Strategic Groups and Competitive Equilibrium:
A Game Theoretic Approach

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Abstract

The study of strategic groups in the industrial organization and strategic management literature has mainly focused on the identification of strategic groups and its relationship with performance. In this paper, questions regarding the existence of strategic groups, limits on the number of strategic groups and measures of inter-group and intra-group relative positions are raised.

A game theoretic model of a competitive industry addresses these questions by examining the long-run structure of the industry through the concept of equilibrium. Using structural stability concepts, a sharp upper bound, akin to a benchmark, for the number of strategic groups is derived. In addition, a number of indices are also proposed to measure the distribution of strategic groups and characterize the distribution of firms within a particular strategic group.
1.0 **Introduction**

Industrial organization researchers have stressed the importance of market structure elements in explaining firm performance [Bain (1968), Caves (1977), Scherer (1980)]. Conduct was ignored partly because there were difficulties in obtaining information about firm conduct but also because it was argued that market structure constrains firms' discretionary conduct. Thus, structure causes all firms in an industry to behave alike and, therefore, suggests that firms are homogeneous except for their size.

Hunt (1972) relaxed the assumption of homogeneity and showed that in the "white goods" industry, the firms differed strongly in terms of their strategic behavior. Firms which were similar in their strategic behavior were clustered into groups which he called strategic groups.

Following Hunt, numerous studies have been performed on the subject of strategic groups [Newman (1973, 1978), Porter (1973, 1979, 1980), Hatten (1974), Patton (1976), Oster (1982), Primeaux (1983a, b), Fiegenbaum and Primeaux (1983), Hergert (1983)]. All these studies attempted to identify strategic groups in various industries. The main differences between them arise from the choice of methods used to identify and define strategic groups (see also McGee (1982), McGee and Thomas (1984)). It is obvious from these studies that there is no one way of defining strategic groups. In fact, the choice of the main strategic dimensions for defining strategic groups should be strongly influenced by the characteristics of the particular industry being studied and the purposes of the study.
This study considers the long-run structure of markets in a competitive environment and makes use of the concept of equilibrium, as a strategic benchmark, to resolve certain specific issues about strategic groups. They are:

1) If there are strategic groups within industries, how many will there typically be?

2) How can one measure the strategic distances between strategic groups?

3) What is the relative position of a firm within a strategic group?

This more focused analysis of strategic groups will be useful to firm strategists since it predicts potential end-game positions (Porter (1980), Harrigan (1980)). In addition, it can identify those strategies which will be dominated in the marketplace. Thus, a firm will be able to consider the gap between its current position and possible equilibrium positions and decide whether or not to make some adjustments to improve its strategic position [as in Karnani (1982)].

Such detailed strategic analysis can also develop an understanding of the fundamental forces affecting market structure. Since strategic groups are a structural element of an industry, just like other elements such as concentration ratios, barriers to entry and product differentiation, the equilibrium characteristics of strategic groups' structure can provide insights about competitive positioning and industry evolution. From a public policy perspective, it can help policymakers (e.g., the FTC Anti-Trust Division) to decide whether or not to intervene in the dynamics of a market. For example, a current market monopoly situation may be analysed as leading to many strategic groups in the long run.
The organization of this paper is as follows: in Section 2, a theoretical model, based on a game theoretic approach, will be developed to describe the industry and its strategic groups. In Section 3, insights about specific grouping issues will be presented. For example, using structural stability analysis, sharp bounds on the number of strategic groups can be determined. In Section 4, an example will illustrate the main ideas of this paper together with a discussion on the implications of the results in this example. Section 5 will summarize and bring out the overall contribution of this study.

2.0 The Model

The basic industry model assumes that there are a number of firms, each of which produces a single product for the market. Each firm's product can be differentiated, but it remains a close substitute for the products of the other firms. For example, in the micro-computer industry, there are a variety of competitors, such as IBM, Apple, NEC, Zenith, Wang and Burroughs with closely substitutable products. The firms act as monopolists, as far as their customers are concerned, and set prices; on the other hand, they compete amongst themselves for the consumer demand since customers can easily switch from buying one firm's product to another's. This economic market model was posited by Chamberlin to capture the essence of product differentiation; this market form falls in between that of pure competition and pure monopoly, and is aptly named monopolistic competition (Mansfield, 1974). When the number of firms in such a market structure becomes small, it is often referred to as an oligopoly.
From an economics viewpoint, this market should be represented by complete descriptions of both the consumer side and the producer side. It is common in the strategic management literature to concentrate on the firm, its aspirations, the variables under its control and those not under its control. We will follow a similar modeling approach, but will incorporate the consumer side by picking controllable and non-controllable variables that appear to affect the customer's consumption decision. For example, the customer may base his purchase on information received about the industry average price, average quality, etc. and these will figure in the demand or sales function of each firm along with the individual firm's price and quality. This provides a more complete approach to modeling the industry than those currently used. Coincidentally, the current literature on strategic groups does consider aggregate industry variables, such as industry advertising, R&D, direct costs, etc., the underlying motivation being to describe the strategic posture of each firm [Schendel and Patton (1978), Galbraith and Schendel (1983)].

In order to model the strategic behavior of individual firms, certain basic elements will be used. They include goals/objectives, possible resource decisions and environmental constraints faced by the firm. Our model assumes that there is a single objective for each firm, namely utility maximization. While it is true that there could be a number of goals, often conflicting, which motivate firms, the contention here is that cost factors can be attributed to all the key goals and a net benefit or utility function could be derived for each firm. For example, the trade-off between market share maximization and return on
investment maximization can be captured by structuring the utility function in an appropriate manner.

The firms' actions or resource decisions involve several controllable variables. These can be both strategic and operational variables. They may include those that pertain to a customer's consumption decision, for example, price, advertising and service as well as those that are a firm's internal variables, such as R&D, raw materials cost and unit manufacturing cost.

The environmental constraints encompass those variables which the individual firm cannot control. These non-controllable variables will be chosen to describe the broad competitive environment of the industry. They will also be chosen to represent those informative variables which are available to the consumer in making his purchasing decision, such as industry average price and average quality.

Thus, the model is slightly more general than Galbraith and Schendel's (1983) formulation. In their performance model, some of the independent variables were structured relative to the industry aggregate value (e.g., relative advertising, which is computed by dividing a firm's advertising level by the industry aggregate advertising level). In this formulation, the utility function includes separate independent variables for the numerator and denominator of each relative value measure (e.g., firm advertising level (controllable) and industry average advertising level (uncontrollable) rather than the aggregate level).

As an example of such a model, let the controllable strategic variables for each firm be research and development (R&D), capital investment
(CI), advertising (AD), price (P), and quality (Q). Let the non-controllable variables be the industry averages of research and development (AR&D), capital investment (ACI), advertising (AAD), price (AP) and quality (AQ). (See Galbraith and Schendel (1983) for more such variables.) Each firm will take these non-controllable variables as given and try to maximize its utility function:

\[ U(P_i, CI_i, AD_i, R&D_i, Q_i, AP, ACI, AAD, AR&D, AQ) \]

with respect to its controllable variables. In this model, each firm will be assumed to be identical in terms of its resources and cost structures. The function \( U \) is assumed to be continuously differentiable in all its variables but not necessarily quasi-concave in its controllable variables. An equilibrium for this market is a set of strategies for each firm such that the outcome of the competitive game is stable, i.e., in equilibrium, firm \( i \) cannot do any better than using its equilibrium strategy given that its competitors are playing their equilibrium strategies. This type of stability concept is called a Nash equilibrium in economics and game theory. Even though this game could have asymmetric equilibria when firms have different profits, the model formulation focusses initially on symmetric equilibria since, a priori, all firms have been assumed to be identical, i.e., the \( U \) function is the same for all firms and so are all the parameters of the function.

3.0 Structural Analysis

In this section, the model outlined in Section 2 will be used to develop insights about the following issues:
1) Can more than one strategic group exist within an industry when firms are modelled as being identical with respect to costs and other parameters?

2) What is the largest number of strategic groups which may exist in equilibrium when the industry matures?

3) Can strategic group concepts provide valuable insights about industry structure?

4) How can the internal structure within a strategic group be described?

3.1 The Existence of Strategic Groups

The issues associated with the existence of strategic groups are first examined. Given the model, it is apparent that various kinds of symmetric Nash equilibria may emerge. The symmetric equilibrium could be a pure strategy equilibrium in which case all firms are following the same strategy. Then, there is only one strategic group, consisting of all firms in this equilibrium, and the strategy, which they all follow, will also represent the industry average. On the other hand, the symmetric equilibrium can be expressed in terms of mixed strategies, which are essentially a probabilistic combination of pure strategies. However, it is not reasonable nor practicable to expect firms to behave as if they are tossing coins in making strategic decisions (and also constantly changing them) in the real world. Therefore, the interpretation of these mixed strategy equilibria will be that firms in the industry will split into different groups, with each group using a distinct strategy specified by the mixed equilibrium. The proportion of firms in each group will approximate the probability assessment assigned to the particular strategy used by the group in equilibrium. So the concept of Nash equilibrium can, in itself, give rise to a
variety of strategic structures. If the utility function is such that there is a symmetric pure strategy equilibrium, then only one homogeneous strategic group emerges. But if the utility function allows only mixed strategies in equilibrium, then the "invisible hand" of market interaction forces several strategic groups to emerge.

3.2 Strategic Groups: Stability and Equilibrium

In strategic analysis, it would be useful to predict the actual number of strategic groups which may occur in equilibrium in a given industry. In general, unless the exact functional form of the utility function and specific values of the parameters are known, no such prediction can be made. However, it is possible to predict an upper bound for the number of strategic groups which may exist in an industry if the number of uncontrollable variables in the utility function is known. The general result is that if there are k uncontrollable variables in the utility function, then there can be at most \((k+1)\) strategic groups in a "structurally stable" equilibrium (Kumar and Satterthwaite, 1983). The version of this result for the specific model presented in Section 2 is stated as follows:

**Instability Theorem:** If the dimension of the uncontrollable variable space \((AP, ACI, AAD, AR&D, AQ)\) is five, then no mixed symmetric Nash equilibrium with more than six strategies can be structurally stable.

and is proved in the appendix.

At this point, it is important to specify the difference between a structurally stable and a Nash equilibrium. The notion of Nash
equilibrium conveys a sense of stability of the following form: given the parameters of the model, once every firm is playing its equilibrium strategy, then no firm individually has any incentive to break away from this equilibrium. The idea of structural stability argues for yet another sense of stability, namely: given small shifts in the parameters of the model, the Nash equilibrium structure changes only slightly. This sort of stability ensures that small random fluctuations in the environment will not make firms change their strategies drastically.

This concept of structural stability is taken from the field of differential topology in mathematics [Guillemin and Pollack (1974), Chapters 1 and 2]. Its relevance to the model presented here is obvious since, in practice, the utility function, which is the fundamental building block, can only be estimated within some non-zero margin of error. Given small perturbations in the utility function, it is important to determine whether the equilibrium, in these perturbed models, is similar and close to that in the unperturbed model. If so, the equilibrium in the unperturbed model is said to be structurally stable.

To understand how structural stability is maintained, the Nash equilibrium structure should be examined. In the model, given the uncontrollable variables (AP, ACI, AAD, AR&D, AQ) each firm chooses values for its controllable variables (P₁, CI₁, AD₁, R&D₁, Q₁) so as to maximize its utility. Consider, for example, a particular Nash equilibrium in mixed strategies with the values of the uncontrollable variables of the equilibrium position being (AP*, ACI*, AAD*, AR&D*, AQ*) and two equilibrium strategies (aggregating to the averages
defined by the uncontrollable variables) given by \((P^*_j, CI^*_j, AD^*_j, R&D^*_j, AQ^*_j)\)

\(j = 1,2\). Due to their optimality, the utility associated with each of the strategies, given the equilibrium value of the uncontrollable variables, is exactly the same or, to put it another way, the difference in utility function values, at the equilibrium positions, is zero.

This equilibrium should be checked to see if it is structurally stable. First, perturb the utility function, structurally, by a very small amount so as to keep the values of the perturbed function, as well as its derivatives, close to the corresponding ones of the original function. Then, if the original equilibrium is structurally stable, the perturbed utility-difference function will assume a value equal to zero close to the original equilibrium specified by the uncontrollable variables \((AP^*, ACI^*, AAD^*, AR&D^*, AQ^*)\).

Further, to understand why there is a bound on the number of equilibrium strategies, consider the following analogy. Consider three straight lines, given by \(y_i = a_i + b_i x\), \(i = 1, 2, 3\), meeting at a point \(x^*\). It is quite obvious that there exist small perturbations such that these perturbed lines do not meet at all. Consider a reformulation of the above phenomenon: look at the difference function \((y_1-y_2, y_2-y_3)\) which is a function of \(x\) only. Then this function takes on a value of zero at \(x^*\) and intuition tells that this happens very rarely. Notice, on the other hand, that if there were only two straight lines, then the intersection \(x^*\) can be very stable. In this case, the difference function \((y_1-y_2)\), which is a function of \(x\), takes on a value of zero at \(x^*\) and small perturbations can result in a zero value close to \(x^*\). The difference between stability and instability lies in the dimensions of
the range and domain space of difference function, i.e., dimension of the range space [space of all values \((y_1-y_2, y_2-y_3)\) can take] = 2 in the case of three lines, 1 in the case of two lines, and the dimension of the domain of the difference function, i.e., dimension [space of all values \(x\) can take] = 1 in both cases. It is true that when the domain dimension is greater than or equal to the range dimension, the zero value could be structurally stable (but not necessarily so: consider a U-shaped curve just touching the x-axis—a perturbation of the curve pulling it up uniformly misses the x-axis and thus loses the zero). On the other hand, if the domain dimension is less than that of the range, then no zero value can be stable.

This intuition directly leads us to the instability result since the dimension of the domain space is 5 (= dimension [space of all values \((AP, ACI, AAD, AR&D, AQ)\) can take]) and the dimension of the range space is \((k-1)\) if there are \(k\) strategies, i.e., \((U_1-U_2, U_2-U_3, \ldots, U_{k-1}-U_k)\) where \(U_i\) represents the value of the utility function at the \(i\)th strategy. The result is that if \(5 < (k-1)\), then equilibrium cannot be stable, i.e., there cannot be more than six strategies in a structurally stable equilibrium.

3.3 Strategic Groups and Industry Structure

Given the existence of an upper bound on the number of strategic groups in a stable equilibrium, the structure of this industry can now be examined. Just like measures such as concentration ratios, several measures which try to describe the nature of the strategic group structure (which is stable in the long run) can be defined as follows:
1) The strategic concentration ratio SCR is defined as

\[ \text{SCR} = \frac{\text{number of strategic groups making up 80\% of market share}}{\text{total number of strategic groups}} \]

It measures the distribution of market share, in a similar manner to the concentration ratio in Industrial Organization literature. However, the major difference in calculation is that the groups are ordered in terms of the total market share for each group and the SCR is then computed—-the closer SCR is to 0, the more concentrated the dominant strategic groups are and SCR being closer to 1 implies market share being spread out among the strategic groups.

2) Following Hirschman (1964), the strategic comprehensive index, SCI, can be defined as

\[ \text{SCI} = 1 - \sum_{i=1}^{k} \text{MSG}_i^2 \]

where \( \text{MSG}_i \) is the market share of strategic group \( i \). The SCI measure is interpreted in the following manner. The smaller the SCI, the more concentrated the market share is between a few groups and a high SCI indicates a more dispersed industry.

3) The strategic distance matrix, SDM, can be defined as being the matrix of distances between each strategic group. For example, if there are three strategic groups, then

\[ \text{SDM} = \begin{bmatrix} 0 & d_{12} & d_{13} \\ d_{21} & 0 & d_{23} \\ d_{31} & d_{32} & 0 \end{bmatrix} \]

which is a symmetric matrix (i.e., \( d_{ij} = d_{ji} \)) and

\[ d_{ij} \equiv \text{distance from strategic group } i \text{ to group } j. \]
The \( d_{ij} \) can be specified in terms of a normalized Euclidean distance measure based on the values of the controllable variables at each of the strategic groups \( i \) and \( j \) and on the industry non-controllable variables. For example, let \((P_i, AD_i, R&D_i)\) \( i = 1, 2 \) be two groups' strategies and the uncontrollable values be \((AP, AAD, AR&D)\). Then, two ways of defining \( d_{12} \) are:

\[
d_{12} = \left| \frac{P_1 - P_2}{AP} \right| + \left| \frac{AD_1 - AD_2}{AAD} \right| + \left| \frac{R&D_1 - R&D_2}{AR&D} \right|
\]

and

\[
d_{12} = \left[ \left( \frac{P_1 - P_2}{AP} \right)^2 + \left( \frac{AD_1 - AD_2}{AAD} \right)^2 + \left( \frac{R&D_1 - R&D_2}{AR&D} \right)^2 \right]^{1/2}
\]

4) Building on the previous measure, we define the aggregate strategic distance, ASD, is defined as

\[
ASD = \sum_{i=1}^{k} \sum_{j=i+1}^{k} d_{ij}
\]

which aggregates the total strategic distances between each group, taken two at a time. A large ASD represents a more strategically spread out industry than a low ASD. If distances were taken as a proxy for mobility, the larger ASD would signify greater mobility barriers than in industries with lower ASD (Caves and Porter, 1977).

In summary, these four measures try to characterize the strategic group equilibrium, which obtains in an industry, with respect to market share distribution (measures 1 and 2) as well as strategic distances (measures 3 and 4). They can divulge important information for would-be entrant firms as well as firms which may want to change their strategies.
3.4 Internal Structure of Strategic Groups

Facing the question of relative positions of firms within a strategic group, the assumption of identical utility functions for all firms leads to the trivial result that all firms within a group behave exactly alike, i.e., they use the same strategies. This assumption is, of course, too strong. An example in the next section shows, as expected, small asymmetric perturbations of the utility function do not significantly alter the strategic group structure. Indeed the groups are close to the corresponding strategic groups with identical firms.

It should be noted, finally, that measures similar to those previously defined for industries can adequately describe the structure of firms within each group.

4.0 An Example

In this section, an example is presented to provide insights into the concepts which were dealt with somewhat abstractly in the previous section. The utility function will be assumed to be the profit function and its functional form given by

\[ PR_i = P_i \cdot S_i - AD_i \]

where \( P_i \) is the price charged, \( AD_i \) is the level of advertising expenditure and \( S_i \) is the sales of firm \( i \). The functional form assumes zero marginal cost of production, but this is just for ease of exposition—the results do not depend on it. The sales function \( S_i \) is assumed to be:

\[ S_i = (a+bAP) - (a+bAP)(\alpha + \frac{1}{\delta+\beta AD_i})(1 - \frac{P_i}{AP}) \]
with AP, representing the industry average price, which each firm takes as a given. This sales function represents the classic downward sloping industry sales function, i.e., when every firm charges the same price, and the individual firm's sales function is more elastic than the industry sales function. The sales of each firm can be thought of as being comprised of two parts: one that is guaranteed to all firms based on general consumer need/industry average pricing and the second which depends on interfirm competitive strategies. The sales increase/ decrease, due to strategic competition, is available only when firms deviate from the industry average price.

Therefore, the firms are evaluated in comparison to the industry average price. As a consequence, within the assumptions of the model, increased advertising expenditure increases sales if the price being charged is lower than the industry average and vice versa. This is typical of situations in which competitive characteristics are similar except for price competition; for example, Datril versus extra-strength Tylenol, Compaq versus IBM PC, MCI versus AT&T and local supermarket brands versus national name brands.

The values of the parameters of the sales function allow for sales variations. For example \( a' > 0, \ b < 0 \) will enforce a downward sales variation with increase in industry average price; \( (a + \frac{1}{c}) < 0, \delta > 0, \beta > 0 \) makes sure that the effect of advertising increases at a decreasing rate and incremental sales can be achieved by pricing below industry average and increasing advertising expenditure. Of course, the level of this increase in revenue will have to be traded off against the incremental advertising cost.

The profit maximizing conditions are:
\[ \frac{\partial PR}{\partial P} = 0 \Rightarrow p^* = \frac{AP}{2} \left[ 1 - \frac{1}{(\alpha + \frac{1}{\delta + \beta AD^*})} \right] \]

and

\[ \frac{\partial PR}{\partial AD} \leq 0 \Rightarrow P^*(a + bAP)(1 - \frac{P^*}{AP}) \frac{\beta}{(\delta + \beta AD^*)^2} - 1 \]
\[ \leq 0, \ AD^* = 0 \]
\[ = 0, \ AD^* > 0 \]

Some differences between this model and that of Karnani (1982) are:

1) This model has two controllable variables which interact while Karnani's model uses only marketing expenditure. 2) This model assumes firms are identical, to start with, regarding cost and proficiency while Karnani's model assumes firms are different. 3) Karnani considers the firm's effect on the industry average while this model assumes that each firm has negligible impact on the industry average. This imposes the additional constraint on the model, namely that the firm prices \( P \), must average out to the same industry average \( AP \) used to calculate the individual strategies.

What are the implications of this model? It can be easily shown that, for certain parameter values, there is no equilibrium where each firm uses the same strategies, i.e., follows the same price and advertising expenditures. If it had existed, this would have meant that there was only one strategic group. A heuristic explanation of the non-existence of the symmetric equilibrium is the following: suppose everybody was charging the same price \( P \), then the industry average would also be \( P \) and from the sales function, we can see that no one will advertise (since it increases costs without increasing sales). On the other hand, consider a firm in such an industry position—by
reducing its price just a little below price P and increasing advertising, it could very well increase its profits and market share (this happens when $\beta/\delta^2$ is large). Then, such an industry position cannot be in equilibrium.

The equilibrium position, which exists and is unique [see Kumar and Satterthwaite (1983) for proof in a similar model] is one which has two strategic groups—one group that prices lower than average and advertises while the second group charges prices that are higher than average and does not advertise. For example, with $a = 26$, $b = -2$, $\alpha = -2.4$, $\delta = \beta = 5/9$, the Nash equilibrium strategies are $(2.25, 3.5)$ for strategic group A and $(4,0)$ for strategic group B, where the first number refers to the price and the second, advertising level. Also, the distribution of firms will be 57.15 percent in strategic group A and 42.85% in strategic group B, supporting an industry average price of 3. Each group achieves the same profit level of 64. Obviously, strategic group A has the larger market share (see Figures 1 and 2).

With respect to the questions on existence of strategic groups and the actual number of them, this example shows how the market demand can force the existence of strategic groups in equilibrium. There are two of them, and it can be shown that they are structurally stable, thus obeying the instability result. The instability result also stipulates that this structure presents the maximum amount of strategic differentiation possible, given that there is only one uncontrollable variable, AP.

How can we describe this strategic equilibrium structure using the measures defined in the previous section? Here two measures are computed, namely, the strategic concentration ratio, SCR, and the strategic
comprehensive index, SCI, to represent the market share distribution; and two further measures, the strategic distance matrix, SDM, and the aggregate strategic distance, ASD, to characterise strategic distances between groups.

The sales level for group A is 30 and that for group B is 16; with 57.15% and 42.85% of firms in groups A and B respectively, the market share for group A is 71.43% and that for group B is 28.57%. Then SCR is given as 1.0 and SCI is equal to 0.41. It is easy to see that SCR is sensitive when the number of groups is small—it indicates no market share domination by a small subset of the total number of strategic groups. Obviously, this is not very accurate since one group has more than twice the market share of the other. On the other hand, an SCI value of 0.41 indicates unevenness in market share since otherwise, it would be close to 0.5.

Using the absolute value measure, rather than the $L_2$-norm (see footnote 2), we obtain

$$SDM = \begin{bmatrix} 0 & 0.58 \\ 0.58 & 0 \end{bmatrix}$$

and obviously, ASD = 0.58 also. Therefore, the strategic distances are quite small, measured relative to the industry averages for the strategies. A possible interpretation is that there may be only a weak barrier for firms contemplating a change to the other group.

The description of the relative positions of a particular firm within each strategic group in this example is quite simple. Due to the assumption of the same parameter values, i.e., $a$, $b$, $\alpha$, $\delta$, $\beta$, for all firms, all of the firms behave in exactly the same way within each
group (see Figure 2). This seems a highly unlikely condition and what one would expect is a small scatter of firms within each strategic group and a larger difference between the strategic groups. And this is exactly what one finds, numerically, if one perturbs the profit functions asymmetrically in a small way (see Figure 3). This perturbation captures the uncertainties in estimating the profit function as well as the slight intrinsic differences between firms.

Insert Figure 3 about here

5.0 Conclusion and Discussion

This study follows the spirit of numerous others on the subject of strategic groups. However, the basic thrust of this study is more fundamental than the identification problem. Four questions regarding the existence of strategic groups, limits on the number of strategic groups, characterisation of an industry equilibrium with strategic groups and posture of individual firms within a strategic group have been raised and answered, subject to the limitations of the assumptions behind the monopolistic competitive model used.

As an answer to the question on existence, the nature of the objectives of firms as well as the interactions between variables in the market can force the firms to split into various strategic groups. There will necessarily be some dominant strategies and firms should compare their existing strategies to those that may persist as dominant ones in a stable equilibrium. Otherwise, they may not achieve the maximum potential utility gains. From a public policy point of view, the existence of strategic groups and dominant strategies can help
policy makers decide whether to intervene or allow the market forces
to direct the industry structure.

Regarding the limit on the number of strategic groups in a struc-
turally stable equilibrium, it has been proven that the extent of stra-
tegic diversification is equal to the number of uncontrollable variables
plus one. The obvious question that arises is: what is the appropri-
ate number of uncontrollable variables to be chosen? Clearly, this
depends upon the industry under investigation. For example, in more
finance oriented industries, industry averages of seven factors namely,
financial leverage, capital turnover, return on investment, inventory
turnover, receivables turnover and liquidity and cash positions, could
be chosen as the uncontrollable variables (Chen and Shimerda, 1981).
Then, given the uncontrollable variables, the instability theorem pre-
dicts a definite upper bound (i.e., 8) on the number of strategic groups.

Given the existence of strategic groups and a limit on the number
of them, how can one describe the industry via aggregate measures?
Four aggregate indices have been suggested here, namely, the strategic
concentration ratio, SCR, the strategic comprehensive index, SCI, the
strategic distance matrix, SDM, and the aggregate strategic distance,
ASD. The first two try to capture the market share distribution among
the groups while the latter two measure strategic distances and poten-
tial mobility barriers that may exist among strategic groups.

The final question relates to the relative position of a firm
within a strategic group. The four indices mentioned above could very
well be used to describe the structure within a group. The example in
Section 4 has also shown that once sufficiently small asymmetries are
introduced into the model, through either estimation differences or structural differences, the essential strategic group equilibrium persists. This is typified in terms of a diversity of strategies scattered around a core group strategy rather than the existence of homogeneity within the group.

In summary, this paper has pursued a more fundamental route in analysing the subject of strategic groups. Perspectives on the four issues raised should prove useful to researchers in the business policy and the industrial organisation area. Clearly there is a lot of scope in using the methodology presented here. In particular, market forces, both consumer and producer, are modelled to develop a theory to explain how and why strategic groups evolve rather than to just describe them (which is important in itself).
Footnotes

1This assumption is a benchmark of symmetry, which is somewhat unrealistic. The consequences of relaxing this assumption to achieve greater asymmetry is addressed at the end of Section 3.

2Both measures are standard norms in Euclidean space and are equivalent in the sense that all properties depicted by one measure will also be depicted by the other. The first measure is called the absolute value norm and the second, the L₂-norm.

3The calculation of the distribution of the firms in each group is as follows: given that the prices charged the two groups A and B are 2.25 and 4 respectively with the average being 3, let q be the proportion of firms in group A and (1-q) that in group B. Then,

\[2.25 \times q + 4 \times (1-q) = 3\]

which implies \(q = 57.15\%\) and \((1-q) = 42.85\%\).

4Another working paper by Kumar et al. (1984) addresses the issue of how the methodology can be adapted to identify and form strategic groups within industries.
References


Figure 1: Equilibrium Strategies

Figure 2: Strategic Group Composition with Symmetry
Figure 3: Strategic Group Composition with Asymmetry
Appendix

For the given model, the strategic variables for the firm are
\((P_i, \text{CI}_i, \text{AD}_i, \text{R&D}_i, Q_i)\), the uncontrollable variables are (AP, ACI, AAD, AR&D, AQ) and the common utility function facing each firm is

\[ U(P_i, \text{CI}_i, \text{AD}_i, \text{R&D}_i, Q_i, \text{AP}, \text{ACI}, \text{AAD}, \text{AR&D}, \text{AQ}) \]

which is continuously differentiable. Each firm maximizes its utility function over its strategic variables taking the industry average variables as given.

**Definition:** A mixed strategy based on \(k\) pure strategies is given by
\((P_i, \text{CI}_i, \text{AD}_i, \text{R&D}_i, Q_i, \omega_i)_{i=1}^{k}\), where \(\omega_i\) is the probability placed on the \(i\)th strategy \((P_i, \text{CI}_i, \text{AD}_i, \text{R&D}_i, Q_i)\) with

\[ \omega = (\omega_1, \ldots, \omega_k) \in \Omega_k \equiv \{(\mu_1, \ldots, \mu_k) \mid \Sigma \mu_i = 1\} \]

**Definition:** The reaction set at (AP, ACI, AAD AR&D, AQ), denoted by \(R(\text{AP, ACI, AAD, AR&D, AQ})\), is the set of \((P_i, \text{CI}_i, \text{AD}_i, \text{R&D}_i, Q_i)\) which globally maximizes the utility function at (AP, ACI, AAD, AR&D, AQ).

**Definition:** A symmetric mixed strategy Nash equilibrium based on \(k\) pure strategies is given by \(((P_i^*, \text{CI}_i^*, \text{AD}_i^*, \text{R&D}_i^*, Q_i^*, \omega_i^*)_{i=1}^{k}, \text{AP}, \text{ACI}, \text{AAD}, \text{AR&D}, \text{AQ})\) satisfying for all \(\hat{\omega} = (\hat{\omega}_1, \ldots, \hat{\omega}_k) \in \Omega_k\),
\[ \sum_{i=1}^{k} \omega_i U(p_i^*, c_i^*, a_i^*, \text{R&D}_i^*, q_i^*, \text{AP}^*, \text{ACI}^*, \text{AAD}^*, \text{AR&D}^*, \text{AQ}^*) \geq \]

\[ \sum_{i=1}^{k} \omega_i U(p_i^*, c_i^*, a_i^*, \text{R&D}_i^*, q_i^*, \text{AP}^*, \text{ACI}^*, \text{AAD}^*, \text{AR&D}^*, \text{AQ}^*), \]

\[ \sum_{i=1}^{k} \omega_i^* p_i = \text{AP}^*, \sum_{i=1}^{k} \omega_i^* c_i = \text{ACI}^*, \sum_{i=1}^{k} \omega_i^* a_i = \text{AAD}^*, \sum_{i=1}^{k} \omega_i^* \text{R&D}_i = \text{AR&D}^*, \sum_{i=1}^{k} \omega_i^* q_i = \text{AQ}^*, \]

where

\[ \omega^* = (\omega_1^*, \ldots, \omega_k^*) \in \Omega_k, \]

\[ (p_i^*, c_i^*, a_i^*, \text{R&D}_i^*, q_i^*) \in \text{R(\text{AP}^*, \text{ACI}^*, \text{AAD}^*, \text{AR&D}^*, \text{AQ}^*)} \]

The above definition implies that there are \( k \) strategies \((p_i^*, c_i^*, a_i^*, \text{R&D}_i^*, q_i^*)_{i=1}^{k}\) which are the best responses given \((\text{AP}^*, \text{ACI}^*, \text{AAD}^*, \text{AR&D}^*, \text{AQ}^*)\) and that each of these strategies has a corresponding probability \( \omega_i^* \) associated with it. This probability \( \omega^* = (\omega_1^*, \ldots, \omega_k^*) \) is the one that gives the highest expected profits compared to all other probability measures and also satisfies the consistency condition that the expected or average value of the best response strategy is indeed \((\text{AP}^*, \text{ACI}^*, \text{AAD}^*, \text{AR&D}^*, \text{AQ}^*)\).

This equilibrium implies that around a small neighborhood of \((\text{AP}^*, \text{ACI}^*, \text{AAD}^*, \text{AR&D}^*, \text{AQ}^*)\), there exist \( k \) local maxima and with the added assumption of positive probabilities associated with each utility maximizing strategy, they are isolated. Also, the utility levels at each of these local maxima, at \((\text{AP}^*, \text{ACI}^*, \text{AAD}^*, \text{AR&D}^*, \text{AQ}^*)\), is the same. Let
\[ x_i \equiv (P_i, CI_i, AD_i, R&D_i, Q_i), \quad i = 1, \ldots, k \]

and \[ y \equiv (AP, ACI, AAD, AR&D, AQ) \]

and \[ D: \mathbb{R}^3 \rightarrow \mathbb{R}^{k-1} \] defined by

\[ D(y) = (U(x_i(y), y) - U(x_{i+1}(y), y), \]

\[ i = 1, 2, \ldots, k-1, \]

\[ x_j(y) \in S(y), \]

\[ j = 1, \ldots, k \]

where \( S(y) \) is the set of local maxima at the point \( y = (AP, ACI, AAD, AR&D, AQ) \). Then, in a small neighborhood of \( y^* = (AP^*, ACI^*, AAD^*, AR&D^*, AQ^*) \), \( D(y) \) is well defined and \( D(y^*) = 0 \).

Now consider all perturbations \( U_\lambda \) of the function \( U \) such that the values of the function \( U_\lambda \), its first and second derivatives can be made as close as needed to those corresponding values of \( U \) by choosing \( \lambda \) small enough. To ensure the stability of the equilibrium using profit function \( U \), it is necessary to demonstrate the existence of a similar equilibrium, close by and unique in a neighborhood of the original equilibrium, for all perturbations \( U_\lambda \), with \( \lambda \) in some open neighborhood of the value 0 (where it is assumed that \( U_0 = U \)). Three conditions have to be satisfied for such an existence, namely

(1) the local maxima sets \( S(y) \) and \( S_\lambda(y) \) must be close to each other in a neighborhood of \( y^* \) and the cardinality of \( S_\lambda(y) \) should be equal to \( k \).
(2) if condition (1) is satisfied, then the profit difference function $D_\lambda$ is well defined in this neighborhood of $y^*$. Then, it is necessary to show the existence of $y_\lambda^* \equiv (AP_\lambda^*, ACI_\lambda^*, AAD_\lambda^*, AR&D_\lambda^*, AQ_\lambda^*)$ close to $y^*$, in the neighborhood were $D_\lambda$ is defined, such that $D_\lambda(y_\lambda^*) = 0$.

(3) Then it is necessary to ensure the existence of a probability vector $\om_\lambda^*$ such that the expected value of the best response strategy $(P_i^*, CI_i^*, AD^*_i, R&D^*_i, Q_i^*)$ is indeed $(AP_\lambda^*, ACI_\lambda^*, AAD_\lambda^*, AR&D_\lambda^*, AQ_\lambda^*)$.

Condition (1) can be shown to be true through an application of the implicit function theorem and using the fact that the local maxima are non-degenerate critical points (Guillerain and Pollack (1974)). It is the second condition which leads us to the limitation on the value that $k$ can take.

Proof of the Instability Theorem:

Consider the case where $k \geq 7$. Then, the continuous map $D$, whose domain is some compact subset $A$ of the neighborhood of $y^*$ and whose range is a subset $B$ of $R^{k-1}$, satisfies $D(y^*) = 0$. It should be noted that $D$ is uniformly continuous and $B$ is a compact set of measure zero in $R^{k-1}$. Now either $0$ is on the boundary of set $B$ or not.

If $0$ is on the boundary, then there exists a sufficiently small vector translation $D_\lambda$ such that $0$ is not contained in the range of this perturbation $D_\lambda$.

If $0$ is in the interior of $B$, then approximate $D$ (which is uniformly continuous) uniformly by means of a piecewise continuous linear function, $\hat{D}$. Note that this is a perturbation of $D$. For this
approximation, 0 is either in its range or not. If 0 is not in its range, then this perturbation suffices and let $D_\lambda = \hat{D}$. If 0 is in its range, then it must be on its boundary (since $\hat{D}$ is piecewise linear) and therefore a sufficiently small vector translation $D_\lambda$ of $\hat{D}$ can get rid of the 0 in its range.

Thus, we can construct a perturbation $D_\lambda$ of $D$ either through pure vector translation or through a uniform approximation or through a combination of both, such that no $y$ exists with $D_\lambda(y) = 0$.  

□