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Contract Enforcement in Sellers' Markets

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Abstract

We present an infinitely repeated game model where a monopolist seller has a contractual obligation with several buyers in each period. If a contract is violated the buyers can collect some compensation and impose a penalty on the seller. The Folk Theorem for infinitely repeated games implies that there are an infinite number of subgame perfect equilibria in this model but we employ a new equilibrium concept called validated equilibrium that picks out a unique equilibrium outcome for the game where the seller is able to dominate the buyers. This model is clearly applicable to supply problems of Soviet-type economies, but we believe that it can explain certain phenomena of western economies as well, in particular it sheds some light on problems of entry deterrence.

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Section One-Introduction

In centrally planned economies bureaucrats face tremendous problems in compiling and implementing plans. Only the most naive and dogmatic textbook accounts of central planning could conceive of people constructing a feasible, let alone optimal, plan that is executed by workers and bureaucrats who need no outside motivation except their instructions. Particularly serious are the problems that arise at the implementation phase in the intermediate product markets. Even if the plan is feasible, the task of monitoring every single delivery centrally to make sure that the right amount of the right type of the right good was delivered to the right factory at the right time would be a phenomenally expensive task. Deliveries of intermediate products in the Soviet economy in one year surely number in the billions.¹

In practice socialist economies employ a sensible decentralized procedure to minimize the information processing burden associated with the implementation stage of planning. The plan defines a set of contracts between

¹ Fedorenko (1985) estimates that their are twenty million products produced in the industrial sector alone in the Soviet economy. But even this measure underestimates the diversity of products because goods are not distinguished according to when or where they are available. Karpov (1972) claims that there are about 72,000 different types of ball bearings produced in the Soviet economy.
buyers and sellers specifying what is to be delivered, when it is to be delivered etc. When a contract has been violated the buyer who has been hurt is supposed to report the encroachment. The seller is then punished and the buyer receives some compensation. The scheme induces buyers to report delinquent sellers while deterring the sellers from breaching contracts in the first place.

However many analysts have called attention to the weak performance of the contract enforcement system in socialist economies. Bornstein writes;

"...these annual contracts are characterized by widespread violations of 'contract discipline', largely attributable to shortages and sellers' market conditions, that reduce the effect of contractual agreements in guiding production and supply far below the extent intended by PIEM[the Program to Improve the Economic Mechanism].... When contracts are not fulfilled, buyers seldom file claims for breach of contract, lest they offend sellers upon whom they will be dependent in the future. When penalties are levied, their effect on sellers is weak because the fines are relatively small and they are paid from profits, without significant reductions in enterprise incentive funds or individual bonuses. In turn, fines provide buyers trivial compensation for their losses, as they receive only 5% of the (small) fines with 95% paid to the budget. As a result, there is a practice of 'mutual amnesty' under which buyers do not claim compensation for non-delivery and sellers do not claim compensation for late payment." (Bornstein (1985).

\(^2\) See particularly the two excellent papers by Heidi Kroll in the references.
According to Nove:

"A key factor here,..., is the sellers' market plus monopoly. In an economy of shortage, the supplier is powerful. He can insist on his own terms, knowing that he can cause great inconvenience." (Nove (1977) page 113).

In this paper we study the issue of the weakness of contract discipline in situations where a monopolist seller\(^3\) is in a long term relationship with some buyers.\(^4\) The seller has a contractual obligation to each buyer in each of an infinite number of periods. These obligations are very difficult (maybe impossible) to fulfill.\(^5\)

The main conclusion is that unless there are very generous rewards offered to buyers for reporting contract violations the seller will be in a position to intimidate his buyers so that they do not report contract violations. However, even though at equilibrium the seller is persistently violating more contracts than necessary while the buyers meekly refrain from complaining, the contract system still plays a valuable role in preventing the seller

\(^3\) One monopoly supplier is the typical situation in Soviet industry (see Nove (1977) pp. 42-44 and 113)) although the recent reforms initiated by General Secretary Gorbachev have the aim of eliminating most of these monopolies and moving toward a system of wholesale trade (see Hewitt (forthcoming)).

\(^4\) One tendency in Soviet reform has been to actually encourage buyers to develop a long term relationship with suppliers (see Schroeder (1979) and Bornstein (1985)). The theory is that then suppliers can specialize in tailoring their production to the very specific needs of their customers. Ironically it is the point of this paper that under these conditions suppliers do anything but tailor their production to the needs of their customers.

\(^5\) Levine states that the Soviet economy "has been marked by a chronic sellers' market; i.e., the situation where demand is consistently pressing upon supply." See Levine (1959), page 151. Also see Levine (1966).
from breaching even more contracts than would be violated without the contract system.

To achieve the results we use an infinitely repeated game model to capture the idea of a long term relationship. While there is now a large literature on how to model reputations in finitely repeated games (see Kreps, Wilson, Milgrom, and Roberts (1982) and Kreps and Wilson (1982)) we do not present such results here.

Unfortunately the infinite formulation is plagued by the problem that almost any feasible payoff vector qualifies as the equilibrium payoff vector in a perfect equilibrium.\(^6\) So an infinitely repeated game model seems unlikely to deliver a result of complete seller dominance. Indeed in our model there can be either buyer or seller dominance at a perfect equilibrium. However, we are able to introduce a new equilibrium concept called a validated equilibrium which is essentially unique and involves seller dominance in this model.

We present the model in section two. In section three the folk theorem for infinitely repeated games is reviewed and applied to our game. Section three contains a discussion of some interesting types of subgame perfect equilibria and some simple comparative statics are performed. We show that in the equilibrium where buyers dominate, the behavior of the seller can be improved only by increasing the penalties for contract violations. In the equilibrium where the seller

\(^6\) This is the Folk Theorem (see Abreu (1983) and Fudenberg and Maskin (1986)).
dominates, the only way to improve his performance is to increase the rewards to buyers for reporting contract violations even though buyers will never report any contract violations in this equilibrium.

In section five we introduce the validated equilibrium concept and illustrate it through examples. Section six presents our main result: that seller dominance is the essentially unique validated equilibrium of the model. We also point out how the same arguments can be applied to an infinitely repeated game of entry to show that entry deterrence in the unique outcome of the game when there are a sufficiently large number of potential entrants.

Section seven draws some conclusions and presents some possibilities for future research.

Section Two-The Model

There are n+1 players in the game: n buyers, B_1,...,B_n indexed by i, and one monopolist seller M. We denote the strategy spaces of the players in the stage game (the one-shot game that is repeated) by S_1,...,S_n, S_M and the payoff functions by P_1,...,P_n, P_M which are functions from n+1-tuples of strategies to the real numbers.

The structure of the game is particularly simple. First the seller chooses a number between 0 and n. Call this number q(s_M) for each s_M in S_M. This is the number of contracts he violates. The seller then decides which
contracts to violate i.e. the monopolist picks his victims. After the seller moves, each victim (a buyer whose contract has been violated) decides either to report or not report the violation. We denote the number of reported violations by $r(s_1,\ldots,s_n,s_M)$ for each possible configuration of strategies. We do not consider mixed strategies in this game.

$$P_M(s_1,\ldots,s_n,s_M)= w(r(s_1,\ldots,s_n,s_M))+v(q(s_M))$$

where $w(.)$ is decreasing and $v(.)$ is increasing. Unpleasant effort is required for the monopolist to fulfill contracts but it is bad to be caught violating a contract. The latter can be true for many reasons. For example, firms or their managers might be fined for breaking the law and managers might diminish their chance of promotion by acquiring a reputation for producing shoddy goods.

For each $i$,

$$P_i(s_1,\ldots,s_n,s_M)=X \text{ if } i's \text{ contract was fulfilled}$$

$$=Y \text{ if } i's \text{ contract was violated and } i \text{ reported the violation}$$

$$=Z \text{ if } i's \text{ contract was violated but } i \text{ did not report the violation}$$

where $X>Y>Z$. The idea is that there is some compensation for reporting contract violations but this compensation is not so generous that buyers actually prefer their contracts to be violated so they can collect compensation.
The stage game is repeated an infinite number of times. The move of player i in period t is denoted \( s_{it} \). The history of the infinite game (supergame) until period t, denoted \( H_t \), is given by \( (s_{11}, \ldots, s_{1t-1}, \ldots, s_{i1}, \ldots, s_{it-1}, \ldots, s_{n1}, \ldots, s_{nt-1}, s_{M1}, \ldots, s_{Mt-1}) \). In other words the history of the supergame up to time t is simply a list of what every player has done through period t-1.

The strategy of player i in the supergame is given by functions \( s_{it}(H_t) \) for \( t=1,2,\ldots \) that give a stage game move as a function of every history of every possible length. The strategy space for player i is the set of all possible such sequences of functions. Supergame strategies for the monopolist are defined in the same way. Denote an \( n+1 \)-tuple of supergame strategies by \( s^\infty=(s_{11}^\infty, \ldots, s_{n1}^\infty, s_{M1}^\infty) \) where \( s_{i1}^\infty=(s_{i1}, s_{i2}(H_2), \ldots, s_{it}(H_t), \ldots) \) for each i and similarly for \( s_{M1}^\infty. \) Since we never have occasion to discuss the one-shot game in this paper we will drop "\( \infty \)" when we refer to supergame strategies.

The supergame payoffs are then computed according to the overtaking criterion which can be explained as follows.\(^7\) Consider two infinite sequences of one-shot payoffs \( x_t \) and \( y_t \) for \( t=1,2,\ldots \) and denote these sequences \( x \) and \( y \). Then \( y \) is preferred to \( x \) iff \( \lim_{T \to \infty} \sum_{t=1}^{T} (y_t-x_t)>0. \)

This is not a discounting formulation but we have checked that all of the qualitative results still hold in this case. But it is slightly more complicated and we feel

\(^7\) See Rubinstein (1979).
that the complications divert attention from the essence of this paper. Also the results can be proved in the case where payoffs are expressed according to a limit of means criterion.

$s_i$ constitutes a Nash equilibrium iff each strategy $s_i$ \(i=1,\ldots,n\) is a best response to 
\[(s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_n,s_M)\] and \(s_M\) is a best response to 
\[(s_1,\ldots,s_n)\]. In other words if player i assumes that the other players are playing strategies 
\[(s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_n,s_M)\], $s_i$ will maximize his payoff.

A (subgame) perfect equilibrium has the additional property that for any history of strategies through time $t-1$, $H_t$, the strategies from $t$ on constitute a Nash equilibrium. This includes histories that could not occur if all players are playing the equilibrium strategies.

We will work with a new refinement of the subgame perfection we call validated equilibria that will be introduced in section five.

Section Three-Subgame Perfect Equilibrium\(^8\)

The so called folk theorem for infinitely repeated games is well known in game theory (although perhaps not so widely known generally as to merit its name).\(^9\) To state it we define the minmax payoff vector as follows. The minmax

\(^8\). Often we will say perfect equilibrium when we mean subgame perfect equilibrium.

\(^9\). Early results on the perfect Folk Theorem were achieved by Aumann and Shapley (1976) and Rubinstein (1979).
payoff for a player in any game is the lowest payoff that
the other players can hold that player at or below. We can
find a player's minmax payoff by ranging over all
combinations of strategies for the other players and then
computing the payoff of the optimal response of the first
player to these combinations. A combination that yields the
lowest payoff to the first player is a minmax combination
and the payoff received by the player is his minmax payoff.
The vector of minmax payoffs is the minmax vector.

Formally let \( S_\sim = S_1 \times \ldots \times S_{i-1} \times S_{i+1} \ldots \times S_M \) where "\( \times \)"
denotes cross product. Then \( s_\sim \in S_i \) is a minmax strategy for
the players other than \( i \) if it solves

\[
\min \left[ \max_{s_i \in S_i} \left( \max_{s_\sim \in S_\sim} \mathcal{P}_i(s_i, s_\sim) \right) \right].
\]

Theorem One (Perfect Folk Theorem) - A vector of payoffs
is the long run average\(^{11}\) payoff vector of a perfect
equilibrium if it is feasible and strictly pareto dominates
the minmax payoff vector of the one-shot game.

Proof (sketch) - Consider a payoff vector \( (p_1, \ldots, p_n, p_M) \) that
strictly dominates the minmax point and supergame strategies
\( (s_1, \ldots, s_n, s_M) \) that yield that payoff vector as the long run
average.

If players play according to \( (s_1, \ldots, s_n, s_M) \) a sequence
of game histories, \( H_1, \ldots, H_T, \ldots \) are generated which we will
call the equilibrium path (although we must still show that
this path can be generated at an equilibrium).

\(^{10}\) \( s_i, s_\sim \) is shorthand for \( (s_1, \ldots, s_{i-1}, s_i, s_{i+1}, \ldots, s_M) \)

\(^{11}\) For an infinite sequence of payoffs \( x_T \) for \( t=1, 2, \ldots \) the
long run average is \( \lim_{T \to \infty} \frac{\sum_{t=1}^T x_T}{T} \).
We modify the strategies' dependence on history off the equilibrium path to sustain a perfect equilibrium. For any history that is off the equilibrium path there must be at least one player who deviated from \((s_1, \ldots, s_n, s_M)\) to make that history possible. Choose one such player and have all the other players join to hold this player down to his minmax payoff for long enough to wipe out any gain the player might have achieved from his deviations.\(^{12}\) The punishments are enforced by punishing any player who fails to participate in the same manner. Any player who does not punish the second deviator is punished and so on in an infinite regress. After a punishment phase ends the game proceeds as if the present game history is correct (i.e. the history that would prevail currently if no deviations had ever taken place).

These strategies will constitute a subgame perfect equilibrium and yield long run average payoffs \((p_1, \ldots, p_n, p_M)\) since on the equilibrium path the play of the game will look like the players were playing \((s_1, \ldots, s_n, s_M)\).\(^{11}\)

In our game the minmax payoff for the monopolist is derived by choosing \(q\) to maximize \(p_M\) subject to the constraint that \(r=q\) in every period, i.e. all violations of contract discipline are reported.\(^{13}\) For the buyers \(Y\) is the minmax payoff. The set of feasible payoffs in the one shot

\(^{12}\) This is where it is convenient to use the overtaking criterion.

\(^{13}\) The worst thing that can happen to the monopolist is to have all contract violations reported.
game is computed by letting \( q \) run between one and \( n \) in each case letting \( r \) run from 0 to \( q \). The set of feasible payoffs is the convex hull\(^{14}\) of the set of possible payoffs in the one shot game. It is clear that the set of equilibria is huge.

Note that in general the folk theorem does not allow us to sustain payoff vectors that only weakly dominate the minmax vector.\(^{15}\) This fact actually causes some technical difficulties for the analysis that follows.

Section four- Buyers' Markets and Sellers' Markets

Although there are an infinite number of equilibria in our game two types of subgame perfect equilibria are particularly interesting.

The first equilibrium involves seller dominance of the buyers. Choose the number \( c \) between 0 and 1 such that \( cZ + (1-c)X = Y \). For each small enough \( \epsilon > 0 \) there will be a perfect equilibrium where each buyer has his contract violated exactly \( 100(c-\epsilon)\% \) of the periods and every buyer never reports that his contract has been violated.

In this equilibrium all the buyers receive a little more than their minmax payoffs so they can not be pushed down much further. Note that if \( Y \) is increased then \( c \) is decreased, i.e. if the compensation given to buyers for

\(^{14}\) Strictly speaking it is only vectors of the convex hull with rational entries.

\(^{15}\) The exception to this is Nash equilibrium with a limit of means payoff criterion.
reporting breaches of contracts is increased than the frequency of contract violations decreases. Increasing the penalty leveled at sellers for violating contracts has no effect on the seller's behavior because this penalty is never applied in the seller's dominance equilibrium. We are left with the rather curious conclusion that although contracts are persistently violated and sellers are never caught and punished, nevertheless the contract system plays a useful role in preventing the monopolist's performance from becoming even worse. Call this equilibrium the ε-seller's market equilibrium.

The second equilibrium involves buyers' dominance. In this type of equilibrium buyers report all but a tiny fraction $\epsilon > 0$ of contract violations and the seller chooses the same $q$ in every period that maximizes his utility subject to $r$ being almost equal to $q$. Furthermore the seller's victims in every period $t$ depend only on $t$. It is easy to check that this is a Nash equilibrium and it can be made into a perfect equilibrium by the argument in section three. Note that raising $Y$ does nothing to improve the seller's performance at this equilibrium. But seller performance can be improved by increasing the penalties for contract violations. Call this equilibrium the ε-buyer's market equilibrium.

Note that the set of subgame perfect equilibrium payoffs is open hence the need to introduce the ε' s above.
Section Five-Validated Equilibrium

The existence of both these equilibria demonstrates how weak the perfect equilibrium concept is in the present context. The qualitative properties of the two equilibria could not possibly be more different. While both equilibria (and mixtures of the two) have a certain plausibility, we find the seller's market situation the more interesting, particularly since it is much more consistent with Soviet reality than the other equilibria. In fact it turns out to be the essentially unique outcome under our new equilibrium concept. The rest of this section is devoted to introducing and illustrating the concept of validated equilibrium.

Consider the normal form of an extensive form game with \( n \) players, strategy spaces \( S_1, \ldots, S_n \) and payoff functions \( P_1, \ldots, P_n \). Suppose that player \( i \) announced that he would play a specific strategy \( s_i^* \) that is part of some subgame perfect equilibrium of the game.\(^{16} \) Suppose further that all the other players believe him provisionally. This announcement would induce a game, \( G(s_i^*) \) between \( N-1 \) players with strategy spaces \( S_2, \ldots, S_n \) and payoff functions \( P_2', \ldots, P_n' \) such that \( P_i'(s_2, \ldots, s_N) = P_i(s_1^*, s_2, \ldots, s_N) \).

Denote the set of subgame perfect equilibria of this game \( \text{SP}(G(s_i^*)) \). The crucial question we ask is would the original announcement be a best response to every induced

\(^{16} \) We are working with the normal form of an extensive form game.
game Nash equilibrium? If the answer is yes then we consider the original announcement to be a credible one.

However we wish to apply a slightly weaker notion of credibility. In particular we stipulate that potential leaders are punished in some manner if they deviate from their original announcement. Remember that $G(s_i^*)$ is constructed in such a way that, in general, many nodes left over from the original extensive form game can not be reached in the extensive form of $G(s_i^*)$. So if $s_i \epsilon SP(G(s_i^*))$ then we can change $s_i$ all we want on the nodes that can not be reached given $s_i^*$ and each such new strategy will still belong to $SP(G(s_i^*))$. We fix a single outcome from this large set as follows. At any node of the original extensive form that is impossible to reach given $s_i^*$ the players of the induced game play strategies that are part of the worst subgame perfect equilibrium for player $i$ in the game with $N$ players that begins at this node. Call this smaller set of induced game equilibria $E(G(s_i^*))$. If there does not exist a worst subgame perfect equilibrium for player $i$ then we need to define $E(G(s_i^*, \epsilon))$ be the set of induced game equilibria that assign strategies to the players besides $i$ that are within $\epsilon$ of being the worst subgame perfect equilibrium for $i$ starting at each node that would be impossible given $s_i^*$.

17. Of course there may exist many worst perfect equilibrium outcomes, but the will always exist at least one as long as the strategy sets are compact and the payoff functions are continuous.
Formally, for a strategy $s_i^*$ to be self-validating for player $i$ it must be part of some subgame perfect equilibrium of the game and it must solve $\max_{s_{-i} \in E(G(s_i^*))} P_i(s_i^*, s_{-i})$ for each $s_{-i} \in E(G(s_i^*))$.

We define the value of a self-validating strategy $v(s_i^*)$ as the payoff resulting from the worst induced game equilibrium. Clearly this is pessimistic conjecture but our analysis will depend on this assumption. Formally we define $v(s_i^*) = \inf_{s_{-i} \in E(G(s_i^*))} P_i(s_i^*, s_{-i})$.

For a set of strategies to constitute a validated equilibrium we will require that the strategies constitute a subgame perfect equilibrium and that no player has a self-validating strategy with a payoff strictly higher than the proposed equilibrium payoff. So we are looking for subgame perfect equilibria that can not be upset through the actions of a credible leader.

Formally, $(s_1, \ldots, s_n)$ is a validated equilibrium if it is a subgame perfect equilibrium and there does not exist a player $i$ with a self-validating strategy $s_i^*$ such the $v(s_i^*) > P_i(s_1, \ldots, s_n)$.

Because the equilibrium set for infinitely repeated games is open it turns out the we have to employ a slightly more general concept of $\epsilon$-validated equilibrium in this paper. First, to define a self-validating strategy we use the induced game equilibrium set $E(G(s_i^*, \epsilon))$ rather than
E(G(s₁*)). Second we assume that players will not block an equilibrium for very small gains. We say (s₁, ..., sₙ) is an \( \epsilon \)-validated equilibrium if it is a subgame perfect equilibrium and there does not exist a player i with a self-validating strategy \( s_i \) such the \( v(s_i) > P_i(s_1, ..., s_n) + \epsilon \). This is a sensible concept if there is some small cost attached to a blocking action.

We now illustrate the concept. Consider the following game:

\[
\begin{array}{c|cc}
\text{player one} & \text{L} & \text{R} \\
\hline
\text{U} & 2,1 & 0,1 \\
\text{D} & 2,0 & 1,2 \\
\end{array}
\]

The unique validated equilibrium of the game is (down, right). This is because the unique self-validating strategy is right for player two. Up is not a self-validating strategy for player one because player two can respond optimally with any of his strategies including mixtures if they are allowed. Up will not be an optimal response to all of these strategies.
(up,left) is the unique validated equilibrium in the game below:

\[
\begin{array}{c|cc}
\text{player one} & \text{L} & \text{R} \\
\hline
\text{U} & 2,2 & -1,-1 \\
\text{D} & -1,-1 & 0,0 \\
\end{array}
\]

In the game below player one chooses the left matrix or the right matrix, player two chooses up or down and player three chooses left or right.

\[
\begin{array}{c|cc|cc}
\text{player two} & \text{L} & \text{R} & \text{player one} & \text{L} & \text{R} \\
\hline
\text{U} & 4,4,4 & 0,0,0 & \text{U} & 0,6,6 & 0,8,8 \\
\text{D} & 0,0,0 & 0,2,2 & \text{D} & 0,8,0 & 5,5,5 \\
\end{array}
\]

There are two pure strategy Nash equilibria: (left,up,left) and (right,down,right). Only the second is a validated equilibrium. To verify that (right,down,right) is a validated equilibrium note that right is a self-validating strategy for player one\(^{18}\) since it induces the game,

\(^{18}\) It is also true that down is self-validating for two and right is self-validating for one.
which has a unique Nash equilibrium of (down,right). Given this induced game equilibrium player one would be satisfied with his announcement of right. This implies that (left,up,left) is not a validated equilibrium since it could be blocked by player one.\(^\text{19}\)

It is straightforward but tedious to verify that neither player two nor player three has a self-validating strategy with a value higher than five. This is true even allowing for mixed strategy equilibria and announcements. Therefore (right,down,right) is a validated equilibrium.

Incidentally it is easy to show that left is not self-validating for player one. An announcement of left by player one induces the game.

\(^{19}\). Player two and player three also can block.
This induced game has equilibria (up, left) and (down, right). But if players two and three settle at (down, right) player one would want to abandon his announcement and play right.

It should be clear from the examples above that in a two person game a strategy may not be self-validating but it can become so if the payoffs for the game are perturbed slightly. This will not be the case in generic games with more than two players so validated equilibrium is robust in the sense that small perturbations in payoffs will lead to small changes in the equilibrium set.

Section Six-Validated Equilibrium of the Repeated Game

We now can state the main result of this paper.

Theorem-Suppose that for some small $\epsilon > 0$:

a) the $\epsilon$-sellers' market equilibrium in section four is preferred by the seller to his minmax payoff plus $\epsilon$:

b) the utility to the seller of violating on average $(c-\epsilon+1)n$ contracts per period and being reported once per period is higher than the utility of his best response to being always reported for any transgression by $n-1$ buyers with the last buyer always being acquiescent.

Then there exist $\epsilon$-validated equilibria for each $\epsilon > 0$ and the seller's long-run average payoff is within $\epsilon$ of his maximum subgame perfect equilibrium long-run average payoff at each of these $\epsilon$-validated equilibria.
Proof-The $\epsilon$-sellers' market equilibrium is a subgame perfect equilibrium in which buyers never report contract violations and each has his contract violated in $100(c-\epsilon)$ percent of the periods where $c$ satisfies $cZ+(1-c)X=Y$. The seller responds to any reported contract violation by minmaxing the squealer, i.e. violating his contract for enough periods to make the buyer regret his transgression. The essentially\textsuperscript{20} unique equilibrium in the induced game generated by this strategy involves complete acquiescence by the buyers. The strategy is self-validating because by condition a the seller prefers the sellers' market equilibrium to his minmax payoff plus $\epsilon$ which he will receive forever if he ever deviates from his announcement.

We can conclude that any combination of strategies where the seller does not receive a long-run average payoff within $\epsilon$ of his maximum subgame perfect equilibrium can not constitute an $\epsilon$-validated equilibrium. Such a configuration would be blocked by the seller.

Now note that no buyer has a self-validating strategy. Proof-Consider any announced strategy by a buyer. There will always exist an induced game equilibrium where each remaining buyer has his contract violated $(c-\epsilon)100$ percent of the time and they never report violations and the announcing buyer has his contract violated in every period. If the seller fails to violate the announcing buyer's

\textsuperscript{20} We need the qualifier essentially because we can obtain many equilibria by varying the action off the equilibrium path.
contract in a single period all players immediately switch to reporting all contract violations (minmaxing the seller) for a sufficient number of periods to make the seller regret his softness. Assumption b on the seller's preferences ensures that this outcome is a subgame perfect equilibrium in the induced game.

There is only one best response for the announcing buyer to having his contract violated in every period. That is to always fight so we can conclude that if the original announcement was self-validating the announced strategy must have involved fighting all contract violations.

In the above induced game equilibria an announcement to always fight yields the minmax payoff for the announcing buyer. This is already sufficient to complete the proof since we have shown that buyers certainly can not receive more than their minmax payoffs in the long run through a blocking action.

But an announcement to always fight is not even self-validating because any deviation from this announcement (i.e. not reporting some contract violation) will yield a long-run average payoff from that point on slightly higher than the minmax level due to the method of completing induced game equilibrium strategies when there is no worst equilibrium payoff for the announcing player.

So the proof is complete.
Condition a of the theorem in completely innocuous. Without it the economic situation would be without interest. Condition b becomes more likely to be satisfied if there are many buyers. The intuition flowing from the proof above is that with many buyers the cost to the monopolist of not clamping down hard on a single aggressive buyer is very large because this weakness will turn the other buyers (who are large in numbers) aggressive.

We now show how the model can be altered to serve as a theory of entry deterrence. Suppose a firm M will definitely be in a market for an infinite number of periods. Several other firms independently take a decision in each period to be in or to be out. If at least one firm enters in a given period then the incumbent must decide either to fight or not to fight. Fighting hurts both the monopolist and any firm that has entered. Also regardless of whether or not he fights the monopolist always likes to have fewer firms in the market.

This infinitely repeated game will have equilibria where all firms enter in every period, equilibria where no firm ever enters and many equilibria in between. But under assumptions analogous to a and b in the theorem above at the essentially unique validated equilibrium no firm ever enters the market.

Condition b is interesting in this context. It states that the incumbent would rather engage in a price war every period with a persistent entrant than to quietly acquiesce
in the entry of all the other potential entrants. So the existence of a plethora of potential competition can actually encourage monopoly by raising the stakes to the monopolist of slightly loosening his grip on the market.

Section Seven—Conclusion

The validated equilibrium notion is not the ultimate solution to the multiplicity problem for repeated games. However I think it has a certain attractiveness for the game studied above. It allows a certain type of leadership pattern to emerge based on the structure of the game without simply designating one player to be a Stackelberg leader. In principle any player can become a leader by announcing he will take a certain course of action but the structure of the game may render his announcement incredible.

Of course the equilibrium concept has the additional virtue that it has allowed us to capture and analyze an important mode of behavior for managers in Soviet-type economies. The model produces several interesting results. First, the system of contracts probably is playing a positive but weak role in disciplining sellers. Second, to improve the performance of suppliers it seems that the authorities should focus their attention on increasing rewards for reporting breaches of contracts rather than increasing penalties applied to breachers.
Some people have concluded based on this model that the problem of contract enforcement in the Soviet context can be completely solved by giving rewards for reporting contract violations that are so generous that the buyers would not care whether their contracts are satisfied or whether they are violated but the buyers report the violation. In other words eliminate the problem by setting $Y$ equal to $X$. In the model presented above that would be an effective strategy but it would be a bad idea in general. This is because if the court system was unable to distinguish perfectly between legitimate complaints and false, giving overly generous rewards for contract violations would encourage too much complaining. While this consideration has been left out of the model we believe it could be incorporated. This however, could be a topic for future research.
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