The Loan Loss Reserve Decision in Large Commercial Banks

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Most of this research was undertaken during the academic year 1987-1988, when Vasconcellos was a Visiting Assistant Professor of Finance at the University of Illinois at Urbana-Champaign. Some ideas developed in this study are taken from Chapter V of Vasconcellos's doctoral dissertation at the University of Illinois. He thanks William R. Bryan and Paul Newbold for their comments on earlier drafts.
ABSTRACT

In the recent past, major increases in the loan loss reserves by several money center banks have focused the attention of academicians, the financial community, and regulators on the implications of such decisions for the earnings of those banking firms as well as on their ability to weather major borrower defaults in the medium and long run.

This study examines the process by which a bank determines the size of its loan loss reserve ratios in a particular point in time, as well as over a period of time. These decisions are influenced by many internal forces as well as competition, regulation, and tax factors. This work approaches the bank's decision with respect to loan loss reserve ratios from a decision-theoretic standpoint. Actual decisions are examined within the framework of a Bayesian decision model, and conclusions are obtained with respect to the relative sizes of the penalties for overestimation and underestimation of the loan loss reserve ratios, given a loss function which implies a normative decision rule consistent with the principle of expected utility maximization.
THE LOAN RESERVE DECISION IN LARGE COMMERCIAL BANKS

1. INTRODUCTION

In 1987, several large commercial banks made headlines by making dramatically large additions to their loan loss reserves. These large provisions for loan losses resulted from management decisions that were reportedly driven by the deterioration in the quality of bank loans to less developed countries. Besides affecting reported accounting earnings, these additions to loan loss reserves affect bank capital as currently defined. American commercial banks now operate under uniform minimum capital requirements (see Gilbert, Stone, and Trebing [1985]) that can act as constraints on a variety of bank decisions, including the decision about the size of the loan loss provision. Recently, the U.S. and 11 other major industrial nations reached a final agreement to produce uniform capital requirements for banks in these countries by the end of 1992 (see Cash [1988]).

Little has been written about the process by which a bank determines the size of its loan loss provision and, by implication, the size of its loan loss reserve, for any particular period. Most of the existing analyses of this problem focus on a regulatory and/or an analysts viewpoint, with a consequent emphasis on the decision output (i.e., the dollar amounts of the provision and the resulting loan loss reserve), as well as on the meaning and measurement aspects of several reserve ratios and the information conveyed by them with respect to the adequacy of the bank's loan loss reserve. A good example of this approach is Cates [1983]. Other studies have examined the tax aspects
relating to the provision for loan losses and the impact of an increase in the loan loss reserve on a bank's stock price, that is, "the informational and economic effects of loan loss reserve changes" (see Henderson [1987] and Grammatikos and Saunders [1988]). The work of George Vojta [1973a, 1973b], however, is an important exception. Vojta attempted to examine the decision process of the banking firm, which ultimately results in choices for the provision for loan losses and, a fortiori, for the loan loss reserve. This study represents one additional step in the direction proposed by Vojta, with the use of empirical decision-theoretic methods.

The process of arriving at a decision concerning the size of the loan loss reserve is of fundamental importance to the banking firm. This is so because this decision has implications for a bank's regulatory capital and because of the asymmetric nature of the penalties associated with making a set of decisions over time which may be either too conservative or too aggressive. A bank that is consistently "conservative" in its decision with respect to the provision for loan losses (i.e., the provision consistently exceeds the actual loss by a large amount) will experience reduced reported earnings and a reduction in its internally sustainable growth rate holding leverage constant. On the other hand, a bank that is consistently "aggressive" (i.e., the provision consistently falls short of actual losses) will experience increased regulatory attention, pressures to increase capital and, if losses are severe enough, ultimate bankruptcy. Notice that the same reasoning applies with respect to the level of the ratio of loan loss reserves to total loans or net loans. A "conservative"
bank is one whose loan loss reserve to total loans ratio is consistently above the peer norm (say, the average of peer banks). Likewise, an "aggressive" bank is one whose loan loss reserve to total loans ratio remains consistently below the norm for its peers. Indeed, one can argue that the level of the loan loss reserve or its ratio to total loans or net loans is the fundamental decision, and treat the decision with respect to the provision for loan losses as a residual decision.²

In this study, we examine the process by which a commercial bank determines the size of its loan loss reserves (or the level of the ratio of loan loss reserves to total loans or net loans) for any particular period. This decision is influenced by many internal bank factors as well as competition, regulation, and tax factors. We will focus the discussion on the theoretical and statistical properties of optimal decisions with respect to the loan loss reserve. In order to develop this firm decision-making point of view, we will approach this problem with the aid of a decision-theoretic or Bayesian statistical model. We will argue that this model produces a normative decision rule which is consistent with the principle of maximization of expected utility while being intuitive and flexible. This model is then used to investigate the loan loss reserve decisions of large U.S. banks in the period 1982-1986.

This paper is organized as follows. Section 2 discusses the major building blocks of this decision-theoretic model: the loss function, the prior distribution, and the likelihood function. In particular, the problem of the appropriate form for the loss function
is discussed, given its pivotal role in the derivation of the decision rule. Section 3 applies Bayesian analysis to the problem at hand. A Bayes estimate for the ratio of the loan loss reserve to total loans is derived in this section. In Section 4, this methodology is applied to a sample of large U.S. commercial banks in the period 1982-1986. Time series data on actual ratios of the loan loss reserves to total loans are used to generate data-based prior distributions for this ratio. Cross-sectional data is then combined with the time series information and a posterior distribution for the ratio of loan loss reserves to total loans results. Actual ratios are then compared with the parameters of the posterior distribution, and inferences result with respect to the relative size of the parameters in the loss function which represent penalties for the overestimation and underestimation of loan loss reserves. Finally, Section 5 presents the conclusions of this study, a discussion of the limitations of this methodology, and some suggestions for future studies.

2. THE MODEL

A schematic view of the logic of Bayesian decisions as applied to the problem under study is presented in Figure 1. The three major building blocks which contribute to the bank's decision with respect to the ratio of loan loss reserves to total loans are (i) its prior information (including, but not limited to, a time series of loan loss reserve ratios), (ii) contemporaneous information (represented by a cross section of the most recent ratios of loan loss reserves to total loans for banks with similar size and other characteristics), and
(iii) a loss function, whose parameters represent the bank's view of the penalties associated with underestimation or overestimation of loan losses. According to Bayesian decision theory, the prior distribution and the likelihood function combine to form a posterior distribution for the ratio loan loss reserves to total loans, which is the source of all inferences. Given the choice of the loss function, it becomes possible to obtain a point estimate for this ratio which minimizes expected posterior loss and therefore maximizes expected utility. In this study, the actual bank decisions with respect to the ratio of loan loss reserves to total loans are then compared to the Bayes decision and inferences are made with respect to the parameters of the hypothesized loss function.

One major advantage of this approach is that it mimics the actual decision process carried out by the banking firm. Clearly, when deciding the dollar amount to be allocated to the provision for loan losses in the next period (quarter or year), the bank's senior management considers the history of its past decisions and the most recent loan loss reserve decisions of its peer banks. In addition, some non data-based prior information, such as taxes and law or regulation, is also relevant (see Figure 1). We now turn to a more specific discussion of the loan function and prior information.

2.1 The Loss Function

A loss function is one of the basic components of a decision-theoretic statistical model. The equivalence of utility maximization and loss minimization is well known. In this study, we have
considered two types of loss functions, namely, the squared error loss and the linear loss. The squared error loss has the form

$$L(\theta,a) = (\theta - a)^2$$

where $\theta$ is the parameter of interest (i.e., the ratio of actual loan losses to total loans) and $a$ is the decision (in this study, the ratio of loan loss reserves to total loans). This is probably the most widely used loss function. In light of the problem under study, however, the symmetry of the squared-error loss becomes a problem. It seems that the penalties associated with overestimating the ratio of loan losses to total loans should be smaller than the penalties resulting from underestimating it (i.e., write-offs of capital and ultimate bankruptcy). Hence our interest in a generalized form of the squared-error loss, known as weighted squared-error loss, which can be written as

$$L(\theta,a) = w(\theta)(\theta - a)^2$$

where the squared error, $(\theta - a)^2$, is weighted by a function of $\theta$, reflecting the fact that the consequences of an estimation error will vary according to the magnitude of the ratio of actual loan losses to total loans.

The second major type of loss function of interest for this research is the linear loss. A particular version of linear loss function, known as the absolute error loss, is perhaps the most common. It can be written as

$$L(\theta,a) = |\theta - a|.$$
We can see that the symmetry of this loss function will cause the same type of problems described above in connection with the squared-error loss. Note, however, that in this case the penalties for large errors will be less severe. This brings us to the generalized linear loss function, which is of particular interest for this study, and has the form

\[
L(\theta, a) = \begin{cases} 
K_0(\theta - a) & \text{if } \theta - a > 0, \\
K_1(a - \theta) & \text{if } \theta - a \leq 0,
\end{cases}
\]

(4)

where \(K_0\) and \(K_1\) are constants, usually with different values, which can be chosen to reflect the relative importance (or perceived penalties) of underestimation and overestimation, respectively.

In principle, the weighted squared error loss and generalized linear loss would be equally qualified to represent the bank's loss function. They imply, however, very different decision rules with respect to feasibility of use and intuitive meaning. This point will be discussed in the next section. But let us first turn our attention to the other fundamental component of this model, the prior distribution.

2.2 Prior Information

There are several techniques available for the subjective determination of a prior density. In light of the purposes of this study, it seems appropriate to use the matching of a given functional form, i.e., to assume that \(\pi(\theta)\), the prior density for the unknown parameter \(\theta\), is of a given functional form, and then choose a particular density
of this form (i.e., its parameters) which most clearly matches prior beliefs.

In the case of the loan loss reserve decisions of commercial banks, both data-based and nondata-based information is available for the construction of the prior distribution (see Figure 1). Data-based prior information comes from a time series of ratios of loan loss reserves to total loans, whereas nondata-based prior information relates to tax and regulatory considerations. In our empirical analysis, we use only data-based prior information. It should be clear that this implies gains in simplicity at the expense of neglecting potentially relevant information. For reasons that will be apparent below, in particular those relating to the use of conjugate priors, the choice of a distribution to represent prior beliefs in this study falls upon the univariate normal distribution, written as $N(\mu, \tau^2)$: $\mathcal{X} = \mathbb{R}^1$, $-\infty < \mu < +\infty$, $\tau^2 > 0$, and

$$f(x|\mu, \tau^2) = \frac{1}{(2\pi)^{1/2} \tau} e^{-\frac{(x-\mu)^2}{2\tau^2}}$$  \hspace{1cm} (5)

where $\mu$ and $\tau^2$ are the mean and variance of the prior distribution and $x$ is any given observation. The normal distribution is especially suited to the use of conjugate distributions, for it produces a known functional form for the posterior distribution. This will be discussed in the next section.
3. THE OPTIMAL BANK LOAN LOSS RESERVE DECISION

3.1 The Posterior Distribution

Bayesian analysis consists basically of combining the prior information \( \pi(\theta) \) and the contemporaneous (sample) information \( (x) \) into the posterior distribution of \( \theta \) given \( x \), i.e., \( \pi(\theta|x) \), from which all decisions and inferences are made. This posterior distribution, then, reflects the updated beliefs about \( \theta \) after observing the sample \( x \).

As explained above, in this work the sample information comes from a cross section of commercial banks of similar characteristics, particularly size. This procedure seems justified on the grounds that it gives a better perspective of any given bank within a group of peer banks, as opposed to focusing only on the present experience of a particular bank under study. On the other hand, by having the latest information coming from a sample as opposed to only one observation, a bias is built into the Bayesian method which gives more weight to the contemporaneous information and less weight to the prior information.

If both the prior distribution and the contemporaneous information come from the same family of distributions, i.e., if we have conjugate families, then in some cases it is possible to obtain simple expressions for the parameters of the posterior distribution as functions of the parameters of the prior and the likelihood function. In particular, if both are normally distributed, it can be shown that

\[
\pi(\theta|x) \sim \mathcal{N}(u(x), \rho^{-1}),
\]

where

\[
u(x) = \frac{\sigma^2/n}{(\tau + \sigma^2/n)} \mu + \frac{\tau^2}{(\tau + \sigma^2/n)} \bar{x},
\]

\[
\rho = \frac{n\tau^2 + \sigma^2}{\tau^2 \sigma^2},
\]

(6) and (7)
and $\pi(\theta) \sim \mathcal{N}(\mu, \tau^2)$, $\mathbf{X} = (X_1, \ldots, X_n) \sim \mathcal{N}(\theta, \sigma^2)$. Note that $\bar{X} \sim \mathcal{N}(\theta, \sigma^2/n)$. In sum, the precision $^{11}$ measures of the prior and the sample information function as weights when computing the parameters of the posterior distribution of $\theta$ given a sample $x = (x_1, \ldots, x_n)$. The use of conjugate priors is appealing because it allows one to start with a prior of a given functional form and end up with a posterior distribution of the same functional form, but with parameters updated by the sample information.

The next step in the Bayesian analysis is predicated on the fact that the posterior distribution supposedly contains all available information about the parameter of interest $\theta$ (i.e., the ratio of loan losses to total loans). Therefore, any inferences or decisions concerning $\theta$ should be made solely through the posterior distribution. At this juncture the loss function plays a crucial role, as shown below.

3.2 The Bayes Rule $^{12}$

The derivation of the Bayes rule involves finding a decision rule which minimizes Bayes risk, defined as the lowest bound for the risk (i.e., the expected loss) of all decisions. Any Bayes decision, therefore, will be an optimal decision because the risk, or expected loss, cannot be smaller for any other decision. It is possible to demonstrate $^{13}$ that minimizing Bayes risk is equivalent to minimizing the expected loss with respect to $\pi(\theta|x)$, the posterior distribution of $\theta$ given $x$. This is to say that a Bayes rule minimizes the posterior expected loss of the action $a$. This quantity to be minimized is the
same as the one which is called (somewhat loosely) "average loss" in Figure 1.

In order to obtain specific Bayes rules, it becomes necessary to spell out the loss function that applies to the problem under study. As discussed above, in this study two types of loss function are of interest: the weighted squared-error loss and the generalized linear loss. The Bayes rules for these loss functions turn out to be the following. Consider first the weighted squared-error loss. If \( L(\theta, a) = w(\theta)(\theta - a)^2 \), the Bayes rule is

\[
\delta(x) = \frac{E_{\pi(\theta | x)}[\theta w(\theta)]}{E_{\pi(\theta | x)}[w(\theta)]} = \frac{\int \theta w(\theta)f(x | \theta) dF(\theta)}{\int w(\theta)f(x | \theta) dF(\theta)}
\]

where \( \pi(\theta | x) \) is the posterior distribution, as before, and \( f(x | \theta) \) is the likelihood function which gives rise to the contemporaneous (sample) information. This Bayes rule can be interpreted as a ratio of weighted averages of the prior distribution. Given the objectives of this study, however, this decision rule presents serious problems: first, because it does not suggest intuitively any particular location parameter or fractile of the posterior distribution; secondly, because of the disturbing presence of the (unknown) parameter of interest in the weight function.

The generalized linear loss offers a more interesting result. For this loss function,

\[
L(\theta, a) = \begin{cases} 
K_0(\theta - a) & \text{if } \theta - a > 0, \\
K_1(a - \theta) & \text{if } \theta - a \leq 0,
\end{cases}
\]
it turns out that \( \frac{K_0}{(K_0 + K_1)} \) fractile of \( \pi(\theta|x) \) is a Bayes estimate of \( \theta \). This result has several appealing features given the problem of estimating the ratio of loan losses to total loans in commercial banks.\(^{15}\) First, it is intuitive: it is relatively easy to conceptualize a fractile of a p.d.f. Second, it allows for asymmetric penalties for overestimation and underestimation to be reflected in the optimal decision rule, since they are captured in the weights \( K_0 \) and \( K_1 \) of the loss function or, more appropriately, in the \( K_0/K_1 \) ratio.\(^{16}\)

To summarize, we have shown that, under reasonable assumptions with respect to the choice of a loss function and of a prior distribution of the ratio of loan losses reserves to total loans, a Bayes decision rule for the optimal ratio of loan loss reserves to total loans emerges which is both theoretically sound and intuitive. In the next section, this methodology is applied to a sample of large U.S. commercial banks in an effort to identify patterns of decisions with respect to loan loss reserves and the perceived weights or penalties associated with underestimation and overestimation. In addition, our empirical results also address the question of optimality of those decisions, given the information sets represented by the prior distribution and sample information, resulting in a posterior distribution for the ratio of loan loss reserves to total loans.
4. DATA AND RESULTS

A sample of the largest 100 banks in 1986 was selected from yearend Reports of Condition filed with federal banking regulators by all insured banks. The final sample consists of the 82 banks that appeared in yearend data with the same bank identifier code for each year 1972 through 1986.

The ratio of each bank's yearend loan loss reserve to its total loans (hereafter loss reserve ratios) was calculated for each of these 15 years. Table 1 presents the mean and standard deviation of the loss reserve ratios and the average total assets for the 82 bank sample for each of the 15 years. It is interesting to observe that the average loss reserve ratio started relatively high in 1972, declined through 1978, and then increased through 1986, approaching its 1972 level. The yearly standard deviations are relatively stable except for the years 1983 and 1986 when they doubled in size. Figure 2 presents frequency histograms of the 82 loss reserve ratios for each of the years 1982 through 1985.

Chi square tests were conducted to determine if the null hypothesis of normality could be rejected for any of the 14 cross section distributions (1972-1985) of loss reserve ratios. Only in 1983 was the null hypothesis rejected at the 1 percent level. A few extreme values along with substantial kurtosis caused rejection of the null hypothesis.

Following the methodology described in the previous section, posterior distributions were constructed for each bank for four consecutive years assuming conjugate normal distributions. For example,
the first set of posterior distributions consisted of the 1972 through 1982 time series of yearend loss reserve ratios for each bank as the prior distributions and the cross section distribution of loss reserve ratios for all 82 banks in 1982 for the likelihood function (contemporaneous information). Therefore, 82 unique posterior distributions constituted the first set. This procedure was repeated to create a second set of 82 posterior distributions using the 1972 through 1983 time series of loss reserve ratios for each bank and the 1983 cross section of loss reserve ratios. The same procedure was repeated two more times, with the time series being lengthened by one year each time and the 1984 and 1985 cross sections being used.

As anticipated in footnote 9, the means of the posterior distributions were dominated by the mean of the cross section or sample information. In terms of Equation 6, the weight placed on \( \bar{x} \) was on the order of .95 to .98 indicating the importance of the contemporaneous information in making the loss reserve decision. In addition the posterior distributions have small variances relative to the cross section and time series distributions. Figure 3 illustrates this result by showing the time series distribution (1972-1982), the cross section distribution (1982), and the resulting posterior distribution for one bank.

The posterior distributions reflect the data-based information available to managers as they make their decisions about the next period's loan loss provision, and hence the loss reserve ratio. The actual loss reserve decision for each bank was located in its posterior distribution. For example, a bank's actual 1983 loss
reserve ratio was treated as a drawing from its posterior distribution which was constructed using the 1972 through 1982 time series data for the bank and the 1982 cross section data for all 82 banks. The location of this actual value was summarized as the percentage of the distribution to the left of the actual value, y percent (see footnote 16).

Following the procedure in footnote 16, four $K_0/K_1$ ratios were calculated for each bank using its actual loss reserve ratios for each year 1983 through 1986 and the associated posterior distributions. These ratios allow us to characterize the actual decisions of each bank. If the $K_0/K_1$ ratio takes on a value of less than 1, then $K_0 < K_1$ which indicates that the bank places greater weight (cost or penalty) on attaining a loss reserve ratio that is "too large" rather than "too small." We have used the label "aggressive" to describe this behavior. If the $K_0/K_1$ ratio takes on a value greater than 1, then $K_0 > K_1$ which indicates the placing of greater penalty on being under reserved. This behavior has been given the label "conservative."

Table 2 provides some summary information on the values of the $K_0/K_1$ ratio. Columns 2 and 3 present the number and percent of sample banks with $K_0/K_1$ ratios between 1 and 200, which encompasses 49.5 percent of the upper tail of the posterior distributions. Columns 4 and 5 present the number and percent of sample banks with $K_0/K_1$ ratios between 0.005 and 1. The tail to the left 0.005 constitutes only 0.5 percent of the mass of the posterior distribution. All other banks
had $K_0/K_1$ values in the tails of their respective posterior distributions, i.e., less than 0.005 or greater than 200.

Many banks demonstrated extreme values for the $K_0/K_1$ ratio, i.e., values that fall outside 99 percent of the posterior distribution, for the 1983-1986 period. Given the choice of the loss function, these loan loss reserve decisions are considered suboptimal in a statistical sense. Note, however, that this result follows basically from the fact that the variance of the posterior distribution is much smaller than the variances of the prior density or the cross sectional sample. This, in turn, is implied by the use of the parameters of the sample mean in the calculation of the parameters of the posterior density. The implication is that the frequency of actual decisions which are located within the posterior mass decreases dramatically.

An alternative procedure would be to use only the most recent loan loss reserve ratio as the contemporaneous information for each individual bank. The weight of this ample information would decrease substantially and the variance of the posterior density would be of about the same magnitude as that of the prior, as can be seen from Equations (6) and (7) since, in this case, $n = 1$. The number of seemingly optimal decisions would be much higher using this alternative procedure; however, this improvement in the results would come at the expense of disregarding an important part of the bank management's information set, that is, the actions of the set of peer banks.

Since $K_0/K_1$ ratios are calculated over a 4-year period it is possible to observe the consistency of bank loss reserve decisions over time. If the value of $K_0/K_1$ switches from greater than 1 to less
than 1 during a period of time, the bank lacks consistency in its decision process. Table 3 addresses the consistency of loss reserve decisions over the 1983-1986 period. Only 37 of the 82 banks made consistent decisions where consistency is defined as continuously aggressive or conservative during each of the four years. The remaining 45 banks had $K_0/K_1$ ratio values that were not consistently on one side of 1.

5. CONCLUSIONS AND LIMITATIONS

The major contribution of this paper is to lay out an intuitively appealing model of the process by which managers of individual banks determine the size of their loan loss reserve ratios. These decisions are approached using the framework of a Bayesian decision model. Using a generalized linear loss function decision rule for the optimal ratio of loan loss reserves to total loans emerges.

Data for 82 large banks for the period 1972-1986 is used to examine actual decisions to infer the relative sizes of penalties for overestimation and underestimation of the loss reserve ratios. Results indicate that between 9 percent and 20 percent of the 82 banks behaved as if the penalty for underestimation was larger than the penalty for overestimation; another 5 percent to 21 percent made decisions as if the penalty for overestimation were the larger penalty. A large number of the loss reserve decisions were not optimal.

Shortcomings of this research include the use of annual data. In fact managers would have more recent than one year old cross section information on the decisions of peer banks. The use of quarterly data
may improve the results of this approach. The model used here was based only on data-based information and did not take into account tax considerations, examination results, and other information that may influence the loss reserve decision.\footnote{17} Finally, in order to construct the posterior distributions, it was assumed that the prior and the cross section distributions were normal. While tests of the cross section distributions indicated that this was generally the case, the prior distributions have too few data points to conduct such tests. These limitations suggest possible extensions of this study.

Overall, perhaps the basic message of this paper is that the application of statistical decision theory allows us to examine the loan loss reserve decision in commercial banks from the individual firm's viewpoint, as opposed to an aggregate point of view. In short, the loan loss reserve decision is treated as a management controlled variable within a framework of tax, regulatory, and market constraints.
NOTES

1 One recent study states that "the [loan] loss provision at large banks cut the industry's returns on assets and equity in 1987 to about one-fifth of their 1986 levels." See McLaughlin and Wolfson [1988], p. 403.

2 Consider the fact some banks actually make negative provisions at times in an effort to adjust their loan loss reserves to their desired levels. Negative provisions are difficult to rationalize. It seems to be more sensible to focus on the loan loss reserve or a ratio of loan loss reserve to total loans or net loans and treat the provision for loan losses as a residual decision.

3 Recall that many observers seemed to agree that Citicorp's decision to allocate $3 billion to its loan loss reserve in the spring of 1987 put a great deal of pressure on other money center banks with a large degree of exposure to Third World debt to follow suit.

4 In general, nondata-based prior information does not change the functional form of the prior distribution but may change its parameters.

5 There are many good references in the econometrics and statistics literature which discuss Bayesian decision theory in detail. This study has relied especially on DeGroot [1970] and Berger [1980]. Mindful of space limitations, we have attempted to keep this portion of the study as short as possible. The reader is referred to the above sources for a comprehensive discussion of the Bayesian methods. Our notation generally follows Berger [1980].

6 A possible extension of this study would be to design a more comprehensive empirical procedure which takes nondata-based prior information into account.

7 This choice is not entirely arbitrary. Normality tests were performed on the cross sectional data and the null hypothesis of normality could not be rejected (see Section 4 below). The small size of the available time series of loan loss reserves to total loans, which goes back only to 1972, prevents any serious attempt at resolving the standard statistical problem of determining a density function from a series of observations from that density. Hence the assumption of normality for the prior distribution.

8 Other possible criteria for defining the sample of peer banks would be location and similarities in the structure of the loans and securities portfolio.
This point becomes immediately apparent below when one realizes that the distribution of the sample mean is utilized. The bigger the sample, the smaller the variance of the sample mean. This improves the precision of the contemporaneous information and adds to its weight in the computation of the parameters of the posterior distribution.

A detailed treatment of this point is given in Berger [1980], pp. 92-98.

Precision is defined as the inverse of the variance.

Bayesian analysis usually makes two assumptions at this point. First, the prior p.d.f. is assumed to be proper or informative. Second, the problem is assumed to have a finite Bayes risk. This method is known as the extensive form of Bayesian analysis.

See Berger [1980], pp. 108-9, for a proof.

A derivation of these Bayes rules is presented in Berger [1980], pp. 111-112.

This result implies that if \( K_0 = K_1 \), i.e., the penalties for overestimating or underestimating the ratio of loan losses to total loans are perceived to be equal by the bank's management, then the median of the posterior distribution is a Bayes decision. If the conjugate distributions are distributed normally, then, of course, this is also equivalent to the mean of the posterior.

Suppose we have a \( y(\%) \) fractile of \( \pi(0|x) \). Then \( (K_0/(K_0+K_1)) = y_\% \). Make \( K_0 = 1 \) to obtain \( K_1 = \frac{1-y_\%}{y_\%} \) where \( y_\% \) gives us the proportion of the posterior distribution below the fractile \( y \). When \( K_0 = K_1 \), then \( y \) is equal to 1/2. This is the median of the posterior. For the fractiles below the median, \( K_0 < K_1 \) and \( \frac{K_0}{K_1} < 1 \). Conversely, for the fractiles above the median, \( K_0 > K_1 \) and \( \frac{K_0}{K_1} > 1 \) (see the diagram below). In our empirical tests, we will be ultimately interested in the \( K_0/K_1 \) ratio.
This portion of the information set might change the parameters of the prior distribution (see footnote 4).
REFERENCES


Figure 1. The Decision Process of the Regulated Banking Firm.
FIGURE 2

FREQUENCY HISTOGRAMS OF LOSS RESERVE RATIOS
1982 - 1985

1982

LOSS RESERVE RATIO

1983

LOSS RESERVE RATIO

1984

LOSS RESERVE RATIO

1985

LOSS RESERVE RATIO
FIGURE 3

DENSITY FUNCTIONS FOR ONE BANK

- - PRIOR

- - CROSS SECTION

-. - POSTERIOR

LOSS RESERVE RATIO
Table 1
82 Bank Sample Values

<table>
<thead>
<tr>
<th>Year</th>
<th>Loan Loss Mean</th>
<th>Reserve Ratio Std Dev</th>
<th>Average Total Assets (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>1.743%</td>
<td>0.3727%</td>
<td>$1,811,314</td>
</tr>
<tr>
<td>1973</td>
<td>1.691%</td>
<td>0.3010%</td>
<td>2,212,782</td>
</tr>
<tr>
<td>1974</td>
<td>1.687%</td>
<td>0.2987%</td>
<td>2,242,175</td>
</tr>
<tr>
<td>1975</td>
<td>1.685%</td>
<td>0.2990%</td>
<td>2,212,040</td>
</tr>
<tr>
<td>1976</td>
<td>1.275%</td>
<td>0.4147%</td>
<td>2,408,552</td>
</tr>
<tr>
<td>1977</td>
<td>1.225%</td>
<td>0.3439%</td>
<td>2,649,277</td>
</tr>
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<td>1978</td>
<td>1.051%</td>
<td>0.2480%</td>
<td>4,741,077</td>
</tr>
<tr>
<td>1979</td>
<td>1.113%</td>
<td>0.2866%</td>
<td>5,442,852</td>
</tr>
<tr>
<td>1980</td>
<td>1.149%</td>
<td>0.3245%</td>
<td>6,057,804</td>
</tr>
<tr>
<td>1981</td>
<td>1.170%</td>
<td>0.3279%</td>
<td>6,933,696</td>
</tr>
<tr>
<td>1982</td>
<td>1.234%</td>
<td>0.3744%</td>
<td>12,815,100</td>
</tr>
<tr>
<td>1983</td>
<td>1.369%</td>
<td>0.6102%</td>
<td>13,230,700</td>
</tr>
<tr>
<td>1984</td>
<td>1.320%</td>
<td>0.3500%</td>
<td>14,096,150</td>
</tr>
<tr>
<td>1985</td>
<td>1.432%</td>
<td>0.3483%</td>
<td>15,187,520</td>
</tr>
<tr>
<td>1986</td>
<td>1.652%</td>
<td>0.6157%</td>
<td>16,448,830</td>
</tr>
</tbody>
</table>
Table 2

Number and Percentage of Banks at the Critical Values of the Posterior Distribution

<table>
<thead>
<tr>
<th>Year</th>
<th>Conservative Banks with $1 &lt; K_0/K_1 &lt; 200$</th>
<th>Aggressive Banks with $0.005 &lt; K_0/K_1 &lt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td></td>
</tr>
<tr>
<td></td>
<td>of Banks</td>
<td>% of Sample</td>
</tr>
<tr>
<td></td>
<td>of Banks</td>
<td>% of Sample</td>
</tr>
<tr>
<td>1983</td>
<td>8</td>
<td>9.8%</td>
</tr>
<tr>
<td>1984</td>
<td>13</td>
<td>15.9%</td>
</tr>
<tr>
<td>1985</td>
<td>16</td>
<td>19.5%</td>
</tr>
<tr>
<td>1986</td>
<td>7</td>
<td>8.5%</td>
</tr>
</tbody>
</table>

Sample size = 82
Table 3
Consistency of Bank Loss Reserve Decisions for the years 1982-1985

<table>
<thead>
<tr>
<th>Consistency of Decision</th>
<th>Number of Banks</th>
<th>Percentage of Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consistently Conservative $1 &gt; \frac{K0}{K1}$</td>
<td>19</td>
<td>23.2%</td>
</tr>
<tr>
<td>Consistently Aggressive $0 &lt; \frac{K0}{K1} &lt; 1$</td>
<td>18</td>
<td>22.0%</td>
</tr>
<tr>
<td>Not Consistent</td>
<td>45</td>
<td>54.9%</td>
</tr>
</tbody>
</table>

Sample size = 82