POWER SYSTEM TRANSIENT STABILITY ANALYSIS OF STIFF SYSTEMS USING THE MULTIRATE METHOD

BY

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THESIS

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ABSTRACT

Power system transient stability studies are complex and can take a substantial amount of time to simulate. The Multirate integration method is one way to speed up simulation studies. This thesis studies the effects of using a type of Multirate simulation method in a software package. In this study, the Multirate method is used mainly on simulating fast transients resulting from excitation models. Proper subinterval values to maintain stability in PowerWorld Simulator are established for exciter eigenvalue ranges. Subintervals are applied to exciter models of a 16,386 bus system based on exciter eigenvalues. Application of the Multirate method on exciters provides a time savings for each time step, with the largest being a 10 percent faster solution when using a one cycle per second time step.
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CHAPTER 1 INTRODUCTION

Modern power system dynamics require the use of computer simulation to ensure correct operation and planning. The simulation of the system in time domain provides critical insight into the security of the overall power system. This insight allows operators to limit the extent of system cascading failures and improve the overall reliability. As modern systems become more and more complicated with the introduction of FACTS device models, dynamic loads, and other power electronic models, the system becomes more susceptible to numerical instability.

Many different numerical methods exist to solve power system dynamic equations in [1]. The basic methods for numerical analysis of transient stability problems are divided into two main categories, implicit and explicit methods. These methods form the basis of computing transient stability response of a system. Each family of methods realizes its own advantages and pitfalls in regards to stability and reliability calculations. The strengths and weaknesses of each method change with the size and type of system in question. The main concern with each integration method is the size of the time step necessary for a numerically stable simulation.

When considering large systems of differential equations, computational savings becomes a key focus. The step size required for a stable simulation is a determining factor in the simulation speed. Faster simulations allow for prompt analysis of a system and any problems that arise from an event. With sufficiently fast simulations, real-time transient stability analysis can be performed by a system operator. A quicker reaction by the operator to any type of major event can limit or even prevent a major cascading failure in the network [2].

Typically, power systems incur a wide range of transients throughout the models. Fast transients are most often only a small fraction of the observed dynamics. Work has been done to combine the short and long term analysis into one method [3]. An integration method that solves these fast dynamics with a smaller time step than the slow transients will provide a more efficient
and sufficiently accurate simulation result. The Multirate method, introduced in [4], provides an integration technique suitable for systems exhibiting a wide variety of time responses.

The Multirate method was first applied to solve power system dynamic equations in [5]. Later this method was improved to include the power system analysis of FACTS devices as well [6]. For the Multirate method to be effective, the system must have a vast difference between the fast and slow transients. In order to use the Multirate method effectively, a partition scheme is developed in [7]. The error analysis of the Multirate method is considered in [8]. The Multirate method will be considered in more detail in Chapter 2.

Multirate simulation provides a promising integration method for large power studies. This thesis looks to analyze transient stability of power systems while focusing on integration methods. The goal of the thesis is to provide insight into the Multirate method and its uses in a power system simulation. The effects of using a type of Multirate method implemented in PowerWorld Simulator are explored. The results of Multirate implementation on exciters will be discussed in Chapter 5.

1.1 General Transient Problem

In power system stability analysis, many studies must be performed. Many stability analysis programs use simple step-by-step integration methods to solve dynamic models. While these methods have been around for a long time, they can still be applied to varieties of models. They are well known and understood by the system operators. This allows easy analysis of the simulation in practice. These methods also contain all the information necessary for the state variables in the system. The basis of many stability studies is how the system will respond to a major event disturbance. Multiple contingency situations may be studied such as variations of system faults. These faults can be modeled at different locations in the system to find the effect
on the system. Stability studies are useful in the planning and operation of the system under the conditions specified.

Power system equations exhibit nonlinear qualities, which become more vexing as the system becomes larger. The equations also include a dynamic behavior in the system itself to further complicate the solution process, necessitating basic algorithms for computer simulation.

The overall system model contains set of differential equations as described in [1]

\[ \dot{y} = f(y, x) \quad (1.1) \]

and a set of algebraic equations

\[ 0 = g(y, x) \quad (1.2) \]

In set (1.1), each machine in the system is represented with its differential equation. A machine’s subset of equations is uncoupled from the other machines in the network. These equations include any dynamically modeled components and the internal dynamics of the generators and their respective control systems. In set (1.2), the stator equations for each machine are represented. Also included in set (1.2) are the equations describing the transmission network.

Equation (1.1) has a specific form associated with it. This form is semi-nonlinear and is shown in (3).

\[ \dot{y} = f(y, u) = A \cdot y + B \cdot u \quad (1.3) \]

The matrix \( A \) is a sparse, block diagonal, square matrix. \( B \) is rectangular, sparse, and block. If saturation is not modeled, both \( A \) and \( B \) remain constant [1].

The algebraic set is divided into the sparse bus admittance matrix, (1.4), and forcing function (1.5).

\[ I(E, V) = Y \cdot V \quad (1.4) \]
\[ u = u(E, V) \]  \hspace{1cm} (1.5)

The vector \( I \) contains the bus current injections of the network. The injections are a function of the bus voltage, and the stator internal and terminal voltage in the network reference frame for a load and generator, respectively. The equation (1.5) is solved and inserted into (1.3) [1].

Figure 1.1 depicts the model of the machines and how they are connected to the network. It is a representation of the aggregated power system components and shows how they interact with each other. More components can be added to the drawing as they are added to the system model.

Interfacing the equations between the state equations and the dynamic equations becomes a necessary task. For interfacing to happen, equations must share the same reference frame. The classical machine model equations and their transformation into the industry standard reference frame are detailed in [9] and [10]. Once all the model equations share the same reference frame, interfacing of the equations becomes possible with interactions between different models shown in Figure 1.1.

Typically a stability analysis is an initial value problem of the equations (1.1) and (1.2). The solution is found through the use of a numerical integration method of choice. In a stability study, the network achieves different states at different times. Initially the system is in steady state equilibrium. This initialization process becomes an important step to ensure stability. If an initial operating point of a machine is not obtained correctly, the integration method may not converge to a proper value [1]. A discussion of the calculation of initial conditions appears in the appendix of [1].
At a specified time a fault or some other major event disturbance acts on the system. The system reacts to the disturbance, and the system structure changes as the network tries to control the event. Depending on what actions are taken by the network, a set of equations changes during this time. Once the fault is cleared, the system of equations reacts once again to find a stable equilibrium point. If the fault is not cleared quickly enough, the system will not find a stable equilibrium point in the post-fault network and the system is considered unstable.

When solving the system during the timeline described above, the simplest integration method necessary is used. In this way computation and solution complexity is preserved. There are many different families of integration methods. Depending on the dynamics of the system, one method may be better than another. Two of the simplest families traditionally used in stability analysis software are explicit and implicit methods. Each has its own advantages and disadvantages which will be discussed in Chapter 2.
CHAPTER 2 NUMERICAL INTEGRATION METHODS

2.1 Explicit Methods

Explicit methods are used in most power system software packages. These integration methods are easy to implement; however, there are many issues with numerical stability of the solution method. Stiff systems create numerical stability of explicit methods. This stability occurs when there is a large difference between the eigenvalues of the Jacobian matrix of a system. This difference reflects a part of the system that reacts faster to a disturbance than other areas of the system.

When determining the stability of the numerical integration scheme, the two error terms are considered. The truncation error concerns the error that occurs from the discrete approximation in the algorithm. Round-off error arises from the limitations of the finite word length of the computer. For the algorithm to be considered numerically stable, the total local error must be decreasing [11].

2.1.1 Euler

The explicit Euler method is the least complex and least accurate solution method. The entire algorithm is shown in equation (2.1).

\[ y_n = y_{n-1} + h\dot{y}_{n-1} \]  (2.1)

This method requires a very small step size on anything but the simplest of models. This method typically takes a long time to converge due to its trivial implementation [1].

2.1.2 Runge-Kutta

The Runge-Kutta algorithm is displayed below in equations (2.2)-(2.6). This method is discussed in detail by Dandeno and Kundur [12]. There are many different variations of this method. One can use as many orders as they deem necessary. Typically a fourth-order is
necessary to handle stiff systems [1]. In this method, the calculated value in (2.6) is the weighted average of the previous equations (2.2)-(2.5).

\[ k_1 = hf(y_{n-1}, u_{n-1}) \]  
\[ k_2 = hf(y_a, u_a) \text{ with } y_a = y_{n-1} + k_1/2 \]  
\[ k_3 = hf(y_b, u_b) \text{ with } y_b = y_{n-1} + k_2/2 \]  
\[ k_4 = hf(y_r, u_r) \text{ with } y_r = y_{n-1} + k_3 \]  
\[ y_n = y_{n-1} + (k_1 + 2k_2 + 2k_3 + k_4)/6 \]  

2.1.3 Stability Study

For a numerical stability study of the two explicit methods above, a small example from [11] is used. In this example, the system includes two coupled equations with an initial value provided. The eigenvalues of the system are changed to allow analysis of various systems. Stiff, moderately stiff, and non-stiff systems are considered. Stiffness is the ratio of the largest to the smallest system eigenvalues, as shown in (2.7). Equation (2.8) gives the general system for the stiff example and Equation (2.9) shows the exact solution of the system.

\[ \text{stiffness} = \frac{|\lambda_{\text{max}}|}{|\lambda_{\text{min}}|} \]  
\[ \begin{bmatrix} \dot{w} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 998 & 1998 \\ -999 & -1999 \end{bmatrix} \begin{bmatrix} w \\ v \end{bmatrix} \text{ with } \begin{bmatrix} w(0) \\ v(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \]  
\[ \begin{bmatrix} w(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} 2e^{-t} - e^{-1000t} \\ -e^{-t} - e^{-1000t} \end{bmatrix} \]  

The main criterion for the determination of stability was for the steady state value to be within a certain error tolerance. Also considered was the peak value and the time at which this value occurred in the simulation run, as well as the time where the steady state value last enters
the range of error tolerance. Different time steps were considered to determine the accuracy and stability of the method as well.

The results of the error analysis for each method can be seen in Table 2.1 and Table 2.2. In each of the tables, one can see that the error increases with the step size. Also, as the stiffness of the system of equations increases, the error increases as well. To help illustrate these trends, the differences in the error of these two methods are displayed in Table 2.3. The negative values in Table 2.3 indicate a smaller error in the Euler solution. As is evident from inspection of Table 2.3, the Euler method is more accurate than the fourth order Runge-Kutta method for the smaller time steps. As the time step becomes larger, the Runge-Kutta solution provides a more accurate representation of the exact solution. The system stiffness seems to affect each solution method in similar ways. For each solution method, the error increases with the system stiffness. With the increased system stiffness, the explicit methods lose numerical stability and fail to converge.

Table 2.1 Solution Error of Euler Method for Different Step Sizes

<table>
<thead>
<tr>
<th>Stiffness</th>
<th>1.00E-05</th>
<th>5.00E-05</th>
<th>1.00E-04</th>
<th>5.00E-04</th>
<th>1.00E-03</th>
<th>5.00E-03</th>
<th>1.00E-02</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.670817</td>
<td>0.670852</td>
<td>0.670896</td>
<td>0.671479</td>
<td>0.67215</td>
<td>0.677573</td>
<td>0.684477</td>
</tr>
<tr>
<td>50</td>
<td>0.805342</td>
<td>0.805374</td>
<td>0.805413</td>
<td>0.80592</td>
<td>0.806505</td>
<td>0.81121</td>
<td>0.817098</td>
</tr>
<tr>
<td>100</td>
<td>0.814389</td>
<td>0.814419</td>
<td>0.814457</td>
<td>0.814949</td>
<td>0.815517</td>
<td>0.852526</td>
<td>0.975166</td>
</tr>
<tr>
<td>500</td>
<td>0.926702</td>
<td>0.927564</td>
<td>0.92865</td>
<td>0.937711</td>
<td>0.950199</td>
<td>Inf</td>
<td>Inf</td>
</tr>
<tr>
<td>1000</td>
<td>0.955152</td>
<td>0.95626</td>
<td>0.957657</td>
<td>0.969927</td>
<td>0.997502</td>
<td>Inf</td>
<td>Inf</td>
</tr>
</tbody>
</table>

Table 2.2 Solution Error of Runge-Kutta Method for Different Step Sizes

<table>
<thead>
<tr>
<th>Stiffness</th>
<th>1.00E-05</th>
<th>5.00E-05</th>
<th>1.00E-04</th>
<th>5.00E-04</th>
<th>1.00E-03</th>
<th>5.00E-03</th>
<th>1.00E-02</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.670822</td>
<td>0.670875</td>
<td>0.670942</td>
<td>0.671249</td>
<td>0.67169</td>
<td>0.675258</td>
<td>0.67981</td>
</tr>
<tr>
<td>50</td>
<td>0.805346</td>
<td>0.805393</td>
<td>0.805452</td>
<td>0.80572</td>
<td>0.806121</td>
<td>0.809278</td>
<td>0.813213</td>
</tr>
<tr>
<td>100</td>
<td>0.814392</td>
<td>0.814438</td>
<td>0.814495</td>
<td>0.814761</td>
<td>0.815141</td>
<td>0.85205</td>
<td>0.97025</td>
</tr>
<tr>
<td>500</td>
<td>0.926702</td>
<td>0.927566</td>
<td>0.928652</td>
<td>0.937697</td>
<td>0.950163</td>
<td>Inf</td>
<td>Inf</td>
</tr>
<tr>
<td>1000</td>
<td>0.955152</td>
<td>0.95626</td>
<td>0.957659</td>
<td>0.969917</td>
<td>0.997002</td>
<td>Inf</td>
<td>Inf</td>
</tr>
</tbody>
</table>
Table 2.3 Error Difference between Explicit Solution Methods

<table>
<thead>
<tr>
<th>Stiffness</th>
<th>1.00E-05</th>
<th>5.00E-05</th>
<th>1.00E-04</th>
<th>5.00E-04</th>
<th>1.00E-03</th>
<th>5.00E-03</th>
<th>1.00E-02</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-4.6E-06</td>
<td>-2.3E-05</td>
<td>-4.6E-05</td>
<td>0.00023</td>
<td>0.00046</td>
<td>0.002315</td>
<td>0.004667</td>
</tr>
<tr>
<td>50</td>
<td>-3.8E-06</td>
<td>-1.9E-05</td>
<td>-3.8E-05</td>
<td>0.000192</td>
<td>0.000385</td>
<td>0.001933</td>
<td>0.003885</td>
</tr>
<tr>
<td>100</td>
<td>-3.8E-06</td>
<td>-1.9E-05</td>
<td>-3.8E-05</td>
<td>0.000188</td>
<td>0.000376</td>
<td>0.001933</td>
<td>0.003885</td>
</tr>
<tr>
<td>500</td>
<td>-2.8E-07</td>
<td>-1.4E-06</td>
<td>-2.8E-06</td>
<td>1.43E-05</td>
<td>3.6E-05</td>
<td>Inf</td>
<td>Inf</td>
</tr>
<tr>
<td>1000</td>
<td>-1.9E-07</td>
<td>-9.6E-07</td>
<td>-2E-06</td>
<td>1.04E-05</td>
<td>0.000499</td>
<td>Inf</td>
<td>Inf</td>
</tr>
</tbody>
</table>

Stiff systems require very small time steps to converge to a solution when using explicit integration methods. When a system is stiff, large changes occur in the solution in a short period of time. This numerical instability becomes apparent in the bottom right corner of Table 2.3, where the system is considered stiff, and the time step has become large. Both explicit methods experience a large error in the first few iterations. This error is too large and the integration solution diverges. For stiff systems, explicit methods become increasingly inefficient and unstable. The fourth order Runge-Kutta method provides a more accurate solution than the Euler method for larger time steps. For large systems, this can provide valuable numerical stability and solution time savings.

2.2 Implicit Methods

In implicit integration methods, the equations are solved simultaneously with the solutions of some equations used within a single iteration. Interpolation is used in implicit methods. This requires the functions to use points that are not known at the time $t$ of the simulation. This requires them to become variables within the simulation [13].

Implicit methods allow for much larger time steps to be taken on stiff problems. This allows for greater computational savings over explicit integration methods. However, on non-stiff problems, the implicit methods provide savings similar to those of higher order explicit methods [1]. Implicit methods are not used often in power system software due to problems
associated with limits. When saturation limits are modeled into the system, the matrices $A$ and $B$ of (1.3) have nonlinearities introduced into them. This prevents the method from having a direct solution and convergence is questionable [1]. Many times the saturation limits are ignored with implicit methods, thus creating some inaccuracies in the solution.

2.3 Multirate Method

The Multirate method uses different time steps in the numerical integration scheme to solve a power system. A small time step is used for the fast changing equations while a multiple of the small time step is used for the relatively slower changing equations. The implementation is shown in Figure 2.1 [7]. As one can see from Figure 2.1, the ratio of the fast time step to the slow time step is 4. Therefore, the equations for the fast changing variable must be solved at points where the larger, slower system has not been solved. The fast equations must use an interpolated solution value, as necessary, from these slow equations.

![Figure 2.1 Multirate Method Implementation](image)

2.3.1 Multirate Algorithm

The algorithm is as follows [7]:

1. Initialize the system with initial conditions.
2. Choose small time steps $h_1$ for the fast equations and a multiple $4h_1$ for the slow equations.
3. For each time step $t$, do the following:
   a. Solve the fast equations using $h_1$ steps.
   b. Use interpolated values from the slow equations.
   c. Update the variables using the fast equations.
   d. Solve the slow equations using $4h_1$ steps.
4. Repeat until the simulation is complete.

This approach allows for efficient simulation of power systems by adapting to the varying dynamics of the system.
1) The slow state variables are predicted at the time $t + H$ as shown in (2.9) where the $p$ superscript denotes a predicted value.

$$y_{t+H}^p = y_t + H\dot{y}_t$$ (2.10)

2) Predict the slow algebraic variables at $t + H$. Equation (2.10) is the linear interpolation.

$$\hat{z}_y(t + ih) = \frac{i}{m}(z_{y,t+H}^p - z_{y,t}) + z_{y,t}$$ (2.11)

3) Integrate fast components for $i=1,2,\ldots,m-1$ in equations (2.12) and (2.13).

$$\dot{x}(t + ih) = f(\hat{y}(t + ih), x(t + ih), \hat{z}_y(t + ih), z_x(t + ih))$$ (2.12)

$$0 = h_x(\hat{y}(t + ih), x(t + ih), \hat{z}_y(t + ih), z_x(t + ih))$$ (2.13)

The integration time step is $h$. The interpolated values are found using (2.14) and (2.15).

$$\hat{y}(t + ih) = \frac{i}{m}(y_{t+H}^p - y_t) + y_t$$ (2.14)

$$\hat{z}_y(t + ih) = \frac{i}{m}(z_{y,t+H}^p - z_{y,t}) + z_{y,t}$$ (2.15)

4) Integrate the fast and slow components together at the large time step using the respective time steps for each component.

5) If $|y_{t+H}^p - y_{t+H}| > \varepsilon$, $y_{t+H}^p = y_{t+H}$; else $t = t + H$.

6) Go to Step 1

2.3.2 Equation Partitioning

For the algorithm described above to be valid, the equations must be properly partitioned into a fast and slow subsystem. If there is too much coupling between the two sets of equations, the integration method becomes inefficient and can eventually become unstable.
The Multirate method achieves speed-up when the ratio of the number of fast variables to the number of slow variables is great. Since most power systems have a large number of buses compared to generators, there will be a greater number of algebraic variables than state variables. This naturally warrants using the Multirate method for a large system because the ratio of slow to fast variables is guaranteed to be large [7].

Besides the ratio of slow to fast components, the local truncation error plays a strong role in determining the partitioning of the state variables. When the local truncation error is large, the state variable is changing rapidly. Therefore, when state variables have a large local truncation error (LTE), they can be considered fast variables. The error analysis of the Multirate method is detailed in [8].
CHAPTER 3 METHODOLOGY

3.1 Procedure

To study the Multirate method, power system transient stability simulations are performed. PowerWorld Simulator is the simulation software package used in the transient analysis studies. This software package includes a feature to use “subintervals” on any control or machine model applied to the system. The subintervals allow the operator to use smaller time steps when the model is solved in the integration method used in the software.

Two main power systems are used in the study. The first system is a small 4-bus system and the other is a large industry modeled power system. The transient stability tool of the simulator is mainly used in the simulation and analysis of the two systems. Within each system, individual control models and generator models with known fast transients are considered. The behavior of the system’s stability due to the nature of the models is a major focus.

To start, a default simulation with default model parameters is run using a small time step. This gives a system solution with which to compare subsequent simulation runs. Bus voltage magnitude and voltage angle data are used as a basis for demonstration of system stability. A simulation is considered stable when the simulation in question converges to within 5% of the default value for the bus voltage and 10% of the default value for the bus voltage angle within a reasonable time frame.

Basic control method analysis is used to help determine which control models have a significant effect on the stability of the system. Three exciter models are studied as they seem to have a large impact on the overall system stability with regard to the large system. These include the EXST1_GE, EXAC1, and EXDC2_GE models. Block diagrams of these models are from PowerWorld Simulator but are publicly available and can be seen in Figures A.1-A.3. Each
control model contributes fast decaying transients. These transients arise from feedback loops
determined as fast using basic control theory.

Parameter variation is executed on the models to determine how the system reacts to
perturbations. The changing of parameters allows the user some insight into the control model.
When a parameter is changed, the eigenvalues of the model will change. The insight from basic
control theory analysis of the model block diagrams helps the user regulate which parameters
have the greatest effect on eigenvalues.

After any parameters are changed, a new simulation is run. Bus voltage magnitude and
voltage angle data of the bus with the largest eigenvalue are considered. With the constraints
mentioned before, the simulation is determined as stable or unstable. For unstable simulations,
the next step is to apply subintervals to the model contributing to the large eigenvalue. The
subintervals considered are 2, 4, 8, 16, 32, 64, and 128. These numbers correspond to the ratio
of the system time step to the smaller time step to be used on the individual model. Results of the
simulations on each system are discussed in the following sections.

3.2 Eigenanalysis

Eigenvalue analysis becomes an important tool when studying large power systems. It
allows an operator to understand the various dynamics related to a generator and its associated
control models. The results of a stability study include the A matrix. Figure 3.1 shows the
general form of equation (1.3) [1]. As seen in Figure 3.1, the A matrix contains the coupling
between the different components of the machine and control models, the state variables.
To see how the relationships of the components in the state matrix affect the eigenvalues of the system, a participation matrix is calculated. A general calculation of the participation matrix is given in [14]. First, a right n-column eigenvector, $\varphi_i$, and a left n-row eigenvector, $\psi_i$, are calculated. The left and right eigenvectors of state matrix $A$ associated with eigenvalue $\lambda_i$ are calculated using (3.1) and (3.2), respectively. Equation (3.3) gives the matrix of the right eigenvectors, while (3.4) shows the matrix of left eigenvectors for state matrix $A$. Both eigenvectors are necessary to perform the participation factor calculations.

![Figure 3.1 General Machine Equation Form [3]](image-url)
\[ A\varphi_i = \lambda_i \varphi_i \]  
(3.1)

\[ \psi_i A = \lambda_i \psi_i \]  
(3.2)

\[ \Phi = [\varphi_1 \varphi_2 \varphi_3 \ldots \varphi_n] \]  
(3.3)

\[ \Psi = [\psi_1^T \psi_2^T \psi_3^T \ldots \psi_n^T]^T \]  
(3.4)

Once the eigenvectors are known, a participation matrix can be calculated. The participation matrix is given by (3.5) with each element calculated by (3.6).

\[ P = [p_1 p_2 p_3 \ldots p_n] \]  
(3.5)

\[ P = \begin{bmatrix} p_{11} \\ p_{21} \\ \vdots \\ p_{n1} \end{bmatrix} = \begin{bmatrix} \varphi_{11} \psi_{11} \\ \varphi_{21} \psi_{12} \\ \vdots \\ \varphi_{n1} \psi_{1n} \end{bmatrix} \]  
(3.6)

Participation factors helps an operator pinpoint the origin of any problematic eigenvalues.

With the block diagrams of the model controls, the operator can adjust parameters of the component to regulate an eigenvalue. Large negative eigenvalues and positive real valued eigenvalues create problems in many power systems. With the insight of the participation factors, eigenvalues can be better controlled to create a more stable system.
CHAPTER 4 SYSTEMS OF STUDY

4.1 Small System

The first system considered is a variation of the four bus case used as example 11.3 in [15]. Figure 4.1 shows the one-line diagram of the system. The case represents a single-machine infinite bus example. The system has a 100 MVA per unit base. The generator is connected to the infinite bus through a transformer and three transmission lines. The transmission lines are lossless, exhibiting only line reactance. The generator is supplying the infinite bus per unit power at unity voltage.

![Figure 4.1 Four Bus Single Machine Infinite Bus System](image)

Multiple control models are applied to the generator for this study. The main machine model used in this system study is the GENROU model. This model is supported by many simulation packages to characterize round rotor synchronous generators. A block diagram for this model is displayed in Figure A.4.

Excitation control models are the main type of control implemented on the generator. Standard model systems are defined in [14]. Excitation systems are used to stabilize the generator bus voltage. Exciters provide voltage stability through control of the field voltage,
With adjustments to the field voltage, the field current changes and the terminal voltage of the generator will adjust as well [10]. For each simulation, the same contingency event is used. At $t=1$ second, a balanced 3-phase fault is applied to bus 3. The fault is cleared by opening the lines from 3 to 4 and 3 to 1 at time $t=1.34$ seconds.

The default case uses the GENROU model with default parameters and exciter model EXST1_GE. The block diagram for the exciter is shown in Figure 4.2. For the default case, the maximum eigenvalue magnitude is 2602. The exciter exhibits a participation factor of one for this eigenvalue and the parameter values are shown in Table 4.1.

**Exciter EXST1_GE**

![EXST1_GE Exciter Block Diagram](image)

Table 4.1 Bus 4 Exciter EXST1_GE Default Parameters

<table>
<thead>
<tr>
<th>$V_{i_{\text{Max}}}$</th>
<th>$V_{i_{\text{Min}}}$</th>
<th>$T_c$</th>
<th>$T_b$</th>
<th>$K_a$</th>
<th>$T_a$</th>
<th>$V_{r_{\text{Max}}}$</th>
<th>$V_{r_{\text{Min}}}$</th>
<th>$K_f$</th>
<th>$T_f$</th>
<th>$V_{a_{\text{Max}}}$</th>
<th>$V_{a_{\text{Min}}}$</th>
<th>$I_{lr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>200</td>
<td>0.01</td>
<td>3.6</td>
<td>0</td>
<td>0.5</td>
<td>0.4</td>
<td>99</td>
<td>-99</td>
<td>99</td>
</tr>
</tbody>
</table>

**4.2 Large System**

To perform a more thorough study of the Multirate method, a large power system is considered. For the large system, a well-known industry model is used. This model consists of
16,386 buses, 3,246 generators, and 8,106 system loads. Two contingency events occur in the system of study. Two generators are opened at time $t = 1.0$ seconds. The generators remain open and the system settles to a steady state. Simulations are run for 35 seconds to obtain a reasonable steady state solution. In this system, three buses are the main focus of the study due to the large eigenvalues and exciter models implemented at the generators. Bus 73285 implements an EXST1_GE exciter with parameters in Table 4.2 and an eigenvalue magnitude of 3770.0168. Bus 43375 uses an EXAC1 exciter with parameters in Table 4.3 and an eigenvalue magnitude of 2571.4131. Exciter model EXDC2_GE is used on bus 33283 with parameters given in Table 4.4. This bus has a maximum eigenvalue magnitude of 1052.6558. Each of the maximum eigenvalues has an exciter participation factor of one for these buses.

Table 4.2 Bus 73285 Exciter EXST1_GE Default Parameters

<table>
<thead>
<tr>
<th>ViMa</th>
<th>ViMi</th>
<th>Tc</th>
<th>Tb</th>
<th>Ka</th>
<th>Ta</th>
<th>Vrma</th>
<th>Vrmi</th>
<th>Kc</th>
<th>Kf</th>
<th>Tf</th>
<th>VaMa</th>
<th>VaMi</th>
<th>Ilr</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.0</td>
<td>-99.0</td>
<td>1.0</td>
<td>1. 0</td>
<td>85</td>
<td>0. 2</td>
<td>5.9</td>
<td>-5.9</td>
<td>0. 0</td>
<td>0. 03</td>
<td>0. 4</td>
<td>5.9</td>
<td>-5.9</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 4.3 Bus 43375 Exciter EXAC1 Default Parameters

<table>
<thead>
<tr>
<th>Tb</th>
<th>Tc</th>
<th>Ka</th>
<th>Ta</th>
<th>Vrmax</th>
<th>Vrmi</th>
<th>Te</th>
<th>Kf</th>
<th>Tf</th>
<th>Kc</th>
<th>Kd</th>
<th>E1</th>
<th>SE(E1)</th>
<th>E2</th>
<th>SE(E2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>1</td>
<td>1</td>
<td>25</td>
<td>13.5</td>
<td>0.5</td>
<td>0.03</td>
<td>0.06</td>
<td>0.06</td>
<td>0.5</td>
<td>0.0</td>
<td>1. 5</td>
<td>0.2</td>
<td>2.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.4 Bus 33283 Exciter EXDC2_GE Default Parameters

<table>
<thead>
<tr>
<th>Tr</th>
<th>Ka</th>
<th>Ta</th>
<th>Vrmax</th>
<th>Vrmin</th>
<th>Te</th>
<th>Kf</th>
<th>Tf1</th>
<th>Tf2</th>
<th>E1</th>
<th>SE(E1)</th>
<th>E2</th>
<th>SE(E2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.022</td>
<td>2974.3</td>
<td>0.1</td>
<td>37.15</td>
<td>0</td>
<td>1.16</td>
<td>0.21</td>
<td>0.6</td>
<td>1.16</td>
<td>0.75</td>
<td>4.327</td>
<td>1.0</td>
<td>5.77</td>
</tr>
</tbody>
</table>
CHAPTER 5 DISCUSSION OF RESULTS

5.1 Eigenvalue Limits

Insight into the subinterval effectiveness of maintaining system stability was gained during the simulations on the small system. During these simulations, parameter variation of all generator and control models provided different insights into the system. In each case, exciter models were used with the generator. The exciter models EXST1_GE, EXAC1, and EXDC2_GE proved to be the most problematic in the system. From the eigenvalue and participation matrix analysis, the exciter was found to cause stability problems in each simulation studied. The participation matrix also gives the location of the large eigenvalue on the exciter model block diagram.

Once the origin of the large eigenvalue of the system is known, the parameters contributing to the large eigenvalue are varied. First, the EXST1_GE exciter was studied. The first feedback loop shown in Figure 4.1 caused the greatest response to the large eigenvalues of the system. Parameters of this feedback loop were varied to change the magnitude of the largest eigenvalue. A subinterval was placed on the exciter system and the feedback loop parameters were changed until the system lost stability. This was repeated for each subinterval value and for the EXAC1 and EXDC2_GE exciter models as well.

In each exciter case, the results were very similar. The exciter contributed a participation factor of 1 to the largest eigenvalue magnitude in each case. Table 5.1 shows the upper eigenvalue limits for stability of the system when using the specified subinterval on the excitation model and integration time step. The eigenvalue stability limits for each of the exciter models were very close to one another and the values in the table are the limit averages.
Table 5.1 Eigenvalue Limits of Subintervals

<table>
<thead>
<tr>
<th>Time Step</th>
<th>Subinterval</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 (cycles/sec)</td>
<td>235.94</td>
<td>480.19</td>
<td>9588.24</td>
<td>1911.06</td>
<td>3852.20</td>
<td>7687.35</td>
<td>15382.42</td>
<td></td>
</tr>
<tr>
<td>0.5 (cycles/sec)</td>
<td>490.23</td>
<td>977.34</td>
<td>1939.41</td>
<td>3852.20</td>
<td>7707.40</td>
<td>16402.42</td>
<td>38202.47</td>
<td></td>
</tr>
<tr>
<td>0.25 (cycles/sec)</td>
<td>956.32</td>
<td>1926.91</td>
<td>3854.20</td>
<td>7702.35</td>
<td>15382.42</td>
<td>30762.46</td>
<td>6102.48</td>
<td></td>
</tr>
</tbody>
</table>

A similar procedure to test eigenvalue limits was used on the large system. Three separate buses, each with a different exciter model were considered. For each exciter, the subinterval eigenvalue limits were tested. In each case, the results of Table 5.1 and the small system were confirmed. When subintervals are used on the integration of excitation control models, well-defined limits exist to determine how large the subinterval can be for a certain time step.

When an exciter model violates the eigenvalue limit found for the specified subinterval time step, a steady state solution is impossible and numerical instability occurs. The coupling of the fast transient due to the exciter allows exciter models to be considered almost separately from the rest of the system. The coupling of the exciter was very weak in each case. The participation factor of the largest eigenvalue was one or very nearly one in each case examined. Due to this weak coupling, stability problems when using subintervals can be attributed to the local truncation error becoming too large during the Multirate integration of the exciter.

5.2 Large System Results

After the eigenvalue limits for the subintervals on exciter models were verified, the performance of the Multirate method on the system as a whole was considered. Once again, the stability procedure described before was used on buses 73285, 43375, and 33283. Along with
the convergence of the bus voltage magnitude and voltage angle, the integration solution details of the system are used to judge the performance of a solution run. The solution results include the overall simulation time, maximum angle difference, time of the maximum angle difference, total number of Newton solutions, number of Jacobian factorizations, and the number of forward and backward substitutions. For each simulation run, the same computer was used so as to keep consistency with the simulation time.

A default simulation run with a time step of 0.1 cycles was run to give a steady state value to prove convergence of the bus voltage magnitude and voltage angle. The eigenvalue limits in Table 5.1 help when determining an appropriate subinterval to place on the exciter model. The same three system time steps were examined. For each time step, a myriad of different combinations were considered when placing subintervals on system components including machine models and stabilizers. No definite conclusion or intuition was gained through these simulations, so the focus of the study returned to studying the effect of the Multirate method on excitation controls.

The default solution details for each time step in question are shown in Table 5.2. For the default case, no subintervals were used and the system showed convergence to a steady state value. As expected, a major solution speed-up is observed between the small 0.25 cycle time step and the 0.5 and 1.0 cycle time steps. The solution time savings between the 0.5 cycle and 1.0 cycle time step is much less, however. The accuracy related to the maximum angle difference seemed similar between the different time steps. The number of substitutions and the number of Newton solutions followed a trend similar to the simulation time savings. Only the large time step cut down on the number of Jacobian factorizations in the simulation solution.
Table 5.2 Default Simulation Solution Details

<table>
<thead>
<tr>
<th></th>
<th>0.25 cycles</th>
<th>0.5 cycles</th>
<th>1.0 cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sim. Time (secs)</td>
<td>1015.753</td>
<td>535.115</td>
<td>450.203</td>
</tr>
<tr>
<td>Max. Angle Diff. (Deg)</td>
<td>397.118</td>
<td>397.010</td>
<td>397.300</td>
</tr>
<tr>
<td>Time of Max Angle Difference (secs)</td>
<td>4.358</td>
<td>4.358</td>
<td>4.367</td>
</tr>
<tr>
<td>Total Newton Solutions</td>
<td>12622</td>
<td>6325</td>
<td>4174</td>
</tr>
<tr>
<td>Jacobian Factorizations</td>
<td>73</td>
<td>73</td>
<td>71</td>
</tr>
<tr>
<td>Forward/Backward Subs</td>
<td>12345</td>
<td>6205</td>
<td>4183</td>
</tr>
</tbody>
</table>

For the next set of results, subintervals were implemented of some, but not all, of the exciter models in the system. Only exciters that exhibited a maximum eigenvalue magnitude of 500 or more had subintervals assigned. The number of the subintervals applied to the individual exciter model was determined from Table 5.1. Only 45 exciters in the system contributed eigenvalue magnitudes of more than 500. Table 5.3 displays the solution results of this case.

Table 5.3 Simulation Solution Details with Subintervals on Top 45 Exciters

<table>
<thead>
<tr>
<th></th>
<th>0.25 cycles</th>
<th>0.5 cycles</th>
<th>1.0 cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sim. Time (secs)</td>
<td>1000.029</td>
<td>523.118</td>
<td>445.008</td>
</tr>
<tr>
<td>Max. Angle Diff. (Deg)</td>
<td>397.956</td>
<td>396.735</td>
<td>397.299</td>
</tr>
<tr>
<td>Time of Max Angle Difference (secs)</td>
<td>4.375</td>
<td>4.350</td>
<td>4.367</td>
</tr>
<tr>
<td>Total Newton Solutions</td>
<td>12623</td>
<td>6327</td>
<td>4174</td>
</tr>
<tr>
<td>Jacobian Factorizations</td>
<td>75</td>
<td>75</td>
<td>71</td>
</tr>
<tr>
<td>Forward/Backward Subs</td>
<td>12342</td>
<td>6342</td>
<td>4170</td>
</tr>
</tbody>
</table>
For each time step, little solution time savings was realized: only 15 seconds for the best case. Also, additional calculations are necessary for the half-cycle and quarter-cycle time step solutions.

One other case considered was placing subintervals on each exciter in the system, regardless of whether it contributed to the largest eigenvalue of its generator. Once again the subintervals followed the limitations of Table 5.1. The results of this simulation are displayed in Table 5.4. Time savings increased again but not as much as one might expect. The full cycle time step simulation experienced the greatest speed-up, almost 10 percent faster than the default case.

Table 5.4 Simulation Solution Details with Subintervals on Each Exciter

<table>
<thead>
<tr>
<th></th>
<th>0.25 cycles</th>
<th>0.5 cycles</th>
<th>1.0 cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sim. Time (secs)</td>
<td>991.168</td>
<td>514.504</td>
<td>407.834</td>
</tr>
<tr>
<td>Max. Angle Diff.</td>
<td>396.830</td>
<td>396.711</td>
<td>397.329</td>
</tr>
<tr>
<td>(Deg)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time of Max Angle</td>
<td>4.350</td>
<td>4.350</td>
<td>4.350</td>
</tr>
<tr>
<td>Difference (secs)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Newton</td>
<td>12620</td>
<td>6327</td>
<td>4174</td>
</tr>
<tr>
<td>Solutions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jacobian</td>
<td>75</td>
<td>75</td>
<td>71</td>
</tr>
<tr>
<td>Factorizations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forward/Backward</td>
<td>12516</td>
<td>6349</td>
<td>4235</td>
</tr>
<tr>
<td>Subs</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

All of these simulations provided accurate and stable results. Figures 5.1-5.3 show the bus voltage magnitude and voltage angle of bus 73285. Figure 5.1 displays the full step cycle solutions at the bus for default case as well as each exciter case used. Figures 5.2 and 5.3 show the bus73285 voltage magnitude and voltage angle for each time step and subinterval case study.
Figure 5.1 Bus 73285 Full Cycle Step Exciter Simulation Bus Voltage Magnitude (Left) and Voltage Angle (Right)

Figure 5.2 Bus 73285 Bus Voltage Magnitude (Left) and Voltage Angle (Right) with Subintervals on 45 Exciters
5.3 Conclusions

The Multirate method can be developed into a useful integration method when applied correctly to power systems. It allows more control over various elements of the system solution. When used correctly, the smaller subinterval time steps can provide a simulation speed-up, especially if one can use subintervals at a large time step that is not stable without their use.

Exciter models lend themselves well to the Multirate method. In many cases, they create a stiff system with their large eigenvalues. However, the large eigenvalues have little to no participation from other machine control models. This demonstrates a nearly decoupled partition in the equations, allowing the exciter to be solved with an individual time step. Since the exciter is nearly decoupled, using the Multirate method for integration provides accurate results and a stable steady state solution.

The results shown in this thesis suggests that the Multirate method can be used on very loosely coupled power system equations. On a large system, with many exciters, use of the Multirate method can increase solution speed while maintaining solution convergence. Small
speed gains were made when using the Multirate method and the same system time step. Greater speed gains are made when using subintervals on exciters, allowing for use of a larger system time step. The eigenvalue limits presented help to decide how large a subinterval is necessary when changing system time steps for solution speed-up.

5.4 Future Work

Additional study of the Multirate method and power system stability is necessary. This study focused mainly on exciters, which can contribute to a stiff system. Other system models can contribute to numerical instability, including machine models, stabilizers, and governors. For stability issues of these models, a study to determine when coupling becomes a problem in the Multirate method is necessary. Also, a study on when using the Multirate method becomes detrimental to a power system is necessary. If the method is overused, it could eventually slow down a simulation due to the extra calculations needed in the smaller intervals.
REFERENCES


APPENDIX A

MODEL BLOCK DIAGRAMS

Figures A.1-A.4 show block diagrams of important machine models used in this study as seen in [16].

Figure A.1 Exciter Model EXST1_GE Block Diagram
Exciter EXAC1

Figure A.2 Exciter Model EXAC1 Block Diagram

Exciter EXDC2_GE

Figure A.3 Exciter Model EXDC2_GE Block Diagram
Figure A.4 GENROU Machine Model Block Diagram