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PROFIT SHARING AND A WAGE EARNERS' INVESTMENT FUND

Hans Brems

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By Hans Brems

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PROFIT SHARING AND A WAGE
EARNERS' INVESTMENT FUND

Hans Brems *

University of Illinois at Urbana-Champaign

To a wage earners' investment fund let all employers contribute compulsorily a fraction of their profits bill. To the employees the fund issues nonnegotiable fund certificates, redeemable after a specified number of years. Within the framework of a simple neoclassical model of steady-state inflation and growth, the article determines the size of such a fund as well as its effects upon the marginal productivity of capital, disposable-income distribution between capital and labor, the propensity to save national output, and the real wage rate. Profit sharing is briefly compared with an investment fund to which all employers contribute compulsorily a fraction of their wage bill.

I. THE SCHEME

Unlike American labor unions, Western European ones are used to influencing national economic policy by working with the government, Barbash [1]. A recent example of such collaboration
is the emergence in Western Europe of the idea of a wage earners' investment fund financed by profit sharing.

Serving the dual purpose of giving labor a share of, first, the capital gains accruing to stockholders in an inflationary economy and, second, the co-determination rights inherent in stock ownership, a wage earners' investment fund would work as follows. Primarily in the form of corporate stock all employers would contribute compulsorily a fraction of their profits bill to the fund. The fund would belong to the employees. To the individual employees, in turn, the fund would issue nonnegotiable fund certificates. A specified number of years after its issue a fund certificate would become redeemable in cash at a price which would include the share of that certificate in all capital gains and dividends made by the fund during the lifetime of the certificate. The fund would be allowed to sell contributed corporate stock at any time and buy other stock.

The purpose of the present article is to examine the macroeconomic effects of such a fund, i.e., the effects upon the marginal productivity of capital, disposable-income distribution, the pro-
penalty to save national output, and the real wage rate.

What would be a suitable theoretical framework for our analysis? From the purposes of the fund it follows that our model should be capable of accommodating inflation and that its capitalists should be stockholders rather than bondholders. We choose the simplest possible one-sector neoclassical model of steady-state inflation and growth. Its capitalists are capitalist-entrepreneurs producing a single good from labor and an immortal capital stock of that good, hence investment is the act of setting aside part of output for installation as capital stock. Capital stock is the result of accumulated savings—voluntary as well as forced. Technology, available labor force, and the money wage rate are growing autonomously.

II. NOTATION

Variables

\( c \equiv \text{propensity to consume national output} \)

\( C \equiv \text{consumption} \)

\( \phi \equiv \text{size of wage earners' investment fund relative to capital stock} \)

\( \Phi \equiv \text{absolute size of wage earners' investment fund} \)
\( g_v \equiv \text{proportionate rate of growth of variable} \ v \equiv c, C, \phi, I, \lambda, \ k, L, P, S, \theta, X, Y_1, Y_2, Y, \text{and} \ Z \)

I \equiv \text{investment}

\( \lambda \equiv \text{internal rate of return} \)

k \equiv \text{present gross worth of a physical unit of capital stock}

K \equiv \text{physical marginal productivity of capital stock}

L \equiv \text{labor employed}

\( \nu \equiv \text{propensity to save national output} \)

P \equiv \text{price of good}

r \equiv \text{discount rate applied by capitalist-entrepreneurs}

S \equiv \text{physical capital stock}

\( \Theta \equiv \text{disposable-income to output ratio, called "the payout ratio"} \)

W \equiv \text{wage bill}

X \equiv \text{physical output}

Y \equiv \text{disposable money income}

Z \equiv \text{profits bill including employers' contribution to fund per year}

\textbf{Parameters}

a, \beta \equiv \text{exponents of production function}

b \equiv \text{employers' contribution to fund as a fraction of profits bill}
\[ c_d \equiv \text{propensity to consume national disposable real income} \]
\[ e \equiv \text{Euler's number, the base of natural logarithms} \]
\[ F \equiv \text{available labor force} \]
\[ g_p \equiv \text{proportionate rate of growth of parameter } p \in F, M, \text{ and } w \]
\[ M \equiv \text{multiplicative factor of production function} \]
\[ \rho \equiv \text{redemption period} \]
\[ w \equiv \text{money wage rate} \]

Parameters listed are stationary except \( F, M, \) and \( w, \) whose growth rates \( g_F, g_M, \) and \( g_w \) are stationary. Time coordinates are \( t \) and \( T. \)
The unit of time is the year.

III. THE EQUATIONS OF THE MODEL

1. Definitions

Fifteen variable growth rates are listed in Sec. II. To all apply the definition

\[ (1) \text{ through } (15) \quad g_v = \frac{dv}{dt} \]

Define investment as the derivative of capital stock with respect to time:
(16) \[ I \equiv \frac{dS}{dt} \]

2. Production

Let capitalist-entrepreneurs apply the production function

(17) \[ X = M L^\alpha S^\beta \]

where \(0 < \alpha < 1; \ 0 < \beta < 1; \ \alpha + \beta = 1; \) and \(M > 0\). Let profit maximization under pure competition equalize real wage rate and physical marginal productivity of labor:

(18) \[ \frac{w}{\partial L} = \frac{\partial X}{\partial L} = \frac{X}{L} \]

Physical marginal productivity of capital is defined:

(19) \[ \kappa \equiv \frac{\partial X}{\partial S} = \frac{X}{S} \]

Multiply (19) by price of output \(P\) to find value marginal productivity of capital. Define money profits earned on each physical unit of capital stock \(S\) as its value marginal productivity. Then multiply by \(S\) to find money profits earned on entire capital stock.
(20) \[ Z = \kappa PS = \beta PX \]

Under full employment available labor force equals labor employed:

(21) \[ F = L \]

Define wage bill as money wage rate \textit{times} employment:

(22) \[ W = wL \]

3. \textbf{Absolute Size of Fund}

At time \( t \), let the employers contribute the amount \( bZ(t) \) to the wage earners' investment fund. Let \( bZ(T, t) \) be the value at time \( T \) of the amount \( bZ(t) \) contributed at time \( t \). How does \( bZ(t) \) grow to become \( bZ(T, t) \)? Assume wage earners to have the same motivation and skill as capitalist -entrepreneurs hence, like the latter, to be making the internal rate of
return 1 on the money value of the capital stock they own, i.e. the wage earners' investment fund. Let the earnings of the fund be compounded continuously, then

\[ bZ(\tau, t) = e^{\int_{\tau}^{t} bZ(t) \, dt} \]

Let all wage earners present their fund certificates for redemption as soon as the latter become redeemable. Redemption at time \( t \) is the accumulated value at time \( \tau \) of the contribution made at time \( \tau - \rho \), where \( \rho \) is the redemption period. The size of the fund at time \( \tau \) is the value at time \( \tau \) of all contributions made from \( t = \tau - \rho \) to \( t = \tau \):

\[ \Phi(\tau) \equiv \int_{\tau}^{\tau} bZ(\tau, t) \, dt \]

The profit bill out of which the contributions to the fund are made, is growing at the proportionate rate \( g_Z \), hence

\[ Z(\tau) = e^{g_Z(\tau - t)} \]

Insert (23) and (25) into (24) and find the size of the fund
\[ \Phi(t) = \int_t^T (1 - g_Z)(\tau - t) bZ(\tau) \, d\tau \]

The integration would be facilitated by assuming the internal rate of return \( l \) and the proportionate rate of growth of the profits bill \( g_z \) to be stationary:

\[ \frac{dt}{dt} = 0 \]
\[ \frac{dg_z}{dt} = 0 \]

The integration will have to be carried out separately for \( l \neq g_z \) and \( l = g_z \). Find all variables in the outcome referring to the same time \( \tau \), purge it of \( \tau \), and write the size of the fund

\[ \Phi = bZ[e^{-(1 - g_z)\rho} - 1]/(1 - g_z) \text{ for } l \neq g_z \]
\[ \Phi = bZ\rho \text{ for } l = g_z \]

4. Disposable-Income Distribution between Labor and Capital

Redemption at time \( \tau \) is the accumulated value at time \( \tau \) of the contribution made at time \( \tau - \rho \). That value we write \( bZ(\tau, \tau - \rho) \)
and define labor's disposable income at time $t$ as the wage bill plus redemption at that time:

$$Y_l(t) = W(t) + bZ(t, t - \rho)$$

Insert (25) into (23), replace $t$ by $t - \rho$, and find redemption

$$bZ(t, t - \rho) = e^{(1 - g_Z)\rho} bZ(t)$$

Use this, (18), (20), and (22) to write labor's disposable income

$$(30) \quad Y_l = \theta_1 PX, \text{ where}$$

$$\theta_1 \equiv \alpha + \beta e^{(1 - g_Z)\rho}$$

The capitalist-entrepreneurs are making the internal rate of return on the money value of the capital stock they own, i.e., all capital stock minus the wage earners' investment fund. The internal rate of return includes profits and capital gains, as we shall see in Eq. (57) in Sec. V. Follow convention and exclude capital gains from disposable income. According to (20) profits are earned at the rate $\kappa$, so define disposable income of the capitalist-entrepreneurs as
their profits on all capital stock minus the fund minus their contribution to the fund

\[ Y_2 = \kappa(PS - \phi) - bZ \]

Remembering the two separate forms of (29), insert (20) and (29) and write the disposable income of the capitalist-entrepreneurs

(31) \[ Y_2 = \theta_2 PX, \text{ where} \]
\[ \theta_2 = \beta - \beta b - \beta b \kappa [e^{(1 - g_Z)\rho} - 1]/(1 - g_Z) \text{ for } i \neq g_Z \]
\[ \theta_2 = \beta - \beta b - \beta b \kappa \rho \text{ for } i = g_Z \]

Add (30) and (31) and find national disposable money income

(32) \[ Y = Y_1 + Y_2 = \theta PX, \text{ where} \]
\[ \theta = \theta_1 + \theta_2 = 1 + \beta b (1 - g_Z - \kappa) [e^{(1 - g_Z)\rho} - 1]/(1 - g_Z) \text{ for } i \neq g_Z \]
\[ \theta = \theta_1 + \theta_2 = 1 - \beta b \kappa \rho \text{ for } i = g_Z \]

\( \theta \) is a disposable-income to output ratio, a "payout" ratio.
5. Consumption

Let the parameter $c_d$ be the propensity to consume national disposable real income:

(33) \[ C = c_d Y/P \]

Define the variable $c$ as the propensity to consume national output:

(34) \[ C = cX \]

Take (32), (33), and (34) together and find

(35) \[ c = c_d \theta \]

Define the variable $\nu$ as the propensity to save national output:

(36) \[ \nu = 1 - c \]

6. Equilibrium

Finally, output equilibrium requires output to equal the sum
of consumption and investment demand for it:

(37) \[ X = C + I \]

IV. SOLUTIONS FOR PROPORTIONATE RATES OF GROWTH

Define, as Hahn and Matthews [3] did, steady-state growth as stationary proportionate rates of growth. Our system (1) through (37) possesses the following set of steady-state solutions:

(38) \[ g_C = 0 \]

(39) \[ g_C = g_K \]

(40) \[ g_\Phi = g_W \]

(41) \[ g_I = g_K \]

(42) \[ g_L = 0 \]

(43) \[ g_K = 0 \]
To convince himself that those are indeed solutions, the reader should take derivatives with respect to time of (16) through (37). He should then use the definitions (1) through
(15), insert the solutions (38) through (52), and convince himself that each equation is satisfied. Thus our auxiliary assumptions (27) and (28)—consistent with (42) and (52), respectively—have paid off handsomely. But there is more to growth theory than finding proportionate rates of growth. Our purpose was to find the effects of a wage earners' investment fund upon the physical marginal productivity of capital, disposable-income distribution between labor and capital, the propensity to save national output, and the real wage rate. Those effects are effects upon levels in a growing economy. In determining such levels our solutions for proportionate rates of growth (38) through (52) will be useful.

V. PHYSICAL MARGINAL PRODUCTIVITY \( K \) AND INTERNAL RATE OF RETURN \( I \)

According to our solution \( (43) \) a physical unit of capital stock added at time \( T \) would have the physical marginal productivity \( K \) at any time from \( t = T \) to \( t = \infty \). What sort of value marginal productivity will it have? Let it be perfectly foreseen by the entrepreneurs that price is growing at the proportionate rate \( g_p \):
But let the capitalist-entrepreneurs be purely competitive ones, hence price is beyond their control. At time \( t \), value marginal productivity is, then

\[
\frac{\partial[P(t)X(t)]}{\partial S(t)} = P(t)k
\]

As seen from the present time \( T \), value marginal productivity at time \( t \) is \( e^{-r(t-T)}P(t)k \), where \( r \) is the discount rate applied by the capitalist-entrepreneurs. Define present gross worth \( k \) at time \( T \) of the physical unit of capital stock as the present worth of all its future value marginal productivities:

\[
k(T) \equiv \int_T^\infty e^{-r(t-T)}P(t)kd t
\]

Let the rate of inflation be less than the discount rate:

\[
\varepsilon_p < r
\]

Insert (53) into (54) and use (55) to carry out the integration.
Since in the outcome all variables refer to the same time \( t \), we may purge it of \( t \):

\[
(56) \quad k = \frac{P_k}{r - g_p}
\]

Define the present net worth of the physical unit of capital stock as gross worth minus price:

\[
n = \frac{k}{r - g_p} - 1]P
\]

Define the internal rate of return \( i \) as that value of \( r \) which makes net worth equal to zero, hence

\[
(57) \quad i = k + g_p
\]

where \( g_p \) stands for (45). In an inflationary economy, then, the internal rate of return of a physical unit of capital stock equals the physical marginal productivity of that unit plus the proportionate rate of capital gain (45).
VI. A TRANSCENDENTAL EQUATION IN PHYSICAL MARGINAL PRODUCTIVITY $\kappa$

Use (34), (36), and (37) to find $I = \nu X$ and (1) through (16) to find that $I = g_S S$, hence $S = \nu X / g_S$. Insert that into (19) and find

$$\kappa = \beta g_S / \nu$$

Insert (45), (46), (52), and (57) into $t - g_Z$ and find

$$1 - g_Z = \kappa - g_S$$

Remembering the two separate forms of (32), insert (32), (35), (36), and (59) into (58) and find the transcendental equation in $\kappa$:

$$\kappa \left\{ \frac{(1 - c_d) / g_S + \beta bc_d}{e^{(\kappa - g_S) \rho} - 1} \right\} = \beta \text{ for } t \neq g_Z$$

$$\kappa \left\{ \frac{(1 - c_d) / g_S + \beta bc_d}{\rho} \right\} = \beta \text{ for } t = g_Z$$

An explicit solution of (60) is beyond reach. But our appendix proves the existence of a unique and positive solution for $\kappa$. And once we had empirical values of the parameters entering (60) we could find that solution numerically. Let us choose such values, then.
VII. STYLIZED EMPirical VALUES OF PARAMETERS

For Northwestern European economies let us adopt

\[ \alpha = \frac{3}{4} \]
\[ \beta = \frac{1}{4} \]
\[ c_d = \frac{7}{8} \]
\[ g_F = 0 \]
\[ g_M = \frac{3}{100} \]

Practical profit-sharing schemes may exempt proprietorships, other noncorporations, small corporations, interest on liabilities, or even interest on equity. Using \( \beta = \frac{1}{4} \) will therefore overstate the volume of contributions resulting from a given contribution fraction \( b \). From (46) it follows that

\[ g_S = \frac{1}{25} \]

VIII. NUMERICAL SOLUTIONS FOR LEVELS

1. The Physical Marginal Productivity of Capital

Insert our stylized empirical parameter values into our transcendental equation (60) and find

\[ (\kappa - 0.04)\rho \]

\[ \kappa\{100 + 7b[e^{\gamma - 1}]/(\kappa - 0.04)} - 8 = 0. \]
Eq. (61) contains the two structural characteristics of a wage earners' investment fund, i.e., \( b \equiv \) employers' contribution as a fraction of the profits bill and \( \rho \equiv \) the redemption period. Let us examine a rather wide range of alternative structural characteristics of such a fund, say

\[
\begin{align*}
h &= \frac{1}{40}, \; \frac{1}{20}, \; \frac{1}{10}, \; \text{and} \; \frac{1}{5} \\
\rho &= 2, \; 4, \; 8, \; \text{and} \; 16
\end{align*}
\]

Inserting these alternative values and using an IBM 360/75 at the University of Illinois, (61) was solved for \( \kappa \). The results are shown in Column 3 of Table 1 and in Figure 1. As one would expect, the higher the employers' contribution fraction \( b \) and the longer the redemption period \( \rho \) are, the lower is the physical marginal productivity of capital stock \( \kappa \). But the elasticities of the latter with respect to \( b \) and \( \rho \)—apparent as the steepness of the curves on their double-logarithmic scale—are modest in the range considered politically. Beyond \( h = \frac{1}{10} \) and \( \rho = 8 \) they become noticeably higher.
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<th>Capitalist Payout Ratio</th>
<th>Overall Payout Ratio</th>
<th>Propensity to Save</th>
<th>National Output</th>
<th>Fund as a Fraction of Capital Stock</th>
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<td>0.143</td>
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</tr>
</tbody>
</table>
2. Disposable-Income Distribution between Labor and Capital

Insert (59) into (30) through (32) and write the payout ratios

\[(\kappa - g_S)\rho \]

\[\theta_1 = \alpha + \beta b e\]

\[(\kappa - g_S)\rho \]

\[\theta_2 = \beta - \beta b - \beta b \kappa [e^{(\kappa - g_S)\rho} - 1]/(\kappa - g_S)\]

\[(\kappa - g_S)\rho \]

\[\theta = 1 - \beta bg_S [e^{(\kappa - g_S)\rho} - 1]/(\kappa - g_S)\]

Here are two opposing forces at work: Rising \(b\) or \(\rho\) will at the same time make (62) rise and (63) and (64) fall (because \(b\) and \(\rho\) are rising) and make (62) fall and (63) and (64) rise, because \(\kappa\) is falling! But the former force wins in the practical range, as seen from Columns 4 through 6 of Table 1 and from Figure 2: The higher the employers' contribution fraction \(b\) and the longer the redemption period \(\rho\) are, the higher is labor's payout ratio and the lower is that of the capitalist-entrepreneurs. Labor wins,
Hans Drems, "Profit Sharing"
FIGURE 2

Hans Brems, "Profit Sharing"
FIGURE 3

Hans Brems, "Profit Sharing"
FIGURE 4
Hans Brems, "Profit Sharing"
and the capitalist-entrepreneurs lose. But labor wins slightly less than the capitalist-entrepreneurs are losing, so the overall ratio is lower. The elasticities of the payout ratio of capitalist-entrepreneurs with respect to b and ρ are considerable, especially beyond $b = 1/10$ and $ρ = 8$. We conclude that the redistributive effects of a wage earners' investment fund may well be considerable.

3. The Propensity to Save National Output

Insert (35) and (64) into (36) and write the propensity to save national output

$$v = 1 - c_d + βbc_d g_s [e^{(κ - g_s)ρ} - 1]/(κ - g_s)$$

Again, two opposing forces are at work: Rising $b$ or $ρ$ will at the same time make (65) rise (because $b$ and $ρ$ are rising) and fall, because $κ$ is falling! But the former force wins in the practical range, as seen from Column 7 of Table 1 and from Figure 3: The higher the employers' contribution fraction $b$ and the longer the redemption period $ρ$ are, the higher is the propensity to save national output. The elasticities of the latter with respect to $b$ and $ρ$ are modest in the range considered politically. Beyond $b = 1/10$ and
\[ \rho = 8 \text{ they become noticeably higher.} \]

4. The Real Wage Rate

Use (34), (36), and (37) to find \( I = vX \) and (1) through (16) to find that \( I = g_S S \), hence \( S = vX/g_S \). Insert that into (17), insert the outcome into (18) and find the real wage rate

\[
\frac{w/P}{c} = \alpha[M(v/g_S)^B]^{1/\alpha}
\]

Using our empirical parameter values we find the elasticity \( \beta/\alpha \) of the real wage rate with respect to the propensity to save national output to be \( 1/3 \)—a particularly simple Wicksell Effect. Consequently, the elasticities of the real wage rate with respect to \( h \) and \( \rho \) would be one-third of those of the propensity to save national output with respect to \( h \) and \( \rho \) shown in Figure 3. In other words, \( w/P-h \) and \( w/P-\rho \) curves would have in any point one-third of the steepness of the corresponding point of the corresponding curve in Figure 3.
5. Relative Size of Fund

Insert (20) and (59) into (29) and write size of fund

\[ \phi = \phi PS, \text{ where} \]
\[ \phi \equiv b\kappa [e^{(\kappa - g_s)\rho} - 1]/(\kappa - g_s) \]

There are two opposing forces at work here: Rising \( b \) or \( \rho \) will at the same time make \( \phi \) rise (because \( b \) and \( \rho \) are rising) and fall, because \( \kappa \) is falling. But the former force wins very easily in the practical range, as seen from Column 8 of Table 1 and from Figure 4:

The higher the employers' contribution fraction \( b \) and the longer the redemption period \( \rho \) are, the larger is the investment fund as a fraction of the value of capital stock. The elasticities of the fraction \( \phi \) with respect to \( b \) and \( \rho \) are considerable.

IX. PROFIT SHARING OR INVESTMENT WAGE: DOES IT MAKE ANY DIFFERENCE?

From (18) and (22) we know that \( W = \alpha PX \) and from (20) that \( Z = \beta PX \). Whether employers contribute a fixed fraction of their wage
bill or their profits bill, then, they will be contributing a
fixed fraction of the value of output. Consequently the differences
between an investment wage and profit sharing cannot\(^2\) be great.
Four differences, rooted inside or outside our model, do seem to
be worth mentioning.

First, under an investment wage one would define—within
our model—the money wage rate \(w\) as used in (18) as including
employers' contribution per man year to the wage earners' investment fund.
Only if raising \(w\) by the investment wage per man year would raise the
price of goods \(P\) in the same proportion could the real wage equal the
physical marginal productivity of labor in accordance with (18) at full
employment in accordance with (21). In this sense the investment wage
would have been shifted to the price of the product—with a modification
made in Sec. VII, 4.

No such shifting would occur under profit sharing: The capitalist
entrepreneurs would be maximizing \((1 - b)Z\) but within the rigid

\(^2\)In detail, the effects of an investment wage within an otherwise
similar model have been examined in [2].
null
framework of the model the employment $L$ which maximizes $(1 - b)Z$ is the same as that which maximizes $Z$. Consequently price $P$ would remain unaffected by profit sharing. This difference between an investment wage and profit sharing is rooted inside our model. The next three differences are rooted outside it.

Second, the rigid framework of our model included the specific provision, made in Sec. I, that capital stock is immortal. That provision eliminated depreciation allowances from our model. But in the real world, profit sharing would introduce an incentive, absent under the investment wage, to magnify depreciation allowances in every way possible. As we know from corporate-income-tax practice and price-control practice there are such ways.

Third, the rigid framework of our model included the specific provision, also made in Sec. I, that capitalists should be stockholders rather than bondholders. That provision eliminated bond financing from our model. But in the real world, with profits defined to mean profits after interest on liabilities, profit sharing would introduce an incentive, also absent under the investment wage, to substitute bond financing for equity financing. To remove that incentive, profits would have to be redefined to mean profits after interest on both equity and liabilities—as done by the French 1967 statute on profit sharing.

Fourth, our solutions were steady-state ones, and others were
ignored. The real world may be steady-state in the long run, but in the short run it has cycles. As we know, the wage bill fluctuates less over the cycle than does the profits bill. Consequently, contributions in the form of an investment wage would fluctuate less than would contributions in the form of profit sharing. However similarly an investment wage and profit sharing would work in the long run, they would thus work differently in the short run, and the difference might be important psychologically: Under profit sharing, labor might feel asked to participate in a risk which would be absent from an investment wage.
APPENDIX

THE EXISTENCE OF A UNIQUE AND POSITIVE SOLUTION OF EQUATION (60)

1. The Function \((e^f_p - 1)/f\)

Define \(f = \kappa - g_s\). Then Eq. (60) has in it a function \(G(f)\) defined as

\[
G(f) = \frac{(e^f_p - 1)}{f} \text{ for } f \neq 0
\]

\[
G(0) = \rho \text{ for } f = 0
\]

Assume \(\rho > 0\). Then \(G(0)\) is positive. And \(G(f)\) is positive for \(f < 0\), because then \(e^f_p < 1\), and also positive for \(f > 0\), because then \(e^f_p > 1\). The limit of \(G(f)\) for \(f \to 0\) is found by L'Hôpital's Rule:

\[
\lim_{f \to 0} G(f) = \rho
\]

But if \(G(f)\) has both the value and the limit \(\rho\) at \(f = 0\), it is continuous at \(f = 0\). The function is shown in Figure 5.

To see how \(G(f)\) varies with \(f\), differentiate with respect to \(f\):

\[
\frac{d[(e^f_p - 1)/f]}{df} = \frac{fpe^f_p - (e^f_p - 1)}{f^2} = e^{-f_p} - (1 - fp)
\]

\[
(68)
\]
The denominator of (68) is nonnegative: It is positive for all values of \( f \) other than \( f = 0 \), for which it is zero. The numerator is also nonnegative. Write it \( u \equiv e^{-x} - (1 - x) \), where \( x \equiv fp \). Take the derivative \( du/dx = -e^{-x} + 1 \), set it equal to zero, and find \( x = 0 \). Take the second derivative \( d^2u/dx^2 = e^{-x} > 0 \). Consequently \( u \) satisfies the first-order and second-order conditions for a minimum at \( x = 0 \): \( u \) is positive for all values of \( f \) other than \( f = 0 \), for which it is zero. For \( f = 0 \) the limit of the derivative (68) can be found by using L'Hôpital's Rule twice:

\[
\lim_{f \to 0} \frac{d[(e^{fp} - 1)/f]}{df} = \frac{\rho^2}{2}
\]

which is positive.

2. The Brace of Eq. (60)

The brace of Eq. (60) may be written

\[
(69) \quad (1 - c_d)/g_s + \beta bc_d G(f)
\]

Realistically assume that

\[
0 < \beta < 1
\]

\[
0 < b
\]
The Expression $\frac{e^{(\kappa-g_s)-1}}{\kappa-g_s}$ for $g_s = 0.04$

Physical Marginal Productivity of Capital Stock, $\kappa$

$\rho = 16$

$\rho = 8$

$\rho = 4$

$\rho = 2$

FIGURE 5

Hans Brems, "Profit Sharing"
Then the brace (69) is positive for all values of \( f \). At \( f = 0 \) it has both the value and the limit \((1 - c_d)/g_S + \beta b c_d \rho\), hence is continuous.

3. The Entire Eq. (60)

But if the brace is always positive, then \( \kappa \) times the brace is negative, zero, and positive for \( \kappa < 0 \), \( \kappa = 0 \), and \( \kappa > 0 \), respectively. Moreover, since the derivative (68) has a positive limit at \( \kappa = g_S \) and is positive at all other values of \( \kappa \), the brace is rising with rising \( \kappa \), and \( \kappa \) times the brace is rising in more than proportion to \( \kappa \). Consequently, if we draw the left-hand side of (60) as a function of \( \kappa \), the function will be continuous, will be located in the third quadrant for \( \kappa < 0 \), will pass through the origin for \( \kappa = 0 \), and will be located in the first quadrant for \( \kappa > 0 \). It is rising without bounds as \( \kappa \) rises without bounds.

The right-hand side of (60) can be drawn as a horizontal line at the positive distance \( \beta \) from the \( \kappa \)-axis. Curve and line must intersect, will do so only once, and will do so in the first quadrant. This proves the existence of a unique and positive solution for \( \kappa \).

\[
\begin{align*}
0 &< c_d < 1 \\
0 &< g_S \\
0 &< \rho
\end{align*}
\]
REFERENCES


FOOTNOTES

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1Profit sharing is on the statute books of France, was proposed by labor in the Netherlands in 1964, and is being proposed by labor in the German Federal Republic. Alternatively, employers would contribute a fraction of their wage bill (an investment wage) or a fraction of their equity (equity sharing). A summary of statutes and proposals in Europe has recently been published by The Economist Intelligence Unit [4]. The present article confines itself to profit sharing. Elsewhere [2] the writer has examined the investment wage.