Creditor Country, Debtor Country and Stability Under Rational Expectations

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Steve Turnovsky made useful comments on a previous draft.
ABSTRACT

This paper provides two counter-examples to the widely-held view that valuation effects do not add to instability under rational expectations. It is shown first that given certain parameter values, the creditor country's equilibrium is stable with static expectations and is a saddle-point with rational expectations, the debtor country's equilibrium could be completely unstable with static and rational expectations. Second a creditor country could be stable under static expectations but completely unstable under rational expectations while the opposite is true for a debtor country.

Key-words: Debtor Country, Creditor Country, Valuation Effects, Rational Expectations, Saddle-Point
I. INTRODUCTION

Practitioners of open-economy macroeconomics have, for long been worried about the implications of a net debtor country. There was a belief that the model of a small open economy which is a net debtor would be characterized by dynamic instability (see e.g., Branson, Halttunen and Masson (1979), Boyer (1977) and Enders (1977)).

This was before the advent of rational expectations. With rational expectations or perfect foresight, several authors (see, Branson and Henderson (1985), Kouri (1983), Henderson and Rogoff (1982)) showed that the long-run equilibrium was always a saddle point and therefore as long as the economy jumped to the stable arm following an unanticipated immediately implemented permanent disturbance then instability need not arise.

To be a little more precise the proposition could be recast as follows: Two economies which are identical in every respect, except their foreign asset positions would have similar dynamics under rational expectations. If the net creditor country has a well-behaved dynamics (i.e., the long-run equilibrium is a saddle point) then so would it be for a net debtor country—merely because of a negative net foreign asset position we should not expect to observe a different pattern of behavior (i.e., instability).

In this paper we show by means of two counter-examples that this proposition is generally not true. In the first model below, for the assumed parameter values, under static expectations the creditor country always possesses a stable long run equilibrium whereas the
debtor country does not. Under rational expectations, given the parameter values, the creditor country's long run equilibrium is a saddle-point whereas the debtor's country's long run equilibrium is completely unstable.

In the second example, it is shown that with static expectations and given parameter values, the creditor country has a stable long run equilibrium but the debtor country does not. However, when the expectations are rational, it turns out that the debtor country has a saddle-point long-run equilibrium whereas the creditor country's long-run equilibrium is completely unstable again for the given parameter values.

The upshot of all this is that rationality of expectations does not always make an unstable equilibrium when expectations are static stable and vice-versa. The first example shows that there are (not very implausible) parameter values for which even with perfect foresight the debtor country's run equilibrium remains unstable. In the second example it is shown while rational expectations certainly makes the debtor country's long-run equilibrium a saddle-point, whereas it was completely unstable under static expectations. It is the creditor country's long-run equilibrium (which was stable under state expectations), now becomes completely unstable given the parameter values.

Before proceeding, it should be pointed out that most of the parameter values below are not wildly unrealistic. Since it is our wish to provide counterexamples rather than prove general results these seem adequate.
The rest of the paper is organized as follows. In Section 2, the first model is presented and examined. In the next section, we analyze the second model—it is actually a slightly altered version of the model in Section 2. Finally Section 4 contains some concluding comments.

2. EXAMPLE 1

The model is an open economy version of a IS-LM-Phillips curve model. Agents have rational expectations. The goods demand function has wealth as an argument, so there is also a wealth accumulation equation.

The economy produces a good which is an imperfect substitute for the imported good which is produced abroad. It takes all foreign variables as given. For simplicity it is assumed all bonds are denominated in the foreign currency. We shall also ignore interest payments on these bonds so that no distinction is made between the trade balance and the current account.

The model is given below. (All variables except interest rates are in logarithms, a dot over a variable denotes a time derivative and all coefficients are positive.)

\[ M - Q = -\alpha_1 i + \alpha_2 Y \]  
(1)

\[ i = i^* + \dot{E} \]  
(2)

\[ Y = \beta_1 (E-P) + \beta_2 W \]  
(3)

\[ \dot{P} = \pi (Y - \bar{Y}) + \dot{E} \]  
(4)
Either \( W = fE + fF + (1-f)M - Q \) \hspace{1cm} (5a)

or \( W = -gE - gD + (1+g)M - Q \) \hspace{1cm} (5b)

\[ Q = \delta P + (1-\delta)E \] \hspace{1cm} (6)

Either \( \dot{F} = \gamma_1(E-P) - \gamma_2w \) \hspace{1cm} (7a)

or \( \dot{D} = -\gamma_1(E-P) + \gamma_2w \) \hspace{1cm} (7b)

where \( M \) is the nominal stock of money (assumed to be constant), \( E \) the nominal exchange rate expressed as the domestic currency price of foreign exchange, \( P \) the price of the domestic good in domestic currency, \( i \) the domestic nominal interest rate, \( i^* \) the foreign nominal (and real) interest rate, \( Y \) is the level of domestic output (\( \overline{Y} \) is its fixed long-run level), \( W \) is real domestic wealth, \( F \) the domestic holding of foreign assets, \( D \) the domestic debt (in foreign currency), \( f(g) \) the share of the foreign asset (debt) in domestic wealth and \( Q \) the price index (the domestic CPI).

Equation (1) is the asset markets equilibrium condition. The real money supply (in terms of the consumption basket) must equal the demand for it. The demand falls as the nominal interest rate rises and rises as output (the transactions proxy) rises.

Equation (2) links the domestic nominal interest rate to the foreign interest rate via the uncovered interest parity condition, i.e., the difference between the former and the latter is the expected rate of depreciation of the domestic currency.

Equation (3) is the domestic goods market equilibrium condition. Output \( Y \) is demand-determined in the short run. Demand for domestic
output depends on wealth and the terms of trade. A rise in wealth raises demand as does a worsening of the terms of trade (a rise in (E-P), the foreign currency price of the foreign good is constant and its logarithm is zero) switching demand towards domestic goods—implicitly we are assuming that the Marshall-Lerner condition is satisfied.

Equation (4) is the expectations-augmented Phillips curve. It can be derived from a wage-Phillips curve and mark-up pricing where the expected increase in the cost-of-living enters with a unit coefficient and agents possess perfect foresight. (See Turnovsky (1981) for similar specification.⁴)

Equations (5a) and (5b) define wealth for the creditor and the debtor country respectively. It is assumed that for the latter also wealth is positive. The deflator is the cost of living index (defined in (6) below).

The price index in equation (6) is a weighted average (δ being the share of the domestic good) of the price of the domestic and foreign good.

Equation (7a) and (7b) give the current account equations for the creditor and debtor country respectively. The current account improves with a real depreciation and worsens with a rise in wealth (because this raises domestic absorption).

Suppose the relevant parameter values are as follows:

\[\alpha_1 = 0.5\quad \alpha_2 = 0.2\quad \pi = 1\quad f = 0.7\]
\[\delta = 0.9\quad \beta_1 = 0.4\quad \beta_2 = 0.7\quad \gamma_1 = 0.7\]
\[\gamma_2 = 0.3\quad \text{and}\quad g = 0.8\]
Now consider the model under static expectations. Here expected $\dot{E}$ is always zero. By substituting (1), (2), (3), (5a) and (6) in (4) and (7a) we obtain the dynamics of the creditor country in terms of two differential equation in $P$ and $F$ as shown in Appendix 1.

The trace of the coefficient matrix is

$$-\left\{\pi(\beta_1 + \beta_2 \delta) + \gamma_1 \alpha_2 \beta_2 f + \gamma_2 f(\alpha_2 \beta_1 +(1-\delta))\right\}/\left[\alpha_2 (\beta_1 + \beta_2 (f-(1-\delta))) + (1-\delta)\right]$$

and the determinant is

$$f \pi (\beta_1 \gamma_2 + \gamma_1 \beta_2)/\left[\alpha_2 (\beta_1 + \beta_2 (f-(1-\delta))) + (1-\delta)\right].$$

A sufficient condition for the trace to be negative and the determinant to be positive is that $f-(1-\delta) > 0$, and this is satisfied for our assumed parameter values. Therefore, under static expectation the creditor country's long-run equilibrium is stable.

For the debtor country under static expectations, we substitute (1), (2), (3), (5b) and (6) in (4) and (7b) to obtain two differential equation in $P$ and $D$. The determinant of the coefficients matrix is

$$\pi g(\beta_1 \gamma_2 + \gamma_1 \beta_2)/\left[\alpha_2 (\beta_1 - \beta_2 (g+(1-\delta))) + (1-\delta)\right]$$

which is positive if the term in the denominator is positive (the term gives the effect of a depreciation on money demand). For our parameter values it is positive (=.05) so the determinant is positive.

The trace of this matrix is

$$-\left\{\pi(\beta_1 + \beta_2 \delta) + g(\gamma_1 \beta_2 \alpha_2 - \gamma_2 \alpha_2 \beta_1 - \gamma_2 (1-\delta))\right\}/\left[\alpha_2 (\beta_1 - \beta_2 (g+(1-\delta))) + (1-\delta)\right].$$

Evaluating this expression for the parameter values above, we get $1.6 > 0$. Since both the trace and the determinant are positive, the model under static expectations is completely unstable.

With rational expectations, we get a third order dynamic system for the creditor country by substituting (1), (3), (5a) and (6) in
(2), (4) and (7a) (in E, P and F). This can be written compactly as follows.

\[
\begin{bmatrix}
\dot{E} \\
\dot{P} \\
\dot{F}
\end{bmatrix} = A
\begin{bmatrix}
E \\
P \\
F
\end{bmatrix}
\] (8)

where A is the coefficients matrix (the value of the elements of A, i.e., the \(a_{ij}\)'s are given in Appendix 2).

The determinant of A is given by

\[\det(A) = \pi f(\beta_1 \gamma_2 + \beta_2 \gamma_1)/a_1 > 0\] (9)

which implies that there are either two stable roots (i.e., roots with negative real parts) and one unstable root, or three unstable roots.

For our model to possess a sensible solution we require that there be two stable roots ("corresponding to" the two backward looking variables P and F) and one unstable root ("associated with" the forward-looking variable E).

A sufficient condition for ruling out the complete instability case is that the sum of the product of two roots at a time

\[\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1 = -\pi a_1^{-1}(\beta_1 + \beta_2 f \delta) + \gamma_2 f(\beta_1 \pi - a_1^{-1}) + \gamma_1 \pi \beta_2 f\] (10)

be negative (\(\lambda_i\)'s are the roots of A). This expression is equal to -1.675 for the given parameter values.

Therefore the long-run equilibrium for the creditor country under rational expectations is a saddle-point.
Turning to the debtor country first recall that we had instability with static expectations with the values of the parameters given above.

With perfect foresight we can express the dynamics of the debtor country in terms of a system of three differential equations by substituting (1), (3), (5b) and (6) in (2), (4) and (7b)

\[
\begin{bmatrix}
\dot{E} \\
\dot{P} \\
\dot{D}
\end{bmatrix} = B
\begin{bmatrix}
E \\
P \\
D
\end{bmatrix}
\]  

(11)

the values of \(b_{ij} \)'s are given in Appendix 2. The determinant of \(B\) is

\[
\frac{\pi g(\beta_1 \gamma_2 + \beta_2 \gamma_1)}{a_1} > 0
\]

(12)

So as for the creditor country there are either three unstable roots or one unstable root and two stable roots.

The trace of \(B\) is given by

\[
-\alpha_2 \beta_2 (1 + g)/a_1 - \pi (\beta_1 + \beta_2 \delta) - \gamma_2 g + 1/a_1
\]

(13)

For the parameter values given above this is equal to 0.23.

The sum of the product of the roots two at a time is given by

\[
= \gamma_2 g (\alpha_1^{-1}) + \pi \beta_2 g \gamma_1 - \pi (\beta_1 - \beta_2 g \delta) \alpha_1^{-1}
\]

(14)

which given our parameter values is equal to 0.21. Since the values of all the three equations (12), (13) and (14) turn out to be positive, \(B\) possesses no stable root. The long-run equilibrium is completely unstable in spite of the fact that we have two sluggishly evolving variables—\(P\) and \(D\).
In this section we saw that in our model given the parameter values, the creditor country is stable both under static and rational expectations, whereas the debtor country is unstable under both kinds of expectations formation. The Branson-Henderson assertion is therefore not true here.

3. **EXAMPLE 2**

The only difference between the model of the last section and this one is that here the money demand function depends on real wealth. This has interesting implications for the problem at hand.

Replace equation (1) by

\[ M - Q = -\alpha_1 i + \alpha_2 Y + \alpha_3 W \quad \frac{1 - \alpha_3 (1 + f)}{1 - \alpha_3 (1 + g)} > 0 \quad (15) \]

Consider two economies with the following parameter values

- \( \alpha_1 = .5 \)
- \( \alpha_2 = .1 \)
- \( \alpha_3 = .6 \)
- \( \beta_1 = 1 \)
- \( \beta_2 = .8 \)
- \( \pi = 1 \)
- \( \gamma_1 = 4.36 \)
- \( \gamma_2 = .2 \)
- \( \delta = .5 \)
- \( f = .7 \)
- \( g = .6 \)

First before turning to an analysis of the dynamics of the models, note that both the restrictions in equation (15) are satisfied and hence, an increase in M creates excess supply in the money market directly. The indirect effect on the money market operates through the goods market and its effect on E and Y (recall the undershooting case in Dornbusch (1976)).

As before, we express the dynamics of the creditor country under static expectations in terms of P and F. The trace of the coefficient matrix is
\[ \pi \left\{ -\beta_1 (1-\alpha_3 (1-f)) - \gamma_1 (\alpha_2 \beta_2 f + \alpha_3 f) - \gamma_2 (f \alpha_2 \beta_1 + f (1-\delta)) \right\} / \Delta_1 \]

and the determinant \( = \pi \left\{ \gamma_1 \beta_2 + \gamma_2 \beta_1 \right\} / \Delta_1 \)

where \( \Delta_1 = \alpha_2 (\beta_1 \beta_2 (f-(1-\delta))) + \alpha_3 (f-(1-\delta)) + (1-\delta) \).

The trace is negative and the determinant positive if \( f-(1-\delta) > 0 \), which is true for our parameter values so under static expectations, the creditor country is stable.

When expectations are rational, we can reduce the dynamics of the creditor country to a third-order system in \( E, P \) and \( F \) as in the last section (using (15) instead of (1)).

\[
\begin{bmatrix}
\dot{E} \\
\dot{P} \\
\dot{F}
\end{bmatrix}
= \begin{bmatrix}
E \\
P \\
F
\end{bmatrix}
\]

(16)

the elements of the \( C \) matrix are given in Appendix 2.

The determinant of \( C \) is

\[ f \pi (\beta_1 \gamma_2 + \gamma_1 \beta_2) / \alpha_1 > 0 \]

(17)

so as for matrices \( A \) and \( B \) in the previous section either there are three unstable roots or one.

The trace of \( C \) is

\[ \alpha_3 f a_1^{-1} - \pi (\beta_1 + \beta_2 \delta) - \alpha_2 \beta_2 (1-f) a_1^{-1} - \gamma_2 f + (1-\alpha_3) a_1^{-1} \]

(18)

and the sum of the product of two roots at a time is

\[ -f \pi (1-\alpha_3) (\delta \beta_2 + \beta_1 f^{-1}) - \alpha_3 f (\beta_1 + \beta_2 \delta) a_1^{-1} - \gamma_2 f a_1^{-1} + f \pi (\beta_1 \gamma_2 + \gamma_1 \beta_2) \]

(19)
For $C$ to possess three unstable roots we require both equation (18) and (19) to be positive (the determinant is unambiguously positive).

With the parameter values given above the trace of $C$ is equal to \(0.05\) and equation (19) is equal to \(0.07\).

The creditor country which was unambiguously stable under static expectations fails to possess a saddle-point equilibrium under perfect foresight and the given parameter values.

Turning to the debtor country and proceeding as in the previous section we can reduce the dynamics with static expectations to a second order system in $P$ and $D$. After doing so the trace of the coefficient matrix is found to be

$$\left\{ \pi(-\beta_1 + \beta_2 (1+g)\alpha_3 + \beta_2 g\delta) - \gamma_1 (\alpha_2 \beta_2 g + g\alpha_3) - \gamma_2 (g(1-\delta)+\alpha_2 \beta_1) \right\}/\Delta_2 \tag{20}$$

where

$$\Delta_2 = (1-\delta) + \alpha_2 (\beta_1 - \beta_2 (g+1-\delta)) - \alpha_3 (g+(1-\delta)).$$

For the parameter values above, this expression has a value of \(1.04\), implying that at least one of the roots of the coefficient matrix is positive and the system is unstable.

Turning to the case of rational expectation we can reduce the debtor country's dynamics to a system of three differential equation in $E$, $P$ and $D$

\[
\begin{bmatrix}
\dot{E} \\
\dot{P} \\
\dot{D}
\end{bmatrix} = 
\begin{bmatrix}
H & 0 & 0 \\
0 & H & 0 \\
0 & 0 & H
\end{bmatrix}
\begin{bmatrix}
E \\
P \\
D
\end{bmatrix}
\tag{21}
\]

The values of $h_{ij}$'s are also given in Appendix 2.
The determinant of $H$ is

$$g^m (\beta_1 y_2 + \beta_2 y_1) a_1^{-1} > 0$$  \hspace{1cm} (22)

again implying that either there are no stable roots or two stable roots.

The trace of $H$ is

$$(1-\alpha_3) a_1^{-1} - \left\{ \pi (\beta_1 + \beta_2) + y_2 g + (\alpha_2 y_2 (1+g) + \alpha_3 g) / a_1 \right\}$$  \hspace{1cm} (23)

which is negative for our assumed parameter values ($=-1.7$) which implies that at least one root must be stable but then we know (from equation (22)) that then two roots are negative (or have negative real parts).

The debtor country which was unstable with static expectations for the given parameter values turns out to be well behaved, i.e., possessing a saddle-point under perfect foresight.

In this section we examined a model where the creditor country goes from being stable under static expectations to completely unstable under perfect foresight. The opposite is true for the debtor country.

4. CONCLUSIONS

In this paper we have re-examined the dynamic stability of two economies identical in every respect but in their net foreign asset position. It was found that the conventional wisdom—that debtor countries which are unstable under static expectations become (saddle-point) stable under perfect foresight—is not generally true.
A corollary that the creditor country's dynamics is always more stable—i.e., instability is usually associated with a debtor country—was also shown to be untrue in general.

In our first example a debtor country remains unstable under perfect foresight. In the second example the debtor country's equilibrium indeed becomes a saddle-point but the creditor country goes from being completely stable to completely unstable.
Footnotes

1 A more complete list of references is to be found in Branson and Henderson (1985).

2 Two quotations will give a flavor of the conclusions reached in the literature:

No matter whether goods prices are fixed or flexible under rational expectations, instability ... (can) not (arise) because of perverse valuation effects associated with negative net foreign asset positions. (Branson and Henderson (1985), p. 782.)

...(W)hen expectations are rational the problem of nonexistence of short-run equilibrium does not arise provided that long-term equilibrium is unique. (Kouri (1983), p. 154.)

3 "... (N)egative net foreign asset positions are not an independent source of instability. Instability can arise only under non-rational expectations" (Branson and Henderson (1985), p. 777).

4 Consider first a wage-Phillips curve.

\[ V = j(Y-\bar{Y}) + \dot{Q} \]

where \( V \) is the log of the money wage rate and \( \dot{Q} \), the expected (percentage) increase in the cost of living. If prices are a (fixed) mark-up on money wages then

\[ \dot{P} = j(Y-\bar{Y}) + \dot{Q}. \]

Using the definition of \( Q \), (4) follows where \( \pi \equiv j/(1-\delta) \).

Note the \( \dot{E} \) term in (4) is the expected rate of depreciation. We have

\[ E(t) = E_1(t,t) + E_2(t,t) \]

where \( E_1(t,t) = \lim_{h \to 0} \epsilon [(E(t+h)-E(t))/I(t)]/h \]

and \( E_2(t,t) = \lim_{h \to 0} \epsilon [E(t+h)/I(t+h)-E(t+h)/I(t)]/h \)

where we have assumed \( E(t,t) = E(t) \). \( \epsilon \) is the expectations operator and \( I(t) \), the information set at \( t \).

When new information arrives it is \( E_2(t,t) \) (the revision in expectations) which jumps and \( E_1(t,t) \) remains finite. Only with this interpretation of \( \dot{E} \) is it possible to maintain that \( P \) is not a forward-looking jump variable.
When this condition holds with equality, i.e., \( f = (1-\delta) \), the country is insulated from a once-and-for-all change in the rest of the world's price level.

The responsiveness of the trade balance to the real exchange rate is high here but this just implies that the domestic and foreign goods are close substitute. Perfect substitutability, i.e., purchasing power parity implies \( \gamma \to \infty \).
Bibliography


Appendix 1

Here we outline a method for analyzing the stability of a model under, static expectations. We use the creditor country's model in Section 2.

Putting \( \dot{E} = 0 \) and substituting for \( W \) and \( P \) in the money market equilibrium, we get

\[
E = E(P, F), \tag{A.1}
\]

\[
E_P = \{ \delta - \alpha_2(\beta_1 + \beta_2) \}/S, \quad E_F = \alpha_2 \beta_2 f/S
\]

where \( S = (1-\delta) + \alpha_2(\beta_1 + \beta_2(f-(1-\delta))). \)

Then substituting for \( W \) in equations (4) and (7a), we have

\[
\dot{P} = \pi[\beta_1(E - 1) + \beta_2(f-(1-\delta)E - \delta)] P + \pi[\beta_1 E_P + \beta_2((f-(1-\delta))E_F + f)]F \tag{A.2}
\]

and

\[
\dot{F} = [\gamma_1(E - 1) - \gamma_2((f-(1-\delta))E - \delta)] P + \gamma_1[ E_F - \gamma_2(f-(1-\delta))E_F + f)]F. \tag{A.3}
\]

(A.2) and (A.3) constitute the second order dynamic system under static expectations.

The values of \( E_P \) and \( E_F \) (or \( E_D \)) differ between models in sections 2 and 3. For the debtor country the relevant dynamic variables are \( P \) and \( D \).
The values of the coefficients of the A, B, C and H matrices are given below:

<table>
<thead>
<tr>
<th>$A_q$</th>
<th>$A_H$</th>
<th>$B_q$</th>
<th>$B_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbf{A}_q + \mathbf{B}<em>q \mathbf{g}</em>{\mu\nu}$</td>
<td>$\mathbf{A}_H + \mathbf{B}<em>H \mathbf{g}</em>{\mu\nu}$</td>
<td>$\mathbf{A}_q + \mathbf{B}<em>q \mathbf{g}</em>{\mu\nu}$</td>
<td>$\mathbf{A}_H + \mathbf{B}<em>H \mathbf{g}</em>{\mu\nu}$</td>
</tr>
</tbody>
</table>

$\mathbf{A}_q = \mathbf{B}_q$, $\mathbf{A}_H = \mathbf{B}_H$, $\mathbf{A}_q + \mathbf{B}_q \mathbf{g}_{\mu\nu} = \mathbf{A}_H + \mathbf{B}_H \mathbf{g}_{\mu\nu}$.