A Modern Analysis of the Effects of Site Value Taxation

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Abstract

Formal analysis is generally absent from the previous literature on the effects of site value taxation. This paper analyzes the impacts of such a system (under which the property tax on improvements is eliminated, with the tax burden shifted toward land) using standard modern methods. Specifically, the analysis derives the long-run impacts on the level of improvements, the value of land, and the price of housing of a shift to a graded tax system (where the improvements tax rate is lowered and the land tax rate is raised). The paper also analyzes the incidence of the short-run windfall gains and losses which result from gradation of the tax system.
The Effects of Site Value Taxation:  
A Modern Analysis

by

Jan K. Brueckner

1. Introduction

Ever since the publication of Henry George's "Progress and Poverty" in 1879, the possibility of using land value taxation as a source of government revenue has intrigued economists and other social commentators. While George's ideas have had little general impact, land value taxation is practiced in Jamaica and in certain cities in Australia and New Zealand, and graded property tax systems (where land is taxed at a higher rate than improvements) are in use in some Canadian provinces as well as in the city of Pittsburgh.¹

The literature dealing with land (or site) value taxation is vast (for an excellent bibliography, see Carmean (1980)). Most writers have been concerned with predicting the effects of a shift from a typical property tax system, where land and improvements are taxed at the same effective rate, to a system of pure site value taxation, where the improvements tax is eliminated and land is taxed at a higher rate (tax revenue is held constant). Others deal with the effects of transition to a graded system (where the improvements tax rate is lowered but remains positive), recognizing that pure site value taxation is simply an extreme case of gradation. Consensus has emerged on a number of points. First, nearly all writers agree that reduction or elimination of the improvements tax will raise the level of improvements in the long run, leading to more intensive land-use. Second, there is
agreement that in the short run, windfall gains and losses will result from a movement to a graded system as tax bills rise for certain properties and fall for others. The list of studies which attempt to quantify such impacts includes Schaaf (1970), Smith (1970), Neuner et al. (1974), Lusht (1975), Killoren and Casey (1981), and Stoddard and Fry (undated). Third, the problem of generating accurate land value assessments in the absence of frequent transactions in vacant land is recognized by most writers (a practicing assessor (Back (1970)) claims that accuracy can be achieved). There is considerable interest (but less agreement) regarding the impact of site value taxation on speculation in vacant land, with some commentators arguing that speculation will be curbed by higher land holding costs while others predict no effect (see Brown (1927) for an early contribution). The best general discussions of these and other points are provided by Becker (1969), Harriss (1970), and Peterson (1978).

What is remarkable about this large literature is the almost complete absence of modern analysis. Most studies rely on verbal arguments or simple diagrams, and the few analytical efforts (McCalmont (1976) and Cuddington (1978)) are marred by ad hoc assumptions or misplaced emphasis. While several correct predictions regarding the effect of site value taxation have been derived without the aid of rigorous methods (the predicted increase in land-use intensity, for example), the lack of precision of past studies has led to substantial confusion on certain points. A prime example is the question of land value impacts. As is shown below, a lower improvements tax raises land value while a higher land tax lowers it. The
net effect is ambiguous but depends in a straightforward way on the magnitudes of various parameters. While the analysis underlying this result is quite intuitive, only two authors (Becker (1969) and Harriss (1970)) correctly grasp the principle involved. Indeed, in his recent contribution, McCalmont (1976) goes so far as to say that "not even the direction, let alone the amount, of the change in land rent can be ascertained from theory alone..." (p. 928). This claim is false, as the ensuing analysis will demonstrate. Another important question on which the literature is virtually silent is the impact of site value taxation on housing prices. Modern analysis gives an immediate answer, as will be seen below.

The remainder of this paper will elaborate on the above points by conducting an analysis of the effects of site value taxation using standard modern methods. Section 2 investigates the long run impacts of a revenue-preserving shift from a standard property tax system to a graded system under two different scenarios. In the first case, the graded tax system is imposed in only a small part of a single city, so that the price of housing is unaffected. In the second case, implementation occurs city-wide, so that price effects emerge. In both cases, the analysis derives the impacts of gradation on the level of improvements and the value of land. The impact on the price of housing is also derived for the second case.

While Section 2 assumes that the price of housing is spatially uniform, Section 3 allows spatial variation. In this setting, improvements and land value vary with location, and short-run windfall gains and losses result from a switch to a graded tax system. The analysis
is devoted to investigating the spatial pattern of gains and losses under the assumption that the housing price contour is exogenous. Section 4 of the paper briefly presents some empirical evidence relevant to the predictions of the analysis, and Section 5 offers conclusions.

2. Long-run effects

2.1 The case of an exogenous price of housing

In reality, property taxes are levied on a wide variety of types of structures: residential, commercial, and industrial. Typically, the interior space in one type of structure is unsuitable for any other use. In the following analysis, this fact is ignored and the property tax base is assumed to consist of a homogeneous class of structures called "housing." In the model, housing floor space is rented at price \( p \) per square foot and is produced using inputs of capital \( N \) and land \( L \) under a neoclassical constant returns technology represented by the production function \( H(N,L) \). Since output is indeterminate under constant returns, the analysis focuses on levels of output and capital input on a per-acre-of-land basis. Housing output per acre is \( H(N,L)/L = H(N/L,1) \equiv h(S) \), where \( S \) is capital per acre of land (hereafter improvements per acre), a measure of land-use intensity, and \( h(S) \equiv H(S,1) \). Note that \( h' = H_1 > 0 \) and \( h'' = H_{11} < 0 \) by the concavity of \( H \).

The net-of-tax rental prices of capital and land are represented by \( i \) and \( r \) respectively, and the tax rates on improvements (capital) and land are \( \tau \) and \( \theta \) respectively. The gross-of-tax capital and land prices are therefore \( (1+\tau)i \) and \( (1+\theta)r \) respectively. Note that since
taxes are expressed as a fraction of net rental price instead of value, conversion to value terms would require multiplication of the tax rates by the discount rate. Note also that $\tau = \theta$ will hold under a standard property tax system.

The shift to a graded property tax system is assumed to occur over a land area of size $\bar{L}$ (referred to subsequently as the "tax zone"). Locational advantages are absent within the tax zone, so that the housing price $p$ is spatially uniform. Furthermore, in the initial analysis, the tax zone is viewed as representing a small portion of the relevant housing market. This means that the change in the tax system will have a negligible effect on the total supply of housing in the market, with the result that the price $p$ can be viewed as exogenous. Finally, given that the analysis deals with the effects of a localized rather economy-wide tax change, the net return to capital is also taken to be exogenous (the locality faces a perfectly elastic supply of capital).

Profit per acre for a housing producer operating in the tax zone is given by $ph(S) - (1+\tau)iS - (1+\theta)r$. The equilibrium conditions for the producer require that profit per acre is maximal and that the maximized value equals zero. The appropriate conditions are

$$ph'(S) = (1+\tau)i$$

(1)

$$ph(S) - (1+\tau)iS - (1+\theta)r = 0.$$  

(2)

Together, eqs. (1) and (2) determine equilibrium values of improvements per acre $S$ and net land rent $r$. The impacts on $S$ and $r$ of changes in the tax rates $\tau$ and $\theta$, which are used to derive the effects
of a shift to a graded tax system, are computed by totally differentiating the system (1)-(2). The results are
\[
\frac{\partial S}{\partial \tau} = \frac{i}{ph''} < 0
\] (3)
\[
\frac{\partial S}{\partial \theta} = 0
\] (4)
\[
\frac{\partial r}{\partial \tau} = \frac{-iS}{1+\theta} < 0
\] (5)
\[
\frac{\partial r}{\partial \theta} = \frac{-r}{1+\theta} < 0
\] (6)

By increasing the cost of capital, an increase in the improvements tax rate \(\tau\) naturally reduces improvements per acre, as seen in (3). In addition, by reducing the profitability of development, a higher improvements tax depresses land rent, as seen in (5) (rent serves to exhaust residual profit). Similarly, (6) shows that a higher land tax rate depresses land rent. In fact, the higher tax is fully capitalized, so that gross-of-tax land cost remains unchanged ((6) yields \(\frac{3}{\partial \theta} (1+\theta)r = 0\)). Finally, (4) indicates that a higher land tax rate has no effect on the level of improvements.

The goal of the analysis is to derive the impacts on \(S\) and \(r\) of a revenue-preserving shift to a graded tax system. Starting with a standard tax system (where \(\tau = \theta\)), gradation results from an increase in \(\theta\) combined with a revenue-preserving change in \(\tau\) (pure site value taxation emerges when \(\tau = 0\)). The first step in the derivation is the computation of the derivative \(\partial \tau/\partial \theta\), which gives the revenue-preserving change in \(\tau\) accompanying an increase in \(\theta\). Noting that tax
revenue originating from the tax zone equals \( \xi(t \xi S + \theta r) \), \( \partial \tau / \partial \theta \) must satisfy \( d(t \xi S + \theta r)/d \theta = 0 \), or

\[
\frac{\partial \tau}{\partial \theta} iS + \tau iS (\frac{\partial S}{\partial \theta} + \frac{\partial S}{\partial \tau} \frac{\partial \tau}{\partial \theta}) + r + \theta (\frac{\partial \tau}{\partial \theta} + \frac{\partial r}{\partial \tau} \frac{\partial \tau}{\partial \theta}) = 0
\]  

(7)

Substituting from (3)-(6) and rearranging, (7) yields

\[
\frac{\partial \tau}{\partial \theta} = \frac{-r}{iS} \left[1 + \frac{(1+\theta)\xi S}{\phi h S} \right].
\]  

(8)

Eliminating \( h'' \) using the relationship

\[
\sigma = \frac{-h'(h-Sh')}{Shh''},
\]  

(9)

where \( \sigma \) is the elasticity of substitution between capital and land in housing production, \(^3\) and noting that land's factor share \( \mu \) (which equals \( (1+\theta)\xi l/p\phi h \)) may be written \( (h-Sh')/h \), (8) reduces to

\[
\frac{\partial \tau}{\partial \theta} = \frac{-r}{iS} \left\{1 - \frac{(1+\theta)\xi \sigma}{(1+\theta)\mu} \right\}^{-1}.
\]  

(10)

Inspection of (11) shows that the sign of \( \partial \tau / \partial \theta \) is ambiguous, so that a revenue-preserving change in \( \tau \) may involve either a decrease or an increase. The outcome depends crucially on the magnitude of the elasticity of substitution, which, for given values of \( \tau \), \( \theta \), and \( \mu \), determines the sign of the expression in braces in (10). Inspection of (10) indicates that for \( \sigma \) sufficiently close to zero, \( \partial \tau / \partial \theta \) will be negative, while for \( \sigma \) sufficiently large, \( \partial \tau / \partial \theta \) will be positive. To gain an intuitive understanding of this result, the first step is to note that (10) equals minus the ratio of the derivative of tax revenue with respect to \( \theta \) \((r+\theta \xi r / \partial \theta)\) and the derivative of revenue with respect to \( \tau \) \((iS + \xi i S \partial \tau + \theta \xi r) / \partial \tau \). Since the first derivative is always positive (revenue is always increasing in \( \theta \)), \(^4\) the sign of \( \partial \tau / \partial \theta \)
depends on the sign of the latter derivative, which depends in turn on two separate effects. First, since $\partial r / \partial t < 0$ by (5), an increase in $t$ indirectly depresses revenue from the land tax, making the last term in the derivative negative. The effect of a higher $t$ on improvements tax revenue (captured by $iS + t\partial S / \partial t$) is ambiguous, however, and depends on the magnitude of $\sigma$. A low (high) value of $\sigma$ means that improvements tax revenue is increasing (decreasing) with $t$ due to weak (strong) substitution away from capital as $t$ rises.\(^5\) When $\sigma$ is large, a higher $t$ therefore depresses revenue from both the improvements and land taxes, cancelling the revenue gain from a higher $\theta$. In this case, $\partial \tau / \partial \theta > 0$ must hold for revenue to remain constant. When $\sigma$ is sufficiently small, however, the increase in improvements tax revenue resulting from a higher $t$ dominates the decline in land tax revenue, and total revenue rises with $t$. In this case, $\tau$ must fall as $\theta$ rises to keep total revenue constant.\(^6\)

Whether $\partial \tau / \partial \theta$ is negative or positive for plausible values of $\sigma$ depends on the magnitudes of the other parameters in (10). To make matters simple, suppose first that $\partial \tau / \partial \theta$ is evaluated under a standard property tax system, so that $\tau = \theta$ holds. In this case, the sign of (10) is the same as the sign of $\sigma - (u_2 / \tau)$. Focusing first on land's share, published (or implied) estimates of $u_2$ range from 20% to 50%, with most values lying in the middle or lower end of this range.\(^7\) In addition, data compiled by the Advisory Council on Intergovernmental Relations (1983, Table 37) show that the average effective property tax rate in the U.S. in 1981 for single family homes with FHA insured mortgages was 1.26%. With a value-to-rent ratio between 10 and 20 (a
discount rate between 5% and 10%), this yields a $\tau$ between 12% and 26% (recall that $\tau$ is the tax rate on net rent, not value). Together, these $u_\ell$ and $\tau$ values imply that $u_\ell/\tau$ lies between .8 and 4.2, with a plausible value falling near the middle of this range. Since published estimates of the elasticity of substitution in housing production are almost always smaller than unity (see McDonald (1981) for a survey), it follows that $\sigma - (\mu_\ell/\tau)$ is almost certainly negative, implying $\partial \tau / \partial \theta < 0$. This implication is welcome given that a positive value for $\partial \tau / \partial \theta$ would be quite unnatural.

While the initial shift away from a standard property tax will typically require a decline in $\tau$, the behavior of $\tau$ as the rate falls below $\theta$ is harder to determine. If it could be shown that $\partial [((1+\theta)\tau/(1+\tau)u_\ell)]/ \partial \theta < 0$ holds, then (assuming a constant $\sigma$) the expression in braces in (10) would remain positive as $\theta$ rises away from $\tau$, and $\partial \tau / \partial \theta < 0$ would hold globally. Unfortunately, the above derivative is ambiguous in sign, so that the initial negativity of $\partial \tau / \partial \theta$ does not guarantee global negativity. Note, however, that for $\tau$ sufficiently close to zero, $\partial \tau / \partial \theta$ must be negative (see (10)). As a result, it is safe to say that $\partial \tau / \partial \theta$ will be negative at the beginning ($\tau$ near $\theta$) and at the end ($\tau$ near zero) of a transition to pure site value taxation. Intermediate cases may show $\partial \tau / \partial \theta > 0$, although, as mentioned earlier, this outcome would be most unnatural.

Having computed $\partial \tau / \partial \theta$, it is now possible to derive the impacts on $S$ and $\tau$ of a shift to a graded property tax system. Since $\partial S / \partial \theta = 0$ by (4), the impact on $S$ is simply
Given that $\frac{dS}{d\theta} < 0$ by (3), $\frac{dS}{d\theta}$ will be positive in the normal case where $\frac{\partial \tau}{\partial \theta}$ is negative. This is the outcome recognized in the earlier literature: a shift in the property tax burden toward land and away from improvements will raise the level of improvements.\(^8\)

While earlier writers were correct on this point, they typically failed (as mentioned above) to appraise correctly a graded tax system's effect on land value. The present analysis gives an immediate answer since it follows (using (5), (6), and (10)) that

$$\frac{dr}{d\theta} = \frac{dr}{\theta} + \frac{dr}{\tau} \frac{\partial \tau}{\partial \theta}$$

$$= \frac{\Lambda \tau}{(1+\theta)(1-\Lambda)},$$

where $\Lambda \equiv (1+\theta)\tau/(1+\tau)\mu$, is the expression inside the braces in (10).

Since $\frac{\partial \tau}{\partial \theta} \leq 0$ as $1 - \Lambda \geq 0$, (12) implies that

$$\frac{dr}{d\theta} \geq 0 \text{ as } \frac{\partial \tau}{\partial \theta} \leq 0.$$  \hspace{1cm} (13)

In other words, land value (which is proportional to r) rises (falls) with $\theta$ when the revenue-preserving change in $\tau$ is negative (positive). In the intuitively sensible case where $\frac{\partial \tau}{\partial \theta}$ is everywhere negative, this result implies that land value rises continuously as $\theta$ increases, reaching a maximum in the case of pure site value taxation.

Some understanding of these results can be gained by inspection of the first line of (12). Since both $\frac{dr}{d\theta}$ and $\frac{dr}{d\tau}$ are negative, it is clear that if a higher $\theta$ requires a higher $\tau$ to maintain tax revenue ($\frac{\partial \tau}{\partial \theta} > 0$), then land value declines unambiguously, as indicated

\[\frac{dS}{d\theta} = \frac{\partial S}{\partial \tau} \frac{\partial \tau}{\partial \theta}. \]  \hspace{1cm} (11)
by the lower inequalities in (13). When $\partial \tau / \partial \Theta < 0$ holds, however, a lower improvements tax raises land value at the same time that the higher land tax depresses it. While the net effect might appear to be ambiguous (and was identified as such by those earlier writers astute enough to recognize the trade-off involved), the mathematics unambiguously indicate that the impact of the lower improvements tax wins out and land value rises. Consideration of the case where $\sigma = 0$ and capital-land substitution does not occur makes this outcome somewhat clearer. First, note from (10) that $\partial \tau / \partial \Theta$ is closer to zero when $\sigma = 0$ than when $\sigma > 0 (\partial \tau / \partial \Theta < 0$ is assumed). The reason is that when $\sigma > 0$, $S$ increases in response to a lower $\tau$, which means that the revenue-preserving decline in $\tau$ must be greater than in the case where $\sigma = 0$ and $S$ is fixed. Next, note from (12) that when $\sigma = 0$, $dr/d\Theta = 0$ holds. Since a revenue-preserving tax change thus leaves $r$ unchanged when $\sigma = 0$, and since constant revenue requires a faster decline in $\tau$ when $\sigma > 0$, the value-increasing effect of a lower $\tau$ must dominate in the latter case, leading to an increase in $r$. This surprising conclusion, which establishes that transfer of the tax burden from improvements to land actually raises land value, shows the usefulness of modern analysis applied to the present problem.

Summing up, the above analysis has shown that for representative parameter values, an initial shift away from a standard property tax system toward a graded system will lead to an increase in both improvements per acre and land value. Provided that $\partial \tau / \partial \Theta < 0$ continues to hold as $\Theta$ increases, the initial impacts are simply magnified as the tax burden on land increases, with improvements per acre and land
value eventually reaching a maximum under a system of pure site value taxation.

2.2 The case of an endogenous price of housing

In this section, the tax zone is assumed to encompass the entire housing market. The assumption that locational advantages are absent is maintained, however. In this case, a change in the property tax system will have an impact on the price of housing, and a market-clearing equation must be added to the previous equilibrium system (1)-(2). Letting \( D(p) \) denote the aggregate demand function for housing, which satisfies \( D' < 0 \), the expanded equilibrium system consists of the earlier equations together with

\[
\bar{\alpha}h(S) = D(p). \tag{14}
\]

The separate impacts of \( \tau \) and \( \theta \) on \( p, S, \) and \( r \) are derived by totally differentiating (1), (2), and (14). The results are given by (4), (6), and

\[
\frac{\partial p}{\partial \tau} = \frac{(\bar{\alpha}h'/D')i}{ph'' + (\bar{\alpha}h'^2/D')} > 0 \tag{15}
\]

\[
\frac{\partial p}{\partial \theta} = 0 \tag{16}
\]

\[
\frac{\partial S}{\partial \tau} = \frac{i}{ph'' + (\bar{\alpha}h'^2/D')} < 0 \tag{17}
\]

\[
\frac{\partial r}{\partial \tau} = \frac{h\partial p/\partial \tau - iS}{1+\theta} \geq 0 \tag{18}
\]

As before, a higher land tax is fully capitalized (leaving \((1+\theta)r\) unchanged) and has no effect on \( S \). As a result, there is no impact on
p, as shown in (16). A higher improvements tax once again lowers the level of improvements (see (17)), but its effect on land rent is ambiguous, as shown in (18). The latter result is due to the fact that the improvements tax is shifted forward, raising the price of housing (see (15)). Since the higher p tends to increase the profitability of development at the same time that the higher τ reduces it, the net impact on r is indeterminate. This outcome makes θr/θτ numerically larger than before (compare (18) and (5)), and comparison of (17) and (3) shows that θS/θτ is also larger (less negative) in the present case. This follows from the stimulating effect of a higher p, which moderates the decline in S resulting from the higher improvements tax.

Substituting the above results in (7) to compute θτ/θθ yields

\[
\frac{\partial \tau}{\partial \theta} = \frac{-r}{iS} \left[ 1 + \frac{(1+\theta)\tau + (\theta E h'h'(D'))^{-1}}{S[A'h' + (\theta h'^2/D')]^{-1}} \right].
\] (19)

To simplify (19), let \(\eta \equiv (p/h)(\partial h/\partial p) = -h'^2/hh > 0\) represent the elasticity of housing supply per acre of land. Also, let \(\varepsilon \equiv pD'/D = pD'/\sqrt{\theta} < 0\) represent the elasticity of housing demand. Substituting \(\eta\) and \(\varepsilon\) together with \(\sigma\) and \(\mu\) into (19), the equation reduces to

\[
\frac{\partial \tau}{\partial \theta} = \frac{-r}{iS} \left[ 1 - \frac{\sigma((1+\theta)\varepsilon + \theta(1+\tau))}{(1+\tau)\mu}(\varepsilon - \eta) \right]^{-1}.
\] (20)

To see that the earlier solution for \(\partial \tau/\partial \theta\) is just a special case of (20), note that (20) reduces to (10) when \(\varepsilon \to -\infty\) (when p is exogenous). While the sign of \(\partial \tau/\partial \theta\) is again ambiguous, it is easy to see that (20) is negative whenever the corresponding derivative in
(10) is negative. To establish this fact, note first that the expression in braces in (20) is greater than the corresponding expression in (10). When the latter expression is positive (implying $\partial \tau / \partial \theta < 0$ in the $p$-exogenous case), it follows that the expression in (20) is a greater positive quantity, implying that $\partial \tau / \partial \theta$ from (20) is negative but less in absolute value than (10). Therefore, when $\partial \tau / \partial \theta$ is negative in the $p$-exogenous case, the $p$-endogenous derivative is also negative but closer to zero. The reason for this result is that the expressions in (10) and (20) are proportional to the derivative of tax revenue with respect to $\tau$. Since $S / \partial \tau$ and $r / \partial \tau$ are both larger in the $p$-endogenous case, tax revenue responds more vigorously to a change in $\tau$. In the case where revenue increases with $\tau$ in the $p$-exogenous case, it increases at a faster rate when $p$ is endogenous, meaning that a smaller decrease in $\tau$ is needed to keep revenue constant as $\theta$ rises.

Computation of the impacts of a switch to a graded tax system proceeds as before. First, since $\partial p / \partial \theta = 0$, it follows that

$$\frac{dp}{d\theta} = \frac{\partial p}{\partial \tau} \frac{\partial \tau}{\partial \theta},$$

which (noting (15)) is negative as long as $\partial \tau / \partial \theta < 0$. Since a higher $\theta$ has no impact on $p$ while a lower $\tau$ reduces it, the total effect of gradation is to reduce the price of housing. In addition, since $S / \partial \theta$ again equals zero, $dS / d\theta$ is again given by (11). Therefore, the level of improvements again increases with $\theta$ provided that $\partial \tau / \partial \theta$ is negative. Upon substituting (6), (18), and (20) into the first line of (12) to compute $dr / d\theta$, simplification yields the result
where \( 1 - \Lambda' \) is the expression in braces in (20). When \( 1 - \Lambda' > 0 \), so that \( \partial \tau / \partial \theta < 0 \), the sign of (22) is the same as the sign of \(-[1+\tau(1+\varepsilon)]\), which depends in turn on the magnitude of \( \varepsilon \). If housing demand is inelastic, so that \(-1 < \varepsilon < 0\), then \( 1 + \tau(1+\varepsilon) > 0 \) and \( dr/d\theta < 0 \).

When demand is elastic, \( 1 + \tau(1+\varepsilon) \) can have either sign, with \( 1 + \tau(1+\varepsilon) \geq 0 \) holding as \( \varepsilon \geq -(1+(1/\tau)) \). Substituting a representative \( \tau \) value of .20 to appraise the effect of an initial shift toward a graded tax system, it follows that \( dr/d\theta \geq 0 \) as \( \varepsilon \leq -6 \). Since most estimates indicate that housing demand is price inelastic (see Mayo (1981) for a survey), \( dr/d\theta < 0 \) seems the likely outcome. Note that as \( \tau \) falls toward zero, the case against \( dr/d\theta > 0 \) is even stronger (demand would have to be even more elastic in order for the inequality to hold). As a result, it appears that \( dr/d\theta \) is globally negative.

This result is clearly the opposite of the outcome in the p-exogenous case (where land value rises), and the explanation lies in the behavior of the housing price. Since \( p \) falls with \( \theta \) in the present situation, a new force which reduces the profitability of development (and hence the value of land) enters the analysis. Since it may be shown that the price of housing falls faster with \( \theta \) the less elastic is the demand for housing (the absolute value of \( dp/d\theta \) is larger when \( \varepsilon \) is closer to zero), the specific results above become clearer. Inelastic or moderately elastic demand leads to a sharp decline in \( p \) and a correspondingly large depressing effect on \( r \). This effect, which was not present before, is sufficient to reverse the previous outcome and lead to a decline in land value. When demand is
highly elastic, the decline in \( p \) is moderate and the depressing effect on \( r \) is not sufficiently strong to reverse the earlier positive impact, so that land value rises.

Summing up, the preceding analysis has shown that when \( \partial r / \partial \theta \) is negative, an increase in \( \theta \) depresses the price of housing, raises the level of improvements, and, as long as housing demand is not highly elastic, depresses the value of land. The maximum impacts will be observed when a system of pure site value taxation is reached. While the sign of \( \partial r / \partial \theta \) is generally ambiguous, so that the above conclusions cannot be viewed as automatic, \( \partial r / \partial \theta < 0 \) seems natural. In any case, since it was shown that \( \partial r / \partial \theta \) evaluated at \( \tau = \theta \) is almost certainly negative in the \( p \)-exogenous case, and since negativity implies that \( \partial r / \partial \theta < 0 \) also holds for \( \tau = \theta \) when \( p \) is endogenous (see above), we can conclude with a high degree of confidence that starting with a standard property tax system, the initial shift toward a graded system will have the impacts listed above.

3. An analysis of temporary gains and losses

In long run equilibrium, housing producers are indifferent to the features of the property tax system since profit is identically zero. Before full market adjustment occurs, however, producers can experience windfall gains or losses from a change in the tax system. While the uniformity of land-use within the tax zone means that such gains and losses will never emerge in the model of Section 2, land-use heterogeneity within the zone (differences in improvements per acre and land values) will generally lead to windfalls. The purpose of this section is to present a stylized analysis of such windfalls using
a model where the price of housing (and thus the levels of S and r) varies within the tax zone.

Without loss of generality, the tax zone is taken to be a rectangle of unit width, and the price of housing $p$ is assumed to be a decreasing function of distance $x$ from one end of the rectangle ($\partial p/\partial x < 0$). This could reflect the impact of differential commuting costs within the tax zone or the influence of an amenity or disamenity near the zone. The housing price function is taken to be exogenous.

The first step in the analysis is to deduce the spatial behavior of $S$ and $r$ implied by spatial variation in $p$. This is done by totally differentiating (1) and (2) with respect to $x$, which yields

$$\frac{\partial S}{\partial x} = \frac{-h'}{ph''} \frac{\partial p}{\partial x} < 0 \quad (23)$$

$$\frac{\partial r}{\partial x} = \frac{h}{1+\theta} \frac{\partial p}{\partial x} < 0, \quad (24)$$

indicating that both improvements per acre and land value decline with $x$ (these results are familiar from urban spatial analysis). The levels of $S$ and $r$ prevailing prior to a change in the tax system will be denoted $\overline{S}$ and $\overline{r}$. While $\overline{S}$ and $\overline{r}$ are implicitly functions of $x$, the $x$ argument will be suppressed for simplicity in the analysis.

The analysis investigates the effect of a shift to a graded property tax system over both short- and medium-run horizons. The feature which is common to both the short and medium run is that improvements per acre at any given location are fixed at the level prevailing prior to the change in the tax system ($S$ is fixed at $\overline{S}$). This is a reflection of the longevity of housing capital, which makes an immediate
response to a tax change uneconomical. What distinguishes the short- and medium-run cases is that in the short run, the assessed value of land will remain at its previous level, whereas in the medium run, the effects of reassessment will emerge. In both cases, however, land cost net of the tax liability will remain fixed at \( \bar{r} \), reflecting the fact that payments on existing mortgages will be unaffected by a change in the tax system.

Since revenue per acre (\( ph(S) \)) and net-of-tax improvements and land costs (\( iS \) and \( \bar{r} \)) will remain fixed in the short and medium run, gains and losses will be due entirely to changes in tax liabilities. To analyze the short-run case, the first step is to note that the tax payment at a given location equals \( \bar{r}iS + \bar{\theta}r \) and that total revenue from the tax zone is given by \( \bar{r}i \int_0^b S dx + \bar{\theta} \int_0^b r dx \), where \( b \) is the length of the zone (recall that its width is unity). Letting \( S_A \equiv \int_0^b S dx/b \) denote the average level of improvements per acre in the zone, with \( \bar{r}_A \) defined analogously, tax revenue is given by \( \bar{r}iS_A + \bar{\theta}\bar{r}_A \) (note \( b = \bar{\ell} \)). Constancy of total tax revenue then requires

\[
\frac{\partial \bar{r}}{\partial \bar{\theta}} = \frac{-\bar{r}_A}{iS_A} < 0. \tag{25}
\]

To find the short-run change in the tax liability at a particular location, (25) is substituted into \( iS \partial \bar{r}/\partial \bar{\theta} + \bar{r} \), yielding

\[
\bar{S}[(\bar{r}/S)-(\bar{r}_A/S_A)]. \tag{26}
\]

Eq. (26) indicates that parcels with high ratios of land value to improvements face higher taxes as \( \bar{\theta} \) rises and \( \bar{r} \) falls, with taxes declining for parcels with low \( \bar{r}/S \) ratios. Note that if land-use is
uniform within the tax zone, so that \( \bar{r} = \bar{r}_A \) and \( \bar{S} = \bar{S}_A \), then (26) is zero and the tax liability is unchanged at each location.

The spatial pattern of gains and losses is deduced by taking the derivative of \( \bar{r}/\bar{S} \) with respect to \( x \). Calculations based on (23) and (24) yield

\[
\frac{\partial \bar{r}/\bar{S}}{\partial x} = \frac{(1-\sigma)h \partial \bar{p}}{1+\theta} \partial x',
\]

indicating that the expression in parentheses in (26) is an increasing (decreasing) function of \( x \) as \( \sigma \geq 1 \). In the realistic case where \( \sigma < 1 \), this implies that (26) is positive (negative) for low (high) values of \( x \) ((26) must change sign in order for tax revenue to remain constant). Therefore, windfall losses (higher taxes) accrue to parcels with low \( x \)'s while windfall gains (lower taxes) accrue to parcels with high \( x \)'s. A shift to a graded property tax system thus imposes short-run losses on the most intensively developed parcels while bestowing short-run gains on the least intensively developed sites. This result might at first appear counterintuitive since parcels with high improvements per acre stand the most to gain from a lower improvements tax. This observation, however, ignores the fact that such parcels also have high land value, which makes an increase in \( \theta \) especially burdensome. When \( \sigma < 1 \), the latter effect dominates and the total tax liability rises.

Although the above analysis applies to a tax zone with a single type of real estate, the conclusions based on (26) apply even in the case of mixed land uses. That is, regardless of what types of property are located in the tax zone, comparison of the \( \bar{r}/\bar{S} \) ratio for a
given parcel to the ratio of average values for the zone tells whether taxes rise or fall for that parcel in the short run. Using this principle, the impact analyses cited in the introduction attempt to predict the short-run incidence of a shift to pure site value taxation for various municipalities. Typical findings show that many commercial and industrial properties would face higher taxes, while single family homes would generally benefit from lower tax bills.

As mentioned above, the difference between the short and medium run is that land will be reassessed for tax purposes in the latter case. Reassessment will reflect land's market value, which is based on the profitability of new development. Specifically, value will be given by the r solution to (1)-(2) with S freely variable, even though improvements are currently frozen at S. With reassessment, the tax liability becomes \( \bar{r}S + \theta r \), and total revenue from the tax zone is given by \( \bar{r}S \frac{e^x}{e^x + \theta} + \theta \int_0^b rdx \). Using (5) and (6) to compute \( \partial \tau / \partial \theta \) yields a solution identical to (25), so that \( \partial \tau / \partial \theta \) is the same regardless of whether or not land is reassessed. The tax change for a given parcel is now \( d(\bar{r}S + \theta r) / d\theta \), which, using (5), (6), and (25), reduces to (26) divided by \((1+\theta)\), implying that the change in the post-reassessment tax liability is proportional to the change in the pre-reassessment liability (absolute post-reassessment changes are somewhat smaller). As a result, the previous conclusions hold for the medium-run case: parcels with high (low) \( \bar{r}/S \) ratios face higher (lower) post-reassessment taxes, with the losers (gainers) being located at low (high) \( x \) values. It is interesting to note that since
dr/dθ may be shown to be exactly the negative of the tax change, post-reassessment land values fall where values are high (at low x's) and rise where they are low (at high x's).

Summing up, the analysis in this section has shown that a shift to a graded property tax system will generate temporary gains and losses in a distinct spatial pattern. Provided that \( \sigma < 1 \), the most intensively developed parcels will suffer windfall losses as their tax liabilities rise while the least intensively developed sites reap windfall gains in the form of lower tax bills. This pattern holds both before and after land is reassessed.\(^{15}\)

4. Empirical evidence

The ideal empirical test of the predictions of the long-run analysis of Section 2 would rely on a cross-section regression relating the level of one of the endogenous variables (\( p \), \( S \), or \( r \)) in various cities to a measure of the extent of gradation of the city's property tax system as well as other explanatory variables. The rarity of graded tax systems, however, means that the data for such an exercise is simply unavailable. A less reliable but nevertheless suggestive test would involve a comparison of the features of two cities whose underlying characteristics are identical except for the presence of a graded property tax system in one and a standard system in the other. Archer (1972) presents what seems to be the only such comparison in the existing literature. The data he offers allow comparison of the level of improvements per acre in the downtown parts of the Australian cities of Sydney (where pure site value taxation has been in use in 1916) and Melbourne (which has a standard property tax system). Archer is careful to
recognize the importance of holding other things equal in making such a comparison, and he notes that the two cities have nearly identical size populations and downtown work forces. Table 1 reproduces Archer's table (1972, p. 32) listing the distribution of downtown building heights in the two cities.

Table 1

<table>
<thead>
<tr>
<th>Building Height</th>
<th>Sydney</th>
<th>Melbourne</th>
</tr>
</thead>
<tbody>
<tr>
<td>vacant site</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>building in construction</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>ground floor only</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 floors</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>3 floors</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>4 floors</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>5 floors</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>6 floors</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>7 floors</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>8 floors plus</td>
<td>64</td>
<td>42</td>
</tr>
<tr>
<td>Total</td>
<td>4 sites and 78 buildings</td>
<td>1 site and 75 buildings</td>
</tr>
</tbody>
</table>

Inspection of the table shows that a greater proportion of Sydney buildings fall in the 8-floors-plus category, confirming the prediction that site value taxation leads to a higher level of improvements per acre. Unfortunately, Archer does not present data which would allow testing of the theory's predictions regarding land values and housing prices.

5. Conclusion

This paper has presented rigorous analysis of two of the principal issues treated by the previous literature on site value taxation: long run effects and the incidence of short term gains and losses. The long-run analysis established that when a graded property tax
system is instituted over an area small enough so that the price of housing is unaffected, two distinct effects emerge in the normal case where $\partial \tau / \partial \theta < 0$: the level of improvements per acre rises, as does the value of land. The increase in land value is a surprising result given that gradation of the tax system increases the tax burden on land. When the tax zone is large enough so that gradation affects the price of housing, the outcome is somewhat different. Improvements per acre still rise when $\partial \tau / \partial \theta < 0$, but this is accompanied by a decline in the price of housing coupled with a decline in the value of land, with the latter effect emerging as long as housing demand is not highly elastic. While the impact of gradation on the level of improvements was well understood in the earlier literature, the predictions relating to land values and the price of housing are entirely new.

The contribution of the short-run analysis is to show that the windfall gains and losses which result from gradation of the tax system have a rather surprising spatial incidence. Contrary to a common impression, the most intensively developed parcels suffer windfall losses in the form of higher taxes, while the least intensively developed parcels benefit from windfall gains.

It must be pointed out that a principal limitation of the analysis in this paper is its partial equilibrium nature. For example, as a result of the implicit assumption that landlords are absentee (see footnote 9), the analysis in Section 2.2 fails to capture the impact of changes in rental income on the demand for housing. Although a general equilibrium model would capture such feedbacks, it is an open question whether the type of analysis done in this paper would be tractable in such a framework.
Finally, it should be noted that the paper has been silent on the normative question of whether a shift to a graded property tax system (or adoption of pure site value taxation) would be desirable. The answer to this question, however, is implicit in the optimal tax literature (see Sandmo (1976)) and affirms Henry George's original claim. Specifically, a principal lesson of optimal tax theory is that efficiency requires tax rates to be inversely related to demand (or in the case of inputs, supply) elasticities. When the relevant elasticity for a given commodity is zero, as in the case of land supply, it is optimal for society's tax burden to be carried by that commodity alone. As a result, modern theory vindicates George's belief in the efficiency of site value taxation.
Footnotes


2 Turvey (1955) also claims that the impact of site value taxation on land values is ambiguous, but his reasoning is unclear.

3 This expression results from rewriting \( \sigma = \frac{H_1 H_2}{H_1 H_2} \) in terms of the function \( h \).

4 Using (6), \( \frac{\partial \tau / \partial \theta} = \frac{\tau}{1+\theta} > 0 \).

5 Using (3), \( \frac{\partial \tau S / \partial \tau} = iS(1-\tau \sigma/(1+\tau)u_\mu) \).

6 Another way of expressing this result is that a necessary (but not a sufficient) condition for \( \partial \tau / \partial \theta < 0 \) to hold is that the relevant range of the improvements tax "Laffer curve" is upward sloping.

7 Direct \( \mu_\mu \) estimates can be found in Richman (1965), Gottlieb (1969), and Harriss (1970). Implied \( \mu_\mu \) estimates can be computed from data contained in the impact studies cited in the introduction.

8 Pollock and Shoup (1977) provide an empirical estimate of the magnitude of the impact on improvements using a model which posits a value for \( \partial \tau / \partial \theta \) and makes use of a particular parameterization of eq. (1).

9 Since \( \bar{T} \) is exogenous, the analysis ignores the possibility that a change in the tax system could affect the spatial size of the city. Analysis of such an effect, which requires use of a monocentric city model, unfortunately proved to be intractable.

10 The fact that housing demand does not depend on net land rent \( r \) (which determines the income of land owners) reflects the implicit assumption that land owners are absentee, living outside the tax zone.

11 Note that \( \partial h / \partial p = h' \partial S / \partial p \), with \( \partial S / \partial p = -h'/p' \) from (1).

12 The difference between the two expressions equals

\[ -(1+\theta)(\eta+1)\sigma/(1+\tau)\mu_\mu (\varepsilon-\eta) > 0. \]

13 When revenue decreases with \( \tau \) in the \( p \)-exogenous case, revenue can either increase or decrease with \( \tau \) when \( p \) is endogenous, yielding an ambiguous sign for \( \partial \tau / \partial \theta \).

14 When \( \Lambda' < 0 \), so that \( \partial \tau / \partial \theta > 0 \), this conclusion is reversed.
In the long run, the outcome in the present framework is similar to previous results. If the expression in the denominator of (10) is positive at each \( x \), \( \partial \tau / \partial \theta < 0 \) holds and \( dS / d\theta > 0 \) is satisfied for all \( x \). Although \( dr / d\theta \) can have either sign at a given location, total land value in the tax zone rises with \( \theta \) provided \( \partial \tau / \partial \theta < 0 \).

Since the analysis requires tax revenue to be held fixed, a substantial public spending difference between the cities (which is unlikely in any case) would invalidate its predictions.
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