Confirmatory and Exploratory Causal Measurement Models Regarding the Underlying Economic Dimensions Inherent in Annual Accounting Data: An Empirical Study Based on Naive Expectations

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Abstract

This study investigates various measurement models depicting the economic dimensions inherent in the reporting of annual accounting data. A theoretical model structure based on four financial dimensions of a firm, liquidity, leverage, profitability, and activity, is estimated and tested using a Full Information Maximum Likelihood confirmatory factor analytic approach.

The overall tests of model fit indicate that alternative model structures may be more representative. Various alternative structures are constructed on an exploratory basis. These models are then estimated and tested.
1.0 Introduction

The linkage between accounting derived measures of firm attributes and the underlying economic dimensions they purport to measure has been given little empirical consideration. This study investigates various model configurations which represent the information characteristics of a firm that result from the issuance of annual accounting data.

Using four fundamental economic dimensions of the firm, various model configurations linking expectation errors of accounting data derived measures to expectation errors of the four underlying dimensions are estimated and tested using a Full Information Maximum Likelihood factor analytic approach. As such, various measurement models are formulated, estimated, and tested.

In previous research, Benston (1967), Ball and Brown (1968), Beaver (1968), Brown (1970), May (1971), Brown and Kennelly (1972), Kiger (1972), Hagerman (1973), Gonedes (1974, 1975), Beaver, Clarke, and Wright (1979) and others have investigated the reactions of the securities market to the announcement of corporate financial accounting information. Evidence indicates the market reacts to the announcement of the earnings per share figure. Earnings per share is recognized as having information content, a statistical dependency between earnings per share expectation errors and abnormal security returns and/or abnormal trading volume.

The earnings per share figure is, in effect, an indicator of the profitability of the firm. It is this profitability dimension of the firm, as well as other economic dimensions of the firm, that is of interest to the investor. Earnings per share and the accounting data
derived measures can be construed as indicators of four underlying firm dimensions; liquidity, leverage, profitability, and activity.

Ohlsen (1979) provides an analytic model relating accounting information to security prices. He examines security valuation relative to the stochastic behavior of accounting numbers and develops this valuation function: (p. 334)

\[ P_t = A + \sum_{i=1}^{N} B_i X_{it} + C D_t \]

where: \( P_t \) is the price of the security at time \( t \).

\( X_t = (X_{it}, X_{2t}, \ldots, X_{nt}, D_t) \) is a vector of datum concerning the economic attributes of the firm at time \( t \).

\( X_{it} \) denotes financial accounting numbers that represent the economic attributes of the firm at time \( t \).

\( D_t \) is dividends paid at time \( t \).

\( A, B_i, \ldots, B_n, C \) are the valuation parameters obtained by solving a system of simultaneous equations.

Ohlsen does not stipulate the accounting numbers to be used. Instead he asserts (p. 318), “the fundamental characteristics of financial variables are their (joint) stochastic time-series behavior ... information variables in this mode of analysis can be any type of variable that affects investors' expectations about future events.”

The number of data items inherent in annual financial reporting is very large. In many cases, these items are highly interrelated and purport to measure the same economic attributes of the firm. The approach of this study, adapted from Ohlsen (1979, p. 317), "stipulates
the existence of 'real' economic variables and then uses accounting data as estimates of the real variables."

2.0 Development of the Hypothesized Measurement Model

Lev (1974, p. 12) and Foster (1978, p. 28) suggest that four different economic dimensions of a firm are considered in evaluating a firm's performance. Van Horne (1980, pp. 710-713) and Weston and Brigham (1972, pp. 17-19) assert that the liquidity, leverage, profitability, and efficiency or activity dimensions are used to evaluate the financial condition and performance of a firm. The investor then uses these four economic dimensions of a firm to help formulate expectations of future returns.

A cue, which may vary in type and intensity, is the link between the perception of a stimulus and the response. An announcement of earnings or other financial data is a stimulus; it produces cues to the extent that expectations of firm attributes, deemed pertinent for investment decisions, change or are realized. According to Beaver (1981a, p. 36), financial datum becomes information when it alters beliefs about security specific parameters.

The expectation errors for the liquidity, leverage, activity and profitability dimensions, prompted by the announcement of accounting data, are the cues investigated in this study. These expectation errors are the differences between expectations of the dimensions prior to the release of the accounting data and the realizations of these dimensions given the publication of the accounting data.

Although these dimensions can be defined, they are unobservable constructs representing the financial and operating aspects of an
economic entity. Mock (1976, p. 27) suggests the use of observable surrogates or indicators as measures of unobservable constructs. The basic model of this approach is: (Mock, 1976, p. 52)

\[ X_t = \xi_t + \delta_t \]

where: \( X_t \) = the observed number of score which is assigned as the magnitude of the attribute of interest.

\( \xi_t \) = the unobservable true magnitude of the attribute.

\( \delta_t \) = an unobservable error component.

\( t = 1, 2, \ldots, T \) represents replications (of the measurement process or of objects measured).

It is assumed:

1. the relationship is stable.
2. the error component is a random variate which is distributed independently of the true score.
3. the measurement errors, \( \delta_t \), are additive to the true score.

Since it is not possible to directly observe the four economic dimensions of a firm, certain measurement devices or surrogates are used. The common measurement devices or surrogates used are financial ratios. Following are the four unobservable financial dimensions and the measures of each used in this project:

**Liquidity**

- **Current Ratio** = Current Assets/Current Liabilities
- **Quick Ratio** = (Cash + Marketable Securities + Receivables)/Current Liabilities
- **Defensive Interval** = (Cash + Marketable Securities + Receivables)/(Expenditures ÷ 365)

**Leverage**

- **Total Debt to Equity Ratio** = Total Debt/Total Equities
- **Long-Term Debt to Equity Ratio** = Long Term Debt/Total Equities
- **Times Interest Earned** = Income before Interest and Taxes/Interest
**Profitability**

Return on Assets = Net Income/Average Total Assets  
Earnings to Sales Ratio = Net Income/Net Sales  
Primary Earnings Per Share  
Return on Common Stock Equity = Net Income after Preferred Dividends/Common Equity

**Activity**

Asset Turnover = Net Sales/Average Total Assets  
Receivable Turnover = Net Sales/Average Net Receivables  
Inventory Turnover = Cost of Goods Sold/Average Total Assets

The degree to which financial accounting ratios are indicators of various underlying economic dimensions of the firm has been researched by Stevens (1973) and Johnson (1979). Stevens (1973) employed twenty financial ratios in an explanatory principal components analysis. His results divided the variables into six groups which represented six underlying factors or dimensions. Four of the factors are the same as employed in this study; liquidity, leverage, profitability, and activity. The other two factors were deemed as "others." His results indicated that the ratios representing leverage, profitability, and (to some extent) the activity dimensions do possess high degrees of concomitant variation. As such, the validity of these ratios as measures of the associated financial dimension is warranted. However, since Stevens omits any loadings less than .7, it is very difficult to assess the degree to which the ratios loaded on multiple factors.

Johnson (1979) conducted an exploratory factor analysis on sixty-one financial ratios using eight factors. His results indicate that some ratios loaded on more than one factor. This implies that some of the ratios may not be good indicators for the underlying financial dimensions.
The results of these studies indicate that a measurement model comprised of ratios as indicators of the four underlying financial dimensions is warranted. However, the existence of some ratios loading on more than one factor indicates that a high degree of covariability may exist between indicators of different dimensions. Both the Stevens (1973) and the Johnson (1979) studies failed to test the adequacy of fit for their factor analytic models. In addition, the use of an oblique factor analytic solution cannot be theoretically defended since one would expect covariation to exist among the underlying dimensions.

The expectation errors regarding the underlying financial or economic dimensions of a firm and the expectation errors regarding the observable measures of the four dimensions comprise the measurement model of this study.

Let: 
\[ \xi_1 = \text{expectation error regarding the liquidity dimension} \]
\[ \xi_2 = \text{expectation error regarding the leverage dimension} \]
\[ \xi_3 = \text{expectation error regarding the profitability dimension} \]
\[ \xi_4 = \text{expectation error regarding the activity dimension} \]
\[ x_1 = \text{expectation error of the current ratio} \]
\[ x_2 = \text{expectation error of the quick ratio} \]
\[ x_3 = \text{expectation error of the defensive interval} \]
\[ x_4 = \text{expectation error of the long term debt to equity ratio} \]
\[ x_5 = \text{expectation error of the total debt to equity ratio} \]
\[ x_6 = \text{expectation error of the times interest earned ratio} \]
\[ x_7 = \text{expectation error of the return on total assets} \]
\[ x_8 = \text{expectation error of the earnings to sales ratio} \]
\[ x_9 = \text{expectation error or primary earnings per share} \]
\( x_{10} \) = expectation error of the return on equity
\( x_{11} \) = expectation error of the total return on equity
\( x_{12} \) = expectation error of the accounts receivable turnover
\( x_{13} \) = expectation error of the turnover ratio

\( \lambda \) = measurement coefficient between the observable measure and the underlying/unobservable financial dimension expectation error

\( \delta_1 \) to \( \delta_{13} \) = the associated measurement error

The hypothesized measurement model is:

\[
\begin{align*}
x_1 &= \lambda_{11} \xi_1 + \delta_1 \\
x_2 &= \lambda_{12} \xi_1 + \delta_2 \\
x_3 &= \lambda_{13} \xi_1 + \delta_3 \\
x_4 &= \lambda_{21} \xi_2 + \delta_4 \\
x_5 &= \lambda_{22} \xi_2 + \delta_5 \\
x_6 &= \lambda_{23} \xi_2 + \delta_6 \\
x_7 &= \lambda_{31} \xi_3 + \delta_7 \\
x_8 &= \lambda_{32} \xi_3 + \delta_8 \\
x_9 &= \lambda_{33} \xi_3 + \delta_9 \\
x_{10} &= \lambda_{34} \xi_3 + \delta_{10} \\
x_{11} &= \lambda_{41} \xi_4 + \delta_{11} \\
x_{12} &= \lambda_{42} \xi_4 + \delta_{12} \\
x_{13} &= \lambda_{43} \xi_4 + \delta_{13}
\end{align*}
\]

Figure 1 is a diagrammatic representation of the hypothesized measurement model. The \( x \)'s represent the observed expectation errors which are surrogates for the expectation errors of the underlying financial dimensions. The \( \delta \)'s represent the measurement errors of the observed expectation error as an imperfect measure of the unobservable financial dimension expectation error. The observed expectation error is a composite of the underlying dimension expectation error and the measurement error.

[INSERT FIGURE 1]
where it is assumed that the $\xi$'s are not orthogonal and may covary.

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Figure 1. Hypothesized Measurement Model
3.0 Constrained Factor Analysis Using Full Information Maximum Likelihood

A model is a "representation of reality to explain some aspect of it" (Miller and Star, 1969, p. 145 and Montgomery and Urban, 1969, p. 9). Representing the underlying conceptual and theoretical structure, a causal model portrays the causal links and chains between the components of the process researched (Abdel-Khalik and Ajinkya, 1979, pp. 20-23). Causal modeling is unique in its effort to develop a structured network of causal relationships built upon theoretical underpinnings. A model structure is developed and the solution is constrained by the parameter requirements of the theoretical structure. In a confirmatory factor analysis, this means that certain variables are constrained to load on certain factors and not load on others.

To estimate the parameters and test the model, Lisrel: Analysis of Linear Structural Relationships by the Method of Maximum Likelihood by Joreskog and Sorbom (1978) is chosen. Appendix A contains a glossary and a description of the notation used in LISREL. Joreskog and Sorbom describe the program: (1978, p. 3)

The LISREL model is particularly designed to handle models with latent variables, measurement errors and reciprocal causation (simultaneously interdependence). In its most general form it assumes that there is a causal structure among a set of latent variables or hypothetical constructs some of which are designated as dependent variables and others as independent variables. These latent variables are not directly observed variables that are related to the latent variables. Thus the latent variables appear as underlying causes of the observed variables.

The hypothesized measurement model of this project,

\[
\begin{align*}
   x_1 &= \lambda_{11} \xi_1 + \delta_1 \\
   x_2 &= \lambda_{12} \xi_1 + \delta_2 \\
   x_3 &= \lambda_{13} \xi_1 + \delta_3 \\
   x_8 &= \lambda_{32} \xi_3 + \delta_8 \\
   x_9 &= \lambda_{33} \xi_3 + \delta_9
\end{align*}
\]
\[ x_3 = \lambda_{13} \xi_1 + \delta_3 \]
\[ x_4 = \lambda_{21} \xi_2 + \delta_4 \]
\[ x_5 = \lambda_{22} \xi_2 + \delta_5 \]
\[ x_6 = \lambda_{23} \xi_2 + \delta_6 \]
\[ x_7 = \lambda_{31} \xi_3 + \delta_7 \]
\[ x_{10} = \lambda_{34} \xi_3 + \delta_{10} \]
\[ x_{11} = \lambda_{41} \xi_4 + \delta_{11} \]
\[ x_{12} = \lambda_{42} \xi_4 + \delta_{12} \]
\[ x_{13} = \lambda_{43} \xi_4 + \delta_{13} \]

is a specified form of the following general model. (Joreskog and Sorbom, 1978, pp. 3-7)

\[ \beta \eta = \Gamma \xi + \zeta \]  
(1)

where:  
\( \eta \) (mxl) is a vector of the latent (underlying/unobservable) endogenous variables \( \xi \) (nxl) is a vector of the latent (underlying/unobservable) exogenous variables \( \beta \) (mxm) is the matrix of causal coefficients relating the endogenous variables to each other \( \Gamma \) (mxn) is the matrix of causal coefficients relating the endogenous variables to the exogenous variables \( \zeta \) (mxl) is a vector of random residuals or prediction errors

\[ Y = \Lambda_y \eta + \xi \]  
(2)

\[ X = \Lambda_x \xi + \delta \]  
(3)

where:  
\( Y \) (pxl) are observations/indicators/measures of the latent endogenous variables \( \eta \) \( X \) (qxl) are observations/indicators/measures of the latent exogenous variables \( \xi \) \( \Lambda_y \) (pxm) is a matrix of regression coefficients of \( Y \) on \( \eta \) \( \Lambda_x \) (qxn) is a matrix of regression coefficients of \( X \) on \( \xi \) \( \xi \) is a vector of measurement errors for \( Y \) as measures of \( \eta \) \( \delta \) is a vector of measurement errors for \( X \) as measures of \( \xi \)
Through assumption that all the variables are mean-deviated:

\[ E(\eta) = 0 \]
\[ E(\xi) = 0 \]
\[ E(\delta) = 0 \]
\[ E(x) = 0 \]
\[ E(\epsilon) = 0 \]

The following are also assumed:

\[ \sigma_{\xi \xi} = 0; \] the prediction errors are uncorrelated with the exogenous variables

\[ \sigma_{\epsilon \eta} = 0; \] the measurement errors of \( y \) as a measure of \( \eta \) are uncorrelated with \( \eta \)

\[ \sigma_{\delta \xi} = 0; \] the measurement errors of \( x \) as a measure of \( \xi \) are uncorrelated with \( \xi \)

\[ \sigma_{\epsilon \xi} = 0; \] the measurement errors of \( y \) as a measure of \( \eta \) are uncorrelated with \( \xi \)

\[ \sigma_{\delta \eta} = 0; \] the measurement errors of \( x \) as a measure of \( \xi \) are uncorrelated with \( \eta \)

\[ \sigma_{\epsilon \xi} = \sigma_{\delta \xi} = 0; \] the measurement errors are uncorrelated with the prediction errors

However, in the general LISREL model it is assumed that the measurement errors may be correlated among themselves.

Let: \( \Phi (n \times n) \) = covariance matrix of the exogenous variables, \( \xi \)

\( \Psi (m \times m) \) = covariance matrix of the prediction errors, \( \xi \)

\( \Omega_\epsilon \) = covariance matrix of the measurement errors of the endogenous variables

\( \Omega_\delta \) = covariance matrix of the measurement errors of the exogenous variables

The variance-covariance matrix of the \( x \) and \( y \) variables created by the specified causal model is (Joreskog and Sorbom, 1978, p. 5):
\[\Sigma((p + q) \times (p + q)) = \\
\begin{bmatrix}
\Lambda_y \left( \beta^{-1} \Gamma \Phi \Gamma' \beta'^{-1} + \beta^{-1} \Psi \beta'^{-1} \right) \Lambda_y + \psi \\
\Lambda_x \Phi \Gamma' \beta'^{-1} \Lambda_y \\
\Lambda_x \Phi \Lambda' + \psi \\
\end{bmatrix}
\]

In application of this general model, the elements of \( \Lambda_y, \Lambda_x, \beta, \Gamma, \Phi \)
\( \Psi, \psi \), and \( \psi \) are specified to be either free, constrained, or fixed, depending upon the hypothesized structure.

The measurement model, equations (2) and (3) can be written in factor analytic form as:

\[ Z = \Lambda f + e \]

where: \( Z = (y, x) \)
\( f = (\eta, \xi) \)
\( e = (\epsilon, \delta) \)

\[ \Lambda = \begin{bmatrix}
\Lambda_y & 0 \\
0 & \Lambda_x \\
\end{bmatrix} \]

Therefore, the measurement model is a restricted factor analysis model in which the factors \( \eta \) and \( \xi \) satisfy a linear structural equation system of the form:

\[ \beta \eta = \Gamma \xi + \xi \]
By specifying $\Phi$, the covariance matrix of the exogenous variables, to be diagonal, an orthogonal solution is derived. If the $\Phi$ matrix is specified as full rank, an oblique solution is obtained. For additional references on the use of factor analytic techniques in causal modeling see Jackson and Borgatta (1981, pp. 179-281), Judge, Griffiths, Hill and Lee (1980, pp. 550-554), Hanushek and Jackson (1977, pp. 302-324).

Before one can estimate the parameters of the model it is necessary to establish that the parameters are identified. For a given model specification, the structure denoted by $\Lambda_y$, $\Lambda_x$, $\beta$, $\Gamma$, $\Phi$, $\Psi$, $\Omega$, and $\Omega_0$ generates one and only one variance-covariance matrix, $\Sigma$, but there may be numerous structures generating the same $\Sigma$ (Joreskog and Sorbom, 1978, pp. 9-11). Two or more structures that generate the same $\Sigma$ are equivalent. A parameter that has the same value for all equivalent structures is identified. The whole model becomes identified when all of the individual parameters are identified.

Let $\mathbf{K}$ be a vector of all the independent, free, and constrained parameters specified by a certain model and let $t$ be the order of $\mathbf{K}$. The problem of identification is whether or not $\mathbf{K}$ is determinable by $\Sigma$. To assess this, consider the equations in (4) of the form:

$$\sigma_{ij} = f_{ij}(\mathbf{K}), \quad i \leq j$$

There are $(1/2)(p + q)(p + q + 1)$ equations and $t$ unknowns elements in $\mathbf{K}$. A necessary condition for identification of all parameters is that:

$$t \leq (1/2)(p + q)(p + q + 1)$$
The number of estimated parameters must be less than or equal to the number of elements in the lower left triangle of the observed variance covariance matrix for the $x$ and $y$ variables.

The specified model matrices for the hypothesized causal model of this study and the number of elements to be estimated are as follows:

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Number of Elements to be Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_x$</td>
<td>13</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>10</td>
</tr>
<tr>
<td>$\Omega_0$</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>36</td>
</tr>
</tbody>
</table>

Constraining $\Omega_0$, such that only the main diagonal elements are estimated and the remaining elements of the lower left triangle are fixed at 0, fulfills the necessary condition for identification. The model is overidentified since $36 \leq \frac{1}{2}[(p + q) \times (p + q + 1)] = 91$.

This constraint or restriction implies that the measurement errors, $\delta_1$ through $\delta_{13}$, do not covary. No covariance among the measurement error terms presumes that the underlying construct is the only systematic source of variation in the observed indicators.

For estimation and testing of the model it is assumed that the distribution of the observed variables can be described by the first two moments, a mean vector and the variance-covariance matrix. The estimation process comprises fitting the $\Sigma$, the covariance matrix constructed by the hypothesized model specifications, to the observed covariance matrix $S$. 
The fitting function:

\[ F = \log |\Omega| + \text{tr} (\Omega^{-1} S^{-1}) - \log |S| - (p + q) \]

is minimized with respect to \( \mathbf{K} \); \( \mathbf{K} \) is the set of free, constrained, or equivalent parameters designated by the hypothesized model. In minimizing the fitting function, one is minimizing the difference between the generalized variance of the created covariance matrix and the generalized variance of the observed covariance matrix. If one assumes that the recreated variance-covariance matrix, \( \Omega \), equals the observed variance-covariance matrix, \( S \), the determinant of \( \Omega \), the generalized variance of \( \Omega \), equals the determinant of \( S \). Hence, \( \log |\Omega| \) equals \( \log |S| \). Since \( \Omega = S \), \((S^{-1} \Omega^{-1})\) is equivalent to an identity matrix of order \((p + q)\). Therefore, the trace of \((S^{-1} \Omega^{-1})\) equals \((p + q)\). The result is \( F = 0 \) when the recreated covariance matrix \( \Omega \) equals the observed covariance matrix \( S \). The hypothesized model structure represents the process which produced the observed covariance matrix.

Maximum likelihood estimates, efficient for large samples, result if the distribution of \((y, x)\) is multinormal (Joreskog and Sorbom, 1978, p. 3 and Hanushek and Jackson, 1977, pp. 314-316). The procedure to select the estimates that minimize the F function involves taking the derivatives of the F function, with respect to each parameter estimated, and solving this set of simultaneous equations for the values.
that equate the derivations to zero (Hanushek and Jackson, 1977, p. 315). For a more complete discussion of the estimation procedure see Joreskog in Goldberger and Duncan (1973, pp. 85-112).

Once the maximum likelihood estimates of the parameters have been obtained, the hypothesized model is tested for goodness of fit. The total model is tested to determine its ability to create a covariance matrix, \( \Sigma \), that replicates the observed covariance matrix, \( \Sigma \). Let \( H_0 \) be the null hypothesis representing the total model as specified. The alternative \( H_1 \) is that \( \Sigma \) is any positive definition matrix. The test statistic, \( NF_0 \), is minus twice the logarithm of the likelihood ratio where \( F_0 \) is the minimum value of \( F \) and \( N \) is the sample size. \( NF_0 \) is asymptotically distributed as \( \chi^2 \) with degrees of freedom \( d \);

\[
d = \frac{1}{2}[(p + q)(p + q + 1) - t]
\]

where \( t \) is the total number of independent parameters estimated \( H_0 \) (Joreskog and Sorbom, 1978, p. 14).

Appendix B contains a discussion of the \( \chi^2 \) difference test for testing alternative model structures.

The hypothesized measurement model is tested for goodness of fit against a null measurement model. A null measurement model fixes the \( \lambda \)'s equal to zero. The next section reports the results of estimating and testing the model as specified. As warranted, the model is respecified and retested using both the \( \chi^2 \) goodness of fit test and the incremental fit index of Bentler and Bonett (1980, pp. 599-600).

4.0 Confirmatory Data Analysis

The firms studied are calendar year firms listed on the New York Stock Exchange. The accounting data releases studies are for the year ended December 31, 1979. These releases are the announcement of
earnings, the annual report issuance, and the submission of the 10-K report. An initial sample of three hundred firms is randomly chosen from firms that made the earnings announcement during February, 1980 and made public the annual report and the 10-K report prior to March 31, 1980. To be included in the data analysis, a sample firm must meet the following conditions:

1. A firm must have complete requisite data on the Compustat yearly data base for 1978 and 1979.


Of the initial three hundred firms, two hundred and nine meet these requirements.

The observable cues to be investigated are the expectation errors regarding the financial ratios that measure the underlying financial dimensions. An expectation error is the difference between the expectation of a ratio prior to the release of the accounting data and the realization of that ratio due to the release of the accounting data.

For the expectations of the year end ratios for the 1979 year, the market realizes the data contained in quarterly earnings announcements and quarterly 10-Q reports for the first three quarters. The 10-Q reports must be filed within forty-five days of the end of the quarter. Therefore, the 10-Q report for the third quarter 1979 is made public by the middle of November. The expectations of the annual accounting data items for 1979 are a composite of the third quarter data and an estimate of what will happen during the fourth quarter.

For the estimate of the results for the fourth quarter the naive model is used:
\[ E(Q_{t=0}) = Q_t - 4 \]

where: \( Q_t \) is the accounting data item in the fourth quarter of 1979 and \( Q_t - 4 \) is the accounting data item in the fourth quarter of 1978.

\( Q_t - 4 \) is determined as the difference between the 1978 annual report and the third quarter report of 1978 for the data item.

The expectation of an annual accounting datum is expressed as:

\[ E(Y_{79}) = Q_{t-3} + Q_{t-2} + Q_{t-1} + E(Q_{t=0}) \]

\[ E(Y_{79}) = Q_{t-3} + Q_{t-2} + Q_{t-1} + Q_t - 4 \]

where: \( Q_{t-3} \) is the accounting data item in the first quarter 1979

\( Q_{t-2} \) is the accounting data item in the second quarter 1979

\( Q_{t-1} \) is the accounting data item in the third quarter 1979.

Recall the hypothesized measurement model:

\[ x_1 = \lambda_{11} \xi_1 + \delta_1 \]
\[ x_2 = \lambda_{12} \xi_1 + \delta_2 \]
\[ x_3 = \lambda_{13} \xi_1 + \delta_3 \]
\[ x_4 = \lambda_{21} \xi_2 + \delta_4 \]
\[ x_5 = \lambda_{22} \xi_2 + \delta_5 \]
\[ x_6 = \lambda_{23} \xi_2 + \delta_6 \]
\[ x_7 = \lambda_{31} \xi_3 + \delta_7 \]
\[ x_8 = \lambda_{32} \xi_3 + \delta_8 \]
\[ x_9 = \lambda_{33} \xi_3 + \delta_9 \]
\[ x_{10} = \lambda_{34} \xi_3 + \delta_{10} \]
\[ x_{11} = \lambda_{41} \xi_4 + \delta_{11} \]
\[ x_{12} = \lambda_{42} \xi_4 + \delta_{12} \]
\[ x_{13} = \lambda_{43} \xi_4 + \delta_{13} \]

Estimation of these parameters produces the parameter estimates, standard errors, and t-values in Table 1. The overall test of goodness of
fit, $\chi^2 = 419.2233$ with 59 degrees of freedom, indicates the hypothesized measurement model may be a poor representation of the structure underlying the observed relationships among the observed exogenous variables, the $x$'s.

[INSERT TABLE 1]

However, as Joreskog (1979) and Bentler and Bonet (1980) point out, a sufficiently large sample will cause the $\chi^2$ value to be significantly large and may lead to incorrect conclusions. Incremental fit tests allow one to determine the proportion of the generalized variance of the observed variance/covariance matrix explained by the hypothesized model configuration.

Let $M_1$ represent the hypothesized measurement configuration and $M_0$ the null measurement model. The null measurement model restricts the $\lambda$'s to be 0. The test of model equivalence, a test of the equality of parameters for the two models, can be made. The $\chi^2$ for the null measurement model is 1234.3698 with 78 degrees of freedom.

Let $H_0$ represent the null hypothesis of model equivalence.

$$H_0 : M_0 = M_1$$

The $\chi^2$ variate for the test of model equivalence is:

$$1234.3698 - 419.2233 - 815.1465$$

degrees of freedom: $78 - 59 = 19$.

The hypothesis of model equivalence is rejected at the $=.001$ level. This implies that the hypothesized model better represents the causal configuration than the null measurement model.
Table 1. Estimates of Parameters for the Hypothesized Measurement Model

<table>
<thead>
<tr>
<th>Parameter Number</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>T-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( (\lambda_{11}^1) )</td>
<td>1.015</td>
<td>.059</td>
<td>17.083</td>
</tr>
<tr>
<td>2 ( (\lambda_{12}^2) )</td>
<td>.864</td>
<td>.063</td>
<td>13.799</td>
</tr>
<tr>
<td>3 ( (\lambda_{13}^3) )</td>
<td>.102</td>
<td>.070</td>
<td>1.453</td>
</tr>
<tr>
<td>4 ( (\lambda_{24}^4) )</td>
<td>.154</td>
<td>.253</td>
<td>.605</td>
</tr>
<tr>
<td>5 ( (\lambda_{25}^5) )</td>
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<th>T-Value</th>
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</table>
The non-normed fit index,

\[ \Delta_{MM} = \left( \frac{\chi^2_{MO}}{DF_M} - \frac{\chi^2_{M}}{DF_M} \right) \times \left( \frac{\chi^2_{MO}}{DF_M} - 1 \right) \]

represents the increment in fit obtained by using the hypothesized measurement model structure rather than the null measurement model structure.

\[ \rho_{MM} = \left[ \frac{1234.3698}{78} - \frac{419.2233}{59} \right] \div \left[ \frac{1234.3698}{78} - 1 \right] \]

\[ \rho_{MM} = \frac{15.8252 - 7.1054}{14.8252} = .58817 \]

The normed fit index is given by:

\[ \Delta_{MM} = \left( \frac{\chi^2_{MO}}{N} - \frac{\chi^2_{M}}{N} \right) \div \frac{\chi^2_{MO}}{N} \]

since \( \chi^2 = -2 \) logarithm of the likelihood ratio = NF

where \( N = \) sample size and \( F = \) the maximum fit

\[ \Delta_{MM} = \left[ \frac{1234.3698}{200} - \frac{419.2233}{200} \right] \div \left[ \frac{1234.3698}{200} \right] = .66037 \]
The hypothesized measurement model is a substantial improvement over the null measurement model. However, the remaining improvement, 

\[ 1 - \rho_{M_oM_1} = .41183 \quad \text{and} \quad 1 - \Delta_{M_oM_1} = .33963, \]

indicates a better fitting model may be feasible.

5.0 Explanatory Data Analysis

An exploratory analysis is undertaken to identify a more representative measurement model. To accomplish this the squared correlation matrix is computed and the variables are aggregated according to concomitant variation. Variables with a high degree of covariation are presumed to be indicators of a common underlying dimension. The squared correlation matrix and the seven identified factors are presented in Table 2.

[INSERT TABLE 2]

This new measurement model, \( M_2 \), has seven underlying dimensions. The expectation error for the liquidity dimension is represented by the expectation errors for the current ratio and the quick ratio. The expectation errors regarding the defensive interval, the long term debt to equity ratio, the total debt to equity ratio, and the times interest earned ratio are indicators of themselves. The expectation errors for the ratios measuring profitability and activity remain the same as the hypothesized measurement model. The dimensions are allowed to covary but no indicator is allowed to measure more than one dimension. Figure 2 is a diagram of the measurement model \( M_2 \).

[INSERT FIGURE 2]

Table 3 presents the estimates, standard errors, and t-values for the parameters estimated for \( M_2 \). The test for goodness of fit,
<table>
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<tr>
<th>x Variable</th>
<th>7</th>
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<td>4</td>
<td>-51</td>
<td>60</td>
<td>40</td>
<td>100</td>
</tr>
</tbody>
</table>
Figure 2. Exploratory Measurement Model $M_2$
\[ \chi^2 = 204.1125 \] with 48 degrees of freedom, implies that \( M_2 \) does not completely fit the data.

Let: \( M_0 \) be the null measurement model

\( M_1 \) is the priori hypothesized measurement model

\( M_2 \) is the seven factor exploratory measurement model

The test of model equivalence, \( M_1 = M_2 \), is:

\[ \chi^2 = 419.2233 - 204.1125 = 215.1108 \]

\[ \text{DF} = 59 - 48 = 11 \]

The null hypothesis of model equivalence is rejected at \( \alpha = .001 \) level.

The incremental fit indices of \( M_2 \) to \( M_0 \) are:

\[
\rho_{M_0M_2} = \frac{15.8255 - 4.2523}{14.8252} = .7806
\]

\[
\Delta_{M_0M_2} = \left[ \begin{array}{c} 1234.3698 - 204.1125 \\ 200 \end{array} \right] \div 6.1718 = .8346
\]

The incremental fit indices of \( M_2 \) to \( M_1 \) are:

\[
\rho_{M_1M_2} = \frac{7.1054 - 4.2523}{14.8252} = .1924
\]

\[
\Delta_{M_1M_2} = \left[ \begin{array}{c} 419.2233 - 204.1125 \\ 200 \end{array} \right] \div 6.1718 = .1742
\]

These indicate that the seven factor exploratory model is a better model than the original hypothesized measurement model. However, a still better fitting representation may be feasible.
Table 3. Parameter Estimates for Exploratory Measurement Model $M_2$

<table>
<thead>
<tr>
<th>Parameter Number</th>
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<th>Standard Error</th>
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Table 3. (cont'd.)

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<td>30 ( \sigma_6 \xi_7 )</td>
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<td>43 ( \sigma_7 \xi_7 )</td>
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</table>
An analysis of the observed correlation matrix and iterative model building produced the following measurement model. Attempts to specify additional factors resulted in either insignificant factor loadings or under-identification of the model. This exploratory measurement model consists of seven factors or dimensions in which indicators load on more than one dimension. This exploratory measurement model, M₄, is:

\[
\begin{align*}
    x_1 &= \lambda_{11} \xi_1 + \lambda_{71} \xi_7 + \delta_1 \\
    x_2 &= \lambda_{12} \xi_1 + \lambda_{22} \xi_2 + \lambda_{62} \xi_6 + \lambda_{72} \xi_7 + \delta_2 \\
    x_3 &= \lambda_{13} \xi_1 + \lambda_{23} \xi_2 + \lambda_{63} \xi_6 + \delta_3 \\
    x_4 &= \lambda_{34} \xi_3 + \delta_4 \\
    x_5 &= \lambda_{45} \xi_4 + \lambda_{65} \xi_6 + \delta_5 \\
    x_6 &= \lambda_{56} \xi_5 + \delta_6 \\
    x_7 &= \lambda_{67} \xi_6 + \lambda_{77} \xi_7 + \delta_7 \\
    x_8 &= \lambda_{18} \xi_1 + \lambda_{28} \xi_2 + \lambda_{68} \xi_6 + \lambda_{78} \xi_7 + \delta_8 \\
    x_9 &= \lambda_{69} \xi_6 + \lambda_{79} \xi_7 + \delta_9 \\
    x_{10} &= \lambda_{610} \xi_6 + \delta_{10} \\
    x_{11} &= \lambda_{611} \xi_6 + \lambda_{710} \xi_7 + \delta_{11} \\
    x_{12} &= \lambda_{112} \xi_1 + \lambda_{612} \xi_6 + \lambda_{712} \xi_7 + \delta_{12} \\
    x_{13} &= \lambda_{113} \xi_1 + \lambda_{713} \xi_7 + \delta_{13}
\end{align*}
\]

where: \( \lambda_{12} = \lambda_{23} = \lambda_{34} = \lambda_{45} = \lambda_{56} = \lambda_{67} = \lambda_{711} = 1.0 \)
Figure 3 is a diagram representation of the exploratory measurement model $M_3$.

\[ \text{[INSERT FIGURE 3]} \]

The $\chi^2$ test of goodness of fit is $91.3119$ with 40 degrees of freedom. Let $M_3$ be the seven factor, multiple loadings exploratory measurement model. The test of equivalence between the seven factor model $M_2$ and the seven factor multiple loadings model $M_3$ is:

\[ H_0: M_2 = M_3 \]
\[ \chi^2 = 204.1125 - 91.3119 = 112.8006 \]
\[ DF = 48 - 40 = 8 \]

$H_0$ is rejected at the $\alpha = .001$ level.

The incremental fit indices are:

\[ \Delta_{M_0M_3} = \begin{bmatrix} 1234.3698 \\ 78 \end{bmatrix} - \begin{bmatrix} 91.3119 \\ 40 \end{bmatrix} \div \begin{bmatrix} 1234.3698 \\ 78 \end{bmatrix} - 1 = .91 \]

\[ \Delta_{M_0M_3} = \begin{bmatrix} 1234.3698 \\ 200 \end{bmatrix} - \begin{bmatrix} 91.3119 \\ 200 \end{bmatrix} \div \begin{bmatrix} 1234.3698 \\ 200 \end{bmatrix} = .93 \]

\[ \Delta M_{1M_3} = \begin{bmatrix} 7.1054 - 2.2827 \\ 14.8252 \end{bmatrix} = .33 \]

\[ \Delta M_{2M_3} = \begin{bmatrix} 4.2523 - 2.2827 \\ 14.8252 \end{bmatrix} = .13 \]

\[ \Delta M_{1M_3} = \begin{bmatrix} 2.0961 - .4566 \\ 6.1717 \end{bmatrix} = .27 \]

\[ \Delta M_{2M_3} = \begin{bmatrix} 1.0205 - .4566 \\ 6.1718 \end{bmatrix} = .09 \]
where:  \( \sigma_{\varepsilon_1 \varepsilon_2} = \sigma_{\varepsilon_2 \varepsilon_5} = \sigma_{\varepsilon_1 \varepsilon_7} = \sigma_{\varepsilon_3 \varepsilon_7} = \sigma_{\varepsilon_4 \varepsilon_7} = 0 \)

(Some of the factors are allowed to covary and some are constrained to be orthogonal.)

Figure 3. Exploratory Measurement Model M_3.
These indices indicate that $M_3$ is a better representation than either $M_1$ or $M_2$. However, the inability to interpret this model makes it much less desirable than $M_2$.

5.0 Interpretation

The hypothesized measurement model, $M_1$, portrays the theoretical underlying constructs however it only recreates approximately 66% of the generalized variance. Exploratory measurement models are developed from the data and they explain a larger portion of the generalized variance. The first exploratory model, $M_2$, explains 83% of the observed generalized variance. Notice, however, that as the model fit increases the interpretability decreases.

In instances such as this, the research can take two very different paths. The first is to regard the hypothesized measurement model as adequate and to assume that the differences between the recreated variance/covariance matrix and the observed variance/covariance matrix factors are not modeled and are relatively insignificant. The second path is that of further exploratory analysis in which the observed data matrix is used to reformulate the proposed model. Any model can almost always be improved by relaxing the model configuration through the introduction of additional parameters. The difficulty is that the model can become so relaxed that it is sample dependent. In these instances the model is not generalizable and some of the parameters may have no meaning.

In applications of causal modeling techniques, the choice of the most appropriate model must be based on a substantive theoretical and conceptual basis. Although the latter two models, $M_2$ and $M_3$, better fit
the data it is apparent that $M_1$, the original theoretical model, provides a much more useful approach which is congruent with theory. The proportion of explained generalized variance may be quite adequate when one considers that regressions with much smaller coefficients of determination are considered appropriate.
Bibliography


LISREL terminology

Types of Variables

η (eta) Dependent (endogenous) variable: true (i.e., unobserved)
ξ (xi) Independent (exogenous) variable: true (i.e., unobserved)
y Indicator of dependent variable (observed)
x Indicator of independent variable (observed)
ε Measurement error in observed dependent variable
δ Measurement error in observed independent variable
ζ Sources of variance in η not included among the ζ's

Counts

m Number of true dependent variables
n Number of true independent variables
p Number of observed dependent variables
q Number of observed independent variables

Data-oriented Matrices

S (p+q \times p+q), Variance-covariance matrix among the observed independent and dependent variables (or correlation matrix)
Σ (sigma) (p+q \times p+q), Model-generated estimates of variances and covariances among observed independent and dependent variables

Basic Parameter Matrices

Λ_y (lambda) (p \times m), Matrix of regression coefficients (λ's) relating true dependent variables to observed dependent variables
Λ_x (lambda) (q \times n), Matrix of regression coefficients (λ's) relating true independent variables to observed independent variables
\[ \mathbf{B} \ (\text{beta}) \quad (m \times m), \text{Matrix of regression coefficients interrelating true dependent variables} \]

\[ \mathbf{\Gamma} \ (\text{gamma}) \quad (m \times n), \text{Matrix of regression coefficients (Y's) relating true independent variables to true dependent variables; indicates direct effect} \]

\[ \mathbf{\Phi} \ (\text{phi}) \quad (n \times n), \text{Variance-covariance matrix among true independent variables (or correlation matrix)} \]

\[ \mathbf{\Psi} \ (\text{psi}) \quad (m \times m), \text{Variance-covariance matrix among zeta variables (or correlation matrix)} \]

\[ \mathbf{\Theta}_e \ (\text{theta}) \quad (p \times p), \text{Variance-covariance matrix among epsilon variables (or correlation matrix)} \]

\[ \mathbf{\Theta}_\delta \ (\text{theta}) \quad (q \times q), \text{Variance-covariance matrix among delta variables (or correlation matrix)} \]

**Supplementary Parameter Matrices**

\[ \mathbf{C} \quad (m \times m), \text{Variance-covariance matrix among true dependent variables} \]

\[ \mathbf{D} \quad (m \times n), \text{Matrix of regression coefficients for reduced form of structural equations--i.e., coefficients which relate each true dependent variables to true independent variables, giving direct and indirect effects combined} \]
Appendix B

\( \chi^2 \) test in the analysis of covariance structures (Bentler and Bonett, 1980)

Let \( M_k \) be a more restrictive model than \( M_t \). In general, the function \( L (\theta) \) is related to the logarithm of the likelihood function of the observations via

\[
L^* (\theta) = -n L (\theta)/2 + c
\]

where \( c \) is independent of \( \theta \). (See Joreskog: Psychometrica, 1967, 32, 443-482).

Let \( L^* (\theta_k) \) be the maximum of \( L^* (\theta) \) under \( M_k \); let \( L^* (\theta_t) \) be the maximum of \( L^* (\theta) \) under \( M_t \). Thus

\[
L^* (\theta_k) \leq L^* (\theta_t)
\]

since the maximum under a space of restricted range cannot exceed the maximum under a space of less restricted range.

Consequently,

\[
\log \lambda = L^* (\theta_k) - L^* (\theta_t)
\]

is negative, with \( 0 < \lambda \leq 1 \).

To test the null hypothesis of model equivalence \( (H_0: \theta_k = \theta_t) \), \((-2 \log \lambda)\) is asymptotically distributed as a chi square variate.

The degrees of freedom is the difference in the number of parameters estimated under \( M_t \) and \( M_k \). This test is a test of the equality of the parameters under the two models. Since the free parameters in \( \theta_k \) are a subset of the free parameters in \( \theta_t \), various applications of the test can be constructed.

The null hypothesis associated with model comparisons has an alternative form. The alternative is that the covariance matrices
generated by the parameter vectors are equivalent under the $M_k$ and $M_t$
structural models. The significance test is the same as previously
described.