w^H - w^L - a}{w^H - w^L} \right] w^H + \pi + \left[ \frac{a}{w^H - w^L} \right] c_1$$

and when

$$p_{i}(c_1) = \left[ \frac{w^H - w^L - a}{w^H - w^L} \right] w^H + \pi - (1 - \theta)[w_H - w_L - a] + \left[ \frac{a}{w^H - w^L} \right] c_1$$

for $i = 2, \ldots, l-1$, the expected payment to the contractor is identical to the payment when signalled cost provides perfect information as to contractor cost reducing effort.

There are two important points to note about the optimal linear compensation scheme. First, the slope of the compensation function is less than one. In particular cost plus pricing is not optimal because, as in standard principal-agent models, if the contractor bears none of the risk then the contractor has no incentive to take cost reducing effort. Second,
the intercept of the compensation function rises over time. Schemes which are rigid in their temporal component are not optimal.\textsuperscript{13}

Contractor Impatience and Project Delays

In the basic model the only way for the sponsor to induce truthful signalling on the part of the contractor during the lock-in phase is by offering payments in the current period which satisfy the self-selection constraint. However, once it is assumed that the contractor exhibits impatience, there is a second method available to the sponsor. The sponsor could encourage the contractor with a hard task to delay completion of the task for one period by paying the contractor's opportunity cost in the current period if the contractor chooses to delay coupled with making a relatively low payment to the contractor if the task is immediately completed. The sponsor could nevertheless encourage the contractor with an easy task to complete the task in the current period. The benefit to the contractor from immediately completing the task is that the expected future rents would be earned one period earlier. In other words, when the contractor discounts the future project delays can emerge in equilibrium as a self-selection device.\textsuperscript{14} It is our purpose here to illustrate when this form of self-selection is apt to emerge.

\textsuperscript{13} The results that Lewis (1986) obtains may very well depend on the fact that Lewis takes the compensation function as parametric and time invariant. Our results strongly suggest that the compensation function in Lewis' model is not optimal. It is not obvious whether Lewis' results are robust to an endogenous determination of the compensation function along the lines of this paper.

\textsuperscript{14} This idea was motivated by Grossman and Perry (1986) who obtain similar results in a bargaining context.
Let $\delta$ denote the contractor's discount factor. We continue to assume that the sponsor does not discount the future. Let $R_i$ now denote the expected future rent accruing to the contractor when there are $i$ tasks remaining discounted to the current period. Suppose at the start of task $i$ the sponsor offers the contractor the payment $p_{in}$ for completing task $i$ immediately or offers the payment $\pi$ for delaying construction for 1 period. Assume for the moment that this scheme does induce self-selection on the part of the contractor so that the contractor with an easy task is paid $p_{in}$ for completing task $i$ while the contractor with a hard task takes $\pi$ and delays construction. Then after the construction delay the sponsor's equilibrium beliefs about task $i$ are that it is hard with probability 1. Since we are assuming that task $i$ occurs during the lock-in stage the sponsor will then offer the contractor the payment $p_{id}$. $p_{id} = -R_{i-1} + w^H + \pi$, in order to satisfy the voluntary participation constraint for the contractor with a hard task. Of course the contractor with an easy task could also delay completion of task $i$, take $\pi$ in the current period and rationally anticipate earning $p_{id}$ when task $i$ is completed in the subsequent period. In order for the contractor with an easy task to find it optimal to complete task $i$ without delay it must be that

$$p_{in} - w^L - \pi + R_{i-1} \geq \delta[p_{id} - w^L - a - \pi + R_{i-1}] = \delta[w^H - w^L - a].$$

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15 The real issue here is the relative impatience of the two parties. We assume that the sponsor does not discount the future, just as we assumed that the sponsor is risk neutral in our discussion of contractor risk aversion, for simplicity.
If the sponsor does find it optimal to utilize project delays then (22) will bind in equilibrium. In this case the no delay payment made after completion of task \( i \) is

\[
p_{in} = w^L + \pi - R_{i-1} + \delta[w^H - w^L - a].
\]

Observe that when \( \delta = 1 \) the value of \( p_{in} \) given in (23) coincides with the value of \( p^*_i(w^L) \) during the lock-in phase for \( i > 1 \). Hence the effect of project delays in conjunction with contractor discounting is to lower the payment to the contractor with an easy task.

The value to the sponsor from pursuing this delaying strategy in period \( i \), \( V_{id} \), is given by

\[
V_{id} = (1 - \theta)[V_{i-1} - p_{in}] + \theta[V_{i-1} - p_{id} - \pi].
\]

The value to the sponsor from having the project completed without delay, regardless of cost, \( V_{in} \), is given by

\[
V_{in} = (1 - \theta)[V_{i-1} + R_{i-1} - w^H + a - \pi] + \theta[V_{i-1} + R_{i-1} - w^H - \pi].
\]

After making the substitutions for \( p_{in} \) and \( p_{in} \) it can be seen that \( V_{id} \geq V_{in} \) if and only if

\[
\theta \leq \frac{w^H - w^L - a}{w^H - w^L - a + \pi/(1-\delta)}.
\]

By comparing (26) to the inequality given in case (iii) of corollary 1 it can be seen that this use of project delays to induce self-selection is only relevant when \( V_{i-1} - w^L + a - \pi + R_{i-1} > w^H - w^L + a + \pi/(1-\delta) \). Thus
project delays are more likely to occur the closer the project is to completion, where $V_{i-1}$ is large. Earlier in the lock-in phase the sponsor will prefer to induce self-selection via the method developed in the basic model.

In our view the assumption that the contractor exhibits impatience is more relevant when the contractor is highly leveraged and may be rationed in borrowing additional funds at market rates. This would seem to be likely when the scale of the project is large relative to the size of the contractor's entire operation. Thus our result can be interpreted as predicting a correlation between project size and the incidence of project delays, giving some credence to the popular view that it is massive public projects for which the 'horror stories' mentioned in the introduction are most likely.

IV. Conclusion

In this paper we have provided a dynamic model of a contract game whose sequential equilibrium has the property that in some circumstances the compensation per task rises as the project nears completion, although the expected cost per task is time invariant. We have interpreted these results as predicting cost overruns as equilibrium outcomes when these circumstances prevail. Based on analysis of our model cost overruns are more likely the larger the benefit accruing to the sponsor and the longer the time from initiation to completion of the project. In other words, project scale matters in predicting cost overruns.

Since actual compensation functions are almost always written on a cost plus basis our model can be interpreted as predicting cost overruns as
equilibrium outcomes only when contractors can falsify upwards their reported costs with little affect on their actual costs. Our model implies that the sponsor goes along with such falsification, even though the sponsor is well aware of the truth, because such falsification is the only way to attain the equilibrium level of compensation. If such falsification is highly costly to the contractor then our model suggests that actual, cost plus compensation functions are not optimal and may then suggest that contractors do not take the appropriate cost reducing effort in actuality. We prefer the first interpretation though we recognize that the second interpretation also has its merits.

Are cost overruns something economists should worry about? Most of the public concern appears to be over the rent extraction issue. It should be noted that our basic model implies that this is not a problem. Except in the case where the sponsor is locked in right from the initiation of the project the equilibrium of our model has the property that the contractor does not earn any rents overall. Moreover, there is nothing to preclude the sponsor from setting up a competitive bidding among candidate contractors for the rights to the project prior to the commencement of construction as a means of extracting the rents, when the sponsor is locked in right at the beginning of construction. We are agnostic as to whether complete rent extraction occurs in actuality. We merely point out here that if our model is to be taken seriously then some form of nonlinearity in the contractor's payoff function must be demonstrated to make incomplete rent extraction a concern. In the absence of such nonlinearities the public uproar over cost overruns may be best attributed to time inconsistent attitudes concerning the value of project completion.
Nevertheless, our basic model does point to a form of inefficiency which, as far as we are aware, has not received any popular attention. Since the equilibrium of our model has the property that it induces self-selection on the part of the contractor with regard to the costs the contractor announces to the sponsor, the only way that inefficiency can be manifest in our basic model is by completion of the project in instances where it would be socially optimal to cancel the project or by cancellation of the project when it would by socially optimal for the project to be brought to completion. It should be noted that in our model the sponsor is assumed incapable of precommitting to the length of the lock-in phase. Hence there is no reason to expect that the equilibrium length of the lock-in phase is optimal. It turns out that when the expected quasirents earned by the contractor during the lock-in phase are large relative to the net expected benefits of the sponsor during this phase then the lock-in phase will be too short from a social welfare view. In such instances it would be beneficial for the sponsor to announce sometime prior to lock-in that it intends to see the project to completion. These instances are more likely to occur the greater the probability that any particular task is easy and the greater the cost wedge between hard and easy tasks since both of these factors contribute to the size of the quasirents. In other words, there are real social costs because the sponsor is incapable of making a credible commitment at project initiation to the compensation schedule it will honor over the lifetime of the project.

This observation provides the basis for an argument that government agencies should be guided by procurement rules concerning the cancellation decision rigid enough to ensure efficient project completion. Ironically
rules appear to guide the choice of compensation schedule where some sponsor discretion, i.e., a departure from cost plus pricing, may be desirable. Yet the cancellation decision appears to be left to the discretion of the project administrator. From our perspective a redefining of the project administrator’s authority would be highly desirable.
References


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