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Equilibrium, Imperfect Competition and the International Wheat Market

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ABSTRACT

The simple model of perfect competition in spatial markets has been widely applied in analyzing international agricultural trade. Yet many markets do not appear to be perfectly competitive. In this paper we offer a new and efficient method for computing spatial equilibrium in oligopolistic or oligopsonistic markets. We apply the technique to an analysis of several hypotheses of market conduct in the international wheat market that have been put forward by other authors. We conclude that a duopsony model with Cournot-Nash behavior (styled after Carter and Schmitz) does not perform well in explaining trade.
I. INTRODUCTION

The simple model of perfect competition in spatial markets has been widely applied in analyzing international agricultural trade. Following the developments of Samuelson and Takayama and Judge, it is relatively easy to formulate and solve detailed and complex spatial competitive equilibrium models. Yet as Thompson points out, analyses based on the simple competitive model have yielded disappointing results. Simple competitive theory seems unable to explain trade in many agricultural markets.

It is often suggested that the poor performance of the simple competitive model is due to the interference of governments in markets or to manipulation by other market participants such as trading companies (Webb, McCalla and Josling, and Morgan). Nearly 20 years ago McCalla proposed that a US-Canada duopoly (with market power exercised through government policy) was the more appropriate market conduct assumption for the international wheat market. Later, Alaouze et al and Carter and Schmitz suggested a triopoly and a duopsony, respectively, as more appropriate market structure and conduct assumptions in this market. Yet such imperfect competition hypotheses have made negligible inroads into applied trade analysis.

The purpose of this paper is to bridge this gap between the apparent structure and conduct of the international wheat market and quantitative analyses of market performance. In doing so, we present a new and efficient method for computing spatial equilibria in oligopolistic markets. The model and solution technique are based on non-linear complementarity and represent a very promising way of formulating
and solving both competitive and noncompetitive spatial equilibrium problems. We then test various hypotheses of market conduct (mentioned above) in the international wheat market using a simple imperfectly competitive wheat trade model. Because of the methodological focus of this paper, we have chosen to utilize costs and demands from Shei and Thompson's previously published and relatively small model of the international wheat market. Thus, in essence we use their model, modifying only their assumptions about market conduct and their computational technique for finding market equilibria.

In the next section we review relevant developments in agricultural trade analysis. In the third section of the paper we present our model of reaction function equilibria and in the fourth section we apply the model to the international wheat market as we attempt to accept or reject several hypotheses about conduct in that market.

II. AGRICULTURAL TRADE MODELS

As in most economic analysis the paradigm of perfect competition is widely used in evaluating agricultural trade and trade policies.\(^4\) In his thorough review of US developments in agricultural trade models, Thompson states that spatial price equilibrium models\(^5\) are "the most common class of agricultural trade models, particularly for comparative statics analysis of the effects of a change in policy." Thompson supports this statement by citing nearly three dozen spatial equilibrium models of international markets for wheat, rice, corn, sugar, pork, beef, oranges, rapeseed, and peanuts.

Despite its popularity, it has been recognized for some time that the paradigm of simple spatial competitive equilibrium suffers from significant deficiencies, most notably poor performance in explaining
trade patterns, particularly for wheat. Characteristically, such models predict fewer bilateral trades than actually occur, although many actual transactions are quite small. This class of models also exhibits a high degree of sensitivity of equilibrium trade levels to small parameter changes. In essence, the simple model of spatial perfect competition does not seem consistent with how international agricultural markets operate.

There have been a number of different attempts to correct these problems. One approach has been to hypothesize that commodities from different suppliers are not perfect substitutes, despite their physical similarities. This differentiation may be due to institutional factors, historic trade preferences or attitudes toward risk. The result is that demand becomes more complex than in the simple competitive model. This type of demand system is best exemplified by the work of Armington.

A major thrust of the literature in correcting the predictive deficiency of spatial equilibrium models is to examine more closely the role of governments in markets. If one accepts that government policy is predominantly responsible for differences between actual trade patterns and those predicted by the simple competitive model, then the focus of the analysis should be upon the determinants of government policy.

"Endogenizing" government policy in trade analysis has taken two directions. One direction has been to assume that government policy is determined by domestic political factors and not by market power considerations. The focus of such work is on governmental objectives
in the policy-setting process (see Rausser et al). Sarris and Freebairn examine the international wheat market and estimate the relative weights governments place on domestic producer surplus, consumer surplus, support program costs, and price stability. In an analysis of the soybean/rapeseed market, Meilke and Griffith estimate government behavioral equations for the setting of tariffs and domestic price supports for these commodities.

A second approach to analysis with endogenous government policy is to assume that policies serve to coordinate consumers or producers so that they may jointly exercise power in the international market. Thus the international market would be expected to operate as any oligopoly/oligopsony with countries as the participants. In 1966, McCalla suggested that two producing countries dominate the world wheat market and produce and price accordingly. Later Alaouze et al proposed expanding the list of oligopolists to three by adding Australia. Carter and Schmitz suggested that an EEC-Japan duopsony is responsible for observed wheat trade and price patterns. On the empirical side, Jeon has developed a non-spatial dynamic model of duopoly in the world wheat market and Karp and McCalla have modeled the world corn market as a difference game.

A simpler form of market manipulation is through the cartelization of producers (or consumers). Schmitz et al have analyzed in depth the possibility of a grain exporter's cartel. Pindyck has estimated the potential gains from cartelization in the bauxite, copper and petroleum markets while Pincus and Johnson have analyzed the potential cartel profits for a number of agricultural commodities.
III. SPATIAL EQUILIBRIA IN IMPERFECTLY COMPETITIVE MARKETS

The purpose of this paper is to explore the hypothesis that the international wheat market is imperfectly competitive. We propose to test this hypothesis by introducing a spatial model of imperfect competition in that market. However, one must be careful in defining a model of imperfect competition. On the face of it, there are far too many individual producers and consumers to apply any conventional model of imperfect competition to the wheat market. However, if producers (or consumers) are organized into a small number of groups and within a group all producer (or consumer) actions are coordinated, then we have an oligopoly (or oligopsony) among a few cartels of producers (or consumers). In this paper the natural grouping is the nation (or, for the EEC, several nations). Governments serve as coordinators of producers and consumers, looking after their interests as groups. The model then is of imperfect competition among nations. A few nations dominate the market from either the producing or consuming side and set tariffs, price supports, quotas or other instruments to maximize the economic welfare of the producers of consumers they represent.

With that introduction we will move to describe our model of imperfect competition. Although the actors in the model are producing and consuming nations, without loss of generality the model is described mathematically in terms of a few producers and consumers interacting in an imperfectly competitive market. To enhance the model description, we will present it in two steps. We first discuss reaction function models and present a simple model of spaceless...
reaction function equilibria. We then present a model of spatial oligopoly and oligopsony.

To understand why the model is presented in the particular form it is, the reader should be aware that the model is formulated and solved as a nonlinear complementarity problem.\(^9\) Let there be some \(n \times 1\) vector of variables, \(\mathbf{x}\), and an \(n \times 1\) vector valued function, \(\mathbf{f}(\mathbf{x})\). The nonlinear complementarity problem is to find an \(\mathbf{x} > 0\) so that \(\mathbf{f}(\mathbf{x}) > 0\) and \(\mathbf{x}^T \mathbf{f}(\mathbf{x}) = 0\) for all \(i\). In other words we seek a vector \(\mathbf{x}\) with nonnegative components so that \(\mathbf{f}(\mathbf{x})\) is also nonnegative. Further, matching \(\mathbf{x}\) with \(\mathbf{f}(\mathbf{x})\), component by component, at least one member of each pair of \(\mathbf{x}_i\) and \(\mathbf{f}_i(\mathbf{x})\) must be zero. As will become evident throughout the remainder of this section, the models of imperfect competition presented here are precisely of this form. Efficient computer algorithms are available for solving the linear and nonlinear complementarity problem.\(^10\)

A. A Spaceless Reaction Function Oligopoly Model

Consider the case of \(I\) producers of a single commodity. Denote by \(q_i\) the output of the \(i\)th producer and denote by \(\pi_i(q)\) the profit of the \(i\)th producer. (It is not necessary to explicitly consider demand since each producer's profit function involves the output of all producers). Each producer (i) has the simple problem of choosing \(q_i\) to maximize \(\pi_i:\)

\[
\max_{q_i} \pi_i(q) \quad \text{(1a)}
\]

\[
\exists q_i > 0 \quad \text{(1b)}
\]
If \( \pi \) is pseudoconvex then necessary and sufficient conditions for profit maximization are:

\[
\frac{d\pi_i}{dq_i} + \lambda^* = 0 \tag{2a}
\]

\[
\lambda^* q_i = 0 \tag{2b}
\]

\[
\lambda^*, q_i \geq 0 \tag{2c}
\]

or equivalently,

\[
\frac{d\pi_i}{dq_i} \leq 0 \tag{3a}
\]

\[
q_i^* \left[ \frac{d\pi_i}{dq_i} \right] = 0. \tag{3b}
\]

\[
q_i^* > 0 \tag{3c}
\]

Note two things about equation (3). First, it is in the form of a nonlinear complementarity problem, and secondly, the two equations implicitly define the optimal output for producer \( i \) as a function of the output of all other producers: \( q_i^*\left(q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_n\right) \).

This of course is producer \( i \)'s reaction function or best reply function (see Friedman). The reaction function model can take on many forms depending on how the producer thinks his profits will change as he changes output (\( d\pi_i/dq_i \)). It is in computing the derivative of profit with respect to output that conjectures about behavior of competitors enter. It is conventional to express such conjectures through a "conjectural variation." Let \( r_{ij} \) be the conjectural
variation of producer \( i \) with respect to producer \( j \); i.e., \( r_{ij} \) is producer \( i \)'s conjecture of how \( q_j \) will change when \( q_i \) is changed (\( r_{ij} = \frac{\partial q_j}{\partial q_i} \), conjectured). Thus Eqn. (3) can be rewritten as

\[
\sum \frac{\partial \pi_i}{\partial q_j} r_{ij} \leq 0 \quad (4a)
\]

\[
q_i^* \left[ \sum \frac{\partial \pi_i}{\partial q_i} r_{ij} \right] = 0 \quad (4b)
\]

\[
q_i^* > 0 \quad (4c)
\]

where of course \( r_{ii} = 1 \). A Nash equilibrium in this market can be determined by finding a vector \( q^* \) which satisfies a pair of Eqn. 4 for each producer (1 inequalities, 1 equalities).

The various models of reaction function equilibrium differ in the assumed conjectural variation. A Cournot-Nash equilibrium corresponds to \( r_{ij} = 0 \), \( i \neq j \) and can be thought of as a maximum market power non-cooperative equilibrium in that each producer behaves as if he were a single monopolist facing a residual demand curve. He takes no account of the response of competitors to his actions. On the other hand, a Bertrand equilibrium corresponding to \( \sum_{k \neq i} r_{ik} = -1 \), is a minimum market power situation. In this case, each producer assumes that any reduction in his own output will be exactly matched by competitors. In the case when outputs from different producers are perfect substitutes, the Bertrand model results in marginal cost pricing.\(^{11}\)

B. Spatial Oligopoly/Oligopsony With A Competitive Fringe

We now extend our model to a spatial market. The model presented here is of a spatial market for a single good at a point in time. The
essence of the spatial aspects of the market is that transportation of
goods from producers to consumers is a costly process. While the model
is presented in terms of producers and consumers, it is producer and
customer nations (or groups of nations) that will be assumed to exer-
cise market power when the model is applied to the international wheat
market. We present two different but similar spatial models, one of
oligopoly and one of oligopsony.

Let $i = 1, \ldots, I$ index producers and $j = 1, \ldots, J$ index con-
sumers, and further, let

- $q_{ij}$ denote quantity shipped from producer $i$ to consumer $j$
- $\tau_{ij}$ denote the unit transport cost from producer $i$ to con-
sumer $j$
- $P_i(\Sigma q_{ij})$ denote the inverse demand function for consumer $j$
- $c_i(\Sigma q_{ij})$ denote the marginal cost function for producer $i$.

The first of these variables is endogeneous, to be computed. The
transport cost and the marginal cost and inverse demand functions are
exogenous. Note that by indexing the shipments by origin and destina-
tion, we are allowing producers or consumers to price discriminate.
Producers and consumers that are price takers will price at marginal
cost or along their demand curve, in equilibrium. Producers exer-
cising market power will price at or above marginal cost. Consumers
exercising market power will offer prices at or below their demand
curve. Although market-manipulating producers and consumers can both
be included in a single model, for clarity of exposition we will treat
these two situations separately.
1. A Model of Spatial Oligopoly

We first consider a model of oligopoly, with some nation-producers exercising market power and all others in a competitive fringe. All consumers can be totally represented by their inverse demand curves. Thus there are two types of producers in the market: oligopolists and the competitive fringe. Denote the set of oligopolists by \( M = \{i| i \text{ is an oligopolist}\} \). Each producer has the same objective, to maximize profits over \( q_{ij} \geq 0 \):

\[
\pi_i = \sum_j \left( P_j (\sum_i p_i q_{ij}) - \tau_{ij} \right) q_{ij} - \int_0^{\Sigma q_{ij}} c_i(x)dx .
\]

(5)

The price the producer faces is the consumer price \( P_j \) less the transport cost \( \tau_{ij} \). Thus the first term in Eqn. 5 represents revenues. The second term, the integral under the marginal cost curve, represents total variable costs. Producers maximize profits by choosing quantity sold to each consumer, \( q_{ij} \). As in equation (3), the first order conditions for a maximum of (5) are

\[
w_{ij} = \left\{ P_j (\sum_i p_i q_{ij}) - \tau_{ij} + q_{ij} \frac{dP_j}{dq_{ij}} - c_i (\sum_j q_{ij}) \right\} \leq 0 , q_{ij} \geq 0 , w_{ij} q_{ij} = 0 , \forall j
\]

(6)

In the expression above, the dummy variable \( w_{ij} \) is used to facilitate indicating that the term in braces must be nonpositive and that its product with \( q_{ij} \), must be zero. The complementarity condition is necessary because we would expect some flows \( (q_{ij}) \) to be zero and thus the corresponding \( w_{ij} \) to be perhaps negative. The first three terms of the expression in braces of course represent marginal revenue. A fundamental distinguishing characteristic of a spatial model of trade is that one cannot determine \( \text{a priori} \) which producer-consumer pairs
will have no trade \( (q_{ij} = 0) \). Thus the necessity for this condition which is more complex than just marginal revenue equals marginal cost.

For a price taker (the competitive fringe), \( \frac{dP}{dq_{ij}} \) is considered to be zero since the price taker assumes the market price is insensitive to changes in his output. Thus for the competitive fringe, first order conditions (6) are quite simple:

\[
\alpha_{ij} \equiv \{P_j(\sum q_{ij}') - \tau_{ij} - c_i(\sum q_{ij}')\} \leq 0, \quad q_{ij} \geq 0, \quad \alpha_{ij}q_{ij} = 0, \quad \forall j, i \in M (7)
\]

This expression merely states that the price net of transport cost is equal to marginal cost unless no transactions take place \( (q_{ij} = 0) \) in which case price can be less than marginal cost.

We turn now to the case of the oligopolist whose perception that he faces downward sloping demand curves is what causes him to try to extract monopoly rent. Oligopolist i's perception of \( \frac{dP}{dq_{ij}} \) depends on (a) the actual slope of the demand curve for consumer j; and (b) his perception of the extent to which competitors will change their sales to consumer j in response to a change in his sales to consumer j, \( q_{ij} \):

\[
\frac{dP}{dq_{ij}}(\sum q_{ij}') = P_j'(\sum q_{ij}')[1 + r_{ij}] \quad \forall j \in M (8)
\]

In Eqn. (8), \( r_{ij} \) is oligopolist i's perception of how sales of all competitors combined, to consumer j, change with \( q_{ij} \): \( r_{ij} = \Delta(\sum q_{ij}')/\Delta q_{ij} \), conjectured. Note that for simplicity this is an aggregate conjectural variation—individual reactions of competitors are aggregated into \( r_{ij} \).

Thus, combining Eqn. (6) and (8), first order conditions for profit maximization for each oligopolist are given by
This is a standard condition, that for any sales to take place from producer \(i\) to consumer \(j\), perceived marginal revenue must be equal to marginal cost. If no sales occur, marginal revenue may be less than marginal cost. Obviously, with \(r_{ij} = -1\) (a Bertrand equilibrium), Eqn. (8) reduces to price equals marginal cost (if sales occur).

The entire spatial oligopoly model consists of \(I \times J\) variables (the \(q_{ij}\)) and a vector valued function \(w\) of dimension \(I \times J\) defined by equations (7) and (9). The form of these equations corresponds precisely to the nonlinear complementarity problem: find \(q \geq 0\) such that \(w(q) < 0\) and \(w_{ij}q_{ij} = 0\).

2. A Model of Spatial Oligopsony

We turn now to the situation where some consumers exercise market power. Other consumers and all producers are price takers. Define the set of oligopsonists as \(N = \{j|j\ \text{is an oligopsonist}\}\). Prices offered by producers are assumed equal to their marginal production costs. Consumers will be assumed to maximize consumer surplus, choosing purchases from each producers, \(q_{ij}^{*}\):

\[
S_j = \int_0^{\Sigma q_{ij}} P_j(x)dx - \Sigma c_i(\Sigma q_{ij}) + r_{ij}q_{ij}
\]  

The above expression consists of two parts. The first is the area under the consumer's demand curve. The second is total outlays for goods, which are given by the product of price paid and quantity purchased. Price is, of course, marginal production cost plus
transport cost. First order conditions for surplus maximization are straightforward:

\[ u_{ij} \equiv \left\{ p_j (\sum q_{ij}) - [c_i (\sum q_{ij}) + \tau_{ij}] - q_{ij} \frac{dc_i}{dq_{ij}} \right\} \leq 0 \]

(11)

\[ q_{ij} \geq 0, u_{ij} q_{ij} = 0, \forall i,j. \]

Once again, it is in how the consumer perceives \( dc_i/dq_{ij} \) that distinguishes oligopsonists from price-taking consumers. A price-taking consumer assumes he cannot affect the price he pays by cutting back on purchases. Thus for the price-taking consumer, the first order condition (11) becomes

\[ u_{ij} \equiv \left\{ p_j (\sum q_{ij}) - [c_i (\sum q_{ij}) + \tau_{ij}] \right\} \leq 0, q_{ij} \geq 0, u_{ij} q_{ij} = 0, j \notin N. \]

(12)

This is the same condition as for competitive producers (equation 7): that for trade to occur, marginal costs including transport costs equal the demand price.

Oligopsonists perceive that marginal costs are upward sloping: surplus can be extracted from producers by reducing purchases and thus driving down price. As in the case of oligopoly, an oligopsonist's perception of \( dc_i/dq_{ij} \) depends on the slope of producer \( i \)'s marginal cost curve as well as the oligopsonist's perception of how his fellow consumers will respond to an effort to drive down the price of producer \( i \)'s product:

\[ \frac{dc_i (\sum q_{ij}^*)}{dq_{ij}} = c_i' (\sum q_{ij}^*) (1 + s_{ij}) \]

(13)
where \( s_{ij} \) is oligopsonist \( j \)'s perception of how purchases from \( i \) of all other consumers combined change with \( q_{ij} \): 
\[
 s_{ij} = \frac{\Delta (\Sigma q_{ij}')}{\Delta q_{ij}},
\]
conjectured. Thus first order conditions (11) for consumer surplus maximization for the oligopsonists are

\[
 u_{ij} = \{p_j(\Sigma q_{i,j}) - [c_j(\Sigma q_{ij}) + \tau_{ij}] - q_{ij}c_j'(\Sigma q_{i,j})(1 + s_{ij})\} \leq 0
\]

(14)

\( q_{ij} \geq 0, u_{ij}q_{ij} = 0, i, j \in N \)

Thus for any sales to occur, the perceived marginal gain in surplus from a reduced price for the good must be equal to the marginal loss in surplus due to reduced consumption.

The spatial oligopsony model consists of \( I \times J \) variables (the \( q_{ij} \)) and a vector valued function \( u \) of dimension \( I \times J \) defined by equations (12) and (14). The model in full form is to find a vector \( q \geq 0 \) such that \( u(q) \leq 0 \) and \( u_{ij}q_{ij} = 0 \). This is precisely the nonlinear complementarity problem.

IV. COURNOT-NASH EQUILIBRIA IN THE INTERNATIONAL WHEAT MARKET

In this section we apply our spatial model to the international wheat market and examine the ability of four market conduct hypotheses to explain 1972-73 trade. Costs and demand relations are taken directly from Shei and Thompson.\(^{14}\) The Shei and Thompson model is a classic spatial equilibrium model with five producing countries or regions and nine consuming countries or regions. They estimate a set of five linear export supply functions and nine linear demand equations.\(^{15}\) With a per-unit transportation cost associated with every producer-consumer pair, the model is completely specified and they find an equilibrium by the conventional method of maximizing consumer plus
producer surplus. Although more current models of wheat trade may be available, the Shei and Thompson model was chosen for its relative simplicity and well-documented performance.

The hypotheses of market conduct we examine are a) a Canada-US duopoly; b) a Canada-US-Australia triopoly; c) a Japan-EEC duopsony; and d) perfect competition or free trade. As we indicated earlier, the first three of these conduct models have been suggested in the literature as plausible for the international wheat market. All agents other than the oligopolists/oligopsonists are assumed to behave competitively. A fundamental assumption for all of these cases is that of Cournot-Nash behavior—the conjectural variations discussed in the previous section are all assumed to be zero. In other words, when importers or exporters determine trade levels, shipment patterns among all other participants are taken as given and fixed. The analysis involves finding equilibrium trade levels for each of these conduct assumptions and comparing the results to actual trade.

For comparison, Table I shows actual trade shares and prices for 1972-73 and Table II shows trade shares and prices under the perfectly competitive (free trade) market conduct assumption. Table II exhibits the classic characteristics of a competitive spatial equilibrium model. Net trade levels for particular countries are in fair agreement with the actual levels; but the trade flows between specific countries are generally inconsistent with the actual flows. The competitive equilibrium model shows far fewer non-zero trades than in actuality. In that model Australia, Argentina, and the EEC each serve only one
consumer as opposed to many in actuality. The predicted export prices are in fair agreement, however.

For comparison, the trade share matrix associated with the duopoly market conduct assumption is presented in Table III. Note that there are many more non-zero trades in this case than in the free trade case (Table II). Also, the trade shares from each of the duopolists are in "reasonable" agreement with actual trade (Table I). However, trade from Australia, Argentina and the EEC are quite different than in actuality (compare Table III and Table I). Export prices presented in Table III are also in "reasonable" agreement with Table I.

We now move to the question of quantifying model performance in explaining trade. Our fundamental goal is to determine which model, if any, can be judged statistically acceptable in explaining trade. To do this we will present several statistics representing the degree to which model-predicted trade corresponds to actual trade. Ideally, we would like a distribution on the forecast trade share matrix so that we could determine if the mean forecast trade matrix differs in a statistical sense from actual trade. Unfortunately, we do not have a distribution on forecast trade shares. For each conduct assumption we have a single deterministic forecast trade matrix to compare to a single actual trade matrix. We do not consider multiple time periods, which might be useful for imputing a distribution for the trade matrices. Nevertheless, it is still possible to qualitatively compare the matrices and, using some moderate assumptions, statistically compare the matrices and test our hypotheses of market conduct. We use two measures to compare forecasted trade shares with the base level of trade: the Theil inequality.
coefficient and the Spearman rank correlation coefficient. The Theil coefficient is not a statistic in the sense of hypothesis testing but can be used to obtain a qualitative estimate of goodness-of-fit. The Spearman coefficient can be used for hypothesis testing if (after Teigen) we pair corresponding elements of the actual and predicted trade matrices and then treat the resulting set of pairs as a sample from a bivariate population.\footnote{19}

The Theil inequality coefficient (as defined in Theil) has been widely used to compare forecasts with actual values. Much insight can be gained using the inequality coefficient but it cannot be used for hypothesis testing in our case because it is not distribution free. The inequality coefficient ($U$) is in essence the root-mean-squared error between elements of the predicted and actual trade matrices, normalized so that the coefficient lies between zero and one. Perfect prediction is associated with a zero inequality coefficient. Further insight is gained by calculating the variance ($U^2$) and covariance ($U^c$) proportions of the inequality coefficient. These two proportions sum to unity.\footnote{20} Suppose predicted values are plotted against actual values. For perfect forecasting, all points would lie along a $45^\circ$ line. The variance proportion indicates the extent to which the slope of a regression line through the points deviates from one. The covariance proportion indicates the spread of points about this regression line. Thus, the closer the covariance proportion is to a maximum of one, the better the forecast since the variance proportion would then be small, and one would expect some random component in forecasts.
A second statistic we use is the Spearman rank correlation coefficient (see Conover), which is a nonparametric statistic. Pairing each element of the actual (A) and predicted (P) trade matrices, we can view these pairs \((a_{ij}, p_{ij})\) as samples from a bivariate distribution.\(^{21}\) Regressing \(p_{ij}\) against \(a_{ij}\) we obtain \(a_{ij} = \alpha + \beta p_{ij} + \epsilon_{ij}\). We can test the null hypothesis that \(\alpha = 0\) and \(\beta = 1\) (i.e., that the model is a perfect predictor). However, the elements of the predicted trade matrix are not independent nor is there a conventional population from which the \((a_{ij}, p_{ij})\) pairs are drawn. Nevertheless, this statistic and the Theil inequality coefficient should give us a strong foundation on which to test our hypothesis of the performance of the four models of market conduct.

With regard to the hypotheses of market conduct, Table IV shows the two measures of consistency between actual trade shares and simulated trade shares under the four market conduct assumptions. Note that for both the Theil and Spearman statistics, the duopoly and triopoly market conduct assumptions perform considerably better than either the free trade or EEC-Japan duopsony conduct assumptions. The Theil inequality coefficient and the variance proportion are considerably lower in the two oligopoly cases. However, the Spearman rank correlation coefficient paints a slightly different picture, suggesting that the free trade conduct assumption behaves fairly well. However, one must interpret the Theil and Spearman statistics in conjunction with one another. The Spearman coefficient suggests that the pairs of actual and predicted shares for free trade lie nicely along the 45° line. In fact, the three models (all except duopsony) perform similarly on this count.
However the Theil inequality coefficient suggests that free trade gives much greater dispersion about this 45° line than do the oligopoly models. Thus taking the two statistics together points to the superiority of the two oligopoly models with the duopoly model performing best.

We turn now to testing hypotheses of market conduct. Our null hypothesis is that a model predicts trade. More specifically, if one regresses actual trade against predicted trade \( a_{ij} = \alpha + \beta p_{ij} + \epsilon_{ij} \), the hypothesis is that \( (\alpha, \beta) = (0,1) \): that actual and predicted trade are the same, except for a zero-mean error. Our goal then is to reject the hypothesis for some of the conduct assumptions. At the 90 percent level we can reject the null hypothesis for all except the duopoly model. At the 95 percent and 99 percent level, we can only reject the hypothesis for the duopsony model. Thus on all counts, the duopsony model performs very poorly.

V. CONCLUSIONS

Probably the most significant contribution of this paper is our development of a model of spatial imperfectly competitive equilibrium. The approach increases the applicability of spatial equilibrium models to markets that can be characterized as reaction-function oligopolies/oligopsonies, and even offers promise for formulating conventional competitive spatial equilibrium models. The methodology is relatively easy to implement and, in terms of computer time, quite efficient.

Another contribution of the paper lies in testing three imperfect competition models of the international wheat market, models that have received considerable attention in the literature. While the
three models (duopoly, triopoly and duopsony) were suggested by McCalla (1966), Alaouze et al (1978) and Carter and Schmitz (1979), respectively, these authors do not necessarily assume Cournot-Nash price discriminating behavior. Nevertheless, we were able to demonstrate that the duopsony conduct assumption is a very poor explainer of trade. The duopoly and triopoly models performed considerably better, with the duopoly model forecasts being slightly closer to the actual values than the triopoly model.
FOOTNOTES

1 Department of Economics, University of Illinois, and Economics Group, Los Alamos National Laboratory, respectively. Work supported by U.S. Department of Energy and an Arnold O. Beckman Award. Research assistance from Gang Yi and comments from Maury Bredahl, Bill Kost, Lars Mathiesen, Susan Offutt and Phil Paarlberg have been appreciated.

2 In fact, by imposing exogeneously specified government policies on competitive equilibrium models, equilibria can be brought into better agreement with actual trade patterns.

3 There are several possible reasons for this, including the variety of behavioral models of imperfect competition, each resulting in a different equilibrium level of trade (see Sarris and Schmitz). Further, there have been no generally available computational methods for computing spatial equilibrium in imperfect markets.

4 There are two classes of models commonly referred to as spatial equilibrium models. True spatial equilibrium models assume that the price difference for a commodity at two different locations only need be exactly equal to transport cost if trade occurs between the two locations. Equilibrium prices and quantities in such models can be determined by a variety of methods although mathematical programming is most frequently used (following methods suggested by Samuelson and developed by Takayama and Judge). In the second class of spatial models, the assumption is made that prices at various locations are related in some fixed manner to a numeraire price. Such models have the advantage that they are more easily estimated by simultaneous equation methods but the fixed structure of prices is usually only an approximation to a true competitive price equilibrium.

5 One might think there is an inconsistency between perfect competition and trade policies or barriers. However, as long as there are many price taking producers and consumers, free entry and exit and trade barriers are considered totally exogenous, then the model of perfect competition is still appropriate.

6 Brock and Magee actually outline the process in the United States whereby an industrial group can induce the government to impose trade policies that are in the group's interest.

7 It should be noted that the duopoly and triopoly models of the wheat market suggested by McCalla and Alouze et al. are really cooperative oligopolies and thus basically cartels. In their models they assume an agreement exists among the oligopolists regarding pricing and sharing of demand. Carter and Schmitz are not very specific about the operation of a wheat duopsony although they too suggest some sort of cooperative arrangement.
It is generally difficult to compute market equilibria in non-competitive situations. Recently, Spence has shown how the computation of equilibrium in some monopolisitically competitive markets can be reduced to the maximization of a single function. Murphy et al. have proposed an iterative method for computing a Nash equilibrium for an oligopoly by way of a sequence of mathematical programs. It has been known for some time that equilibrium in the case of a pure monopoly (or monopsony) can be determined by maximizing producer (or consumer) surplus (Takayama and Judge).

The idea of using a nonlinear complementarity algorithm to find an equilibrium in an imperfectly competitive market originated in discussions between Lars Mathiesen and the first author. Mathiesen was actually involved in the early stages of formulating the structure of the model presented here and subsequently went on to develop his own model of international steel trade (Mathiesen, 1983). His contributions are appreciated.

The principal algorithm for solving the linear complementarity problem is due to Lemke and has been implemented by many including Tomlin. A number of algorithms have been proposed for solving the nonlinear complementarity problem (e.g., Mathiesen 1982). It should be noted that in general one is not assured of a unique solution to the complementarity problem.

Bresnahan's consistent conjectures equilibrium and limit pricing equilibria involve conjectural variations that are themselves variables and thus endogenous to the problem.

Note that we have assumed $\partial P_k/\partial q_{ij} = 0$ for $j \neq k$, which means that by changing shipments to one consumer a producer cannot affect another consumer's price.

As in equation (6), here we assume that $\partial c_k/\partial q_{ij} = 0$ for $i \neq k$. The interpretation is that cutbacks in purchaser from one producer cannot affect the price of any other producer.

The transport costs which were used by Shei and Thompson were not readily available. However, the model resulting from the use of the transport costs from Sharples closely approximated the results reported by Shei and Thompson.

Thus the resulting imperfectly competitive model is actually a linear complementarity problem, easier to solve than the general nonlinear complementarity problem.

The duopoly, triopoly and duopsony cases are examined because they have been suggested by the other authors mentioned earlier in the paper. However, these authors do not necessarily assume Cournot-Nash behavior. Thus our models may not correspond exactly to theirs.

The resulting models are quite small, with solution time of up to 20 CPU seconds on a DEC VAX-11/780 computer (for the triopoly case).
Teigen assumes that, for a producer-consumer pair, the actual trade \( a_{ij} \) and predicted \( p_{ij} \) are sample pairs from a bivariate distribution and then calculates the Pearson correlation between \( a_{ij} \) and \( p_{ij} \). However, the Pearson correlation coefficient is not a distribution-free statistic and thus is not applicable for testing the relationship between \( a_{ij} \) and \( p_{ij} \). The Spearman rank correlation coefficient is distribution free and thus corrects this problem.

Trade share matrices for the duopsony and triopoly cases are available upon request from the authors.

The bias proportion will be close to zero due to the fact that elements of the share matrices sum to one.

Let \( A \) and \( P \) be the actual and predicted trace matrices, respectively, with elements \( a_{ij} \) and \( p_{ij} \). Assume each \((a_{ij}, p_{ij})\) pair is independent (a strong assumption in our case). Let \( \bar{a} \) and \( \bar{p} \) be the mean values of \( a_{ij} \) and \( p_{ij} \) computed from the sample. Given the relationship \( a_{ij} - \bar{a} = \alpha + \beta (p_{ij} - \bar{p}) + \varepsilon_{ij} \), we can estimate \( \alpha \) and \( \beta \). Since \( \alpha = 0 \) and \( \bar{a} = \bar{p} \), we wish to test the null hypothesis that \( \beta = 1 \). To do this, we let \( u_{ij} = a_{ij} - p_{ij} \) and test the extent of correlation between \( u_{ij} \) and \( p_{ij} \) using the Spearman rank correlation coefficient \( \rho \). For our case of a sample of size 45, the null hypothesis can be rejected if \( |\rho| > .248 \) (at the 90% level) or \( |\rho| > .295 \) (at the 95% level).
REFERENCES


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Source: Shei and Thompson
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Total Exports (10^6 tonnes) | Export Price (US$/tonne)
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TABLE IV: COMPARISON OF TRADE MATRICES FOR SIMULATED MARKET CONDUCT WITH ACTUAL TRADE

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<td>Spearman Rank Correlation Coefficient&lt;sup&gt;b&lt;/sup&gt;</td>
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<td>-0.242</td>
<td>-0.292</td>
<td>-0.407</td>
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</table>

<sup>a</sup>The closer to zero, the better.

<sup>b</sup>Reject at 90% (95%) level the null hypothesis that the model predicts trade if the absolute value of the coefficient exceeds 0.248 (0.295).