An Evolutionary Analysis of Product-Preference Structure: Toward Managerial Control

Zvi Ritz
D. Sudharshan

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign
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Zvi Ritz, Professor
Department of Business Administration

D. Sudharshan, Professor
Department of Business Administration
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by

Zvi Ritz
and
D. Sudharshan

Department of Business Administration
University of Illinois at Urbana-Champaign

Abstract

An important managerial problem is the choice of the optimal new product concept for introduction into a market. This depends on the positions of existing and expected new products as well as the consumer preference structure.

In this paper we develop a number of analytical models, each based on different behavioral assumptions, to describe the dynamic interactions between product positions and consumer preference structures. These models are then used to determine the optimal new product position.

1. Introduction

An important managerial problem is the choice of the optimal new product position for a market.

The main purpose of this article is to introduce and demonstrate the potential of analytical models of the dynamic interactions between product positions and consumer preference structures to aid in this choice.

In recent years a number of analytical models useful for generating new product positions have been developed in which products are represented by point locations in a multi-attribute perceptual space. Customers are locatable in the same product space by their most preferred (ideal) attribute combinations. Relative liking by any customer for the products is represented in these models, by a multi-attribute (conjoint) model measuring "proximity" of each existing product to that consumer's ideal product. Each consumer is presumed to choose among products located
closest to its ideal. (See Shocker and Srinivasan (1979) and May, Shocker and Sudharshan (1981) for a review of such approaches. See Green and Srinivasan (1978) for a review of conjoint measurement, and Cattin and Wittnik (1982) for a review of the managerial applications of conjoint analysis. See Sudharshan, May, and Shocker (1986) for a comparison of analytical new product positioning algorithms.)

In these models, optimal new product positions are chosen given a fixed and static distribution of consumer preferences. This may be a reasonable assumption for mature, stable markets. Recent empirical evidence, however, indicates that consumer preferences shift with the introduction of a new product (see Huber and Puto (1983) and Ratneshwar, Shocker and Stewart (1986)). This highlights the need for understanding the changes in consumer preferences.

Given that there is often a substantial time lag between choice of the new product design and the time it is introduced into a market, and given that products exist in the market for a reasonably long period, it is imperative to be able to predict the shifts in consumer preferences.

Four models for incorporating product position-preference interactions are discussed in this article. Both products and consumers are represented in these models as points in an attribute product space, and product attraction and consumer resistance to change are modeled as forces influencing each consumer's location. These models incorporate increasingly sophisticated assumptions regarding consumer behavior.

Given the pioneering and preliminary nature of this article, the models discussed here are limited to one dimensional attribute spaces. Although the limitations of one dimensional models are clear, nevertheless, substantial insight into the subject matter can be gained even from such simple cases, which is perhaps the reason that one
dimensional attribute space models are used quite often in the literature. (See Lane (1980), Hauser and Shugan (1983) and Kumar and Sudharshan (1986)).

2. The Models

In all of the following models a single attribute space is assumed with a known distribution of consumers in this space at time t=0. Let R be the random variable with a known probability density function \( f_R(r) \), representing this distribution. Thus for example, the portion of the population which is between any two given points \( r_0 \) and \( r_1 \) at time t=0, can be found by calculating \( P[r_0 \leq R \leq r_1] \).

Since the distribution of the consumer population at time \( t > 0 \) will usually differ from the distribution of \( R \), let \( X(t) \) be the random variable representing this distribution.

Characterizing \( X(t) \) amounts to understanding and predicting consumer preference shifts. These characterizations will therefore serve as the cornerstones for developing optimal product positioning strategies.

2.1 Model 1: Constant Rate of Change

The first model is based on the behavioral assumption that every consumer's preference location continues to change at a constant rate and direction upon exposure to a new product or upon an awareness of an existing one, irrespective of the original location of the consumer and the position of this product. For example, consider the portability attribute of micro-computers. The introduction of the first "portable" micro-computer led many consumers to increasingly prefer more portability regardless of the degree of portability of this first computer.
Let \( r \) be the original position (at \( t=0 \)) in the attribute space of a consumer who changes her preferences at a constant rate of \(-c\), and let \( x(t) \) stand for her position at time \( t \geq 0 \), then the above amounts to assuming:

\[
\dot{x} = -c \quad \text{and} \quad x(0) = r . \tag{1.1}
\]

As a result:

\[
x(t) = r - ct \quad \text{for} \quad t \geq 0 . \tag{1.2}
\]

Clearly, consumers may react differently to the same product. Some may be attracted to it at a faster rate than others, and some may even be repulsed by it. Let \( C \) be a random variable with a known probability density function \( f_C(c) \) representing the possible reactions of a typical consumer to the product. Thus:

Result 1:

If consumers independently change their respective preferences according to rule (1.1), and \( R \) and \( C \) are independent random variables, then the distribution of consumer preferences at time \( t \) is represented by the random variable \( X(t) = R - Ct \) with probability density function \( f_X(x;t) \) given by:

\[
f_X(x;t) = \int_{-\infty}^{\infty} f_R(x+ct) \cdot f_C(c) \, dc , \tag{1.3}
\]

or alternatively

\[
f_X(x;t) = \frac{1}{t} \int_{-\infty}^{\infty} f_C(\frac{x}{t}) \cdot f_R(r) \, dr , \tag{1.4}
\]

and expected value and variance:

\[
E[X(t)] = E[R] - E[C]t , \tag{1.5}
\]

\[
\text{Var}(X) = \text{Var}(R) + \text{Var}(C) \cdot t^2 . \tag{1.6}
\]
Notice that even a cursory analysis of the model will indicate that as time progresses:

a. the longer it takes to introduce a new competing product, the farther it should be positioned in the direction of the average trend. This is because the bulk of the population moves in the direction of the average trend (or average change rate) and it becomes less concentrated, and

b. after a certain point in time a firm may be better off in introducing a number of new products rather than a single new product. This is due to increasing diversity in consumer preferences. Namely, product proliferation and niching become desirable, which is an observed practice with many products (cereals, television programs, etc.).

Example 1.

Assume that a product is positioned at the origin, \( x_0 = 0 \), and the consumer preferences are uniformly distributed between 0 and \( R_0 \),

\[
I_{[0,R_0]}(r) = \frac{1}{R_0} (r) \quad \text{where } I_{[a,b]}(x) \text{ equals 1 for } a \leq x \leq b \text{ and 0 otherwise).}
\]

Let also assume that consumers can be either attracted to or repulsed by the product at a constant rate which is uniformly distributed between \(-C_1 \) and \( C_2 \) (\( C_1 \geq 0, C_2 \geq 0 \)),

\[
f_c(c) = \frac{1}{C_1 + C_2} I_{[-C_1,C_2]}(c).
\]

Then:

2.1. If \( t \leq \frac{R_0}{C_1 + C_2} \), then

\[
f_X(x,t) = \begin{cases} 
\frac{x + C_2 t}{R_0(C_1 + C_2)t} & \text{for } -C_2 t \leq x \leq C_1 t \\
\frac{1}{R_0} & \text{for } C_1 t < x \leq R_0 - C_2 t \\
\frac{R_0 + C_1 t - x}{R_0(C_1 + C_2)t} & \text{for } R_0 - C_2 t < x \leq R_0 + C_1 t \\
0 & \text{otherwise}
\end{cases}
\]
2.2. If \( t > \frac{R_0}{C_1+C_2} \), then

\[
f_x(x;t) = \begin{cases} 
\frac{x+C_2t}{R_0(C_1+C_2)t} & \text{for } -C_2t \leq x \leq R_0-C_2t \\
\frac{1}{(C_1+C_2)t} & \text{for } R_0-C_2t < x \leq C_1t \\
\frac{R_0+C_1t-x}{R_0(C_1+C_2)t} & \text{for } C_1t < x \leq R_0+C_1t \\
0 & \text{otherwise}
\end{cases}
\]

Model 2:

In the first model the actual position of the product wasn’t essential. In the following, we assume that the direction of the attraction depends on the relative position of the consumer to the existing product. For example, a political candidate may cause a shift in the political convictions of both voters to his right and his left in the political spectrum.

Again consider a single product positioned at \( X_0 \) in a single attribute space with a known distribution of the consumers in this space at time \( t=0 \). Without loss of generality assume that \( X_0 = 0 \).

Assume that at time \( t=0 \), a consumer positioned at point \( r \), starts to change his preference at a constant rate according to the following rule:

\[
\dot{x} = \begin{cases} 
-C & \text{for } r \leq 0 \\
0 & \text{for } r > 0
\end{cases}
\]

Then

\[
x(t) = (r + ct) I_{(-\infty,0)}(r) + (r - ct) I_{(0,\infty)}(r)
\]

(2.2)

where \( I_{[a,b]}(x) \) equals 1 for \( a \leq x \leq b \) and 0 otherwise.

Let \( C \) be the random variable representing the rate at which a consumer may change her or his preferences. Then:
Result 2.

If consumers independently change their respective preferences according to rule (2.1), and R and C are independent random variables, then the distribution of consumer preferences at time t is represented by the random variable:

\[ X(t) = R + Ct \mathbb{I}_{(-\infty,0]}(R) - Ct \mathbb{I}_{(0,\infty)}(R) \]

with probability density function \( f_x(x,t) \) given by:

\[
 f_x(x,t) = \frac{1}{t} \int_{-\infty}^{0} f_c\left(\frac{x-r}{t}\right) f_R(r) dr + \frac{1}{t} \int_{0}^{\infty} f_c\left(\frac{r-x}{t}\right) f_R(r) dr
\]

and expected value:

\[
 E[X(t)] = E[R] + f_R(0) E[C] t - (1-f_R(0)) E[C] t,
\]

where \( F_R(r) \) is the cumulative probability distribution function of R.

Example 2.

Suppose the product is positioned at the origin, \( x_0=0 \); the consumer preferences are uniformly distributed between \(-R_0\) and \( R_0\). Namely

\[
 f_R(r) = \frac{1}{2R_0} \mathbb{I}_{[-R_0,R_0]}(r)
\]

and the consumers are attracted to the product at a rate which is uniformly distributed between 0 and \( C_0 \),

\[
 f_c(c) = \frac{1}{C_0} \mathbb{I}_{[0,C_0]}(c).
\]

Then the probability density function of \( X(t) \) is given by:

2.1 If \( t \leq R_0/2C_0 \) then:

\[
 f_x(x,t) = \begin{cases} 
 \frac{x+R_0}{2R_0 C_0 t} & \text{for } -R_0 \leq x \leq -R_0 + C_0 t \\
 \frac{1}{2R_0} & \text{for } -R_0 + C_0 t < x \leq -C_0 t \\
 \frac{x+2C_0 t}{2R_0 C_0 t} & \text{for } -C_0 t < x \leq 0 \\
 \frac{1}{2R_0 C_0 t^2} & \text{for } 0 < x \leq C_0 t \\
 \frac{R_0 - x}{2R_0 C_0 t} & \text{for } C_0 t < x \leq R_0 - C_0 t \\
 \frac{R_0 - x}{2R_0 C_0 t} & \text{for } R_0 - C_0 t < x \leq R_0 \\
 0 & \text{otherwise}
\end{cases}
\]
2.2 If \( \frac{R_0}{2C_0} < t \leq \frac{R_0}{C_0} \), then

\[
f_x(x;t) = \begin{cases} 
\frac{x+R_0}{2R_0 C_0 t} & \text{for } -R_0 \leq x \leq -C_0t \\
\frac{R_0 C_0 t + 2x}{2R_0 C_0 t} & \text{for } -C_0t < x \leq -R_0 + C_0t \\
\frac{2R_0 C_0 t + x}{2R_0 C_0 t} & \text{for } -R_0 + C_0t < x \leq 0 \\
\frac{2R_0 C_0 t}{2R_0 C_0 t} & \text{for } 0 < x \leq R_0 - C_0t \\
\frac{R_0 + C_0 t - 2x}{2R_0 C_0 t} & \text{for } R_0 - C_0t < x \leq C_0t \\
\frac{R_0 - x}{2R_0 C_0 t} & \text{for } C_0t < x \leq R_0 \\
0 & \text{otherwise}
\end{cases}
\]

2.3 If \( \frac{R_0}{C_0} < t \leq \frac{2R_0}{C_0} \), then

\[
f_x(x;t) = \begin{cases} 
\frac{C_0 t + x}{2R_0 C_0 t} & \text{for } -C_0t \leq x \leq -R_0 t \\
\frac{R_0 C_0 t + 2x}{2R_0 C_0 t} & \text{for } -R_0 < x \leq -C_0t + R_0 \\
\frac{2R_0 C_0 t}{2R_0 C_0 t} & \text{for } -C_0t + R_0 < x \leq R_0 \\
\frac{C_0 t - x}{2R_0 C_0 t} & \text{for } R_0 < x \leq C_0t \\
0 & \text{otherwise}
\end{cases}
\]

2.4 If \( \frac{2R_0}{C_0} < t \) then

\[
f_x(x;t) = \begin{cases} 
\frac{x + C_0 t}{2R_0 C_0 t} & \text{for } -C_0t \leq x \leq -C_0t + R_0 \\
\frac{1}{2C_0 t} & \text{for } -C_0t + R_0 < x \leq -R_0 \\
\frac{2R_0 C_0 t - x}{2R_0 C_0 t} & \text{for } -R_0 < x \leq 0 \\
\frac{2R_0 C_0 t}{2R_0 C_0 t} & \text{for } 0 < x \leq R_0 \\
\frac{C_0 t - R_0}{2R_0 C_0 t} & \text{for } R_0 < x \leq C_0t - R_0 \\
\frac{C_0 t - x}{2R_0 C_0 t} & \text{for } C_0t - R_0 < x \leq C_0t \\
0 & \text{otherwise}
\end{cases}
\]
Model 3:

In the second model the assumption was that the position of the product influences the direction but not the rate in which the consumer preferences are changing.

In the following we add the assumption that the rate at which a consumer changes her preferences also depends on her "distance" from the existing product. When aerobics was first introduced, the initial participants were those already engaging in considerable physical exercise. This is a behavior pattern captured by this assumption.

Let \( X_0 \) be the position of the product in the attribute space and without loss of generality assume \( X_0 = 0 \). Assume that a consumer positioned at point \( r \), changes her preferences at a constant rate of \(-ch(r)\) where \( c \) is a constant and \( h(r) \) is a continuous, decreasing function of the absolute value of \( r \). Then:

\[
\dot{x} = -c h(r) \quad \text{and} \quad x(0) = r .
\]

(3.1)

and at time \( t \), this consumer will be positioned at

\[
x(t) = r - ch(r)t \quad \text{for} \ t \geq 0 .
\]

(3.2)

Let \( C \) be the random variable representing the distribution of the constant factor \( c \). Then:

Result 3.

If consumers independently change their respective preferences according to rule (3.1), and \( R \) and \( C \) are independent random variables, then the distribution of consumer preferences at time \( t \) is represented by the random variable: \( X(t) = R - C h(R)t \)

with the probability density function \( f_X(x;t) \) given by:

\[
f_X(x;t) = \int_{-\infty}^{\infty} \frac{1}{h(r)t} f_c\left(\frac{r-X}{h(r)t}\right) f_R(r)dr .
\]

(3.3)

and expected value of \( \text{E}[X(T)] = \text{E}[R] - \text{E}[C]E[h(R)]t \)
Example 3.
Let $R$ be uniformly distributed between $[0,R_0]$, $C$ uniformly distributed between $[0,C_0]$ and
\[
h(r) = \begin{cases} 
\frac{1}{r} & \text{for } 0 < r \leq r_0, \\
\frac{1}{r_0} & \text{for } 0 \leq r \leq r_0,
\end{cases}
\]
where $r_0 < R_0$.

We compute $f_x(x;t)$ only for $t \leq \frac{r_0^2}{C_0}$ and $t \leq \frac{(R_0-r_0)R_0}{C_0}$,
\[
f_{x}(x) = \begin{cases} 
\frac{r_0}{R_0C_0t}(x + \frac{C_0t}{r_0}) & \text{for } -\frac{C_0t}{r_0} \leq x \leq 0, \\
\frac{1}{R_0} & \text{for } 0 < x \leq r_0 - \frac{C_0t}{r_0}, \\
\frac{r_0}{R_0C_0t}[(r_0-x) + \frac{1}{2}r_0 - \frac{x^2+2C_0t+x\sqrt{x^2+4C_0t}}{2} - r_0^2)] & \text{for } r_0 - \frac{C_0t}{r_0} < x \leq r_0, \\
\frac{2C_0t+x\sqrt{x^2+4C_0t}}{4R_0C_0t} & \text{for } r_0 < x \leq R_0 - \frac{C_0t}{R_0}, \\
\frac{R_0^2-x^2}{2R_0C_0t} & \text{for } R_0 - \frac{C_0t}{R_0} < x \leq R_0.
\end{cases}
\]

Model 4:
In the last model we incorporate the assumption that the farther the consumers move away from their original position the more they develop a resistance to additional changes.

Thus, if $x(t)$ represents the position at time $t$ of a consumer who was originally at point $r$ and started to change her preferences at a constant rate of $c$, then we assume that:
\[
\dot{x} = k(r-x), \quad (k>0), \quad \dot{x}(0) = -c \quad \text{and } \quad x(0) = r,
\]
which leads to:
\[
x(t) = r - \frac{c}{\sqrt{k}} \sin(\sqrt{k}t),
\]
(4.2)
In addition to the previous assumptions on R and C, we also assume that k is a universal constant common to all consumers (an assumption made because of technical reasons). Then the distribution of the consumer population at time t is:

Result 4.

If consumers independently change their respective preferences according to rule (4.1), and R and C are independent random variables, then the distribution of consumers preferences at time t is represented by the random variable

\[ X(t) = R - \frac{C}{\sqrt{k}} \sin(\sqrt{k} t), \]

with the probability density function \( f_x(x; t) \) given by:

\[ f_x(x; t) = \int_{-\infty}^{\infty} f_R(x + \frac{c}{\sqrt{k}} \sin(\sqrt{k} t)) f_c(c) \, dc \quad (4.3) \]

and expected value and variance of:

\[ E[X(t)] = E[R] - \frac{E[C]}{\sqrt{k}} \sin(\sqrt{k} t), \quad \text{and} \]

\[ \text{Var}(X(t)) = \text{Var}(R) + \frac{\text{Var}(C)}{k} \sin^2(\sqrt{k} t). \quad (4.4) \]

Corollary 5.

For any \( t \geq 0 \),

1. \( E[R] - E[C] \leq E[X(t)] \leq E[R] + E[C] \),

2. \( \text{Var}(R) \leq \text{Var}(X(t)) \leq \text{Var}(R) + \text{Var}(C) \). \quad (4.5)

3. Applications

Both the approach and the models developed in this article can assist in several decisions related to product management.

In the following we discuss the application of the above models to the decision problem of choosing a product position for a market. (Note that because of the introductory nature of this work, these examples do not take into consideration the effect of competing products.)
Product Positioning

The decision of where to position a new product is dependent on how consumers choose among competing products, the evolution of consumer preferences and on managerial objectives.

For illustration purposes we assume that any consumer will choose the product "closest" to her location, provided that this distance doesn't exceed a pre-specified maximum. (This rule has its roots in Hotelling (1929).) This implies that each product will be considered only by consumers located within a certain maximum distance from it, termed the "range of attraction".

We consider product positioning under three different managerial objectives: first, under the assumption that the objective of management is to maximize the number of consumers within the range of attraction (NCRA) at a pre-specified time, a special case of which is the time of introduction; second, under the assumption that the objective is to maximize the average NCRA over the life of the product; and last, under the assumption that the objective is to maximize the discounted NCRA during the life of the product.

Let \( d \) stand for the pre-specified "range of attraction", and let \( y \) be the new product position. Then \( m(y,d;t) \) the NCRA at time \( t \) (where \( t=0 \) is the introduction time) is:

\[
m(y,d;t) = \int_{y-d}^{y+d} f_X(x;t) \, dx .
\] (5.1)

Each of the above objectives translates into one of the following problems.
Problem 1:
Let management’s objective be to maximize the NCRA at time $t_0$, which is either the new-product introduction time or any other pre-specified date.

Then management’s problem is to find a position $y_0$ such that:

$$m(y_0,d;t_0) = \text{Max } m(y,d;t_0) \quad (5.2)$$

$$-\infty < y < \infty$$

Problem 2:
Assume that $\theta$, the (expected) life of the product is known, and that management’s objective is to maximize the average NCRA over the life of the product.

Let:

$$m_1(y,d;\theta) = \frac{1}{\theta} \int_0^\theta m(y,d;t) \, dt \quad (5.3)$$

Then management problem is to find a position $w_0$ such that:

$$m_1(w_0,d;\theta) = \text{Max } m_1(y,d;\theta) \quad (5.4)$$

$$-\infty < y < \infty$$

Problem 3:
Assume that $\theta$, the expected life of the new product is known, and that management’s objective is to maximize the discounted NCRA during the life of the product.

Let $k$ be the discount coefficient and:

$$m_2(y,d;\theta) = \int_0^\theta e^{-kt} m(y,d;t) \, dt \quad (5.5)$$

Then management’s problem is to find a position $z_0$ such that:

$$m_2(z_0,d;\theta) = \text{Max } m_2(y,d;\theta) \quad (5.6)$$

$$-\infty < y < \infty$$

(It should be noticed that the above problems can be stated with additional (technological) constraints on $y$.)
Since the three functions \( m, m_1, \) and \( m_2 \) defined above are continuous and bounded functions they all have maximum points.

For illustrative purposes let us consider the following simple example.

**Example 4.**

Suppose, in the first model, the consumer preferences at time \( t=0 \) are uniformly distributed between 0 and \( R_0 \),

\[
f_R(r) = \frac{1}{R_0} I_{[0,R_0]}(r)
\]

and the consumers are attracted to the product at a rate which is uniformly distributed between 0 and \( C_0 \), \( f_c(c) = \frac{1}{C_0} I_{[0,C_0]}(c) \).

Then the probability density function of \( X(t) \) is given by:

\[
f_X(x;t) = \begin{cases} 
\frac{x+C_0 t}{R_0 C_0 t} & \text{for } -C_0 t \leq x \leq 0 \\
\frac{1}{R_0} & \text{for } 0 < x \leq R_0 - C_0 t \\
\frac{R_0 - x}{R_0 C_0 t} & \text{for } R_0 - C_0 t < x \leq R_0 . 
\end{cases}
\]

Let \( d = \frac{R_0}{8} \) and \( \theta = \frac{R_0}{2C_0} \). Then it is not difficult to demonstrate that:

a. for \( t_0 \leq \theta \), to achieve the first objective, the new product can be positioned at any point \( y_0 \) such that

\[
\frac{R_0}{8} \leq y_0 \leq \frac{7R_0}{8} - C_0 t_0 ;
\]

b. to achieve the second objective the new product can be positioned at any point \( w_0 \) such that \( \frac{R_0}{8} \leq w_0 \leq \frac{3R_0}{8} \) and similarly,

c. to achieve the third objective the new product can be positioned at any point \( z_0 \) such that \( \frac{R_0}{8} \leq z_0 \leq \frac{3R_0}{8} \).

4. Conclusions

In this article, taking a novel approach, we have introduced and analysed four analytical models representing the evolution of consumer
preferences as a function of the position of a pioneering product. Each model embodies a different conceptualization of consumer preference change tendencies and each is likely to be relevant to a different set of markets. We also use these models in determining the position of a new product based on a variety of managerial objectives, thus demonstrating their potential usefulness.

The models developed are parsimonious, yet embody the essence required to realistically portray the dynamics of consumer preference shifts with product entries. Clearly this is just the beginning of a new approach to this important area.
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