A Generalized Linear Combination Approach to Investigate the Relationship Between APT and CAPM

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ABSTRACT

A Generalized Linear Combination Approach is used to investigate the relationship between APT and CAPM. A generalized regression model for testing capital asset pricing is derived. The daily return and prices of Dow Jones thirty firms are used to test three hypotheses related to the generalized CAPM, the APT and traditional multi-index model. The interrelationship among alternative capital asset pricing processes are analyzed in detail.
I. Introduction

Roll (1977) has shown that the capital asset pricing model (CAPM) developed by Sharpe (1964), Lintner (1965), and Mossin (1966) can never empirically be tested since the completed specification and measurement of market rates of return is not possible. Roll and Ross (1980) used factor analysis technique to test the arbitrage pricing theory (APT) derived by Ross (1976, 1977). Jobson (1982) derived a multivariate regression approach to test the APT; Cheng and Grauer (1980) derived a multivariate regression approach to test the CAPM. However, there are weaknesses associated with these studies in both theoretical and empirical aspects in testing CAPM and APT.

The main purposes of this paper are: (i) to derive a generalized regression model for testing the APT and CAPM by using rates of return instead of prices or return premium. (ii) to show the sources of ambiguity associated with linear model for testing either APT or CAPM. In the second section previous studies of testing CAPM and APT are reviewed and examined. In the third section new alternative models for testing the CAPM, the APT and the multi-index model are derived. In the fourth section the theoretical relationship among the CAPM, the APT and the multi-index model are derived. In the fifth section two alternative methods for testing the linear asset pricing model are explored. In section VI the empirical result of testing the APT, the CAPM and the multi-index model are demonstrated. Finally in section VII results of this paper are summarized and concluding remarks are indicated.
II. Review and Critique of Previous Studies

The CAPM has been provided us a framework to determine the market price of risky assets in an equilibrium capital market. One of the most valuable properties of the CAPM is its testability. After having extensively tested the CAPM, Black, Jensen and Scholes (1972), Miller and Scholes (1972), and Fama and MacBeth (1973) and others have concluded that even though the empirical evidence does not support the traditional Sharpe-Lintner-Mossin's [SLM] CAPM, it does support Black's zero-beta version CAPM. The results of these studies were examined and criticized by Fama (1976) and Roll (1977).

Most of the early empirical tests of the CAPM use a two-step approach. First, the time-series security rates of return is used to estimate beta. Secondly, cross sectional regressions of estimated average rates of return and estimated beta are used to test the CAPM. However, there are two potential problems in testing the CAPM, i.e., (i) market portfolio specification and measurement problems and (ii) the errors-in-variables problem associated with using estimated beta as regressor.

Since the beta estimated from the first pass regression may differ from the true beta due to estimation errors, Miller and Scholes (1972), Fama (1976) and others have mentioned that the regression coefficients estimated in the second pass regression are generally inconsistent unless the estimated betas are free from measurement errors. To deal with the errors-in-variables problem, most of the previous studies use the grouping technique to reduce this problem. Because the grouping technique will reduce some important information associated with individual
securities. Gibbons (1982) introduces a new multivariate approach to test the CAPM. Even though Gibbons' multivariate approach can solve the errors-in-variable problem without losing the information associated with individual firms, there still exists the unsolved problem of measuring the market portfolio. Actually, the testing hypothesis proposed by Gibbons is only used to test the Black CAPM against the market model. To avoid the problem of market portfolios, Cheng and Grauer (1980) derived a new and unambiguous test model as indicated in equation (1) for testing the CAPM without relying upon market portfolio information.

\[
\tilde{P}_{it} = b_{ijk} \tilde{P}_{jt} + c_{ijk} \tilde{P}_{kt} + \tilde{\varepsilon}_{it}
\]

where \( \tilde{P}_{it} \) = the \( i \)th security price at time \( t \),
\( b_{ijk} \) and \( c_{ijk} \) = the time independent coefficients, and
\( \tilde{\varepsilon}_{it} \) = the random error term with \( E(\tilde{\varepsilon}_{it}) = 0 \) and \( \text{Var}(\tilde{\varepsilon}_{it}) = \sigma^2_{\varepsilon} \).

This new alternative model of testing the CAPM is based upon the information of security prices and the stationary assumption on the vector of expected returns, \( \tilde{E} \), and the variance-covariance matrix of returns, \( \tilde{\Sigma} \). Nevertheless, in their comment on the Cheng and Grauer alternative test of the CAPM, Turnbull and Winter (1982) pointed out that: (a) The assumption of stationarity requires only three periods of observation on three security prices and this also implies that the adjusted coefficient of determination should be identically equal to one; (b) if equation (1) can be used to describe the structure of
asset prices at each discrete point in time, then $E$ and $\Sigma$ cannot in general be stationary; (c) non-rejection of the null hypothesis does not necessarily imply acceptance of the CAPM and the assumption of stationarity.

The reasons that $E$ and $\Sigma$ cannot in general be stationary are that the variables used in equation (1) are security prices (not returns) and that the security prices don't adjust for the cash-dividend, the stock-dividend, or the stock-split.

Because of some anomalous empirical evidence on the CAPM found by Ball (1978), Basu (1977), Banz (1981), and Reinganum (1981), and the empirical issue of the CAPM's testability questioned by Roll (1977), the arbitrage pricing theory (APT) proposed by Ross (1976, 1977) offers a testable alternative. The APT is an appropriate alternative for CAPM because it is consistent with the capital asset pricing theory. Furthermore, the assumptions of the APT are less restrictive than those of the CAPM, i.e., the APT does not require the identification and measurement of the market portfolio. To test the APT, Roll and Ross (1980) used a two-step procedure which is similar to that of testing the CAPM. First, the variance-covariance matrix of returns is used to estimate the $k$ factors and their related loadings. (The loadings correspond to the beta coefficient(s) in the CAPM.) Secondly, a cross sectional regression is then used to test the linear APT model. Even though there are several advantages of using the APT to test the capital asset pricing, there exist some weaknesses in using the factor analysis procedure to test the APT. They are: (1) There does not generally exist a unique solution for estimating factor
loadings; (2) there is a computer capacity limitation problem, that is, no computer can handle over 200 securities; (3) if grouping data is used to test the APT as done by Roll and Ross, then there exist problems associated with the determination of the number of factors and the comparison of the factors among different groups; and (4) there exists the errors-in-variables problem. Although Chen's (1982) method can be used to obtain the same set of common factors for each security and solve the problem of the computer capacity limitation, it cannot be used to solve other problems mentioned above.

A bilinear approach for testing the APT, proposed by Brown and Weinstein (1983), is designed to solve the errors-in-variables problem. However, there are two ambiguities associated with their bilinear approach. (1) The probability of accepting a predetermined k-factor APT is dependent on the number of securities included in the group. The larger the number of groups, the larger the probability for accepting the APT. (2) The probability of accepting the APT is dependent upon the number of factors used in the test. The more the factors are used, the less the probability for accepting the APT. Because the probability of accepting a k-factor APT is dependent upon the number of securities and the number of factors, Brown and Weinstein's conclusion is potentially questionable. Most recently Shanken (1982) has also discussed the problem of using the factor analysis technique in testing the APT. He argued that the market portfolio plays a prominent role in Connor's (1982) "Equilibrium APT."

To overcome some of the above-mentioned weaknesses, Jobson (1982) used a linear combination technique to derive a multivariate linear
regression approach to test the APT. Jobson shows that an existing sample of return premia for a set of N assets including a subset of k linearly independent portfolios can be used to test APT, i.e., the k-factor APT hypothesis is accepted if the intercept term is zero in the multivariate regression of the (N-k) returns on the k portfolios. Because the zero-beta portfolio is unobservable and it is difficult to estimate, the return premium will be hard to obtain. Later, we will show that the condition of not rejecting the k-factor APT proposed by Jobson is not enough for testing the APT. Therefore, in section III, we will use the return itself instead of return premia to show the conditions of not rejecting the k-factor APT. It will be shown that not only the intercept term in multiple regression is zero, but also the sum of the regression coefficients on the explanatory portfolio returns is one if the explanatory portfolios are greater than or equal to k + 1.

III. New Alternative Models for Testing the CAPM, the APT, and the Multi-index Model

A. CAPM (Alternative derivation is shown in Appendix A)

The generalized CAPM with k-state variables, introduced by Merton (1973), extended by Richard (1979) and Breeden (1979) is simply a multiple linear model which is expressed in terms of the expected return and risks. In equilibrium, the expected return can be given by

\[ E = \gamma \frac{\varepsilon_N}{\varepsilon_N} + \beta^n (\bar{R}_m - \gamma_\mu) + \beta \lambda \]  

(2)
where

\[ E = \text{the Nx1 vector of expected returns}, \]

\[ \beta^m = \text{the Nx1 vector of the sensitivity of returns on assets to} \]

\[ \text{the fluctuations on market portfolio}, \]

\[ b = \text{the Nk matrix of the sensitivities of returns on assets} \]

\[ \text{to the fluctuations on state variables}, \]

\[ R_m = \text{the expected return on market portfolio}, \]

\[ \lambda = \text{the kx1 vector of market risk premia corresponding to state} \]

\[ \text{variables}, \]

\[ \gamma_z = \text{the expected return on zero-beta portfolio, or the riskless} \]

\[ \text{rate if it exists}, \]

\[ \mathbf{1}_N = \text{the Nx1 column vector of unities}. \]

The return generating process for the generalized CAPM with

k-state variables can be written as

\[ \tilde{\gamma}_t = E + \tilde{\beta}^m \tilde{\gamma}_m + b \tilde{\delta}_t + \tilde{\epsilon}_t \tag{3} \]

where

\[ \tilde{\gamma}_t = \text{the Nx1 vector of returns on assets at } t, \]

\[ \tilde{\delta}_t = \text{the kx1 vector of returns on state variables minus their} \]

\[ \text{expected returns at } t, \]

\[ \tilde{\gamma}_m = R_{mt} - R_m, \]

\[ \tilde{\epsilon}_t = \text{the Nx1 vector of error terms}. \]

\[ \tilde{\gamma}_m \text{ and } \tilde{\delta}_t \text{ have mean zero and are assumed to be independent of } \tilde{\epsilon}_t. \]

The elements of \( \tilde{\epsilon}_t \) are assumed to have mean zero and to be mutually independent.
Denote \((\mathbf{R}^m, \mathbf{b})\) as \(\mathbf{B}\) and \((\gamma_{mt}', \delta_t')\) as \(\Delta_t'\). Let portfolio (or security) \(1\), be the base portfolio. And define \(\gamma_{it} = \gamma_{it}', E_i = E_i', \beta_i = \beta_i', b_i = b_i', \) and \(\varepsilon_{it} = \varepsilon_{it}'\), \(i \neq 1\). In addition, assume that there exists a subset of \(N(I)(\geq k+1)\) linear equations with \(N(I)\times 1\) portfolio (or security) return vector \(\gamma_{it}'\), \(\forall i \in I\) and \(i \neq 1\), and the rank of \(\mathbf{B}_I^*\) to be \(k+1\). \(N(I)\) is the number of securities in the set \(I\). The model of equation (3) can be partitioned into three mutually exclusive equations systems as follows:

\[
\gamma_{it} = E_i + B_i \Delta_t + \varepsilon_{it} 
\]  
(4)

\[
\gamma_{Ii} = E_I + B_{Ii} \Delta_t + \varepsilon_{Ii}, \quad i \in I, \text{ and } i \notin \{1,J\} 
\]  
(5)

\[
\gamma_{Ji} = E_J + B_{Ji} \Delta_t + \varepsilon_{Ji}, \quad i \in J, \text{ and } i \notin \{1,I\} 
\]  
(6)

Subtracting (4) from (5) and (6) respectively, we arrive at

\[
\gamma_{Ii} = \gamma_{It} = E_I + B_{Ii} \Delta_t + \varepsilon_{It} 
\]  
(7)

\[
\gamma_{Ji} = \gamma_{Jt} = E_J + B_{Ji} \Delta_t + \varepsilon_{Jt} 
\]  
(8)

Applying matrix operations to (7), we have

\[
\Delta_t = (B_{Ii}^* B_{Ii} - 1) \gamma_{It}' - E_I + \varepsilon_{It}' 
\]  
(9)

Substituting (9) into (8) and rearranging, we obtain

\[
\gamma_{Jt} = (E_J - B_{Ji}^* (B_{Ii}^* B_{Ii} - 1) B_{Ii} E_I) + B_{Ji}^* (B_{Ii}^* B_{Ii} - 1) B_{Ii} \gamma_{It} 
\]  
(10)

\[- B_{Ji}^* (B_{Ii}^* B_{Ii} - 1) B_{Ii} \varepsilon_{It} + \varepsilon_{Jt} \]
By partitioning the set of equations in (2) as in the case of $\tilde{\gamma}_t$, it can be seen that

$$E_j^* = \frac{B_J^*(B_I^{-1} B_I^*)^{-1} B_I^* E_I^*}{B_I^* E_I^*}$$  \hspace{1cm} (11)$$

Therefore, the intercept terms in (10) are zero. Explicitly expressing the base portfolio $l$ in (10), we obtain,

$$\tilde{\gamma}_{jt} = \frac{B_J^*(B_I^{-1} B_I^*)^{-1} B_I^* \gamma_{jt} + (\tilde{\ell}_J - \frac{B_J^*(B_I^{-1} B_I^*)^{-1} B_I^* \ell_I}{B_I^* E_I^*}) \gamma_{1t}}{B_J^*(B_I^{-1} B_I^*)^{-1} B_I^* \tilde{\gamma}_{jt} + (\tilde{\ell}_J - \frac{B_J^*(B_I^{-1} B_I^*)^{-1} B_I^* \ell_I}{B_I^* E_I^*}) \tilde{\gamma}_{1t}}$$

Summing the slope coefficients on $\tilde{\gamma}_{jt}$ and $\gamma_{1t}$ for each security $j \in J$, we arrive at

$$B_J^*(B_I^{-1} B_I^*)^{-1} B_I^* \tilde{\gamma}_{jt} + (\tilde{\ell}_J - \frac{B_J^*(B_I^{-1} B_I^*)^{-1} B_I^* \ell_I}{B_I^* E_I^*}) \gamma_{1t} = \tilde{\ell}_J$$  \hspace{1cm} (12)$$

That is, the sum of slope coefficients for the security $j \in J$ and $N(I) > k+1$ would be unity. If $N(I)$ is chosen to be $k+1$, then $(B_I^{-1} B_I^*)^{-1} B_I^* = (B_I^*)^{-1}$, and (12) and (13) can be simplified as follows:

$$\tilde{\gamma}_{jt} = B_J^*(B_I^{-1} B_I^*)^{-1} \gamma_{jt} + (\tilde{\ell}_J - \frac{B_J^*(B_I^{-1} B_I^*)^{-1} B_I^* \ell_I}{B_I^* E_I^*}) \gamma_{1t}$$

$$- \frac{B_J^*(B_I^{-1} B_I^*)^{-1} \tilde{\gamma}_{jt} - (\tilde{\ell}_J - \frac{B_J^*(B_I^{-1} B_I^*)^{-1} B_I^* \ell_I}{B_I^* E_I^*}) \tilde{\gamma}_{1t} + \tilde{\gamma}_{jt}}{B_J^*(B_I^{-1} B_I^*)^{-1} \tilde{\gamma}_{jt} - (\tilde{\ell}_J - \frac{B_J^*(B_I^{-1} B_I^*)^{-1} B_I^* \ell_I}{B_I^* E_I^*}) \tilde{\gamma}_{1t} + \tilde{\gamma}_{jt}}$$

$$B_J^*(B_I^{-1} B_I^*)^{-1} \tilde{\gamma}_{jt} + (\tilde{\ell}_J - \frac{B_J^*(B_I^{-1} B_I^*)^{-1} B_I^* \ell_I}{B_I^* E_I^*}) \gamma_{1t} = \tilde{\ell}_J$$  \hspace{1cm} (12')$$

(13')$$

By carefully choosing portfolios $l$ and $i$, $\forall i \in I$ such that the resultant error terms $\tilde{E}_{1t}$ and $\tilde{E}_{1t}$ can be negligible, (12) can be expressed as

$$\frac{1}{2}$$
This implies that any security (or portfolio) return \( j \in J \) is a linear combination of other \( k+2 \) or more portfolios' returns plus an error term without the intercept term. Assuming that the variance-covariance of returns on securities and state variables are stationary, the test of the generalized CAPM with \( k \)-state variables is equivalent to the test of the following hypothesis from the ordinary least square (OLS) results in (14).

\[
\gamma_{jt} = \alpha_{j0} + \alpha_{j1} \gamma_{1t} + \ldots + \alpha_{jL} \gamma_{Lt} + \epsilon_{jt}, \quad j \in J \text{ and } N(I) \geq k+1 \tag{14}
\]

Under the null hypothesis, the intercept term is insignificantly different from zero, and the sum of the slope coefficients is insignificantly different from one no matter how many explanatory portfolios are included if the explanatory portfolios are greater than or equal to \( k+2 \).

We call the hypothesis, \( \alpha_{j1} + \ldots + \alpha_{jL} = 1 \), as a strong-form hypothesis. Empirically, this strong-form hypothesis is difficult to be held for most of the securities because of the stochastical property of the data. Alternatively, we propose another hypothesis called weak-form hypothesis, which will be appropriately acceptable in an empirical test. The weak-form hypothesis states that the sum of slope coefficients would be constant if the number of regressors is greater than or equal to \( k+2 \). That is,
Let $H_0: \alpha_{j0} = 0$ and $\alpha_{j1} + \alpha_{j2} = C_j = \text{constant}, \quad \forall j \in J$ and $N(I) \geq k+1$

Let $R_j^2(K)$ be the R-square for security $j$ on $K$ explanatory portfolios. From (12), it can be seen that the explanatory portfolios beyond $k+2$ cannot explain more than the first $k+2$ explanatory portfolios if the first $k+2$ portfolio returns are independent. As a result, the test of $\alpha_{j1} + \alpha_{j2} = 1$ (or constant) is equivalent to the test of

$$R_j^2(K) = R_j^2(K+1) = \ldots \quad j \in J, \quad K = k + 2$$

Furthermore, the multivariate test technique, such as likelihood ratio test (LRT) can be used to test the equation system of $j \in J$ as a whole. We will discuss this technique in section V.

The CAPM, introduced by Sharpe-Lintner-Mossin ($\gamma_z$ is the riskless rate) and extended by Black ($\gamma_z$ is a return on the zero-beta portfolio), is a special case of Merton's generalized CAPM without state variables. That is, the $b$ matrix in equation (2) and (3) is assumed to be zero.

This new approach to test for the CAPM does not have the problems of (a) and (b) in section II which were mentioned by Turnbull and Winter (1982).

Now, suppose that the base portfolio $1$ is not used to derive the model and the return premia are used in the model instead of returns, the equation, which is correspondent to equation (12), can be expressed as follows:
\[
\begin{align*}
(\tilde{Y}_{jt} - \tilde{Y}_{zt}^j) &= B_J (B_I B_I^\prime)^{-1} \tilde{B}_I (\tilde{Y}_{it} - \tilde{Y}_{zt}^i) \\
&- B_J (B_I B_I^\prime)^{-1} B_I \varepsilon_{it} + \varepsilon_{jt}; \quad j \in J \text{ and } N(I) \geq k+1
\end{align*}
\] (15)

This implies that any security (or portfolio) return premium \( j \in J \) can be expressed as a linear combination of (other) \( k+1 \) (not \( k+2 \)) or more portfolio's return premia. However, this model has two major weaknesses in the empirical test: (a) the zero-beta portfolio return, \( \tilde{Y}_{zt} \), is very difficult to estimate; (b) the sum of the slope coefficients is no longer equal to one. That is, \( B_J (B_I B_I^\prime)^{-1} B_I \varepsilon_{it} \) may not be equal to \( \varepsilon_{it} \) for \( N(I) \geq k+1 \).

B. APT

A \( k \) factor APT, formulated by Ross (1976, 1977) and intensively tested by Roll and Ross (1980), is a simple multiple linear model which is described in terms of the expected return and \( k \) risks. The equilibrium expected returns is given by

\[
\bar{E} = \tilde{Y}_{zt} \bar{\lambda}^N + B \bar{\lambda}
\] (16)

\( B \), a \( N \times k \) matrix of factor loadings, is the sensitivity of the returns on assets to the fluctuations on factors. \( \bar{\lambda} \) is a \( k \times 1 \) vector of risk premia corresponding to the risk factors. Suppose that \( \lambda_j \) is the \( j^{th} \) component of \( \bar{\lambda} \). \( \lambda_j \) can be interpreted as the expected excess return or market risk premium on portfolios with unit systematic risk on factor \( j \) and no risks on other factors. Other notations are the same as in III. A.
As presented by Roll and Ross, a \( k \) factor model, which is
describes the return generating process, can be written as

\[
\tilde{\gamma}_t = \mathbf{E} + B \tilde{\delta}_t + \tilde{\varepsilon}_t
\]  

(17)

\( \tilde{\delta}_t \), a \( k \times 1 \) vector of scores on the systematic factors, is the factors' returns minus their expected returns. The factor scores \( \tilde{\delta}_t \) have mean zero and are assumed to be linearly independent of each other and to be independent of \( \tilde{\varepsilon}_t \). \( \tilde{\varepsilon}_t \) is a \( N \times 1 \) vector of random terms with 
\[ E(\tilde{\varepsilon}_t | \tilde{\delta}_t) = 0 \]
and are assumed to be mutually independent. Note that \( \tilde{\delta}_t \)
in the generalized CAPM are not assumed to be independent of each other.

Following the same procedures done in section III. A., we can have
equations (12), (13), and (14) with \( N(I) \geq k \) instead of \( N(I) \geq k+1 \).
Assuming that the covariance between returns on assets and returns on
factors are stationary, the test of the \( k \)-factor APT is equivalent to
the test of the hypothesis described in section III. A. with \( N(I) \geq k \)
instead of \( N(I) \geq k+1 \).

The not rejecting of the \( k \)-factor APT implies that any security
(or portfolio) return \( j \in J \) is a linear combination of any other \( k+1 \)
or more portfolio returns plus an error term without an intercept term.

Comparing our approach with Jobson's, it can be seen that Jobson's
approach has two major weaknesses in doing empirical test: (i) the zero-
beta portfolio return \( \tilde{\gamma}_z \) is difficult to estimate, so is the return
premia, if not impossible; (ii) empirically, the number of independent
portfolios, \( K \), is not easy to determine. But, in our case there exist
no problems in determining the number of independent variables and the null hypothesis is very clear.

C. **Multi-Index Model**

The equilibrium expected returns of a k-index model can be given by

\[ E = \alpha + B \bar{\mathbf{I}} \]  \hspace{1cm} (18)

\( B \), a \( N \times k \) matrix of multi-beta coefficients, are the sensitivity of returns on assets to the fluctuations on indexes. \( \alpha \) is a \( N \times 1 \) vector of intercept terms. \( \bar{\mathbf{I}} \) is a \( k \times 1 \) vector of expected returns on indexes.

The \( k \)-index return generating process can be written as

\[ \bar{\gamma}_t = \alpha + B \bar{\mathbf{I}}_t + \bar{\varepsilon}_t \]  \hspace{1cm} (19)

or

\[ \bar{\gamma}_t = E + B \bar{\delta}_t + \bar{\varepsilon}_t \]  \hspace{1cm} (19)'

where

\[ \bar{\delta}_t = (\bar{\mathbf{I}}_1 - \bar{\mathbf{I}}_1, \bar{\mathbf{I}}_2 - \bar{\mathbf{I}}_2, \ldots, \bar{\mathbf{I}}_k - \bar{\mathbf{I}}_k)' \]

\( \bar{\delta}_t \) have mean zero and are assumed to be independent of \( \bar{\varepsilon}_t \). \( \bar{\varepsilon}_t \) is a \( N \times 1 \) vector of random terms with \( E(\bar{\varepsilon}_t | \bar{\delta}_t) = 0 \) and are assumed to be mutually independent.

Following the same procedure done in section III. A with \( N(I) \geq k \) instead of \( N(I) > k+1 \), we arrive at

\[ \bar{\gamma}_{jt} = (a_j - B_J (B_I B_I^{-1} B_I a_I)) + B_J (B_I B_I^{-1} B_I \bar{\gamma}_I t) \]

\[ + (\bar{\gamma}_J - B_J (B_I B_I^{-1} B_I \bar{\gamma}_I t) \gamma_{I t} - B_J (B_I B_I^{-1} B_I \bar{\varepsilon}_I t) \]

\[ - (\bar{\gamma}_J - B_J (B_I B_I^{-1} B_I \bar{\varepsilon}_I t) \varepsilon_{I t} + \bar{\varepsilon}_{J t}, \text{ for } N(I) \geq k \]  \hspace{1cm} (20)
It can be seen that equation (20) is the same as equation (12) except that the intercept terms in (20) may not be zero. Assuming that the covariance between returns on assets and returns on indexes are stationarv, the test of a k-index model is equivalent to the test of the following hypothesis from the OLS results in (14):

\[
\begin{align*}
\text{Strong-form} & \quad H_0: \alpha_j + \sum_{i=1}^{k} \beta_i j = 1 & j \in J \text{ and } N(I) \geq k \\
\text{Weak-form} & \quad H_0: \alpha_j + \sum_{i=1}^{k} \beta_i j = C_j & j \in J \text{ and } N(I) \geq k+1
\end{align*}
\]

Therefore, the k-index model does not require zero intercept terms in the multiple regression while the APT and the CAPM do. Using the same argument made in IV. A, the test of \( \alpha_j + \sum_{i=1}^{k-1} \beta_i j = 1 \) (or \( C_j \)) is equivalent to the test of \( \bar{R}_j^2(k) = \bar{R}_j^2(k=1) = \ldots \).

IV. Theoretical Relationships among the CAPM, the APT and the Multi-Index Model

In view of our preceding discussion, (14) can be used to test for the CAPM, the APT, or the multi-index model simultaneously. Let us repeat (14) as follows:

\[
\tilde{\gamma}_{jt} = \alpha_{j0} + \alpha_{jl} \tilde{\gamma}_{lt} + \alpha_{JI} \tilde{\gamma}_{It} + \tilde{\varepsilon}_{jt} \quad j \in J \quad j \in J \quad \text{N(I) = 1, 2, ...}
\]

\[
t = 1, \ldots, T
\]

Note that portfolios 1 and 0 need to be chosen so as to have small error terms. Denote \( K = N(I) + 1 \) be the number of explanatory portfolios and \( S_j(K) \) be the sum of slope coefficients for \( j \in J \) with the number of explanatory portfolios = \( K \). In terms of the regression equation (14) with \( K \geq 2 \), the following hypotheses can be formulated.
HYPOTHESIS 1: The intercepts equal zero: $\alpha_{j0} = 0_j$

STRONG-FORM HYPOTHESIS 2: The sums of slope coefficients equal unities: $\alpha_{j1} + \alpha_{j2} = \ell_j \quad K = 2, 3, ...$

WEAK-FORM HYPOTHESIS 2: The sums of slope coefficients are constant for $K \geq 2$: $S(K) = S(K+1) = ... \quad K = 2, 3, ...$

HYPOTHESIS 3: The adjusted coefficients of determination are constant for $K \geq 2$: $R^2(K) = R^2(K+1) = ... \quad K = 2, 3, ...$

Denote $K^*$ be minimum number to accept HYPOTHESES 2 and 3. Now, we have the following situations:

1. **If HYPOTHESIS 1, STRONG-FORM HYPOTHESIS 2 and HYPOTHESIS 3 with $K^* = 2$ are accepted then one of the following model is valid in a strong-form sense.** However, if HYPOTHESIS 1, WEAK-FORM HYPOTHESIS 2 and HYPOTHESIS 3 with $K^* = 2$ are all accepted, then any one of the following models is accepted in a weak-form sense.
   a. the Sharpe-Lintner-Mossin CAPM,
   b. the Black CAPM,
   c. the single factor APT.

2. **Everything is the same as 1 except $K^* > 2$.** Then, anyone of the following models is correct.
   a. the $K^*-1$ factor APT,
   b. Merton's $K^*-2$ state variable CAPM.

3. **Everything is the same as 1 except that Hypothesis 1 is rejected and $K^* \geq 2$.** Then only $(K^*-1)$ Index Model is acceptable.
4. If WEAK-FORM HYPOTHESIS 2 and HYPOTHESIS 3 cannot be accepted with any value of \( K^* \), then either none of the linear asset pricing models could explain historical data very well, or the stationary assumptions are violated.

Even though this linear combination approach can be used to test the linear asset pricing models without measurement-error associated with market portfolio, factors, or indexes the conclusion we can make is not unambiguous. For instance, the linear combination approach cannot be distinguished from Black's CAPM, or S-L-M's CAPM, or the single factor APT. However, this approach is able to investigate the linear relationship between returns and risks. Furthermore, this approach is able to test the specific equilibrium models (such as the CAPM and the APT) against the non-specific linear model (Index Model) as done by Gibbons (1982). In this case, the null hypothesis is the generalized CAPM or the APT, while the alternative hypothesis is the Index Model, and only HYPOTHESIS 1 is needed. That is, if HYPOTHESIS 1 is not rejected with \( K^* \), we can say that the \( K^*-1 \) factor APT or the \( K^*-2 \) state variable CAPM is insignificantly different from the \( K^*-1 \) Index Model.

V. Likelihood Ratio Tests and T Tests for the Linear Asset Pricing Models

The return vector \( \tilde{\gamma}_t \), \( t = 1, 2, \ldots, T \) is assumed to be multivariate normal with mean \( \tilde{E} \) and variance-covariance matrix \( \tilde{\Sigma} \). \( \tilde{\gamma}_t \) is partitioned \( \tilde{\gamma}_{1t}, \tilde{\gamma}_{2t}, \text{ and } \tilde{\gamma}_{3t} \). Now, in terms of testing the three HYPOTHESES (HYPOTHESIS 2 is the strong-form one) done in Section IV, the multivariate linear models required are given by
\[ \tilde{\gamma}_{Jt} = \alpha_{J0} + \alpha_{J1} \tilde{\gamma}_{lt} + \alpha_{JI} \tilde{\gamma}_{lt} + \tilde{\varepsilon}_{Jt} \]  
(U)

\[ \tilde{\gamma}_{Jt} - \tilde{\gamma}_{lt} \tilde{\varepsilon}_{J} = \alpha_{J0}^{*} + \alpha_{JI}^{*} (\tilde{\gamma}_{lt} - \tilde{\gamma}_{lt}^{L}) + \tilde{\varepsilon}_{Jt} \]  
(R1)

\[ \tilde{\gamma}_{Jt} - \tilde{\gamma}_{lt} \tilde{\varepsilon}_{J}^{*} = \alpha_{J1}^{*} (\tilde{\gamma}_{lt} - \tilde{\gamma}_{lt}^{L}) + \tilde{\varepsilon}_{Jt}^{*} \]  
(R2)

Assume that

\[ E(\varepsilon_{J} \varepsilon_{k}) = \sigma_{jk} I_{T} \]  
\[ j \neq k \]

\[ = \sigma_{j}^{*} I_{T} \]  
\[ j = k \]

\[ E(\varepsilon_{J}^{*} \varepsilon_{k}^{*}) = \sigma_{jk}^{*} I_{T} \]  
\[ j \neq k \]

\[ = (\sigma_{j}^{*})^{2} I_{T} \]  
\[ j = k \]

\[ E(\varepsilon_{J} \varepsilon_{k}^{**}) = \sigma_{jk}^{**} I_{T} \]  
\[ j \neq k \]

\[ = (\sigma_{j}^{**})^{2} I_{T} \]  
\[ j = k \]

\[ E(\varepsilon_{J}^{*} \varepsilon_{J}^{*}) = \Sigma_{u} \otimes I_{T} \]

\[ E(\varepsilon_{J}^{*} \varepsilon_{J}^{*}) = \Sigma_{t1} \otimes I_{T} \]

\[ E(\varepsilon_{J} \varepsilon_{J}^{**}) = \Sigma_{t2} \otimes I_{T} \]

where \( \otimes \) indicates a kronecker or direct product operator.

\[ \Sigma = J \times J \text{ contemporaneous variance-covariance matrix with} \]

\[ \text{typical element equal } \sigma_{jk}, \sigma_{jk}^{*} \text{ or } \sigma_{jk}^{**} \]

\[ \varepsilon_{J} \sim N(0, \Sigma_{u} \otimes I_{T}) \]

\[ \varepsilon_{J}^{*} \sim N(0, \Sigma_{t1} \otimes I_{T}) \]

\[ \varepsilon_{J}^{**} \sim N(0, \Sigma_{t2} \otimes I_{T}) \]
With identical regressors across equations in equation systems (U), (R1), and (R2), the Seemingly Unrelated Regression (SUR) model cannot get any gain over the OLS model. Through this paper, the OLS model is used. Even though there are several multivariate test statistics available, the statistic used here is a LRT which compares the statistical fit of the restricted model with that of the unrestricted model. Let \( \hat{\Sigma}_{r(k)} \) be the determinant of the contemporaneous variance-covariance matrix estimated from the residuals of the restricted (unrestricted) model with \( k \) explanatory variables. Then, the appropriate LRT statistic test for the test of HYPOTHESIS 1 is given by

\[
-2 \ln \lambda = T[1 \ln \lvert \hat{\Sigma}_{r2(k)} \rvert - 1 \ln \lvert \hat{\Sigma}_{r1(k)} \rvert] \sim \chi^2_j \tag{21}
\]

For the test of STRONG-FORM HYPOTHESIS 2,

\[
-2 \ln \lambda = T[1 \ln \lvert \hat{\Sigma}_{r1(k)} \rvert - 1 \ln \lvert \hat{\Sigma}_{u(k)} \rvert] \sim \chi^2_j \tag{22}
\]

For the joint tests of HYPOTHESIS 1 and STRONG-FORM HYPOTHESIS 2,

\[
-2 \ln \lambda = T[1 \ln \lvert \hat{\Sigma}_{r2(k)} \rvert - 1 \ln \lvert \hat{\Sigma}_{u(k)} \rvert] \sim \chi^2_{2J} \tag{23}
\]

For the test of HYPOTHESIS 3,

\[
-2 \ln \lambda = T[1 \ln \lvert \hat{\Sigma}_{u(k)} \rvert - 1 \ln \lvert \hat{\Sigma}_{u(k+1)} \rvert] \sim \chi^2_j \tag{24}
\]

The WEAK-FORM HYPOTHESIS 2 can be done by t test. Denote

\[ d_j(k_1,k_2) = S_j(k_2) - S_j(k_1), j \in J. \]

The t test is
I \frac{d_j(k_1,k_2)/J}{d(k_1,k_2)^3}\]

where \(d(k_1,k_2)\) is the standard deviation of the mean, \(d_j\).

The alternative to test HYPOTHESIS 3 can be done by \(t\) test. Denote

\[D_j(k_1,k_2) = R_j^2(k_2) - R_j^2(k_1), j \in J.\]

The \(t\) test for HYPOTHESIS 3 is

\[t_{D}(k_1,k_2) = \frac{\Sigma D_j(k_1,k_2)/J}{d(k_1,k_2)}\]

(26)

VI. Some Empirical Results

A. Data

Based upon our preceding arguments, the three multiple regression equations (U), (R1), and (R2), the four LRT statistics in (21)-(24), and the 2 \(t\)-tests in (25)-(26) are used to test our three hypotheses discussed in section IV. In our model, the explanatory portfolios (securities) are better to have small error terms. One way to reduce the error terms is forming a well-diversified portfolios. Another method is to choose widely-held securities. The later approach is due to the argument made by Levy (1978) as follows:

For securities which are widely held we expect that Beta will provide a better explanation for pricing behavior, while for most securities, which are not held by many investors we would expect that the variance \(\sigma_1^2\) would provide a better explanation for price behavior (p. 657).

This statement implies that a widely-held security is expected to have a very small error term. As a result, Dow Jones thirty companies are
chosen as the sample to test our three hypotheses. Eight hundred observations of daily prices from July 1, 1977 to August 29, 1980 are taken from the Media General Tape.

B. Results

To reduce the multicollinearity problems across the explanatory securities, the ideas proposed by Lloyd and Lee (1976, 78) are employed. First of all, the variance-covariance matrix of the returns is estimated, then factor analysis and rotation are used to obtain eight clusters. The number of the cluster is the same as that of Lloyd and Lee. Finally, the company with the highest factor loading of the first cluster is chosen to be the first explanatory security. The second explanatory security is chosen from the highest factor loading of the second cluster, and so on. Because none of the previous studies done in testing the APT found that the number of factors is over five, it is sufficient that the explanatory securities run from 2 to 8. For convenience, twenty of the twenty-two securities are randomly selected to be the dependent variables. After running the multiple regressions on (U), (R1), and (R2), the LRT are calculated for K from 2 to 8. The LRT results is shown in Table 1.

(Table 1 about here)

From Table 1, it is easily seen that only HYPOTHESIS 1, \(\alpha_{10} = 0\), with K from 2 to 8, is not rejected. This means that the intercept terms from the multiple regressions are insignificantly different from zero. And this implies that the market equilibrium models of the APT or the generalized CAPM are insignificantly different from
the non-specific linear model, Index Model, from a single index to seven indices. The $\chi^2$ for cases 2-4 are significant at less than 1% level. This means that the sums of the slope coefficients are significantly different from one with K from 2 to 8 and $\bar{R}^2$ increases as K increases. The acceptance of null hypothesis in case 1 and the rejection of the null hypothesis in case 4 imply that the more factors or state variables, the more the description of the historical data by the APT or the generalized CAPM. Even though the APT or the generalized CAPM seems to be able to explain the historical data, the factors are greater than seven or the state variables are greater than six in a strong-form sense.

Under the alternative hypothesis of Index Model, the $\chi^2$ values in cases 2 and 3, show a sharp reduction from K = 5 to K = 6. This means that the sums of slope coefficients for K = 6, 7, and 8 are closer to one than those for K = 1, ..., 5. In a weak sense, a five factor APT or a four state variable CAPM seems the best model up against the Index Model. As a result, if there exists a linear asset pricing model, a five factor APT or a four state variable CAPM is an appropriate model.

To better highlight the above arguments, the sums of the slope coefficients and the adjusted $\bar{R}^2$ for each company with K from 1 to 29 are shown in Tables 2 and 3, respectively. It can be seen that the

(Tables 2 and 3 about here)

sum of the slope coefficients and the $\bar{R}^2$ are an increasing function of explanatory securities K. As a consequence, HYPOTHESES 2 and 3 are
rejected with $K \leq 29$. From a strong sense, either none of the linear pricing models with $K \leq 29$ can explain the historical data, or stationarity assumptions are violated. The t-values for the intercept terms show that the intercept terms are insignificantly different from zero for 29 out of 30 companies with $K$ from 1 to 29 (for savings of space, we did not show it here). This evidence confirms that if there exists a linear asset pricing model, the market equilibrium model of the APT or the generalized CAPM would be insignificantly different from the Index Model.

The t-tests of the WEAK-FORM HYPOTHESIS 2 and HYPOTHESIS 3 are represented in Table 4. All of the t-value in Table 4 are positively significant at the 1% level except one negatively significant at the 1%

(Table 4 about here)

level. These confirm the argument drawn from Tables 2 and 3. That is, in a strong sense, none of the linear pricing models with $K \leq 29$ can explain the historical data very well. However, the $t_R$-value with $k_1 \geq 2$ are much smaller than those for $k_1 < 2$. This implies that the first three explanatory securities explain the most part of the pricing behavior. From a very weak sense, a two factor APT or a one state variable CAPM can explain a lot part of the pricing behavior.

To show that it is more reasonable by using returns instead of price levels to run the regressions, Alcoa is selected to illustrate. The others are not represented due to savings of space. The t-values for the intercept terms and the adjusted $R^2$ in terms of returns, price levels, and price changes for Alcoa Company are shown in Table 5.

(Table 5 about here)
From Table 5, it can be seen that the $t$-values for the intercept terms, adjusted $R^2$ and DW statistics for both the return and price change cases are almost the same. The intercept terms are insignificantly different from zero and DW is around 2.00 in both cases. However, most of the intercept terms are significantly different from zero in the price level case. Even though the $R^2$ is very high, the DW remains quite low. This implies that the high $R^2$ is due to autocorrelation and is spurious as mentioned by Granger and Newbold (1975). Therefore, using price levels to run regression is inappropriate.

VII. Conclusion Remarks

To test the generalized CAPM, the APT, and the Index Model, a new testing model is derived in this paper by using a linear combination approach. Under this new approach, three hypotheses are created. To test the models empirically, the daily returns and prices of Dow Jones' thirty companies are selected. The empirical evidence shows that market equilibrium models, such as the APT and the generalized CAPM, are always insignificantly different from the Index Model. In the weak sense, this means, under Gibbons' framework, that S-L-M's and Black's CAPM or single factor APT is not rejected under the alternative hypothesis of the Index Model. In addition, if the deviation of the sum of slope coefficients different from one is used to be the criteria, then a five factor APT or a four state variable CAPM seems to best describe the data from the weak sense. The evidence also shows that the first three explanatory securities explain most part of the pricing behavior. This implies that a two factor APT or a
single state variable CAPM explain most part of the pricing behavior. However, in a strong sense, even though linear market equilibrium models may best describe the historical data, either the factors or state variables are very large (over 25), or the stationarity assumptions are violated.

In sum, we have derived a new testing model to test alternative linear asset pricing models simultaneously. However, the conclusion is not unambiguous. It depends upon what the alternative hypothesis is. Implications of this paper to Shanken's (1982) most recent findings on the testability of APT will be explored in the future research.
Footnotes

1 In their footnote 13, Cheng and Graner (1982) remark that since the components of $\tilde{\Sigma}_{lt}$ and $\tilde{\epsilon}_{lt}$ in equation (12) involve variables with mixed signs, the problem of heteroskedasticity appears minor.

2 If the base security and the securities in I are not assumed to have negligible error terms, then the model of equation (14) must be redefined as follows:

$$\gamma_{jt} = \alpha_j + \sum_{i=1}^{K+1} b_{ji} \gamma_{it} - \sum_{i=1}^{K+1} b_{ji} \epsilon_{it} + \tilde{\epsilon}_{jt}$$

$$= \alpha_j + \sum_{i=1}^{K+1} b_{ji} \gamma_{it} + \epsilon_{jt}$$

The variance of $\tilde{\epsilon}_{jt}^*$ in (F-2) is as follows:

$$\sigma_{\epsilon_j^*}^2 = \sum_{i=1}^{K+1} b_{ji}^2 \sigma_i^2 + \sigma_j^2$$

where we assume that the $\tilde{\epsilon}_j$ and $\tilde{\epsilon}_i$ are independent of each other. Therefore, we can use the following procedures to estimate $b_{ij}$ in (F.1) (see Theil p. 614).

First of all, we select one security from set J, and K+1 securities from set I. Then, we use this K+2 securities to run the regression of equation (F.2). Each time, one of the K+2 securities as the dependent variable, and the remaining as the independent variables, the OLS can be used to estimate $b_{ij}$ and $\sigma_{\epsilon_j^*}^2$. Next, we can use K+2 simultaneous equations system of equation (F.3) to estimate $\sigma_{\epsilon_i^2}$, $i \in I$. 
Secondary, we can use the following equation to estimate $b_{ij}$ unbiased.

$$b_j = (y'_I - T\hat{\Sigma})^{-1} y'_I y_j$$

where $\hat{\Sigma}$ is a diagonal matrix with the $i$th entry as $\hat{\sigma}^2_{\epsilon_i}$ estimated from (F.3), $T$ is the time period.
Reference


Table 1

Likelihood Ratios Test Results

$$\gamma_{jt} = a_{j0} + a_{j1} y_{it} + a_{j2} y_{it} + e_{jt}$$

\(j = 1, 2, \ldots, 20 \in J\)
\(K = 2, \ldots, 8\)
\(t = 1, 2, \ldots, 799\)

Case 1: \(H_0: a_{j0} = 0\)
Case 2: \(H_0: a_{j1} + a_{j2} = 0\)
Case 3: \(H_0: a_{j0} = 0\) and \(a_{j1} + a_{j2} = 0\)
Case 4: \(R^2(K) = R^2(K+1)\)

| \(K\) | \(T[1|E]\) | \(1|E|\) | \(\chi^2_{DF}\) |
|------|----------|----------|--------|
| Case 1 | Case 2 | Case 3 | Case 4 |
| 2 | 0.004 | 483.32* | 483.33* | 105.30* |
| 3 | 0.004 | 471.26* | 471.27* | 207.39* |
| 4 | 0.006 | 412.05* | 412.06* | 314.70* |
| 5 | 0.009 | 427.05* | 427.06* | 51.69* |
| 6 | 0.010 | 344.63* | 344.64* | 121.83* |
| 7 | 0.024 | 341.85* | 341.86* | 133.36* |
| 8 | 0.028 | 352.39* | 352.40* | --- |
| DF | 20 | 20 | 40 | 20 |

*Significant at 1% level

$$\chi^2(40, 0.01) = 63.69; \chi^2(20, 0.01) = 37.57$$
Table 2

The Sum of the Slope Coefficients with Regressors = K

<table>
<thead>
<tr>
<th>K</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>9th</th>
<th>10th</th>
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<td>1</td>
<td>0.384</td>
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<td>0.158</td>
<td>0.112</td>
<td>0.104</td>
<td>0.395</td>
<td>0.320</td>
<td>0.327</td>
<td>0.281</td>
<td>0.241</td>
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<tr>
<td>2</td>
<td>0.523</td>
<td>0.526</td>
<td>0.242</td>
<td>0.202</td>
<td>0.198</td>
<td>0.630</td>
<td>0.541</td>
<td>0.508</td>
<td>0.414</td>
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<td>0.726</td>
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The column title, such as 1st, 2nd, ..., represents the company. It is an alphabetic sequence. The first company is Allied Chemical Company.
Table 2 (cont.)

The sum of the Slope Coefficients with Regressors = K

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<td>0.190</td>
<td>0.360</td>
<td>0.340</td>
<td>0.385</td>
<td>0.353</td>
<td>0.402</td>
<td>0.387</td>
<td>0.394</td>
<td>0.368</td>
<td>0.198</td>
</tr>
<tr>
<td>25</td>
<td>0.204</td>
<td>0.379</td>
<td>0.347</td>
<td>0.434</td>
<td>0.406</td>
<td>0.407</td>
<td>0.386</td>
<td>0.399</td>
<td>0.380</td>
<td>0.195</td>
</tr>
<tr>
<td>29</td>
<td>0.205</td>
<td>0.381</td>
<td>0.353</td>
<td>0.432</td>
<td>0.408</td>
<td>0.409</td>
<td>0.385</td>
<td>0.401</td>
<td>0.385</td>
<td>0.197</td>
</tr>
</tbody>
</table>
Table 4

T-Test for the WEAK-FORM HYPOTHESIS 2 and HYPOTHESIS 3

Case 1: \( H_0: S(K) = S(K+1) = \ldots \)  \( K = 1, \ldots, 28 \)

Case 2: \( H_0: R^2(K) = R^2(K) = R^2(K+1) = \ldots \)  \( K = 1, \ldots, 28 \)

<table>
<thead>
<tr>
<th>( k_1 )</th>
<th>( k_2 )</th>
<th>Case 1: ( t_d ) - Value in (25)</th>
<th>Case 2: ( t_R ) - Value in (26)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>20.026*</td>
<td>13.499*</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>15.067*</td>
<td>89.872*</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>11.946*</td>
<td>5.752*</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>7.997*</td>
<td>7.185*</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>-3.545*</td>
<td>3.282*</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>6.800*</td>
<td>4.422*</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>5.058*</td>
<td>3.381*</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>5.751*</td>
<td>2.684*</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>10.359*</td>
<td>5.049*</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>9.096*</td>
<td>4.286*</td>
</tr>
<tr>
<td>15</td>
<td>20</td>
<td>2.589*</td>
<td>5.534*</td>
</tr>
<tr>
<td>20</td>
<td>25</td>
<td>3.427*</td>
<td>3.033*</td>
</tr>
<tr>
<td>25</td>
<td>29</td>
<td>2.742*</td>
<td>1.725*</td>
</tr>
</tbody>
</table>

* Significant at 1% level.
Table 5

The t-Value for Intercept and the $R^2$ in Terms of Return, Price Level, and Price Change: Alcoa

<table>
<thead>
<tr>
<th>Return</th>
<th>Price Level</th>
<th>Price Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>t for Intercept</td>
<td>$R^2$</td>
<td>t for Intercept</td>
</tr>
<tr>
<td>-0.4106</td>
<td>0.1219</td>
<td>22.4178</td>
</tr>
<tr>
<td>-0.1126</td>
<td>0.1475</td>
<td>17.9624</td>
</tr>
<tr>
<td>-0.3102</td>
<td>0.1688</td>
<td>10.8959</td>
</tr>
<tr>
<td>-0.5084</td>
<td>0.1991</td>
<td>5.0459</td>
</tr>
<tr>
<td>-0.6539</td>
<td>0.2303</td>
<td>9.112</td>
</tr>
<tr>
<td>-0.5980</td>
<td>0.2786</td>
<td>11.1663</td>
</tr>
<tr>
<td>-0.5991</td>
<td>0.2792</td>
<td>12.8000</td>
</tr>
<tr>
<td>-0.5082</td>
<td>0.2877</td>
<td>8.7753</td>
</tr>
<tr>
<td>-0.5527</td>
<td>0.3014</td>
<td>8.3879</td>
</tr>
<tr>
<td>-0.6009</td>
<td>0.3095</td>
<td>8.2821</td>
</tr>
<tr>
<td>-0.6679</td>
<td>0.3154</td>
<td>7.9884</td>
</tr>
<tr>
<td>-0.6977</td>
<td>0.3159</td>
<td>8.7942</td>
</tr>
<tr>
<td>-0.7026</td>
<td>0.3153</td>
<td>2.2567</td>
</tr>
<tr>
<td>-0.7706</td>
<td>0.3282</td>
<td>2.6876</td>
</tr>
<tr>
<td>-0.8182</td>
<td>0.3445</td>
<td>3.1484</td>
</tr>
<tr>
<td>-0.9101</td>
<td>0.3557</td>
<td>3.4564</td>
</tr>
<tr>
<td>-0.9829</td>
<td>0.3571</td>
<td>2.0595</td>
</tr>
<tr>
<td>-0.9339</td>
<td>0.3575</td>
<td>1.7857</td>
</tr>
<tr>
<td>-0.9167</td>
<td>0.3571</td>
<td>1.4818</td>
</tr>
<tr>
<td>-0.9573</td>
<td>0.3617</td>
<td>1.3621</td>
</tr>
<tr>
<td>-0.9897</td>
<td>0.3728</td>
<td>1.5186</td>
</tr>
<tr>
<td>-1.0440</td>
<td>0.3729</td>
<td>1.6218</td>
</tr>
<tr>
<td>-0.9443</td>
<td>0.3767</td>
<td>0.9007</td>
</tr>
<tr>
<td>-0.9442</td>
<td>0.3760</td>
<td>1.0072</td>
</tr>
<tr>
<td>-0.9511</td>
<td>0.3797</td>
<td>1.0330</td>
</tr>
<tr>
<td>-0.9300</td>
<td>0.3796</td>
<td>1.0630</td>
</tr>
<tr>
<td>-0.9094</td>
<td>0.3791</td>
<td>1.1625</td>
</tr>
<tr>
<td>-0.8764</td>
<td>0.3803</td>
<td>0.9005</td>
</tr>
<tr>
<td>-0.8737</td>
<td>0.3796</td>
<td>0.5303</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Return</th>
<th>Price Level</th>
<th>Price Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.90 - 2.03</td>
<td>0.01 - 0.32</td>
<td>1.89 - 2.03</td>
</tr>
</tbody>
</table>
Appendix A

I. The Traditional CAPM

The CAPM can be written by

\[ E_{jt} = \gamma_{ft} + \beta_j (E_{mt} - \gamma_{ft}) \]

and

\[ E_{kt} = \gamma_{ft} + \beta_k (E_{mt} - \gamma_{ft}) \]

Subtracting (A.2) from (A.1), we obtain

\[ E_{jt} - E_{kt} = (\beta_j - \beta_k) (E_{mt} - \gamma_{ft}) \]

where we assume \( \beta_j \neq \beta_k \). Solving \( E_{mt} - \gamma_{ft} \) from (3), we have

\[ E_{mt} - \gamma_{ft} = \left( \frac{1}{\beta_j - \beta_k} \right) (E_{jt} - E_{kt}) \]

For security i, we can get the equation similar to (A.3) as

\[ E_{jt} - E_{kt} = (\beta_i - \beta_k) (E_{mt} - \gamma_{ft}) \]

Substituting (A.4) into (A.5), we arrive at

\[ E_{jt} - E_{kt} = \frac{\beta_i - \beta_k}{\beta_j - \beta_k} (E_{jt} - E_{kt}) \]

or

\[ E_{it} = \frac{\beta_i - \beta_k}{\beta_j - \beta_k} E_{jt} + (1 - \frac{\beta_i - \beta_k}{\beta_j - \beta_k}) E_{kt} = a_{ijk} E_{jt} + b_{ijk} E_{kt} \]

For security j, we can obtain
\begin{align*}
\text{(A.7)} \quad E_{jt} &= \frac{\beta_j - \beta_q}{\beta_p - \beta_q} E_{pt} + (1 - \frac{\beta_j - \beta_q}{\beta_p - \beta_q}) E_{qt} \quad \beta_p \neq \beta_q
\end{align*}

Substituting (A.7) into (A.6), we get
\begin{align*}
\text{(A.8)} \quad E_{it} &= \left( \frac{\beta_i - \beta_k}{\beta_j - \beta_k} \right) \left( \frac{\beta_j - \beta_q}{\beta_p - \beta_q} \right) E_{pt} + \left( \frac{\beta_i - \beta_k}{\beta_j - \beta_k} \right) (1 - \frac{\beta_j - \beta_q}{\beta_p - \beta_q}) E_{qt} \\
&+ (1 - \frac{\beta_i - \beta_k}{\beta_j - \beta_k}) E_{kt} = a_{ijkpq} E_{pt} + b_{ijkpq} E_{qt} + c_{ijk} E_{kt}
\end{align*}

Summing up the coefficients of $E_{pt}$, $E_{qt}$, and $E_{kt}$, we arrive at
\begin{align*}
\text{(A.9)} \quad \sum \text{slope coefficients in (A.8)} &= 1
\end{align*}

Using chain-substitution, we can obtain
\begin{align*}
\text{(A.10)} \quad E_{it} &= \sum_{j=1}^{n} a_{ij}(\beta) E_{jt}, \quad n \geq 2
\end{align*}

where $\sum_{j=1}^{n} a_{ij}(\beta) = 1$

The return generating process can be expressed by
\begin{align*}
\text{(A.11)} \quad \gamma_{jt} &= E_j + \beta_j \delta_{mt} + \varepsilon_{jt} \quad E(\delta_m) = 0 \\
&\quad j = i, j, K, \ldots
\end{align*}

Substituting (A.11) into (A.10) we have
\begin{align*}
\gamma_{it} &= \beta_i \delta_{mt} + \varepsilon_{it} + \sum a_{ij}(B)(\gamma_{jt} - \varepsilon_{jt}) \\
&- \sum a_{ij}(B) \beta_j \delta_{mt} \\
&= \sum a_{ij}(B) \gamma_{jt} + \varepsilon_{it} - \sum_{j=1}^{n} a_{ij}(B) \varepsilon_{jt} \\
&- (\sum a_{ij}(B) \beta_j - \beta_i) \delta_{mt},
\end{align*}
where we can show that \( \sum_{j=1}^{n} a_{ij}(B) \beta_j - \beta_i = 0 \), and

\[
(A.12) \quad \gamma_{it} = \sum_{j=1}^{n} a_{ij}(B) \gamma_{jt} + \varepsilon_{it} \quad \beta_j \neq \beta_k, j \neq k, \quad \text{and } K \geq 2
\]

II. Multi-beta CAPM (two-beta case as illustration)

The two-beta CAPM can be written as equations (A.13), (A.14) and (A.15)

\[
(A.13) \quad E_{jt} = \gamma_{ft} + \beta_{jm}(E_{mt} - \gamma_{ft}) + \beta_{jn}(E_{nt} - \gamma_{ft})
\]

\[
(A.14) \quad E_{kt} = \gamma_{ft} + \beta_{km}(E_{mt} - \gamma_{ft}) + \beta_{kn}(E_{nt} - \gamma_{ft})
\]

\[
(A.15) \quad E_{lt} = \gamma_{ft} + \beta_{lm}(E_{mt} - \gamma_{ft}) + \beta_{ln}(E_{nt} - \gamma_{ft})
\]

Subtracting equation (A.15) from equations (A.13) and (A.14), we obtain

\[
(A.16) \quad E_{jt} - E_{lt} = (\beta_{jm} - \beta_{lm})(E_{mt} - \gamma_{ft}) + (\beta_{jn} - \beta_{ln})(E_{nt} - \gamma_{ft})
\]

\[
(A.17) \quad E_{kt} - E_{lt} = (\beta_{km} - \beta_{lm})(E_{mt} - \gamma_{ft}) + (\beta_{kn} - \beta_{ln})(E_{nt} - \gamma_{ft})
\]

Solving \( E_{mt} - \gamma_{ft} \), and \( E_{nt} - \gamma_{ft} \), we have

\[
(A.18) \quad \begin{bmatrix} E_{mt} - \gamma_{ft} \\ E_{nt} - \gamma_{ft} \end{bmatrix} = \begin{bmatrix} (\beta_{jm} - \beta_{lm}), (\beta_{jn} - \beta_{ln}) \end{bmatrix}^{-1} \begin{bmatrix} E_{jt} - E_{lt} \\ E_{kt} - E_{lt} \end{bmatrix}
\]
where

\[
\begin{vmatrix}
(\beta_{jm} - \beta_{lm}), (\beta_{jn} - \beta_{ln}) \\
(\beta_{km} - \beta_{lm}), (\beta_{km} - \beta_{ln})
\end{vmatrix} \neq 0
\]

Following equation (A.16) for security 1 and substituting the result in equation (A.18), we have

\[
(A.19) \quad (E_{it} - E_{lt}) = [(\beta_{im} - \beta_{lm}), (\beta_{in} - \beta_{ln})] \begin{bmatrix} E_{mt} - \gamma_{ft} \\ E_{nt} - \gamma_{ft} \end{bmatrix}
\]

\[
= [(\beta_{im} - \beta_{lm}), (\beta_{in} - \beta_{ln})] \begin{bmatrix} (\beta_{jm} - \beta_{lm}), (\beta_{jn} - \beta_{ln}) \\ (\beta_{km} - \beta_{lm}), (\beta_{km} - \beta_{ln}) \end{bmatrix}^{-1} \begin{bmatrix} E_{jt} - E_{lt} \\ E_{kt} - E_{lt} \end{bmatrix}
\]

Solving $E_{it}$, we have

\[
(A.20) \quad E_{it} = a_{ijkl} E_{jt} + b_{ijkl} E_{kt} + c_{ijkl} E_{lt},
\]

where we can show that $a_{ijkl} + b_{ijkl} + c_{ijkl} = 1$. Similarly,

\[
(A.21) \quad E_{jt} = a'_{jqpl} E_{pt} + b'_{jqpl} E_{qt} + c'_{jqpl} E_{lt}
\]

if

\[
\begin{vmatrix}
(\beta_{pm} - \beta_{lm}), (\beta_{pn} - \beta_{ln}) \\
(\beta_{qm} - \beta_{lm}), (\beta_{qn} - \beta_{ln})
\end{vmatrix} \neq 0
\]

Substituting (A.21) into (A.20), we have

\[
(A.22) \quad E_{lt} = a^*_{ip} E_{pt} + b^*_{iq} E_{qt} + c^*_{ik} E_{kt} + d^*_{il} E_{lt},
\]
where we can prove that \( a_{ip}^* + b_{iq}^* + c_{ik}^* + d_{il}^* = 1 \). In addition, by chain-substitution, we can obtain

\[
(A.23) \quad E_{it} = \sum_{j=1}^{K} a_{ij}(B)E_{jt}, \quad K \geq 3
\]

The return generating process can be described by

\[
(A.24) \quad \gamma_{jt} = E_j + \beta_{jm} \delta_{mt} + \beta_{jn} \epsilon_{nt} + \epsilon_{it}, \quad j=1, \ldots
\]

Substituting (A.24) into (A.23), we have

\[
(A.25) \quad \gamma_{it} = \sum_{j=1}^{K} a_{ij}(B)\gamma_{jt} + \epsilon_{it} - \sum_{j=1}^{K} a_{ij}(B)\epsilon_{jt}
\]

\[
- (\sum_{j=1}^{K} a_{ij}(B)\beta_{mj} - \beta_{mi}) \delta_{mt} - (\sum_{j=1}^{K} a_{ij}(B)\beta_{nj} - \beta_{ni}) \epsilon_{nt}
\]

\[
= \sum_{j=1}^{K} a_{ij}(B)\gamma_{jt} + \epsilon_{it}, \quad K \geq 3
\]

where, we can show that \( \sum_{j=1}^{K} a_{ij}(B)\beta_{mj} - \beta_{mi} = 0 \), and \( \sum_{j=1}^{K} a_{ij}(B)\beta_{nj} - \beta_{ni} = 0 \). Therefore, this result can be generalized to \( n \)-beta CAPM.

\[
(A.26) \quad \gamma_{it} = \sum_{j=1}^{K} a_{ij}(B)\gamma_{jt} + \epsilon_{it}, \quad K \geq n+1
\]

This is the equation (12) in the text.

We can use this approach to derive the testing models for the APT and the index model.