CENTRAL CIRCULATION AND BOOKSTACKS
The person borrowing this material is responsible for its renewal or return before the Latest Date stamped below. You may be charged a minimum fee of $75.00 for each non-returned or lost item.

Theft, mutilation, or defacement of library materials can be causes for student disciplinary action. All materials owned by the University of Illinois Library are the property of the State of Illinois and are protected by Article 16B of Illinois Criminal Law and Procedure.

TO RENEW, CALL (217) 333-8400.
University of Illinois Library at Urbana-Champaign

When renewing by phone, write new due date below previous due date.
How Tax Agency Audit Policies, Tax Rates, and Uncertainty Affect Taxpayer Compliance

Paul J. Beck
Jon S. Davis
Woon O. Jung

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign
How Tax Agency Audit Policies, Tax Rates, and Uncertainty Affect Taxpayer Compliance

Paul J. Beck, Associate Professor
Department of Accountancy

Jon S. Davis, Assistant Professor
Department of Accountancy

Woon O. Jung, Assistant Professor
Department of Accountancy
How Tax Agency Audit Policies, Tax Rates, and Uncertainty Affect Taxpayer Compliance

Abstract

The present study investigates the effects of tax audit policies on taxpayers' reporting strategies. Previous research is extended by incorporating taxpayers' uncertainty about their tax liabilities and by developing two alternative models of tax agency audit policies. An evaluation of the comparative statics properties of each model indicates that taxpayers' expectations about audit policies can have an important role in mediating the effects of the tax rate structure and uncertainty on taxpayer compliance. A noteworthy result is that increasing the tax rate will strengthen compliance incentives when taxpayers expect the tax enforcement agency to modify its audit policies in response to individual taxpayers' reporting, but have no effect when audit policies are expected to be fixed. We also show that uncertainty about taxable income and penalties affects compliance even when taxpayers are assumed to be risk-neutral.
1. Introduction

Numerous studies [e.g., Allingham and Sandmo (1972), Srinivasan (1973), Singh (1973), Yitzhaki (1974), Kakwani (1978), Landsberger and Meilijson (1982), Koskela (1983), Greenberg (1984), Reinganum and Wilde (1985, 1986), and Graetz, Reinganum, and Wilde (1986) and Alm (1988)] present models of tax evasion and taxpayer reporting behavior. To date this literature has focused primarily on the effects of tax and penalty rate structure, while largely ignoring the impact of taxpayer perceptions of tax agency audit policies on reporting behavior.

Tax agency audit policies have been represented in two ways in the literature. Traditionally, most modelling studies have adopted a partial equilibrium framework in which taxpayers viewed the probability of audit as fixed and unresponsive to reported income (i.e., the selection of returns for audit by random sampling). The assumption of a fixed audit probability has recently been criticized [see Greenberg (1984), Graetz and Wilde (1985)] for ignoring strategic interplay between tax auditing and taxpayer reporting decisions. More recently, several studies [e.g., Greenberg (1984), Graetz, Reinganum and Wilde (1986), and Reinganum and Wilde (1985, 1986)] have incorporated such interplay by adopting a game-theory framework in which taxpayers and the tax agency concurrently make their respective reporting and auditing decisions.

While empirical evidence is somewhat limited with respect to taxpayers' beliefs regarding the tax agency's audit decision, one recent survey [Aitken and Bonneville (1980)] commissioned by the Internal Revenue Service (IRS) suggests that taxpayer perceptions are mixed.
Aitken and Bonneville report that 29.3% of respondents indicated that they believe the IRS randomly selects tax returns for audit. An additional 30.4% of the respondents reported that they believe that attributes of their tax return (e.g., reported income level, the level of deductions taken, irregularities, etc.) provide the basis for audit selection. The remainder of respondents either stated that they did not know how the IRS selected returns for audit or responded with some other miscellaneous audit selection rule. These results provide limited empirical support both for the traditional modelling approach which assumes taxpayers view the audit selection decision as random and for the more recent game-theoretic models which represent the tax agency as responsive to taxpayer reporting decisions.

In the present study, we investigate how expectations about the tax agency's audit policies affect taxpayer compliance. Consistent with the past analytical work and the Aitken and Bonneville (1980) survey evidence discussed above, two economic models of taxpayer reporting are developed. The first is representative of an environment in which taxpayers expect the tax enforcement agency's audit policies to be based upon random selection within audit classes (e.g., groups of taxpayers sharing observable characteristics such as occupation and income sources) rather than strategically directed at individual taxpayers. In the second model, the tax agency is assumed to interact strategically with individual taxpayers. Hence, each taxpayer must anticipate the effect which his (her) reporting decision has on the probability of receiving an audit. The framework underlying both models differs from most previous studies in that we
assume that taxpayers have uncertainty about their tax liabilities and the imposition of penalties.¹ Such uncertainty could arise due to the complexity of the tax laws applicable to taxpayers' transactions and/or frequent changes in the tax code. Our analysis shows that the effects of the tax rate structure and uncertainty about taxable income on taxpayers' reporting incentives depend crucially upon their expectations regarding the tax agency's audit policies. Hence, policy makers' predictions regarding the compliance effects of changes in the tax rate structure and tax simplification should take into account concurrently the role of the tax enforcement agency.

Our study is organized in four sections. In Section 2 we present a benchmark model of taxpayer reporting under the assumptions that audits are random and the audit probability is independent of the amount of taxable income reported. Consideration of the simple model facilitates comparisons with traditional taxpayer reporting models and, thus, highlights the effects of taxpayer uncertainty as well as providing a foundation for subsequent extension. Section 3 incorporates the tax agency as a strategic player in an extensive form game with taxpayers. In particular, the tax agency is assumed to determine its tax audit policies based upon a cost-benefit analysis. Hence, as the audit benefit to the tax agency is directly related to the amount of reported income, tax audit policies are maximally responsive, given the attendant costs of performing audits. Another desirable feature of the sequential equilibrium framework is that the natural time ordering of the reporting/audit process is modelled explicitly.
Furthermore, the tax agency's actual audit policies are fully consistent with the audit policies that taxpayers anticipated when making their reporting decisions. By considering both the fixed and strategic tax agency models, we can determine the extent to which the effects of other environment factors are mediated by expected tax auditing policies. Section 4 presents the conclusions and policy implications of our study.

2. Taxpayer Model

Consistent with previous research, the tax reporting decision is modelled for a representative taxpayer. Furthermore, we temporarily assume that the audit probability (p) depends only upon the taxpayer's audit class membership and, thus, does not vary with the amount of reported taxable income. A fixed audit probability assumption is consistent with an environment in which the tax enforcement agency randomly selects taxpayers belonging to a given audit class that shares observable characteristics. As many previous studies have also relied upon this assumption, a direct comparison of results is facilitated.

Another modelling assumption throughout the study is that taxpayers are risk-neutral. This assumption is based upon several considerations. First, previous studies that investigated the effects of taxpayer uncertainty [e.g., Alm (1988) and Scotchmer and Slemrod (1988)] assumed taxpayer risk aversion. Hence, analysis of risk-neutral taxpayers serves to fill an existing void in the literature. A second, but related reason is that while casual intuition might
suggest that changes in the uncertainty level would either not affect the reporting decisions of risk-neutral taxpayers or would be the same as for risk-averse taxpayers, our results indicate the opposite. A third reason for assuming risk-neutrality is to provide comparability with the assumptions required by the game-theory analysis and to permit the taxpayer's decision problem to be formulated as the minimization of the expected value of the tax liability and penalties.

A distinguishing feature of our characterization is that the taxpayer is assumed to be uncertain about his (her) taxable income and the associated tax liability. Such uncertainty could arise due to the complexity of the tax laws applicable to the taxpayer's particular circumstances and/or changes in the tax statutes. Uncertainty about the post-audit taxable income \( x \) is modelled by means of a probability density function, \( f(x) \). The density is assumed to have a finite interval support, \([L,H]\) and a mean denoted by \( \mu \).

Assuming that the taxpayer files a return on which a taxable income of \( R \) is reported, there are four possible events which result in distinct liabilities. The first is that the taxpayer's return is not audited, in which case, the taxpayer's liability remains \( T(R) \), where \( T(\cdot) \) denotes the tax rate structure. Alternatively, when \( x < R \), the taxpayer's post-audit tax liability will be revised downward from \( T(R) \) to \( T(x) \), thereby resulting in a refund. However, if \( x > R \), the taxpayer will have to pay additional taxes. Furthermore, a monetary penalty could be imposed for underpayment of taxes.

Most tax reporting models have employed two different penalty structures. Under the first, the monetary penalty rate \( q \) is applied
directly to the reported income deficiency \((x-R)\) [see Allingham and Sandmo (1972), McCaleb (1976), Srinivasan (1973), Koskela (1983), Greenberg (1984), and Alm (1988)]. Other studies [e.g., Yitzhaki (1974), Scotchmer (1987a), and Reinganum and Wilde (1987)] have assumed that the penalty is proportional to the tax deficiency, \(T(x) - T(R)\). As the penalties for underpayment of taxes in the United States are based on the tax deficiency, we employ the second penalty structure.\(^3\)

In the United States, the Internal Revenue Code provides a monetary penalty for the substantial underpayment of taxes. However, such penalties can be avoided if the taxpayer has substantial authority for the tax treatment taken.\(^4\) Since the taxpayer (and the tax agency) could have uncertainty with respect to whether substantial authority exists [see Ayres, Jackson, and Hite (1987), and Chow, Shields, and Whittenburg (1987)], we assume that the imposition of penalties is uncertain and denote the associated probability by \(\phi\). The liabilities together with their probability of occurrence are summarized below:

<table>
<thead>
<tr>
<th>Event</th>
<th>Liability</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No Audit</td>
<td>(T(R))</td>
<td>((1-p))</td>
</tr>
<tr>
<td>2. Audit/No Deficiency</td>
<td>(T(x))</td>
<td>(p \int_{L}^{R} f(x)dx)</td>
</tr>
<tr>
<td>3. Audit/Penalty Waived</td>
<td>(T(x))</td>
<td>(p(1-\phi) \int_{R}^{H} f(x)dx)</td>
</tr>
<tr>
<td>4. Audit/Penalty Imposed</td>
<td>(T(x) + q[T(x)-T(R)])</td>
<td>(p\phi \int_{R}^{H} f(x)dx)</td>
</tr>
</tbody>
</table>
The expected tax and penalty liability can be written as:

\[
ET = (1-p)T(R) + p\int_L^H T(x)f(x)dx
\]

\[
+ \phi\int_R^H [T(x)+q(T(x)-T(R))]f(x)dx
\]

\[
+ (1-\phi)\int_R^H T(x)f(x)dx.
\]

(1)

Differentiating (1) with respect to \( R \), we obtain the following optimality condition:

\[
\frac{dET}{dR} = (1-p)T'(R^*) - p\phi q T'(R^*)\int_{R^*}^H f(x)dx = 0,
\]

(2)

where \( R^* \) denotes the solution to (2).

The first group of terms on the RHS of (2) represents the expected marginal benefit to the taxpayer from reducing reported taxable income, while second group represents the expected marginal cost (penalty). Hence, the optimality condition indicates that the taxpayer will have an incentive to increase reported taxable income up to the point where the expected marginal benefits (tax savings) are equated with expected marginal costs. Simplifying (2), the following analogous optimality condition is obtained:

\[
F(R^*) = 1 - (1-p)/(p\phi q).
\]

(3)

Since \( F(*) \) represents the cumulative probability distribution for post-audit taxable income, the first-order condition has a direct interpretation. In particular, (3) indicates that the taxpayer's optimal strategy is to report at the \( 1 - (1-p)/(p\phi q) \) fractile of the
post-audit income distribution. As $F(\cdot)$ is a strictly increasing function, it is apparent that the taxpayer's reported taxable income is an increasing function of the audit probability and the expected monetary penalty rate ($q$) for underpayment of taxes. Proposition 1 presents an analysis of the effects of the tax rate structure.

Proposition 1:

Assuming that the audit probability is independent of the amount of taxable income reported, tax compliance will be the same under proportional and progressive tax regimes. Furthermore, changes in the tax rate have no effect upon compliance.

Proof:

Since assumptions were not made about the tax rate structure (other than differentiability), any strictly convex tax rate structure $T_1(\cdot)$ can be substituted for $T(\cdot)$ in (1) to obtain the same first-order condition as in (3). Similarly, the first-order condition corresponding to the proportional rate structure: $T_2(x) = tx$ also will be the same as (3). Hence, as both first-order conditions are identical under $T_1(\cdot)$ and $T_2(\cdot)$, taxpayers' optimal reporting fractiles and reported taxable income levels will be the same. Also note that, as $T(\cdot)$ does not appear in the first order condition, changes in the tax rate itself have no effect upon taxpayer compliance.

Q.E.D

The result in Proposition 1 concerning the proportional tax rate contrasts with Yitzhaki's [1974] findings that risk-averse taxpayers' compliance increased with the tax rate. An explanation for this difference is that, under decreasing absolute risk aversion, a tax rate
increase has a compliance-enhancing income effect. However, under risk-neutrality, the income effect is absent. Further, when penalties are proportional to the amount of the tax deficiency (as assumed in Proposition 1), the substitution effect is neutralized since penalties increase concurrently with the tax rate [see Yitzhaki (1974)]. Therefore, with both the income and substitution effects absent, taxpayers' compliance incentives with respect to changes in tax regimes or the rate structure are unaffected.

Given the absence of tax regime and rate effects, one might be tempted to conclude that changes in the uncertainty level also would have no effect upon a risk-neutral taxpayer's compliance. However, we show that changes in the uncertainty level will generally affect compliance. Furthermore, in contrast with Alm (1988) who found that increased uncertainty will either have ambiguous effects or enhance a risk-averse taxpayer's compliance, we are able to identify conditions under which compliance will decline.

A taxpayer's uncertainty about taxable income is likely to depend upon the taxpayer's particular circumstances (e.g., sources of income) and the complexity of applicable tax laws and their susceptibility to change. For modelling purposes, we now assume that the taxpayer has a uniform distribution for taxable income. Given a uniform distribution, uncertainty can be manipulated in our model by introducing a second uniform distribution, \( J(x) \) having the same mean \( (\mu) \) as \( F(x) \), but a larger range, \([L-\Delta,H+\Delta]\), where \( 0 < \Delta < L \) represents a perturbation parameter. Before comparing the reporting decisions under distribution \( J(x) \) with those under \( F(x) \), several additional features
of the distributions should be noted. First, as both uniform distributions are symmetric and have the same mean, it is apparent that
\[ J(s) = F(s) = 0.5 \text{ for } s = \mu. \]
A second feature is that \( J(s) > (\leq) F(s) \) for all \( s < (\geq) \mu \). Proposition 2 indicates how the amounts of taxable income reported are affected by changes in the taxpayer's uncertainty about taxable income.

**Proposition 2:**

Assuming a uniform income distribution, a risk-neutral taxpayer's compliance will increase (decrease) in response to changes in the uncertainty level depending upon whether \( p/(1-p) > (\leq) 2/\phi q \). Only in the special case in which \( p/(1-p) = 2/\phi q \) will compliance be unaffected.

**Proof:** (See the Appendix.)

The results in Proposition 2 are in contrast with Alm (1988) and Scotchmer and Slemrod (1988) who found that (decreasingly) risk-averse taxpayers will report higher levels of income. The differences in results appear to be based upon differences in technical modelling assumptions and by the presence of a compliance-enhancing income effect. Given our assumption of risk-neutrality, uncertainty level changes influence reporting incentives through the expected penalty. In particular, increasing the range of the income distribution can be shown to increase the taxpayer's expected marginal penalty when the initial level of reported income is above the mean. Such a penalty increase creates incentives for the taxpayer to report a higher income level to maintain the previous equilibrium relationship between expected benefits (tax savings) and marginal costs (penalties). A
parallel argument will show that, when the initial level of reported income is below the mean, increased uncertainty will reduce the expected marginal penalty, thereby resulting in a lower reported income. Hence, elevating the taxpayer's uncertainty level will have a mean-diverging effect on reported income, except when the taxpayer's optimal decision is to report at the mean of the income distribution. In the following section, we determine whether these results can be extended to an environment in which the tax agency operates strategically.

3. Game Theory Model

Strategic tax auditing policies are now modelled by incorporating the tax enforcement agency as an active player in an extensive form game. The taxpayer is assumed to make the reporting decision first and then the tax enforcement agency decides whether or not to audit upon receipt of the taxpayer's return. Consistent with the concept of a sequential equilibrium, each player in the game is assumed to employ optimal strategies given the previous moves of the other players. An important advantage of this approach is that the tax agency will have no incentive to deviate ex post from the audit strategy anticipated by taxpayers when making their reporting decisions. Reinganum and Wilde (1987) also employ the sequential equilibrium framework in modelling the interplay between taxpayer reporting and tax agency audit policies. The present model differs from theirs in that, prior to the audit, the taxpayer and the tax agency are assumed herein to be uncertain about the taxpayer's actual taxable income and associated tax and penalty liabilities.
The tax agency's audit decision is assumed to be based upon a cost-benefit analysis in which the expected values of incremental revenues from tax collections and penalties are balanced against audit costs. Given our results in Proposition 1, we simplify by assuming a proportional tax rate structure. Such an assumption is arguably a reasonable approximation of the present income tax rate structure in the U.S. following the Tax Reform Act of 1986. Assuming a proportional tax rate, the incremental expected total revenue that would be collected from a taxpayer known to have an income distribution \( f(x) \), but reporting a taxable income of \( R \), is given by:

\[
B = \int_{L}^{H} txf(x)dx + \int_{R}^{H} (x-R)f(x)dx - Rt, \tag{4}
\]

where \( t \) denotes the proportional tax rate.

The first term in (4) is the expected value of tax collections, while the second represents the expected value of penalties. As audits are costly, however, a risk-neutral tax agency will have incentives to perform an audit only when the expected benefit exceeds the cost (i.e., \( c < B \)). Note that the expected audit benefit \( B \) is monotone decreasing in \( R \), having a maximum value when \( R \) equals the lowest possible income level \( L \).

Since \( t \) and \( q \) are assumed to be known by taxpayers, the expected benefit \( B \) from an audit can be computed. Thus, if the tax agency's cost (cutoff point) were known, there would be no uncertainty about whether or not an audit would be performed. Taxpayers in our model, however, are uncertain about audit costs. Hence, the tax agency's
audit decision is uncertain from the perspective of taxpayers. Taxpayers, however, are assumed to assess a uniform probability density function for audit costs, \( g(c) \), defined over the interval, \([C_L, C_H]\). Throughout the study, the maximum expected audit benefit (i.e., \( B \) when \( R = L \)) is assumed to be greater than the lowest possible audit cost \( C_L \). Without this assumption a trivial equilibrium would exist in which taxpayers report the lowest possible income level \( L \) and the tax agency never performs an audit.

Letting \( G(\cdot) \) denote the cumulative probability distribution for audit costs (as assessed by a representative taxpayer), the probability of an audit is given by \( G(B) \), where \( B \) is a function of \( R \) as defined in (4). Accordingly, the taxpayer's expected liability from reporting taxable income of \( R \) is given by:

\[
ET = G(B)[\int_{C_L}^{C_H} \frac{txf(x)dx}{R} + \phi q \int_{C_L}^{C_H} (x-R)f(x)dx] + (1-G(B))tR. 
\]

(5)

Rearranging terms and making use of the definitions: \( G(B) = \frac{B-C_L}{(C_H-C_L)} \) and \( u = \int_{C_L}^{C_H} xf(x)dx \), (5) can be rewritten as:

\[
ET = \frac{(B-C_L)}{(C_H-C_L)} [\mu t + \phi q \int_{C_L}^{C_H} (x-R)f(x)dx - Rt] + Rt. 
\]

(6)

Upon simplification, the first-order condition for the taxpayer's optimal reporting decision is given by:

\[
\frac{(2B-C_L)}{(C_H-C_L)} [1 + \phi q(1-F(R^*))] = 1, 
\]

(7)

where \( B \) is defined in (4).
Two important comparative statics properties of the model concern the effects of changes in the penalty and tax rate. Proposition 3 verifies that, consistent with the results obtained in the fixed audit probability model, taxpayer compliance increases concurrently with the penalty rate for underpayment of taxes.

Proposition 3:

Taxpayer compliance is an increasing function of the expected penalty ($\phi q$) on the tax deficiency.

Proof: (See the Appendix.)

A somewhat more interesting result concerns the effects of changes in the tax rate. While we previously found that tax rate changes had no effect upon compliance when taxpayers expected a fixed audit probability, such is not the case when the tax agency is expected to behave strategically.

Proposition 4:

Taxpayer compliance is an increasing function of the tax rate.

Proof: (See the Appendix.)

Propositions 3 and 4 arise from the impact that changes in the tax and penalty rates have on the audit benefit. In particular, increasing the tax and/or penalty rate results in a larger audit benefit and higher equilibrium audit probability. Such an increase in the equilibrium audit probability enhances taxpayers' compliance incentives vis-à-vis the previous environment in which the audit probability was
fixed. Hence, the amount of taxable income reported by the taxpayer increases even when both the substitution and income effects are absent.

A further issue concerns the role of uncertainty about taxable income. We show in Proposition 5 that uncertainty effects are identical to those in Proposition 2 for cases in which the initial reporting level is above the mean. When the initial report is below the mean, however, the effects are not necessarily the same as in Proposition 2.

**Proposition 5:**

Assuming a uniform income distribution, when taxpayer's initial reported income level is above the mean, increasing taxpayer uncertainty (range of possible taxable income levels) will enhance compliance. Otherwise, the effects are potentially ambiguous.

**Proof:** (See the Appendix.)

Once again, the intuition underlying this result is that mean-preserving changes in the dispersion of audit outcomes can affect the audit benefit and equilibrium audit probability. Unlike penalty and tax rate changes, however, the specific effects depend upon the initial reporting level. Note that when the taxpayer's initial reporting level is above the mean, for a given R value, the expected value of the audit penalty (partial expectation over the interval, [R,H+Δ]) will increase with the range of taxable income levels since the lower endpoint remains fixed, while the upper endpoint is raised. Since this will lead to an increase in the audit probability, the enhanced compliance incentives that were already present with a fixed audit probability will be further reinforced. When the initial reported
taxable income level is below the mean, however, the expected value of penalties will increase, but to a lesser extent as low income realizations also become more likely. Consequently, when the initial reporting level is low, the marginal effect upon the audit benefit and equilibrium audit probability will be smaller than in the previous case. Since the incentives which would otherwise exist for taxpayers to reduce their compliance levels (see Proposition 2) under a fixed audit probability may not be offset, the effects of increased income uncertainty become ambiguous.

4. Conclusions

The present study has provided an analysis of taxpayer reporting decisions under alternative tax auditing policies. With the exception of penalty rate increases that were found consistently to enhance taxpayer compliance, the other comparative static properties were found to be sensitive to taxpayer expectations regarding the tax agency's audit policies. Under the assumption of a fixed audit probability, tax rate increases had no effect upon risk-neutral taxpayer compliance. Alternatively, when taxpayers expected the tax agency to behave strategically in response to reported income, tax rate increases induced greater compliance.

The effects of changes in taxpayers' uncertainty about taxable income also were found to be sensitive to taxpayer expectations about the tax agency's audit policies and to be dependent upon the initial reporting level. In particular, when the audit probability and the penalty rate were sufficiently high to induce taxpayers to report initially above the mean of their taxable income distribution, increased
uncertainty further enhanced compliance under both fixed and strategic tax audit policies. However, under the complimentary circumstances in which taxpayers initially reported below the mean, taxpayer compliance was shown to decline in response to increased uncertainty under fixed audit policies, but have ambiguous effects under strategic audit policies.

The above results have potential implications for policy-makers. First, frequent changes in tax statutes that create or elevate taxpayer uncertainty can have direct effects upon taxpayer compliance. Second, predictions regarding the specific effects on compliance must take into account taxpayer expectations regarding tax audit policies. Thus, tax policy and tax enforcement issues should be analyzed concurrently.

Several simplifying assumptions were introduced in modelling taxpayer compliance. One important assumption is that taxpayers are risk-neutral. As noted previously, incorporating taxpayer risk aversion would create an income effect. In an unpublished study, Beck and Jung (1988a) demonstrated that tax rate increases will predictably enhance risk averse taxpayers' compliance. Another key assumption is that taxpayers and the tax agency have symmetric uncertainty about taxable income. In some cases, however, the information sets of taxpayers and the tax agency could differ. To the extent that taxpayers have additional information about their actual tax liabilities, reported income will become a potential signal as in the Reinganum and Wilde (1986) and Beck and Jung (1988b) models. However, based upon
their findings, we would not expect the presence of information asymmetry to have a qualitative effect upon the results of the present study.

An interesting extension of the present study would be to incorporate a tax advisor as a means of reducing taxpayer uncertainty. Reinganum and Wilde (1988) have already obtained some preliminary results in a tax advisor model. Another extension would be to test experimentally the model-based implications in an experimental economics context. Such testing would be particularly useful in determining the robustness of results to various modelling simplifications and permit additional evidence to be gathered in cases where model-based predictions are ambiguous.
Footnotes

1Alm (1988) and unpublished studies by Scotchmer (1987b), Scotchmer and Slemrod (1988), and Beck and Jung (1988a) also have modelled taxpayer uncertainty about taxable income. Our models differ from theirs in that we incorporate the interplay between the tax agency's audit policies by means of a sequential equilibrium model. The present study also differs from Beck and Jung (1988b) in several respects. First, they assume asymmetric information between taxpayers and the tax agency, but utilize a discrete probability distribution, while we assume symmetric information and model income as being continuous [see Slemrod (1988) for a discussion of the contrast between the discrete and continuous income models]. A further difference is that they define compliance in terms of the taxpayer's type (cut off probability for reporting the high income level), whereas we define compliance in terms of the amount of income reported by the taxpayer.

2Examples of such characteristics could include the taxpayer's occupation, sources of income, residential location (zip code), and the particular schedules filed together with the tax returns.

3Section 6661(a) of the United States Internal Revenue Code (IRC) provides a penalty equal to 20 percent of additional taxes due when the tax liability is substantially understated. This occurs when the reported tax liability is understated by $5,000 or 10 percent of the post-audit tax liability, whichever is larger. Several other penalties that are proportional to additional taxes due may also be assessed. IRC Section 6653(a)(1) imposes a penalty equal to 5 percent of any underpayment of tax together with a non-deductible interest charge (essentially a penalty) equal to 50 percent of the interest relating to any underpayment attributable to taxpayer negligence. In addition, IRC Section 6651(a)(2) provides for a maximum penalty of 25 percent of additional taxes due for failure to pay. If the balance due after filing is more than 10 percent of the tax shown on the taxpayer's return, such a penalty will be imposed unless reasonable cause can be shown. When the taxpayer can be shown to have de-liberately understated taxable income by failing to report income or by knowingly taking inappropriate deductions, criminal penalties also can be imposed for fraud. Since taxpayers are assumed to be uncertain about their taxable income, criminal issues are not addressed in our study.

4In the United States, Section 6661(b)(2)(B) of the Internal Revenue Code (1987) states that the amount of understatement [of taxes] under subparagraph (A) shall be reduced by that portion of the understatement which is attributable to the tax treatment of any item by the taxpayer if there is or was substantial authority for such treatment. When the various sources of authority (e.g., judicial, statutory, and administrative systems) conflict, however, taxpayers could have uncertainty about whether there is substantial authority
for their position. A recent study by Chow, Shields, and Whittenburg (1987) examined the judgments of experienced tax practitioners and found a high level of consistency, but only a moderate level of consensus regarding the presence of substantial authority in the specific cases analyzed.

Differentiating (1) a second time with respect to R, one can verify that the second-order (sufficient) condition for an interior solution will be satisfied provided that:

\[ [p \phi q(1-F(R))-(1-p)]T''(R)/T'(R) < p \phi qf(R). \]

Since the terms inside the brackets on the LHS of the inequality are zero due to the first-order condition, while the terms on the RHS are positive, the latter inequality is clearly satisfied.

The assumption of a uniform distribution is not essential to our analysis. Our proof requires that the cumulative distributions cross at only one point \( (x_C) \) (i.e., \( G(x) > F(x) \) for \( x < x_C \), with the inequality reversed for \( x > x_C \)). The uniform distribution family is particularly convenient to employ as the crossing point occurs at \( x_C = u \equiv (H+L)/2 \). We have also analyzed other income distributions having the single crossing point property and have verified that taxpayer compliance will increase (decrease) depending on whether the initial reporting fractile is above (below) the crossing point.

Alm found that, in general, mean-preserving changes in the dispersion of the distribution of evaded income will have ambiguous effects upon taxpayer reporting. However, under the assumptions of decreasing absolute risk aversion and non-increasing relative risk aversion of less than one, Alm showed that increased uncertainty would increase declared income. While we focus on mean preserving changes in the dispersion of the total income distribution, Alm (1988) considers mean preserving changes in the distribution of evaded income. In our model this would correspond to the truncated income (penalty) distribution defined over the interval \([R,H]\) whose expectation can (and in fact usually will) change with increased uncertainty.

Scotchmer and Slemrod (1988) also focus on mean preserving changes in the total income distribution, but assume a discrete income distribution. Thus, changes in uncertainty in their model will change the expected value of penalties in their model, but not affect the probability of penalty occurrence as in our model. Slemrod (1988) provides a further discussion.

The taxpayer's optimality condition function under distribution \( J(x) \) is given by:

\[
(l-p)T'(R**) - \phi qT'(R**) \int_{R**}^{H+A} j(x)dx = 0, \tag{A}
\]

where \( j(x) = J'(x) \) and \( R** \) denotes the optimal solution to (A).
As the right hand sides of (A) and (2) are equal, it follows that:

\[
\frac{H+\Delta}{(1-p)T'(R^*) - p\phi q T'(R^*)} \int_{R^*}^{H} f(x)dx = \int_{R^*}^{H} j(x)dx.
\]  

(B)

We will now show that if \( R^* > u \), then \( R^{**} > R^* \). Suppose to the contrary that \( \text{R}^{**} = R^* \). Given this supposition, it follows from (B) that

\[
\int_{R^*}^{H+\Delta} j(x)dx = \int_{R^*}^{H+\Delta} f(x)dx.
\]  

(C)

Making use of the facts that \( f(x) = 1/(H-L) \) and \( j(x) = 1/(H-L+2\Delta) \), one can verify that (C) is equivalent to:

\[
[(R^*-L)(H-L+\Delta) - (H-L)(R^*+\Delta-L)] / [(H-L+2\Delta)(H-L)] = 0.
\]  

(D)

Upon simplification, (D) can be rewritten as:

\[
\frac{\Delta}{2} (R^*-u) = 0.
\]  

(E)

Note that when \( R^* > u \), (E), (D), and (C) imply by transitivity that (B) will be violated. Hence, we have obtained a contradiction. In particular, the RHS of (B) will be smaller than the LHS since the expected marginal penalty is larger under distribution \( j(x) \). In order to restore the optimality condition in (B) the taxpayer must, therefore, reduce the expected marginal penalty by increasing reported income (i.e., \( R^{**} > R^* \)). A parallel argument will show that if \( R^* < u \), the expected marginal penalty term under \( j(x) \) will be smaller than the penalty under \( f(x) \), so the taxpayer will have an incentive to reduce the reported income level.

An implicit assumption is that taxpayers and the tax agency have symmetric information about taxable income. In practice, one might argue that taxpayers and the tax agency could have different sources of uncertainty. The taxpayer could be uncertain about the appropriate tax treatment, while the tax enforcement agency could be uncertain about factual circumstances. Under such conditions, the taxpayer's report and the tax agency's audit decision both could provide signals regarding their respective private information. Reinganum and Wilde (1986) and Beck and Jung (1988b) have developed models in which taxpayers have private information that is communicated by their reported income. In the present study, we suppress the signalling value of the report to simplify the model and to facilitate a more direct comparison with the fixed audit probability model in the previous section.
References


Appendix

Proof of Proposition 2:

In proving Proposition 2, it is useful to make a preliminary observation. From the optimality condition in (3), one can verify that a risk-neutral taxpayer will report at the .5 fractile (mean) of his (her) income distribution when \( \frac{p}{1-p} = \frac{2}{(\phi q)} \) and above (below) the mean when \( \frac{p}{1-p} > (\leq) \frac{2}{(\phi q)} \).

Now suppose that the taxpayer's uncertainty is elevated so that \( J(x) \) represents the taxpayer's income distribution. The optimality condition corresponding to \( J(x) \) is given by:

\[
J(R^{**}) = 1 - \frac{(1-p)}{(p\phi q)}, \tag{1A}
\]

where \( R^{**} \) denotes the solution to (1A).

Since the RHS of (1A) and (3) are equal, it follows that:

\[
J(R^{**}) = F(R^*). \tag{2A}
\]

Observe that when \( \frac{p}{1-p} > \frac{2}{(\phi q)} \), \( R^* > \mu \). Therefore, as \( F(R^*) > J(R^*) \) for \( R^* > \mu \), by transitivity, (2A) implies that \( J(R^{**}) > J(R^*) \). As \( J(*) \) is a strictly increasing function, \( R^{**} > R^* \) as claimed.

A parallel argument will establish that when \( \frac{p}{1-p} < \frac{2}{(\phi q)} \), \( R^* < \mu \) and that: \( J(R^*) > F(R^*) = J(R^{**}) \). Only in the special case in which \( \frac{p}{1-p} = \frac{2}{(\phi q)} \) will \( R^* = R^{**} \).

Q.E.D.

Proof of Proposition 3:

Differentiating the first-order condition in (7) with respect to \( \phi q \), we obtain:
Collecting the terms involving $\frac{\partial R^*}{\partial (\phi q)}$, substituting (4A) into (3A), and rearranging terms, (3A) is equivalent to:

$$\frac{\partial R^*}{\partial (\phi q)} \left\{ 2 \left[ -t - \phi t (1-F(R*)) \right] \left[ (1+q(1-F(R*))) - (2B-C_L)\phi f(R^*) \right] \right\}$$

$$+ \left\{ 2 \left[ \int_R^{(x-R^*)} f(x) dx \right] \left[ (1+q(1-F(R*))) + (2B-C_L)(1-F(R*)) \right] \right\} = 0. \quad (5A)$$

As $0 \leq F(R^*) < 1$ and $B > C_L$, it follows that the second term enclosed by braces in (5A) is positive. Since the terms inside the first set of braces are negative, it must be the case that $\frac{\partial R^*}{\partial (\phi q)} > 0$ in order to satisfy (5A).

**Q.E.D.**

**Proof of Proposition 4:**

Differentiating (7) with respect to $t$,

$$2 \left( \frac{\partial B}{\partial t} \right) (1+q(1-F(R*))) + (2B-C_L) \left[ -\phi f(R^*) \frac{\partial R^*}{\partial t} \right], \quad (6A)$$

where:

$$\frac{\partial B}{\partial t} = \mu - R^* - \phi t \frac{\partial R^*}{\partial t} \cdot (1-F(R*)) + \phi \int_R^{(x-R^*)} f(x) dx. \quad (7A)$$

Substituting (7A) into (6A) and simplifying, one can verify that
\[
\frac{3R^*}{3t}\{-2qt(1-F(R^*))(1+q(1-F(R^*))) - qf(R^*)(2B-C_L)\} \\
\frac{H}{R^*} + \left\{2[u-R^*+q\int_{R^*}^x (x-R^*)f(x)dx](1+q(1-F(R^*))) \right\} = 0. \quad (8A)
\]

Note that: \(u - R^* + q\int_{R^*}^x (x-R^*)f(x)dx = B/t > 0\). Hence, as the terms inside the first set of braces are negative, while those inside the second set of braces are positive, (8A) requires that \(\frac{3R^*}{3t} > 0\).

Q.E.D.

Proof of Proposition 5:

Proposition 5 is established by perturbing both the lower and upper support of \(f(x)\) by \(\Delta\) to obtain the distribution, \(J(x) = (x-L+\Delta)/(H+\Delta-(L-\Delta))\). Substituting into (7), the revised optimality condition becomes:

\[
\frac{(2B-C_L)}{(C_{H-C_L})} \left[ 1 + \phi q \frac{H-R^*+\Delta}{(H-L)+2\Delta} \right] - 1 = 0, \quad (9A)
\]

where \(B = t(u-R^*) + q\int (H-R^*+\Delta)^2/[2(H-L+2\Delta)]\).

Differentiating (9A) with respect to \(\Delta\), we obtain:

\[
2\frac{\partial B}{\partial \Delta} [1 + q\frac{(H-R**+\Delta)}{(H-L)+2\Delta}] + \frac{(2R**-H-L)-(H-L+2\Delta)}{(H-L+2\Delta)^2} \frac{3R**}{\partial \Delta} = 0, \quad (10A)
\]

where:

\[
\frac{\partial B}{\partial \Delta} = \frac{\phi q t}{(H-L+2\Delta)^2} (H-R**+\Delta)((R**-L+\Delta)-(H-L+2\Delta)) \frac{3R**}{\partial \Delta} - t \frac{3R**}{\partial \Delta} \quad (11A)
\]

Upon rearranging terms, (10A) can be rewritten as:
\[
\frac{\partial R^*}{\partial \Delta} \left\{ -2t(H-L+2\Delta)[1+ \frac{(H-L+2\Delta)}{\phi q}] \left[ 1 + \phi q \frac{(H-R^*+\Delta)}{(H-L+2\Delta)} \right] - (H-L+2\Delta)(2B-C_L) \right\} \\
+ 2t(H-R^*+\Delta)(R^*-L+\Delta)[1 + \phi q \frac{(H-R^*+\Delta)}{(H-L+2\Delta)}] + (2B-CL) \cdot (2R^*-H-L) = 0. \quad (12A)
\]

Note that, as the coefficient of \( \frac{\partial R^*}{\partial \Delta} \) is negative, the sufficient condition for \( \frac{\partial R^*}{\partial \Delta} > 0 \) is that the remaining terms in (12A) be positive. While these terms could be either positive or negative, observe that they will be positive when

\[
2R^*-H-L \geq 0. \quad (13A)
\]

Simplifying, (13A) is equivalent to

\[
R^* \geq \frac{(H+L)}{2} = \mu.
\]

Thus, a sufficient condition for \( \frac{\partial R^*}{\partial \Delta} > 0 \) is that \( R^* \geq \mu \). Q.E.D.