Estimation of Risk Aversion Separated from Intertemporal Substitution

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Abstract

Why do estimates of the constant relative risk aversion coefficient in the von Neumann-Morgenstern utility function vary over a wide range of values (from 1 to 100 or more)? The vNM utility function constrains risk aversion to be equal to the inverse of the elasticity of intertemporal substitution. The non-expected utility function, developed by Epstein and Zin, and Weil, separates risk aversion from intertemporal substitution. If consumption is a constant fraction of wealth, however, the risk aversion coefficient in the vNM utility function is still identical to that in the non-expected utility function. We find that (i) when we use consumption data, the magnitude of the estimated risk aversion coefficient subject to the constraint of the vNM utility function is much smaller than that separated from intertemporal substitution, and (ii) when we circumvent the vNM constraint, the “consumption-based” Euler equation yields much larger estimates of risk aversion than does the “market portfolio-based” Euler equation.
Introduction

Estimation of the magnitude of the constant relative risk aversion (CRRA) coefficient has been a focal issue in recent years' empirical research of the consumption-based Capital Asset Pricing Model (the consumption CAPM henceforth) developed by Rubinstein (1976), Lucas (1978), Breeden and Litzenberger (1978), and Breeden (1979). Hansen and Singleton (1982 & 1983), using stock return data, suggest that the magnitude of the CRRA coefficient is approximately 1; Ferson (1983), using bond return data, reports a range from -1.4 to 5.4; and Harvey (1988), using interest yield spread data, presents results similar to those of Hansen and Singleton. Grossman, Melino and Shiller (1987), however, find that the magnitude of the CRRA coefficient could be 100 or more. It is an enigma that estimates of the CRRA coefficient vary over such a wide range.

The aforementioned studies assume that the representative consumer (the consumer henceforth) has a time additive, von Neumann-Morgenstern (vNM) power utility function, in which the elasticity of intertemporal substitution is identical to the inverse of the CRRA coefficient. Hence, it is not clear whether these studies estimate the CRRA coefficient or the intertemporal substitution parameter. This vagueness may well be epitomized by Hall (1988, p. 345):

I will refer to the parameter $\sigma$ as the intertemporal elasticity of substitution; I do not think this interpretation is at all controversial. Readers who have a stronger prior belief that the utility function is additively separable and that consumers follow the principle of maximizing expected utility will also interpret $\sigma$ as
the reciprocal of the coefficient of relative risk aversion. Others, such as myself, will avoid drawing any conclusions about risk aversion...

In contrast to the vNM utility function, the non-expected utility function (the EZ utility function henceforth) separates explicitly the consumer's preferences between risk aversion and intertemporal substitution. This non-expected utility function has been developed by Epstein and Zin (1989) and Weil (1990) upon the axiomatic work of Kreps and Porteus (1978). As we show below, if consumption is a constant fraction of wealth, the risk aversion coefficient in the vNM utility function is still identical to that in the EZ utility function. We can circumvent the constraint of the vNM utility function in estimating the risk aversion coefficient. In brief, we find, using consumption data, that estimates of the risk aversion coefficient without the constraint of the vNM utility function are much larger than those subject to the constraint of the vNM utility function.

Our study can be contrasted with the empirical work of Epstein and Zin (1990), which estimates the risk aversion and intertemporal substitution coefficients in the EZ utility function. First, we are primarily concerned with the effect of the constraint of the vNM utility function on the estimated risk aversion coefficient. Second, an important assumption in Epstein and Zin's empirical study is that a stock market index can serve as a surrogate of the market portfolio (i.e., the aggregate wealth). As has been well recognized since Roll's (1977) critique of this assumption in the empirical testing of the CAPM, our estimates of the risk aversion coefficient do not require a proxy of the unobserved market portfolio.
The remainder of this paper is organized as follows: Section 1 presents the Euler equation derived under the EZ utility function, Section 2 compares the risk aversion coefficients of the earlier studies, Section 3 presents our empirical results, and Section 4 concludes the paper.

1 The Model

Consider a pure exchange economy in which perishable consumption goods are produced by one productive unit. An infinitely lived, representative consumer makes a decision in each period, $t$, on the allocation of his wealth, $w_t$, between current consumption, $c_t$, and investment (in common stocks, bonds, and other assets) for the period. The consumer at time $t$ is assumed to maximize the EZ recursive utility, $U_t$, in equation (1):

$$U_t = \left[ c_t^{1-\delta} + \beta \{ E_t[U_{t+1}^{1-\gamma}] \}^{\frac{1-\delta}{1-\gamma}} \right]^{\frac{1}{1-\gamma}}$$

for all $t \geq 0$ (1)

where $E_t$ is the expectations operator conditional upon information available at time $t$, $\beta(0 < \beta < 1)$ is the discount factor, $\gamma(0 < \gamma \neq 1)$ is the risk aversion coefficient, and $\delta(0 < \delta \neq 1)$ is the intertemporal substitution parameter.$^2$ $^3$

$^1E_t[U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}$ is a certainty equivalent to next period’s random utility. As $\gamma$ increases, the certainty equivalent decreases; hence, $\gamma$ is interpreted as the measure of risk aversion.

$^2$If there is no uncertainty,

$$U_t = \left[ \sum_{r=0}^{\infty} \beta^r c_{t+r}^{1-\delta} \right]^{\frac{1}{1-\gamma}}$$

where the elasticity of intertemporal substitution is $1/\delta$. Hence, $\delta$ is interpreted as the degree of aversion to intertemporal substitution.

$^3$For the special case where $\gamma = 1$ and $\delta = 1$, see Epstein and Zin (1989).
The Euler equation, a necessary condition for the consumer’s optimal consumption-savings decision, derived from the EZ utility function is (see Epstein and Zin (1989)):

\[ \beta^\lambda E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\delta} (1 + r_{m,t+1})^{\lambda-1} \right] = 1 \quad \text{for all } i \]  

(2)

where \( \lambda \equiv (1 - \gamma)/(1 - \delta) \); \( r_{m,t+1} \) is the rate of return on the unobserved market portfolio from time \( t \) to time \( t + 1 \); and \( r_{i,t+1} \) is the rate of return on asset \( i \) from time \( t \) to time \( t + 1 \).

If \( \lambda \) is constrained to be equal to 1, i.e., \( \gamma = \delta \), equation (2) reduces to the conventional Euler equation derived under the time additive, vNM power utility function, \( U(c) = \frac{1}{1-\gamma} c^{1-\gamma} \), equation (3):

\[ \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} (1 + r_{i,t+1}) \right] = 1 \quad \text{for all } i. \]  

(3)

If it is assumed, following Epstein (1988) and others, that asset returns are i.i.d. over time,\(^4\) optimal consumption becomes a constant (time-invariant) fraction of wealth; \( c_t = kw_t(k \text{ is a constant}) \) for all \( t \). One can easily show that \( c_{t+1}/c_t = (1 - k)(1 + r_{m,t+1}) \), given that \( w_{t+1} = (w_t - c_t)(1 + r_{m,t+1}) \). By replacing \( 1 + r_{m,t+1} \) with \( (1 - k)^{-1}(c_{t+1}/c_t) \) in equation (2), we derive equation (4):

\[ \beta^\lambda \left( \frac{1}{1 - k} \right)^{\lambda-1} E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} (1 + r_{i,t+1}) \right] = 1 \quad \text{for all } i. \]  

(4)

Estimation of the parameters, \( \gamma, \delta, \) and \( \beta \), from equation (4) may require solving for the endogenous variable \( k \) as a function of these parameters. To

\(^4\)Researchers appear to believe that the i.i.d. return assumption is “not blatantly counterfactual” (Weil (1989, fn. 13, p. 409)).
avoid this requirement, we express equation (4) for the common stock, equation (4-a), and the bond, equation (4-b):

\[
\beta^\lambda \left( \frac{1}{1 - k} \right)^{\lambda - 1} E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} (1 + r_{s,t+1}) \right] = 1 \tag{4-a}
\]

\[
\beta^\lambda \left( \frac{1}{1 - k} \right)^{\lambda - 1} E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} (1 + r_{b,t+1}) \right] = 1 \tag{4-b}
\]

where subscripts \( s \) and \( b \) denote the common stock and the bond, respectively.

By subtracting equation (4-b) from equation (4-a), we derive the Euler equation for the excess return of the common stock over the bond, equation (5):

\[
E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} (r_{s,t+1} - r_{b,t+1}) \right] = 0. \tag{5}
\]

Equation (5) has several important implications for the empirical estimation of the risk aversion coefficient in the Euler equation. First, equation (5) can also be derived when the consumer has the time additive, vNM power utility function. Therefore, when we employ the excess return form of the conventional Euler equation (3), (i) the time additive, vNM power utility function yields the risk aversion coefficient of the EZ utility function, and (ii) the CAPM derived under the EZ utility function reduces to the conventional consumption CAPM.\(^5\)

Second, while equation (5) separates risk aversion (\( \gamma \)) from intertemporal substitution (\( \delta \)), equation (3) constrains \( \gamma \) to be equal to \( \delta \). Among the

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\(^5\)These results are in agreement with Kocherlakota’s (1990) conclusion that if consumption growth rates are i.i.d., (i) CAPMs derived under the EZ utility function have no more explanatory power than those derived under the vNM power utility function, and (ii) estimates of the CRRA coefficient in the vNM preferences are not the intertemporal substitution parameter.
earlier studies that estimate the risk aversion coefficient using consumption data, to our knowledge, only the study of Grossman, Melino and Shiller (1987) employs the excess return form of the Euler equation. The Euler equation tested by Hansen and Singleton (1982 & 1983), Ferson (1983), and Harvey (1988) is equation (3). We conjecture that the observed wide discrepancy in the magnitudes of the risk aversion coefficients (between Grossman, et al. and Hansen-Singleton and others) can be attributed to the constraint of the vNM utility function that requires \( \gamma \) to be equal to \( \delta \) (the vNM constraint henceforth).

Third, since equation (5) holds for any pair of risky assets, estimation of \( \gamma \) from equation (5) does not require a surrogate for the market portfolio.

Fourth, if the market portfolio can be measured by a stock market index, \( c_{t+1}/c_t \), in equation (5), is replaced by \((1 - k)(1 + r_{m,t+1})\), and, then, \( r_{m,t+1} \) is replaced by \( r_{s,t+1} \). This series of substitutions yields:

\[
E_t[(1 + r_{s,t+1})^{-\gamma}(r_{s,t+1} - r_{b,t+1})] = 0. \tag{6}
\]

Equation (6) is identical to that estimated by Brown and Gibbons (1985). We refer to equation (5) as the consumption-based Euler equation, and to equation (6) as the market portfolio-based Euler equation. Both equation (5) and equation (6) circumvent the vNM constraint; but, the consumption-based Euler equation generates larger estimated risk aversion than does the market portfolio-based Euler equation.
2 A Selective Comparison among Alternative Euler Equations

We assume that, conditional upon the information set at time $t$, rates of the stock return and consumption growth are jointly log normally distributed such that:

$$1 + r_{st} \equiv e^{\theta_s},$$
$$c_{t+1}/c_t \equiv e^{\theta_c},$$

where $\theta_i$, $i = s$ or $c$, is normally distributed with mean $\mu_i$ and variance $\sigma_i^2$; $E[e^{\theta_i}] = e^{\mu_i + \frac{1}{2}\sigma_i^2}$, and $\text{cov}(\theta_c, \theta_s) \equiv \sigma_{cs}$.

For the simple analysis, we further assume that $r_b$ is risk-free.$^6$ With these distribution assumptions, we solve each of equations (3), (5), and (6) for $\gamma$.

2.1 $\gamma$ subject to the vNM constraint

From equation (3), we derive equation (7):

$$\frac{1}{2} \sigma_c^2 \gamma^2 - [\mu_c + \sigma_{cs}] \gamma + [\mu_s + \frac{1}{2}\sigma_s^2 + \ln \beta] = 0. \quad (7)$$

There are potentially two solutions for $\gamma$. From the conventional Euler equation, suppose that $P_t > E_t \left[ \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} (P_{t+1} + D_{t+1}) \right]$, where $P_t$ is the ex-dividend price and $D_t$ is the dividend. This inequality, i.e., that the fundamental value is less than the market value, means that the left hand side of equation (7), $f(\gamma)$, is less than zero. The fundamental value decreases monotonically when $\gamma$ increases. Hence, if $f(\gamma)$ is less than zero, $\gamma$ should

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$^6$This assumption will be relaxed in our empirical analysis.
further decrease for the fundamental value to increase toward the market value. For this reason, we take the smaller of two solutions for equation (7), which is (denoted by $\gamma_1$):

$$
\begin{align*}
\gamma_1 &= \frac{\mu_c + \sigma_{cs} - \sqrt{(\mu_c + \sigma_{cs})^2 - 2\sigma_c^2(\mu_c + \frac{1}{2}\sigma_s^2 + \ln \beta)}}{\sigma_c^2} \\
&\approx \frac{\mu_s + \frac{1}{2}\sigma_s^2 - \ln(\frac{1}{\beta})}{\sigma_{cs} + \mu_c}
\end{align*}
$$

(8)

where the second equation is the Taylor approximation (i.e., $(x - z)^{1/2} \approx x^{1/2} - (1/2)x^{-1/2}z$).

2.2 $\gamma$ without the vNM constraint

The solution of equation (5) for $\gamma$ (denoted by $\gamma_2$) is:

$$
\gamma_2 = \frac{\mu_s + \frac{1}{2}\sigma_s^2}{\sigma_{cs} - \mu_b}.
$$

(9)

If the magnitude of the discount rate, $\ln(\frac{1}{\beta})$, is close to that of the interest rate, $\mu_b$, then, one can expect $\gamma_1$ to be much smaller than $\gamma_2$. Note that $\mu_c$, which is relatively much larger than $\sigma_{cs}$, appears in the denominator of equation (8) but not in that of equation (9).

2.3 $\gamma$ without the vNM constraint and with the market proxy assumption

The solution of equation (6) for $\gamma$ (denoted by $\gamma_3$) is:

$$
\begin{align*}
\gamma_3 &= \frac{\mu_s + \frac{1}{2}\sigma_s^2 - \mu_b}{\sigma_s^2}.
\end{align*}
$$

(10)

The numerators of $\gamma_2$ (equation 9) and $\gamma_3$ (equation 10) are the same, while their denominators are $\sigma_{cs}$ and $\sigma_s^2$, respectively. Since the stock market price index is historically much more volatile than consumption (i.e., $\sigma_s^2$ is much larger than $\sigma_c^2$), one can expect $\gamma_3$ to be much smaller than $\gamma_2$. 
3 Empirical Analysis

3.1 Data Base

The consumption data base, obtained from the CITIBASE data, consists of two alternative monthly, seasonally adjusted real consumption measures, nondurables (ND) and nondurables plus services (NDS), from January 1959 through December 1982. We divide consumption observations by corresponding population totals to derive per capita consumption. In order to avoid time aggregation bias in the consumption growth rates, we compute the quarterly growth rate of consumption, following Mankiw, Rotemberg, and Summers (1985), as the ratio of the monthly consumption measure in the last month of the quarter to that of the previous quarter. Our sample periods are from February 1959 through December 1982 for the monthly analysis, and from the second quarter of 1959 through the fourth quarter of 1982 for the quarterly analysis.

Stock returns, obtained from the CRSP Tape, are the value-weighted averages of the rates of return of the stocks listed on the New York Stock Exchange. Bond returns, obtained from the Salomon Brothers Bond Price Quotations, are one- and three-month Treasury bill rates at the beginning of each month and quarter. These nominal stock and bond returns are converted to real returns, using the implicit price deflators corresponding to the measures of consumption.

See, for example, Grossman, et al. (1987), Hall (1988), and Breeden, et al. (1989) for a discussion of the time aggregation bias in the growth rate of consumption using discrete time data. Time aggregation bias in consumption data is not likely to affect the discrepancy in the estimated risk aversion coefficients from alternative Euler equations.
3.2 Empirical Findings

Table 1 presents sample means, covariances, and correlations for our data.

Table 2 reports our computations for $\gamma_1$, $\gamma_2$, and $\gamma_3$, using sample means and covariances of consumption growth rates and asset rates of return reported in Table 1. $\gamma_1$ would represent the risk aversion coefficient estimated by Hansen and Singleton; $\gamma_2$ would represent that estimated by Grossman, et al.; and $\gamma_3$ would represent that estimated by Brown and Gibbons. As would be anticipated by our earlier analysis, the magnitude of $\gamma_2$ is much larger than those of $\gamma_1$ and $\gamma_3$. While the magnitude of $\gamma_2$ is in the range of 46 to 88, those of $\gamma_1$ and $\gamma_3$ are in the range of 0.9 to 2.1. The magnitudes of estimated $\gamma_1$, $\gamma_2$, and $\gamma_3$ are similar to those reported by Hansen-Singleton, Grossman, et al., and Brown-Gibbons, respectively.

Our findings confirm:

1. When we use consumption data, the magnitude of the estimated risk aversion coefficient subject to the vNM constraint is much smaller than that separated from intertemporal substitution.

2. When we circumvent the vNM constraint, the consumption-based Euler equation yields much larger estimates of risk aversion than does the market portfolio-based Euler equation.

Our computations in Table 2 assume log normal distributions for the rates of stock return and consumption growth. To avoid the distribution assumptions, Table 3 reports Hansen’s (1982) generalized method of moments (GMM) estimates of $\gamma$ in equation (5); the statistical results are
consistent with those for $\gamma_2$ in Table 2.\footnote{Our GMM estimates of $\gamma$'s in equation (3) and equation (6) are similar to those of Hansen-Singleton and Brown-Gibbons, respectively. To save space, we do not report these results.}

4 Summary and Conclusion

This paper addresses the enigma that the vNM expected utility-based estimates of risk aversion vary over a wide range of values. Since the vNM utility function constrains risk aversion to be equal to the inverse of the elasticity of intertemporal substitution, we use the non-expected utility function that separates risk aversion from intertemporal substitution. If consumption is a constant fraction of wealth, in its excess return form, the Euler equation derived under the non-expected utility function is equivalent to that derived under the time additive, vNM power utility function; thus, we can estimate the risk aversion coefficient from the vNM utility function. When we use consumption data, estimates of risk aversion separated from intertemporal substitution are much larger than those estimates of risk aversion constrained to be identical to aversion to intertemporal substitution. These results reinforce the claims of Weil (1989) and Kocherlakota (1990) that separating risk aversion from intertemporal substitution is unlikely to resolve the equity premium puzzle posed by Mehra and Prescott (1985).

Our findings should be interpreted cautiously; we do not suggest what the magnitude of risk aversion should be. However, if the use of the market portfolio-based Euler equation (e.g, Brown and Gibbons' use of only return
data) yields a “plausible” magnitude of estimated risk aversion, the use of consumption data and the constraint of the vNM utility function together appear to disguise a plausible estimate of risk aversion.
References


Table 1a
Description of Data: Sample Means and Covariances


<table>
<thead>
<tr>
<th></th>
<th>ND</th>
<th>NDS</th>
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<tbody>
<tr>
<td>$\mu_c$</td>
<td>$0.93 \times 10^{-3}$</td>
<td>$1.60 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\mu_s$</td>
<td>$2.74 \times 10^{-3}$</td>
<td>$2.53 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\mu_b$</td>
<td>$0.58 \times 10^{-3}$</td>
<td>$0.37 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\hat{\sigma}_c^2$</td>
<td>$6.30 \times 10^{-5}$</td>
<td>$2.00 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\hat{\sigma}_{cs}^2$</td>
<td>$5.40 \times 10^{-5}$</td>
<td>$3.50 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\hat{\sigma}_s^2$</td>
<td>$1.89 \times 10^{-3}$</td>
<td>$1.85 \times 10^{-3}$</td>
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<table>
<thead>
<tr>
<th></th>
<th>ND</th>
<th>NDS</th>
</tr>
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<tbody>
<tr>
<td>$\mu_c$</td>
<td>$2.68 \times 10^{-3}$</td>
<td>$4.73 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\mu_s$</td>
<td>$8.13 \times 10^{-3}$</td>
<td>$7.48 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\mu_b$</td>
<td>$1.74 \times 10^{-3}$</td>
<td>$1.09 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\hat{\sigma}_c^2$</td>
<td>$9.10 \times 10^{-5}$</td>
<td>$3.70 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\hat{\sigma}_{cs}^2$</td>
<td>$2.20 \times 10^{-4}$</td>
<td>$1.80 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\hat{\sigma}_s^2$</td>
<td>$7.65 \times 10^{-3}$</td>
<td>$7.44 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

a: ND is nondurables, and NDS is nondurables plus services.
Table 2*
Computing $\gamma$'s in Alternative Euler Equations

<table>
<thead>
<tr>
<th>consumption measure</th>
<th>data frequency</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\gamma_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ND</td>
<td>monthly</td>
<td>1.64</td>
<td>57.5</td>
<td>1.71</td>
</tr>
<tr>
<td>NDS</td>
<td>monthly</td>
<td>1.67</td>
<td>88.2</td>
<td>0.89</td>
</tr>
<tr>
<td>ND</td>
<td>quarterly</td>
<td>1.34</td>
<td>46.4</td>
<td>2.05</td>
</tr>
<tr>
<td>NDS</td>
<td>quarterly</td>
<td>1.36</td>
<td>56.2</td>
<td>1.06</td>
</tr>
</tbody>
</table>

a: $\gamma_1$ represents the risk aversion parameter estimated by Hansen and Singleton (1982 & 1983); $\gamma_2$ represents that estimated by Grossman, Melino and Shiller (1987); and $\gamma_3$ represents that estimated by Brown and Gibbons (1985). In computing $\gamma_1$, we assume that monthly $\beta$ is 0.998 and quarterly $\beta$ is 0.994.
\[
E_{t-1} \left[ \left( \frac{c_t}{c_{t-1}} \right)^{-\gamma} (r_{st} - r_{bt}) \right] = 0
\]  

(5)

Table 3a
GMM Estimates of $\gamma$ in Equation (5)


<table>
<thead>
<tr>
<th>NLAG</th>
<th>$\gamma$ (std. error)</th>
<th>$\chi^2$ (P-value)</th>
<th>NLAG</th>
<th>$\gamma$ (std. error)</th>
<th>$\chi^2$ (P-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80.82 (52.56)</td>
<td>0.46 (0.50)</td>
<td>1</td>
<td>118.77 (67.11)</td>
<td>0.24 (0.63)</td>
</tr>
<tr>
<td>2</td>
<td>61.72 (43.10)</td>
<td>2.38 (0.50)</td>
<td>2</td>
<td>121.50 (64.67)</td>
<td>0.54 (0.91)</td>
</tr>
<tr>
<td>4</td>
<td>33.77 (41.92)</td>
<td>5.87 (0.56)</td>
<td>4</td>
<td>59.75 (56.46)</td>
<td>6.05 (0.53)</td>
</tr>
<tr>
<td>6</td>
<td>25.24 (33.30)</td>
<td>11.16 (0.43)</td>
<td>6</td>
<td>48.34 (49.23)</td>
<td>12.37 (0.34)</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>NLAG</th>
<th>$\gamma$ (std. error)</th>
<th>$\chi^2$ (P-value)</th>
<th>NLAG</th>
<th>$\gamma$ (std. error)</th>
<th>$\chi^2$ (P-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>72.85 (46.97)</td>
<td>1.63 (0.20)</td>
<td>1</td>
<td>53.93 (43.83)</td>
<td>4.21 (0.04)</td>
</tr>
<tr>
<td>2</td>
<td>88.71 (37.49)</td>
<td>1.78 (0.62)</td>
<td>2</td>
<td>67.54 (42.54)</td>
<td>4.98 (0.17)</td>
</tr>
<tr>
<td>4</td>
<td>55.42 (30.29)</td>
<td>5.01 (0.66)</td>
<td>4</td>
<td>59.93 (38.74)</td>
<td>6.81 (0.45)</td>
</tr>
</tbody>
</table>

a: NLAG is the number of lagged variables of \((c_t/c_{t-1})\) and \((r_{st} - r_{bt})\) included in the instrumental variables. $\chi^2$ is the minimized value of the GMM criterion function. P-value is the probability that a $\chi^2$ random variate exceeds the sample statistic.