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INVESTIGATION OF MUTUAL COUPLING EFFECTS ON PATTERN NULL RECONFIGURABLE ANTENNAS IN SMALL ADAPTIVE ARRAYS

BY

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THESIS

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Adaptive arrays composed of fixed pattern elements have been studied extensively in the past. In these arrays, adaptive processing algorithms have control over the magnitude and phase applied to each array element to modify the array pattern. Recently, pattern reconfigurable elements have been introduced into adaptive arrays, establishing a new form of array pattern control. Using a reconfigurable element model, null reconfigurability was shown to be particularly advantageous in small adaptive arrays. Unfortunately, strong mutual coupling between array elements, often a result of their close proximity, can significantly distort each element pattern and substantially affect the predicted adaptive array performance. In this work, the coupling between two null reconfigurable elements with different null tilt combinations has been characterized as a function of element spacing through simulation and measurement. The effect of coupling on element pattern null magnitude and location has also been investigated. Furthermore, the effect of coupling on the performance of reconfigurable adaptive arrays has been studied using the signal-to-interference-plus-noise ratio metric. Results show that although the coupling between elements is low, it has a noticeable impact on the depth and position of each pattern null and even results in the formation of an additional null in some instances. This leads to differences between the performance predicted by the adaptive algorithm and the performance obtained from practical null reconfigurable elements in the presence of coupling.
I want to thank my adviser, Professor Jennifer Bernhard, for her guidance throughout this project. I would also like to thank the other antennas research group members for the thoughtful discussions on this work and their guidance in the laboratory. Finally, I want to thank my family for their support and encouragement throughout my studies.
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CHAPTER 1

INTRODUCTION

As a growing number of wireless technologies are introduced and must share finite electromagnetic spectrum, the need for higher performance wireless systems is increasing. Modern wireless communication devices are already requiring increased signal performance while demanding lower power consumption. Current wireless circuit designs, however, are often limited by their analog-to-digital converters (ADCs). Cognitive radios, in particular, are examples of wireless devices whose power consumption reduction is currently limited by the dynamic range requirement of their ADCs. Cognitive radios use the electromagnetic spectrum more efficiently by sensing and compensating for other users and interference sources. In order to perform this task well, a cognitive radio must be able to tolerate multiple signals of various strengths at once. This requires high dynamic range ADCs which consume more power and have become a limiting factor for widespread adoption of modern cognitive radio [1].

Spatial filtering as proposed by van den Heuvel and Cabric can significantly reduce the ADC dynamic range requirement necessary for high-performance cognitive radio systems [1]. Spatial filtering is a method to reduce or eliminate undesired signals incident from particular angular directions. Spatial filtering relies on antenna pattern manipulation to improve reception of desired signals and reduce reception of undesired signals. This removes the effects of undesired signals before they reach the receiver’s front end, which reduces the performance requirement of the radio’s circuitry. In cognitive radios this allows the use of ADCs with low dynamic range and therefore low power consumption. Adaptive arrays have been shown to be an effective method to achieve spatial filtering and the benefits associated with it.

Adaptive arrays provide spatial filtering by combining signal processing with antenna arrays. They use an algorithm implemented in a signal processor to generate the optimal magnitude and phase to apply to each array
element to maximize the desired signal reception. Typically this involves placing nulls in the direction of interferers and placing the pattern maximum in the direction of the desired signal. The process occurs in a control feedback loop to continuously update the array pattern to maximize the desired and minimize the interference signals, as they change position and strength. Adaptive arrays, as described here, were first explored by Howells and Applebaum in the late 1950s. Their work was later published in [2, 3]. In addition to the works by Howells and Applebaum, an adaptive technique developed by Shor [4], and the LMS (least-mean-square) implementation investigated by Widrow [5] form the foundations of modern adaptive array technology.

Adaptive arrays are an effective way to provide spatial filtering. However, adaptive array technology has significant limitations in the areas of portable and mobile communications which are those largely targeted by cognitive radio applications. Adaptive arrays have traditionally been composed of elements with fixed patterns such as dipoles. Fixed pattern elements in large phased arrays are well understood, straightforward to analyze, and relatively easy to construct. Numerous works have laid the groundwork for the implementation of adaptive arrays with fixed pattern elements. Notable is the detailed work by Compton [6]. Similar to their non-adaptive counterparts, adaptive arrays composed of elements with fixed patterns have control over the magnitude and phase applied to each individual element. For adaptive arrays composed of a large number of elements, magnitude and phase control has been more than satisfactory to achieve excellent interference signal cancelation. After all, large arrays have been shown to have a considerable number of degrees of freedom as a direct result of their significant number of elements [6]. Additionally, large arrays possess extremely high directivity and steerability as a direct result of each array’s large aperture size and large number of elements. Combined with adaptive technologies, they can mitigate large numbers of interference signals. However, their large size and extremely high cost have limited their uses to relatively few applications that can afford the space and expense associated with these antennas. Mobile applications such as those required for a small vehicle or a backpack have limited space for antenna array communication systems. Limited space makes it increasingly essential to get maximum performance out of an array for its size and cost.

Arrays have been mostly ignored for the portable and mobile markets. After all, traditional arrays historically have required a large number of elements.
to achieve sizeable performance gains above a single element. This paradigm is being challenged by advances in reconfigurable antenna technology. Work performed by Roach showed that reconfigurable antennas can be modeled and introduced into adaptive array algorithms [7]. Furthermore, pattern reconfigurable antennas, in particular pattern null reconfigurable antennas, were shown to provide a promising solution to improve the performance of small adaptive arrays [7]. A small increase in array complexity by utilizing a reconfigurable antenna can increase the degrees of freedom of an adaptive array and substantially improve small array performance. Pattern reconfigurable antennas can provide antenna pattern performance improvements that would otherwise be unattainable with a single element. Combining reconfigurable antennas with adaptive array technology provides a promising solution to significantly improve the performance of small arrays for portable and mobile applications. This makes reconfigurable antenna elements a highly attractive option. Besides the work performed by Roach, previous work addressing adaptive array element patterns include the work by Ishide and Compton which studied the effect of element patterns on adaptive array grating nulls [8]. Additionally, Compton described a method to select element patterns to achieve maximum adaptive array performance [9].

Although reconfigurable antennas in adaptive arrays have shown promise, the practical implementation of these antennas in an array configuration has not been studied and poses many challenges. It is well known that mutual coupling can have a significant impact on the performance of fixed pattern element adaptive arrays [10]. Reconfigurable antennas will similarly suffer from mutual coupling, which will impact each element’s electrical characteristics and radiation pattern. The performance of adaptive arrays composed of reconfigurable antennas depends heavily on the ability of each element to change its pattern to that specified by the adaptive algorithm. Any fluctuations in the electrical characteristics or pattern of an element as a result of mutual coupling will degrade the performance of the array. Although small spacing between array elements is important in achieving a small form factor, the mutual coupling between elements is known to increase as the element spacing decreases. As a result, an understanding of the limitations imposed by mutual coupling on element spacing is critical for dependable adaptive array performance.

In this work, the practical considerations for the integration of a pattern
null reconfigurable element into an adaptive array are investigated. Chapter 2 begins with an overview of adaptive array technology and describes in detail previous work on integrating reconfigurable elements into adaptive arrays. In addition, Chapter 2 describes a pattern null reconfigurable element that is investigated in the remainder of the work. Chapter 3 goes on to introduce the array configurations in which the pattern null reconfigurable element described in Chapter 2 will be introduced. Using these array configurations, the mutual coupling between elements is characterized through simulation and measurements. After the mutual coupling is characterized in Chapter 3, Chapter 4 investigates the effects of coupling on the radiation pattern of each element in the array configurations. Finally, Chapter 5 explores the effects of mutual coupling on adaptive array performance. First, the least mean square (LMS) algorithm developed by Roach to work with reconfigurable elements is modified for pattern null reconfigurable elements with switched null states. The signal-to-interference-plus-noise ratio (SINR) is then used to compare the performance between the model null reconfigurable elements and the simulated, practical null reconfigurable element design. Finally, Chapter 6 ends with conclusions from this study and possible directions for future research.
CHAPTER 2

PATTERN RECONFIGURABILITY IN
ADAPTIVE ARRAYS

Before the practical considerations of implementing a pattern null reconfigurable antenna into an adaptive array can be examined in detail, preceding research in this area must be introduced in order to provide a framework for this study. In this chapter, previous work on integrating pattern reconfigurable antennas into adaptive arrays is described in detail. Because the integration of pattern reconfigurable elements into adaptive arrays is an extension of traditional fixed pattern element adaptive array theory, this chapter begins by introducing fixed pattern element adaptive arrays. After the traditional adaptive array has been introduced, previous work performed by Roach in integrating pattern reconfigurable elements into adaptive algorithms is described [7]. Furthermore, important insight from Roach’s study that provides the groundwork for the current work is emphasized. Next, a pattern null reconfigurable element that was previously designed by Yong and Bernhard is introduced [11]. This element was designed to take advantage of the results of Roach’s work which suggests that a pattern null reconfigurable element is highly effective in adaptive arrays. This element’s performance in an array configuration will be studied in detail in future chapters.

2.1 Overview of Traditional Adaptive Arrays

An adaptive array is an antenna that controls its own pattern using feedback control [6]. These arrays consist of a group of antennas that use a signal processor and feedback control to continuously generate optimal magnitudes and phases to apply to each array element for a particular signal environment. The optimal magnitudes and phases create an array pattern that improves reception of a desired signal and reduces reception of undesired signals. Modern adaptive arrays typically consist of a set of fixed pattern antenna elements
in an array configuration each driven by a phase shifter and amplifier. In addition, behind the antenna array is signal processing hardware that detects incident signals and runs an algorithm that generates the optimum element weights. The signal processing hardware and antenna array are combined in a feedback control loop that will converge on an optimal array pattern and will continuously update the pattern as the incoming signals change.

For example, consider the scenario when a desired signal, the signal intended to be received, and a set of interfering signals are incident on the fixed pattern element array from arbitrary angles as shown in Fig. 2.1.

Figure 2.1: Components of a traditional adaptive array [7].

The signals received by each element will be monitored by the signal processor along with the array output. The signal processor uses these signals with a particular algorithm to formulate the optimal weights to apply to each element. The array must constantly readapt to account for changes in the incident signals. For instance, signal properties including the number of signals incident on the array, the angle of arrival of each signal, or the power level of each signal might change over time. Each one of these incident signal changes requires the array to readapt in order to maintain the optimal array pattern for the given signal environment. The antenna array and signal processor form a control loop that will continuously maintain the optimal array pattern.

An adaptive algorithm runs on the signal processing hardware and is an
essential part of any adaptive array. The adaptive algorithm generates the optimal element weights that form an optimal array pattern for the particular signal environment. Because the algorithm is critical to the adaptive array operation, a variety of different algorithms have been developed. Popular algorithms include the least mean square (LMS), Applebaum, and Shor algorithms [6]. Each algorithm has particular strengths and weaknesses. For instance, an attractive feature of the least mean square array is that it does not require knowledge of the direction of arrival of the incoming signals. However, it does require a reference signal correlated with the desired signal. Each algorithm type is based on a particular optimization criterion. For instance, the optimization criterion for the LMS array is the minimization of the mean square error, while the Applebaum array maximizes the signal-to-interference-plus-noise ratio [6]. The LMS array will be described more rigorously in Chapter 5 when the algorithm used for switched null reconfigurable elements is described.

2.2 Reconfigurable Elements in Adaptive Arrays

Traditional adaptive arrays use fixed pattern elements and an adaptive algorithm to generate optimal element weights. The optimal weights produce an array pattern that improves the reception of the desired signal while rejecting interfering signals. The adaptive algorithms used in arrays with fixed pattern elements only have control over each element’s magnitude and phase. However, it was recently shown by Roach that pattern reconfigurable antennas can be integrated into adaptive arrays [7]. By using reconfigurable elements in the array, adaptive algorithms now have control over each individual element pattern, which provides an additional degree of control over the array pattern. Reconfigurable elements, in effect, give the adaptive array more flexibility in reducing interference signals and improving desired signal reception than traditional fixed pattern elements.

2.2.1 Reconfigurable Element Model

A reconfigurable antenna must be accurately modeled before it can be integrated into an adaptive array and have its radiation pattern controlled by an
adaptive algorithm. In addition to generating the optimal weights applied to each element, the adaptive algorithm must select the optimal reconfigurable antenna pattern for a particular signal environment. Therefore, the algorithm requires an understanding of the reconfigurable element’s capabilities and constraints. To satisfy this algorithm requirement, a two-element subarray model for a reconfigurable antenna was developed by Roach which could be easily integrated into an adaptive algorithm [7].

The subarray model developed to represent a reconfigurable element consists of two isotropic elements as shown in Fig. 2.2 and provides a series of benefits for use in adaptive algorithms.

Figure 2.2: Subarray model for a reconfigurable element [7].

First, the subarray allows for a unique pattern to represent each reconfigurable element in the array. Each two-element subarray can have a steerable mainbeam or null whose position is controlled separately from other neighboring subarrays. Second, each subarray can be designed to accurately capture the pattern of a practical reconfigurable element. This includes having a null or pattern maximum in a particular direction. Finally, the subarray model can be integrated into already present adaptive algorithms. The current algorithms that exist are already designed to generate weights for fixed pattern elements. In the subarray model, each reconfigurable element is represented by two fixed-pattern isotropic elements. Therefore, the algorithms do not have to fundamentally change to accept the reconfigurable element model. However, they do have to expand to take into account more elements with different element spacings and certain element constraints.

In order for the two-element subarray model to accurately represent a practical reconfigurable antenna, certain constraints were applied to the model’s isotropic elements. First, a complex conjugate constraint was applied to the two-element model. This means that the weight applied to the second ele-
ment in the subarray is the complex conjugate of the weight applied to the first element. This forces each subarray element to have the same magnitude, which simplifies subarray beam tilt operations and ensures the pattern tilt is dependent only on the phase between subarray elements. However, this constraint does force a two-element limit on the number of subarray elements in the model. The complex conjugate constraint also aids in forming a single null in the reconfigurable element model pattern when a practical null reconfigurable element is desired.

In addition to the complex conjugate constraint, a beam steering constraint was applied to the subarray model. This constraint forces the real and imaginary parts of the complex subelement weights to be related to each other in such a way as to force the mainbeam of the subarray to be limited to a certain angular range. This helps represent a practical reconfigurable antenna which has limitations in the angle over which it can change its pattern. With the constraints applied, the pattern of the modeled reconfigurable element is shown in Fig. 2.3 for a null at broadside and 14 degrees.

![Subarray model patterns](image)

**Figure 2.3:** Two-element subarray model patterns for a null at broadside and 14 degrees.

### 2.2.2 Reconfigurable Adaptive Array Case Study Results

In order to gain insight into the capabilities of reconfigurable elements in adaptive arrays, Roach performed a number of case studies using the two-element isotropic reconfigurable element model described above. Each differ-
ent scenario consisted of a unique incident signal environment that challenged the adaptive array capabilities. In each case study the adaptive algorithm was free to select the best magnitude, phase, and radiation pattern to apply to each element of the adaptive array. The results of the study into integrating pattern reconfigurable elements into adaptive arrays produced valuable insight.

Roach studied a situation in which four signals were incident on an array of five reconfigurable elements from different angles. The desired signal was incident from 45 degrees and the interference signals were incident from 118, 120, and 122 degrees as indicated in Fig. 2.4. Each particular reconfigurable element pattern that the algorithm generated along with the total array pattern is shown in Fig. 2.4a. The angle of each pattern maximum and pattern null are listed in the accompanying table in Fig. 2.4b.

![Figure 2.4: Case study result for a desired signal at 45 degrees and interference signals from 118, 120, and 122 degrees [7].](image)

- $\theta_d = 45^\circ$, $\theta_{int} = \{118^\circ, 120^\circ, 122^\circ\}$

The results show that the algorithm tends to direct the null of each reconfigurable element toward the interference sources, producing an array pattern null in the direction of the interfering signals. The results of this particular case study and other case studies showed that often the best reconfigurable element pattern combinations had each element’s null pointing toward an interference source instead of having each element’s main beam directed toward the desired signal. This indicated that a reconfigurable null tilt, in many cases, may be more advantageous than a reconfigurable beam tilt in small adaptive arrays.
2.3 Pattern Null Reconfigurable Element

A pattern null reconfigurable microstrip element was designed to take advantage of the findings showing that null reconfiguration is advantageous in small adaptive arrays [11]. The element designed is shown in Fig. 2.5.

![Figure 2.5: Microstrip pattern null reconfigurable element.](image)

The configuration consists of a probe-fed driven patch surrounded by four parasitic patches. Each patch is connected to the driven patch by two switches. The switches can be opened and closed in particular combinations to tilt the element’s single null in the yz-plane (E-plane). The patches can be configured for a null at broadside or a null at ±14 degrees from broadside where the positive angles are toward the y-axis. For instance, the null appears at broadside when all eight switches are connected. However, the null tilts to +14 degrees from broadside toward the y-axis when the four switches on the right half of the patch are opened. Similarly, the element’s symmetry suggests that the −14 degree null tilt occurs when only the four switches on the left side of the patch are open. The element patterns for the 0 and 14 degree null tilt states are shown in Fig. 2.6.

The design achieves a null at broadside with all eight switches closed by exciting a TM_{02} mode on the antenna. The TM_{02} mode has electric field maxima at both ends of the patch. Using the surface equivalence principle, the antenna can be viewed as having two radiating slots, each with equivalent magnetic surface currents directed in opposite directions. Because the radiating slots are separated by close to one free-space wavelength, a null is present at broadside, consistent with array theory. On the other hand, the null tilts to 14 degrees when the switches connecting the two right parasitic patches to the driven patch are opened. This is a result of the TM_{01} modes strongly excited on the parasitic patches. The fields from the parasitic
patches combine with the TM$_{02}$ mode excited on the driven patch to tilt the null toward 14 degrees.

The element was designed using a Rogers Duroid 5880 substrate. The dimensions of the microstrip patch are shown in Fig. 2.5. For this study, the element was arbitrarily chosen to operate at 3.5 GHz, but the element is scalable to operate at other frequencies. Additionally for this study, the switches connecting the parasitic patches to the driven patch will be hard-wired copper tape connections. This simplifies the element construction and allows the study to focus on mutual coupling effects in array configurations without the difficulties associated with the introduction of bias networks and solid state device nonlinearities.

2.4 Summary

This chapter introduced in detail the work previously done to incorporate pattern reconfigurable elements into adaptive arrays. A novel two-element subarray model was presented that is instrumental for integration of reconfigurable elements into adaptive algorithms. Previous studies that used the subelement model were shown, and they suggest that a pattern null reconfigurable antenna is highly advantageous in small adaptive arrays. As a result, a pattern null reconfigurable antenna whose performance matches closely
with the subarray model was described. The following chapters will build on the foundations presented here. The null reconfigurable element will be introduced into an array configuration and the effects of mutual coupling on element and adaptive array performance will be studied extensively.
Before the pattern null reconfigurable element that was introduced in Chapter 2 can be integrated into a small adaptive array, the extent of the coupling experienced by this element in an array must be well understood. Mutual coupling can significantly alter antenna array characteristics which can lead to unpredictable adaptive array performance. This chapter quantifies the coupling between two of the pattern null reconfigurable elements as a function of element spacing. The chapter begins by introducing the array configurations studied and goes on to detail the coupling between elements using $S$-parameter results derived from simulations. The results give an indication not only of the coupling between elements, but the effects of coupling on the impedance of each element. Finally, the simulation results are verified through the construction and measurement of an experimental array.

3.1 Array Configurations

In order to investigate the coupling performance of the null reconfigurable element, it must first be introduced into an array configuration. For this study the element is explored in a two-element array setup and is designed for an operating frequency of 3.5 GHz. This work targets using reconfigurable elements to improve the performance of small arrays. As a result, a two-element array provides an appropriate framework to understand the small array coupling situation. The two-element configuration provides the simplest coupling environment and will provide the most insight into element coupling behavior. Moreover, two-elements allow for a straightforward, tractable implementation in a commercial electromagnetic simulator.

As described in Chapter 2, the pattern null reconfigurable element is designed for switched null operation and can be operated in three different
states: a null at $-14$, $0$ or $+14$ degrees. Because the null tilts in the E-plane of the antenna, the elements in the array will be positioned collinearly along this plane. Each element null state has unique current and field distributions on its patches. Different combinations of array element null tilts will produce unique coupling situations and must be examined independently. The array configurations in which the element’s null is at $0$ or $+14$ degrees will be considered in this study. Besides quantifying the coupling for the $0$ and $+14$ degree null element states, these results will provide insight into the coupling behavior for the $-14$ degree null tilt state as well as for elements of this design topology with continuous null tilts.

The four array configurations investigated are listed in Table 3.1. The first configuration has the null of both elements positioned at $0$ degrees. Element 1 in configuration 2, on the other hand, has a null tilt of $14$ degrees. An example of the array setup for configuration 2 is shown in Fig. 3.1. The null tilt is denoted on the element in the figure. For this study, the right element in the array will be referred to as element 1 and the element on the left will be referred to as element 2. For this configuration, the switches on the right-hand side of element 1 are open which produces a null tilt at $14$ degrees. On the other hand, all the switches are closed in element 2 which produces a null at broadside as described in Chapter 2.

Table 3.1: Two-element array configurations

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<th>Null Position (°)</th>
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<tr>
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<td>Element 2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
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<td>3</td>
<td>14</td>
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The distance between elements for this study was chosen to be measured from the geometrical center of each element which is also the position of the element’s feed. The coupling will be observed as the center to center distance between elements for a particular configuration is increased. Because the element is close to one free-space wavelength in length and the element spacing was chosen to be from center to center, the element spacing in the remainder of this study will begin at one free space wavelength so the patches do not overlap.
3.2 Simulation Results

The mutual coupling between elements was studied as a function of element spacing using Ansoft’s HFSS. The array configurations listed in Table 3.1 in section 3.1 were considered. The coupling was observed as the center to center distance between elements, $d$ in Fig. 3.1, for a particular configuration was increased. The coupling was studied using the $S$-parameter results derived from full-wave simulation. The coupling between elements is shown in Fig. 3.2. As expected, the coupling between elements, $S_{21}$, decreased as the element spacing increased for all configurations. Additionally, the coupling between elements is relatively low for all configurations and spacings.

Figure 3.2: Mutual coupling between two pattern null reconfigurable elements where $d$ is the distance between elements as indicated in Fig. 3.1.

Fig. 3.3 shows the $S_{11}$ and $S_{22}$ results which give an indication of the
passive driving impedance of elements 1 and 2, respectively. This provides insight into the effect of mutual coupling on each element’s impedance match. Both elements in all configurations and spacings have a reflection coefficient below $-10$ dB, which is below the generally accepted VSWR = 2 boundary. Additionally, the $S_{11}$ and $S_{22}$ for any particular configuration did not vary substantially as the spacing between elements was increased. This shows that the coupling between elements is not affecting the $S_{11}$ and $S_{22}$ results substantially. However, some $S_{11}$ and $S_{22}$ results are substantially lower than others. For example, the $S_{11}$ of configurations 1 and 3 are close to $-20$ dB while the $S_{11}$ for configurations 2 and 4 are just under $-10$ dB. Element 1 in configurations 1 and 3 has a null at broadside while configurations 2 and 4 have a null at 14 degrees. The isolated element with a null at 14 degrees is not as well impedance-matched to $50\Omega$ as when the element is configured for a null at broadside. As a result, the isolated element null tilt impedance match differences are reflected in the array $S_{11}$ and $S_{22}$ values.

![Figure 3.3: Simulated reflection coefficients (given by $S_{11}$ and $S_{22}$) for elements 1 and 2, respectively.](image)

The $S_{11}$ and $S_{22}$ results aid in understanding the $S_{21}$ results seen in Fig. 3.2. Configuration 1 has the highest coupling of all the configurations while configuration 4 has the lowest coupling. The levels of coupling experienced by configurations 2 and 3 fall between the magnitude of coupling experienced by configurations 1 and 4. Configuration 1 has two elements with nulls at
broadside and both are well matched as shown in Fig. 3.3. As a result, most of the power incident on element 1 is not reflected back to the source and is available to couple to the other element. The second element is well matched as well so the power can be transmitted through to be measured at port 2. Both elements in configuration 4, on the other hand, are not as well matched as those in configuration 1. As a result, more of the power incident on the elements is reflected and is not available to couple to the other element. Configurations 2 and 3 each have an element that is well matched and an element that is not as well matched. As a result, the coupling experienced in these arrays falls between that experienced by array configurations 1 and 4.

3.3 Measured Results

In order to verify the full-wave simulation results, a two-element array with 1.23\(\lambda\) element spacing was constructed and measured. Both elements were constructed on a common Rogers 5880 duroid substrate with a relative permittivity of 2.2. The substrate was first routed to size (1.5\(\lambda\) x 2.7\(\lambda\)) using a milling machine and then the patches were etched using ferric chloride. Over a quarter free-space wavelength of ground plane was left surrounding the microstrip patches. The constructed array setup for configuration 2 is shown in Fig. 3.4.

![Figure 3.4: Constructed two-element array setup in configuration 2.](image)

S-parameter measurements were taken for array configurations 2 and 4 over a frequency range of 3.2 to 3.8 GHz. The measured coupling between
elements alongside the simulated result is shown in Fig. 3.5. Additionally, Fig. 3.6 shows the measured and simulated reflection coefficient for both elements 1 and 2. Figs. 3.5 and 3.6 show that the trends in the measured data agree well with those of the simulation. The measured results, however, are shifted approximately 50 MHz higher in frequency than the simulation predicts. The frequency shift experienced is a result of the finite element discretization used by HFSS in the simulation. Similarly, the measured and simulated results for configuration 4 are shown in Figs. 3.7 and 3.8. Once again, the trends between the measured and simulated data match well with the measured results shifted 50 MHz higher in frequency.

3.4 Summary

In this chapter, the coupling between two null reconfigurable elements was quantified as the spacing between elements was increased. The simulation and measured results show that the coupling between elements is relatively low for two elements positioned collinearly along the E-plane of the antennas. Although S-parameters give important insight into the coupling experienced by the elements in the array, further study is necessary to quantify the effects of coupling on each element’s radiation pattern. In Chapter 4 the effects of coupling on each element’s radiation pattern will be shown and will provide intuition into the effect of coupling on adaptive array performance.
Figure 3.5: Coupling between elements in array configuration 2.

Figure 3.6: Measured and simulated reflection coefficients over frequency for array configuration 2.
Figure 3.7: Coupling between elements in array configuration 4.

Figure 3.8: Measured and simulated reflection coefficients over frequency for array configuration 4.
CHAPTER 4

EFFECTS OF COUPLING ON ELEMENT RADIATION PATTERNS

Pattern null reconfigurable elements have the potential to significantly improve the performance of small adaptive arrays. The performance improvements come about from being able to steer a pattern null in the direction of an interference source. However, the pattern null reconfigurable elements will experience coupling when introduced into an array. In order to place a pattern null toward an interfering source, the effects of coupling on each element's radiation pattern must be well understood. The adaptive algorithm expects each element to be able to deliver a null in a particular direction, that of the isolated element. However, coupling will alter each element’s radiation pattern and resultantly each null’s location and characteristics. Unexpected alterations from mutual coupling in each element pattern will change the adaptive array performance to some extent. As a result, an understanding of the coupling effects on each radiation pattern is imperative in order to deliver predictable adaptive array performance. In this chapter, the effects of coupling on each element’s radiation pattern in the two-element array configurations shown in Chapter 3 are presented and analyzed.

4.1 The Active Element Pattern

The active element pattern (AEP) will be studied to gain insight into the effects of coupling on each element’s radiation pattern. In introductory array theory, the effects of mutual coupling within an array are ignored. As a result, each element’s radiation pattern in an array configuration is identical to its isolated radiation pattern. The total array pattern can be found through the summation of each individual isolated element pattern multiplied by the weight applied to that particular element as shown in Eqn. 4.1, where $g_n$ is the isolated element pattern for element n, $A_n$ is the magnitude of the weight
applied to element \( n \), and \( \varphi_n \) includes both the phase applied to the element and the spatial phase delay arising from the element’s position [12].

\[
F_{tot}(\theta, \phi) = \sum_{n=1}^{N} g_{n,iso}(\theta, \phi)A_n e^{j\varphi_n}
\]  

(4.1)

However, the effects of mutual coupling can be integrated into array theory by using the active element pattern (AEP) concept [13]. The active element pattern is an array element’s pattern in the presence of mutual coupling. To find the AEP, the element for which the radiation pattern is desired is driven with a signal. All other elements in the array are terminated with a load equal to the characteristic impedance of the system. The currents induced on the other elements in the array as a result of mutual coupling radiate. The fields from the induced currents combine with the driven element’s radiation pattern. The result is the radiation pattern of the driven element with the effects of coupling included. The concept of the active element pattern is useful because the effects of coupling on the array’s radiation pattern are accounted for in each element’s AEP. Therefore, the total array pattern in the presence of coupling can be determined through the summation of each element’s AEP multiplied by the weight applied to the particular element as described by Eqn. 4.2, where \( g_n \) is now the AEP of element \( n \).

\[
F_{tot}(\theta, \phi) = \sum_{n=1}^{N} g_{n,AEP}(\theta, \phi)A_n e^{j\varphi_n}
\]  

(4.2)

4.2 Coupling Effects on Array Configuration Element Patterns

Using the concept of the active element pattern, the effects of mutual coupling on the radiation pattern of each element in an array can be found. In particular, the coupling effects on the position, magnitude, and other general characteristics of each element null can be determined for each reconfigurable element in an array. In this study, the effects of coupling on each pattern null reconfigurable element’s null characteristics will be quantified as the spacing between elements increases. Once again, the AEPs will be studied for the four array configurations listed in Table 3.1 in Chapter 3.
HFSS simulations were performed to obtain the AEP of each element in the array for many element spacings. A MATLAB program was then designed to extract the null locations and magnitudes from each AEP. The program searches the AEP for a concave upward dip in the radiation pattern which will be defined here as a null. After all the nulls are found, the deepest, dominant null and the next deepest, secondary null are plotted for that particular element spacing. When the nulls are extracted from each AEP, the effect of coupling on the null location and magnitude is quantified. The following sections will display the results of the coupling effects on each element’s radiation pattern null for each configuration. AEP measurement results are provided for configurations 2 and 4 to verify the simulations.

4.2.1 Array Configuration Two

The effect of coupling on the null characteristics of each element in array configuration 2 was quantified as the spacing between elements increased. The elements in configuration 2 are configured to have the null of element 1 directed toward 14 degrees and the null of element 2 positioned at broadside as shown in Fig. 4.1. Coupling will alter the position and magnitude of the null from these desired isolated element null locations. The null locations and magnitudes in the presence of coupling are quantified below for both elements 1 and 2.

Figure 4.1: Two-element array setup in configuration 2.

Element 1 Active Element Pattern Null Characteristics

The angular position and magnitude of the active element pattern null for configuration 2, element 1, is plotted in Fig. 4.2 as the distance between
elements is increased. To better visualize the data plotted in Fig. 4.2, the measured and simulated AEPs for a spacing of $1.23\lambda$ are shown in Fig. 4.3. The measured and simulated patterns match well. The simulated pattern shows an example of the AEP that the null in Fig. 4.2 was taken from. The element null is at 18 degrees in the AEP, which can be seen plotted in the top graph of Fig. 4.2 for a spacing of $1.23\lambda$.

Fig. 4.2 shows that the position and magnitude of the dominant null change as the spacing between elements increases. Additionally, a shallow secondary null forms in the active element pattern as the element spacing becomes larger. As the elements continue to get further apart, the secondary null becomes deeper and eventually becomes the deepest null while the previously dominant null becomes shallower and eventually disappears.

![Figure 4.2: Active element pattern null locations and magnitudes for element 1 in configuration 2 as a function of element spacing $d$.](image)

Fig. 4.4 shows the simulated active element pattern for an element spacing of $1.7\lambda$ and provides a better visualization than Fig. 4.2 of the secondary null characteristics. In this pattern, two concave upward nulls are present each having similar magnitudes. The AEP shown in Fig. 4.4 for this spacing differs substantially from the isolated element pattern shown in Fig. 2.6. Although the present approach describes this pattern as having two nulls, an alternative interpretation is a single null whose characteristics have been
Figure 4.3: Configuration 2, element 1, co-polar simulated and measured active element pattern for an element spacing of $d = 1.23\lambda$.

Figure 4.4: Configuration 2, element 1, co-polar simulated active element pattern for an element spacing of $d = 1.7\lambda$. Nulls located at $9^\circ$ and $24^\circ$. 

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substantially altered by mutual coupling. Nevertheless, this significant alteration from the isolated element pattern will have an impact on the adaptive array performance. Certain spacings such as the 1.7λ spacing highlighted above will not have a very distinct, deep null and may not be suitable for an adaptive array utilizing this particular array configuration. On the other hand, an element spacing of 1.23λ, whose pattern is shown in Fig. 4.2, has a more distinct null but is positioned away from the ideal 14 degrees. This spacing may be a better choice if a distinct, deep null is desired and the deviation from the ideal position can be tolerated by the algorithm.

Element 2 Active Element Pattern Null Characteristics

Similar to that shown for element one, the angular position and magnitude of the active element pattern null for configuration 2 element 2 is plotted in Fig. 4.5 as the distance between elements is increased. The desired null of element 2 in configuration 2 is located at broadside or 0 degrees. For an element spacing of 1.8λ, the single dominant null location in the presence of coupling is positioned at 0 degrees. However, adjacent spacing values from 1.4 to 2λ produce a null that deviates slightly from broadside. This small deviation of the null from 0 degrees may be tolerable in an adaptive array.

![Figure 4.5](image)

Figure 4.5: Active element pattern null locations and magnitudes for element 2 in configuration 2 as a function of element spacing $d$. 

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Upon first inspection, the data for element spacings of 1 to 1.4λ in Fig. 4.5 indicate dominant and secondary nulls that are positioned relatively far from the desired 0 degrees. However, the AEP for the 1.23λ element spacing shown in Fig. 4.6 provides more insight into the behavior of the nulls. Two concave upward dips in the pattern are seen at the positions indicated by Fig. 4.5. However, it is apparent that two well defined nulls are not present. Instead, this particular situation can best be viewed as a single null whose characteristics have been altered by the mutual coupling between elements. In this case, a ripple with two concave upward dips has been introduced into the bottom of the null. Additionally, the width of the null has been substantially increased from that of the isolated null at broadside previously shown in Fig. 2.6. These alterations as a result of mutual coupling will impact the adaptive array performance to some extent and must be kept in mind when selecting an element spacing.

![Configuration 2, element 2, co-polar simulated and measured active element pattern for an element spacing of d = 1.23λ.](image)

Finally, in Fig. 4.5 a secondary null appears in the magnitude graph when elements are spaced from 1.4 to just over 2λ. However, the secondary nulls do not appear on the null position graph. These ripples in the pattern are very shallow and appear well away from the desired null location. These nulls are not a result of mutual coupling, but instead are a result of the truncated ground plane used in the simulations and measurements. They serve as a
reminder that ground plane effects will have a small impact on the radiation pattern observed.

4.2.2 Array Configuration Four

Similar to configuration 2, the effects of coupling on the null characteristics of each element in array configuration 4 were quantified and analyzed as the spacing between elements was increased. The elements in configuration 4 are setup to have the nulls of both element 1 and element 2 directed toward 14 degrees as shown in Fig. 4.7. Once again, the coupling experienced by each element in the array will alter the position and magnitude of the null from the desired isolated element null locations. The null locations and magnitudes are quantified below for both elements 1 and 2.

Element 1 Active Element Pattern Null Characteristics

The angular position and magnitude of the active element pattern null for configuration 4 element 1 are plotted in Fig. 4.8 as the distance between elements is increased. Both the null position and magnitude fluctuate as the element spacing increases. The measured and simulated active element pattern for a spacing of 1.23\(\lambda\) is shown in Fig. 4.9. A comparison of this AEP with the AEP shown in Fig. 4.3 for element 1 in configuration 2 shows that the pattern characteristics are very similar. The magnitudes of the nulls in both patterns are approximately the same. Additionally, a comparison of Figs. 4.8 and 4.2 indicates that the trends in the null position as a function of element spacing match closely with the trends seen for configuration 2, element 1. The effect of coupling on this element’s radiation pattern is very
similar to that seen in array configuration 2 and the analysis for configuration 2, element 1, can be applied.

Element 2 Active Element Pattern Null Characteristics

In array configuration 4, both elements 1 and 2 should have the same null tilt. However, the effect of coupling on each element’s radiation pattern is very different. The angular position and magnitude of the active element pattern null for configuration 4 element 2 is plotted in Fig. 4.10 as the distance between elements is increased. Although both elements have a 14 degree null tilt, the results seen in Fig. 4.10 are very different from those seen for element 1 in Fig. 4.8. The AEP for a spacing of $1.23\lambda$ shown in Fig. 4.11 shows considerable difference not only from the isolated element pattern shown in Chapter 2 but also from the other AEPs previously shown for elements with a 14 degree null tilt. Unlike the AEPs seen in Figs. 4.9 and 4.3, this element at this spacing does not have a well-defined null. This is shown by the simulation results in Fig. 4.11. Instead, the entire pattern surrounding where the null is located is significantly distorted to the extent in which the null is only a shallow ripple in the pattern. However, as the spacing between elements increases, a dominant, well defined null begins to appear. Therefore, this element will not operate correctly with relatively small element spacing. This is because for small element spacings the coupling eliminates any distinct null. This eliminates any possibility of the adaptive algorithm placing a null with this element in this configuration. These results indicate that when array configuration 4 is utilized, it is desirable to separate the elements by at least $1.4\lambda$ to ensure a well-defined null in the presence of coupling.

4.2.3 Array Configuration One

Next, the effects of mutual coupling on the radiation patterns of the elements in configuration 1 were examined. Similar to that shown for configurations 2 and 4, the null locations and magnitudes from the active element patterns are plotted as the spacing between elements increased. The position and magnitude of the null for configuration 1, element 1, are shown in Fig. 4.12. For this element, there is a single well-defined null very close to the desired angle at broadside. The secondary nulls present near broadside for element
Figure 4.8: Active element pattern null locations and magnitudes for element 1 in configuration 4 as a function of element spacing $d$.

Figure 4.9: Configuration 4, element 1, co-polar simulated and measured active element pattern for an element spacing of $d = 1.23\lambda$. 

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Figure 4.10: Active element pattern null locations and magnitudes for element 2 in configuration 4 as a function of element spacing $d$.

Figure 4.11: Configuration 4, element 2, co-polar simulated and measured active element pattern for an element spacing of $d = 1.23\lambda$. 

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spacings near 1.2 and 2.2λ form a ripple in the bottom of an overall larger null located near broadside. This effect is very similar to that described in the analysis for configuration 2, element 2. The secondary nulls that arise far from broadside such as for spacings above 2.2λ are a result of ground plane effects and are negligible magnitude fluctuations in the radiation pattern. Additionally, the null characteristics for configuration 1, element 2, are shown in Fig. 4.13. The null locations and magnitudes of element 2 exhibit very similar behavior to that of element 1 in this configuration. This is an expected result since the array setup in this configuration is symmetric.

4.2.4 Array Configuration Three

Finally, the effects of mutual coupling on the radiation patterns of the elements in configuration 3 were examined. In configuration 3, the first element has a single, deep null very close to broadside for most spacings as plotted in Fig. 4.14. The secondary nulls present above 1.2λ are shallow fluctuations from ground plane effects and will not significantly impact the array performance. The null location and magnitude characteristics for element 2 are shown in Fig. 4.15. Similar to configuration 4, element 2, the configuration 3, element 2, AEPs have a well-defined null above 1.4λ yet have no well-defined null below 1.4λ similar to that seen in Fig. 4.11. A closer investigation of the AEP for a larger spacing of 1.98λ in Fig. 4.16 shows a well-defined, distinct null at 17.5 degrees. The secondary dip in the pattern near 50 degrees is a result of ground plane truncation effects.

4.3 Large Element Spacing Analysis

The analysis of the effects of coupling on the radiation patterns of each element in various array configurations has shown that coupling can have a significant impact on each element. However, as the spacing between elements becomes large, the effect of coupling should decrease. In order to verify this behavior, simulations were performed at an arbitrary, large spacing of 5.5λ. The active element pattern result plotted with the isolated pattern for element 1 in configuration 2 is given in Fig. 4.17 and shows that the AEP begins to converge to that of the isolated element in terms of null placement.
and characteristics. However, large element spacing leads to an increasingly asymmetric ground plane and, as a result, ground plane effects begin to dominate. A significant amount of ground plane effect ripple appears in the pattern shown in Fig. 4.17. Therefore, when using large element spacing to reduce coupling effects, an appropriately sized ground plane should be selected.

4.4 Discussion

From the analysis results, an important question arises. Is there an optimal spacing for this element to reduce the mutual coupling effects while maintaining relatively small element spacing? This is a complicated question because there is no relatively small spacing (below $2\lambda$) that allows, in the presence of coupling, a well-defined null to be located in the same position as that of the isolated element for all elements in all configurations simultaneously. In practice, however, the best element spacing for small arrays is the smallest spacing that allows the adaptive array in the presence of coupling to achieve its performance goals. For example, it was shown in the active element pattern of configuration 4, element 2, in Fig. 4.11 that a spacing less than $1.4\lambda$ eliminated a well-defined null from the AEP. Although unlikely, if the signal environment is such that the adaptive algorithm never selects array configuration 4, a spacing less than $1.4\lambda$ may be tolerable. As a result, the best element spacing for a particular adaptive array will be related to the signal environment expected and the extent to which pattern changes alter adaptive array performance. Chapter 5 will investigate the extent to which coupling alters the adaptive array performance from that predicted by the adaptive algorithm. This will provide more insight into how much pattern alteration from coupling can be tolerated.
Figure 4.12: Active element pattern null locations and magnitudes for element 1 in configuration 1 as a function of element spacing $d$.

Figure 4.13: Active element pattern null locations and magnitudes for element 2 in configuration 1 as a function of element spacing $d$. 

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Figure 4.14: Active element pattern null locations and magnitudes for element 1 in configuration 3 as a function of element spacing $d$.

Figure 4.15: Active element pattern null locations and magnitudes for element 2 in configuration 3 as a function of element spacing $d$. 
Figure 4.16: Configuration 3, element 2, co-polar simulated active element pattern for an element spacing of $d = 1.98\lambda$. Null located at $17.5^\circ$.

Figure 4.17: Configuration 2, element 1, co-polar simulated active element pattern for an element spacing of $d = 5.5\lambda$ with the isolated element pattern.
In earlier chapters the effects of mutual coupling were characterized for the pattern null reconfigurable elements in a two-element array. The results showed that mutual coupling can have a significant impact on the radiation pattern of each reconfigurable element. This will impact the performance of an adaptive array to some extent. Although the effect of coupling on each individual antenna element provides insight into how the element might operate in an adaptive array, it does not provide a quantitative metric to characterize the effect of coupling on adaptive array performance. The signal-to-interference-plus-noise ratio (SINR) provides the metric that quantifies the performance of an adaptive array for a particular signal environment. Roach used SINR figures to study the performance of reconfigurable elements represented by the ideal subarray model introduced in Chapter 2 [7]. However, the practical reconfigurable element’s radiation pattern is not perfectly represented by the subelement model. Furthermore, mutual coupling effects on the elements in an array, as seen in previous chapters, will further alter an arrayed element’s pattern from that predicted by the subarray model. These differences between the model and a practical element’s pattern will lead to changes in the real adaptive array performance from that predicted by the subelement model in the adaptive algorithm.

In this chapter, the SINR performance of the experimental pattern null reconfigurable element in a two-element adaptive array will be compared to that generated by the subelement model. A variety of case studies will be explored to gain an understanding of the limitations of the subarray model when representing a practical element in the presence of coupling. The chapter will provide a detailed overview of the methods and procedure used to obtain the SINR. This includes providing an introduction to the least mean square (LMS) adaptive algorithm used to generate the optimal element weights and patterns. New modifications to Roach’s LMS algorithm to work with the
switched null elements used throughout this study will be described along with a detailed explanation of the SINR methods used. Finally, case studies will be shown to highlight the impact of mutual coupling on adaptive array performance.

5.1 SINR Comparison Methodology

The SINR will be compared for two-element arrays using four different element implementations. The element implementations that will be investigated include the ideal subarray model, the designed switched null element used throughout this work without coupling included, and the designed element in the presence of mutual coupling. Furthermore, the SINR will be generated for an array of two noncoupled isotropic elements to provide a reference for comparison. The comparison of the SINR between arrays composed of these elements is not a trivial task, however. In order to compare the SINR between the subarray model and the practical element patterns, the relationship between the model parameters and the real null reconfigurable element must be established. Each reconfigurable element is modeled as a two-element subarray of isotropic radiators with particular weight constraints. This formulation allows the LMS algorithm to generate weights for each subarray element in the same way it does for a nonreconfigurable array. However, this results in the algorithm producing twice as many weights as there are real reconfigurable elements. Therefore, the relationship between the four model weights and the two practical weights must be established. Additionally, the model’s radiation pattern must be extracted from the subelement weights. Only then can a practical measured or simulated radiation pattern be substituted in place of the model’s pattern for comparison.

The process used to generate the SINR for each different element is shown in Fig. 5.1. In the flowchart, a particular signal environment is first applied to the LMS algorithm implemented in Mathematica. The signal environment consists of a desired signal incident on the adaptive array with magnitude, $A_d$, from angle, $\theta_d$, along with an interference signal with magnitude, $A_i$, from angle, $\theta_i$. The signal environment also consists of a thermal noise component, $\sigma_{sub}$, present for each element in the array. The algorithm represents each
reconfigurable element using the two-element subarray model introduced in
Chapter 2 and generates an optimal weight for each isotropic element in the
model. The optimal weights maximize the SINR for the particular signal
environment while adhering to complex conjugate and switched null con-
straints. The SINR is then computed using the optimal weights generated
by the algorithm for a subarray model element. Both the optimal weights
and the subarray model SINR are output to a file.

Next, the algorithm output file is read by a MATLAB program that will
use the results along with practical antenna pattern information to generate
the SINR obtainable from the practical antenna design. The HFSS simulated
radiation patterns for the isolated null reconfigurable element were generated
for null tilt states of −14, 0, and 14 degrees from broadside. Additionally, the
radiation pattern for each element in the array configurations previously used
in Chapters 3 and 4 was generated along with those for other configurations
that include the −14 degree null tilt element state.

Before the practical patterns can be used in an SINR calculation, the
subarray model’s radiation pattern must be isolated from the subelement
weights. Isolating the element pattern from the subelement weights produces
a practical element weight that can be applied to a real experimental element.
Additionally, a normalized subarray pattern is found that can be replaced
by a simulated element pattern in the SINR calculation. Using the isolated
element patterns and then the coupled element patterns, the SINR of both

Figure 5.1: The process used to generate the SINR for a particular signal
environment using four different array element implementations.
element implementations can be computed. Finally, the SINR for a two-
element isotropic array is generated as a reference to compare the subarray
and practical element SINR results. The reference array SINR results are
generated using the same signal environment as the practical elements.

The process outlined here will be described in more detail in the follow-
ing sections. First, the LMS algorithm used will be examined including how
the optimal weights are generated. Additionally, the new switched null con-
straints applied to the algorithm will be described. Next, the SINR formu-
alation will be established and will be applied to determine the subarray model
SINR. The separation of the radiation pattern from the subarray pattern
will then be explained. Additionally, the use of the practical patterns in the
SINR calculation will be demonstrated. Finally, differences present between
the thermal noise generated in the practical subarray model and a practi-
cal element are resolved. This section will provide the detailed background
needed to understand how the SINR case study results were generated.

5.1.1 LMS (Least-Mean-Square) Array

The adaptive algorithm developed by Roach for use with reconfigurable el-
ements utilizes the LMS (least-mean-square) adaptive algorithm [7]. The
LMS algorithm details presented in this section will provide a general intro-
duction to the algorithm and will follow Compton’s description in [6]. The
LMS algorithm indirectly maximizes the signal-to-interference-plus-noise ra-
tio (SINR) by minimizing the mean-square error between a reference signal
and the array output. Consider the output of an array $s(t)$ composed of
a desired signal, $s_d(t)$, an interference signal, $s_i(t)$, and a noise component,
$n(t)$, given by

$$s(t) = \alpha s_d(t) + \beta s_i(t) + \gamma n(t).$$

(5.1)

Next, assume the array has a reference signal, $r(t)$, that is a replica of the
desired signal or, at the very least, a signal that is correlated with the desired
signal.

$$r(t) = s_d(t)$$

(5.2)
The difference between the reference signal and the array output is the error and is denoted by $\epsilon$ as

$$\epsilon(t) = (1 - \alpha)s_d(t) - \beta s_i(t) - \gamma n(t). \quad (5.3)$$

The desired signal, interference signal, and noise are assumed to be zero mean processes that are uncorrelated with each other. As a result, the expectations of the cross product terms are zero and the mean-square error (MSE) is given as follows where $E$ denotes expectation.

$$E[\epsilon^2(t)] = (1 - \alpha)^2 E[s_d^2(t)] + \beta^2 E[s_i^2(t)] + \gamma^2 E[n^2(t)] \quad (5.4)$$

Eqn. 5.4 shows that the minimum mean-square error (MMSE) is obtained by maximizing $\alpha$ while minimizing both $\beta$ and $\gamma$. By maximizing $\alpha$ and minimizing both $\beta$ and $\gamma$, the desired signal is maximized in the array output while the interference signal and noise component are minimized as can be seen from Eqn. 5.1. Therefore, the LMS algorithm indirectly achieves the highest SINR by minimizing the MSE.

### 5.1.2 LMS Optimal Weights

The LMS algorithm will generate the optimal element weights that will be used to calculate the SINR for a particular signal environment. The formulation will be considered here using real valued signals. This means that the incoming signal incident on each antenna element is broken into an in-phase ($x_I$) and quadrature ($x_Q$) component. Although complex signals are typically used, using real-valued signals allows certain beam and null steering constraints to be applied to each element by limiting the relationship between the weights applied to the I ($w_I$) and Q ($w_Q$) components. The real valued signals are then combined into a complex signal in the array output, $s(t) = w_I x_I - iw_Q x_Q$, where a negative sign was chosen by Compton to simplify calculations.

The optimal element weights are found using the foundations of the LMS technique described earlier in section 5.1.1. The mean-square error is given in matrix form in terms of the weight vector ($W$), reference correlation vector
(S), and covariance matrix (Φ) as follows.

\[
E[\epsilon^2(t)] = E[r^2(t)] - 2W^T S + W^T \Phi W \tag{5.5}
\]

This equation is equivalent to Eqn. 5.4 shown earlier but is now in terms of antenna array parameters. The values of W, S, and Φ are given below.

\[
W = [w_I, w_{Q1}, w_{I2}, w_{Q1}, ...]^T \tag{5.6}
\]

\[
S = E \left[ \begin{array}{c} x_I(t)r(t) \\ x_Q(t)r(t) \\ x_I(t)r(t) \\ x_Q(t)r(t) \end{array} \right] \tag{5.7}
\]

\[
\Phi = E \left[ \begin{array}{cccccc} x_I(t)x_I(t) & x_I(t)x_{Q1}(t) & x_I(t)x_{I2}(t) & x_I(t)x_{Q2}(t) & \cdots \\ x_{Q1}(t)x_I(t) & x_{Q1}(t)x_{Q1}(t) & x_{Q1}(t)x_{I2}(t) & x_{Q1}(t)x_{Q2}(t) & \cdots \\ x_{I2}(t)x_I(t) & x_{I2}(t)x_{Q1}(t) & x_{I2}(t)x_{I2}(t) & x_{I2}(t)x_{Q2}(t) & \cdots \\ x_{Q2}(t)x_I(t) & x_{Q2}(t)x_{Q1}(t) & x_{Q2}(t)x_{I2}(t) & x_{Q2}(t)x_{Q2}(t) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{array} \right] \tag{5.8}
\]

The optimal weights are determined by finding the minimum mean-square error, and this is first accomplished by taking the gradient of Eqn. 5.5 with respect to the weights.

\[
\nabla_W \{E[\epsilon^2(t)]\} = -2S + 2\Phi W \tag{5.9}
\]

At the minimum, the gradient of the mean-square error equals zero and the optimal weights become

\[
W_{opt} = \Phi^{-1} S. \tag{5.10}
\]

Although adaptive arrays use a feedback control loop to continuously maintain the optimal element weights, the adaptive portion of the array and the associated control system aspects will not be addressed in this study. Instead, this study focuses on the potential of null reconfigurable elements to improve adaptive array SINR performance. The optimal weights and the resulting optimal array pattern for a particular signal environment will yield the maximum SINR performance that can be obtained from these elements. As a result, only the optimal element weights and the resulting SINR will be
considered in this study.

5.1.3 Subarray Constraints

Roach developed a subarray model to represent a reconfigurable antenna in the LMS algorithm [7]. In the model, each reconfigurable element in an array is represented by two isotropic antennas forming a subarray as previously shown in Fig. 2.2. Roach’s algorithm places constraints on the subarray to ensure that it accurately models a practical reconfigurable element. These constraints include a complex conjugate constraint and a beamsteering constraint previously described in Chapter 2. The beamsteering constraint in Roach’s algorithm was designed to limit the mainbeam between an upper and lower angle around broadside. However, the results of the study showed that null reconfiguration was often more advantageous than mainbeam reconfiguration. As a result, the switched null reconfigurable element used throughout this work was developed.

The beamsteering constraint used by Roach has to be modified in order to integrate the switched null design into Roach's adaptive algorithm. Instead of limiting the mainbeam of the subarray model, the null needs to be constrained to 76, 90, or 104 degrees from endfire (14, 0, -14 degrees from broadside) to represent the practical switched null design from Section 2.3. Constraining the subarray null begins with the expression for the subarray pattern given as follows, where $d_{\text{sub}}$ is the distance between subarray elements, $w_n$ is the angle of the weight applied to element n, and k is the wavenumber. Subelement 1 is positioned at the origin consistent with the algorithm implementation.

\[
T_{\text{sub}}(\theta) = w_n + w_n^* e^{jkd_{\text{sub}} \cos(\theta)}
\]

\[
= |w_n| [e^{-j(kd_{\text{sub}} \cos(\theta) - \angle w_n)} + e^{j(kd_{\text{sub}} \cos(\theta) - \angle w_n)}] e^{jkd_{\text{sub}} \cos(\theta)}
\]

\[
= 2|w_n| \cos \left( k \frac{d_{\text{sub}}}{2} \cos(\theta) - \angle w_n \right) e^{jkd_{\text{sub}} \cos(\theta)} \quad (5.11)
\]

Next, the constraint relationship between magnitudes of the real ($w_{I_n}$) and imaginary portion ($w_{Q_n}$) of the weight applied to the subarray must be determined. A null occurs when the argument of the outer cosine term in
Eqn. 5.11 is an odd multiple of \( \frac{\pi}{2} \). The derivation of the relationship between the real and imaginary parts of the subelement weight for a null at angle \( \theta_{\text{null}} \) is shown below, where \( m \) is an integer.

\[
\begin{align*}
kd_{\text{sub}} \frac{d_{\text{sub}}}{2} \cos(\theta_{\text{null}}) - \angle w_n &= (2m + 1) \frac{\pi}{2} \\
\angle w_n &= kd_{\text{sub}} \frac{d_{\text{sub}}}{2} \cos(\theta_{\text{null}}) - (2m + 1) \frac{\pi}{2} \\
\arctan \left( \frac{w_{Q_n}}{w_{I_n}} \right) &= kd_{\text{sub}} \frac{d_{\text{sub}}}{2} \cos(\theta_{\text{null}}) - (2m + 1) \frac{\pi}{2} \\
w_{Q_n} &= w_{I_n} \tan \left( kd_{\text{sub}} \frac{d_{\text{sub}}}{2} \cos(\theta_{\text{null}}) - (2m + 1) \frac{\pi}{2} \right) \\
w_{Q_n} &= -w_{I_n} \cot \left( kd_{\text{sub}} \frac{d_{\text{sub}}}{2} \cos(\theta_{\text{null}}) \right)
\end{align*}
\] (5.12)

However, for a null at 90 degrees (broadside), the cotangent argument becomes undefined. This issue can be resolved by allowing a situation in which the imaginary portion of the weight is much larger than the real portion. This allows the algorithm to select a null that is very close to 90 degrees. By only allowing subarray weights that will place a null in the 76, 90, and 104 degree directions, the switched null reconfigurable element behavior can be modeled and integrated into the algorithm.

5.1.4 Subarray Model Pattern Isolation

In this section the total model subarray pattern is separated into two components: a weight and a normalized pattern. This formulation will allow the normalized pattern from the model to be replaced by the normalized pattern of a simulated or measured practical antenna element. Consider a subarray composed of two isotropic elements. The first subelement is driven by a weight \( w_1 \) while the second subelement is driven by the complex conjugate of \( w_1 \) consistent with the algorithm’s complex conjugate constraint. Additionally, the second subelement has an additional phase term, \( \phi_{\text{sub}} = \frac{2\pi}{\lambda} d_{\text{sub}} \cos(\theta) \), resulting from its spatial location relative to subelement one, where \( d_{\text{sub}} \) is the distance between subarray elements. These weights driving the array will produce a total pattern with a magnitude and phase.
varying with $\theta$ given by

$$F_{\text{tot}}(\theta) = w_1 + w_1^* e^{j\phi_{\text{sub}}}$$  \hspace{1cm} (5.13)

$$= |w_1| e^{j\phi_{\text{sub}}} + |w_1| e^{j(\phi_{\text{sub}} - \phi_{\text{sub}})}.$$  \hspace{1cm} (5.14)

The magnitude and phase applied to subelement 1 can be pulled out along with a factor of 2 using the distributive property as shown.

$$F_{\text{tot}}(\theta) = 2|w_1| e^{j\phi_{\text{sub}}} \left( \frac{1}{2} + \frac{1}{2} e^{-j(2\phi_{\text{sub}} - \phi_{\text{sub}})} \right).$$  \hspace{1cm} (5.15)

This results in a weight term and a normalized pattern term that depends only on the angle of the algorithm generated subelement weight. Using Eqn. 5.15, the two-element subarray can now be viewed as a single element with a complex normalized pattern, $f_{\text{norm}}(\theta)$, and an element weight, $W_{\text{ele}}$, given below. $W_{\text{ele}}$ is the weight that could be applied to a real reconfigurable element with normalized pattern $f_{\text{norm}}(\theta)$.

$$f_{\text{norm}}(\theta) = \left( \frac{1}{2} + \frac{1}{2} e^{-j(2\phi_{\text{sub}} - \phi_{\text{sub}})} \right)$$  \hspace{1cm} (5.16)

$$W_{\text{ele}} = 2|w_1| e^{j\phi_{\text{sub}}}. \hspace{1cm} (5.17)$$

5.1.5 SINR Formulation

Variations between the subarray model and a practical element’s pattern will lead to differences in the adaptive array performance. The signal-to-interference-plus-noise ratio (SINR) provides a metric to compare the performance of adaptive arrays. Using this metric, the performance of arrays with different elements can be compared. The effect of element pattern changes on adaptive array performance can then be quantified. The SINR is defined as follows, where $P_d$, $P_i$, and $P_n$ are the powers of the desired, interference, and noise components received, respectively.

$$\text{SINR} = \frac{P_d}{P_i + P_n}.$$  \hspace{1cm} (5.18)
The desired, interference, and noise powers can be found from the element weight vector $W$ and the signal vectors $X_d$ and $X_i$ as shown.

\[
P_d = \frac{1}{2} E\{|W^T X_d|^2\} = \frac{A_d^2}{2} |W^T U_d|^2 \tag{5.19}
\]

\[
P_i = \frac{1}{2} E\{|W^T X_i|^2\} = \frac{A_i^2}{2} |W^T U_i|^2 \tag{5.20}
\]

\[
P_n = \frac{1}{2} E\{|W^T X_n|^2\} = \frac{\sigma_n^2}{2} W^\dagger W \tag{5.21}
\]

where $\dagger$ denotes the Hermitian and

\[
X_d = A_d e^{j(\omega_d t + \phi_d)} U_d, \tag{5.22}
\]

\[
X_i = A_i e^{j(\omega_i t + \phi_i)} U_i. \tag{5.23}
\]

$A_d$ and $A_i$ are the magnitudes of the desired and interference signals while $\phi_d$ and $\phi_i$ are arbitrary phase offsets. $U_d$ and $U_i$ are vectors that include the array element $n$’s complex voltage pattern, $f_n(\theta)$, and the element phase shift from the origin for the desired signal, $\phi_{dn} = \frac{2\pi}{\lambda} d \cos(\theta_d)$, and the interference signal, $\phi_{in} = \frac{2\pi}{\lambda} d \cos(\theta_i)$. The value $d$ is the distance of the element’s phase center from the origin. $U_d$ and $U_i$ are given by

\[
U_d = \begin{bmatrix}
f_1(\theta)e^{j\phi_{d1}} \\
f_2(\theta)e^{j\phi_{d2}} \\
\vdots \\
f_n(\theta)e^{j\phi_{dn}}
\end{bmatrix}, \tag{5.24}
\]

\[
U_i = \begin{bmatrix}
f_1(\theta)e^{j\phi_{i1}} \\
f_2(\theta)e^{j\phi_{i2}} \\
\vdots \\
f_n(\theta)e^{j\phi_{in}}
\end{bmatrix}. \tag{5.25}
\]

Alternatively, the SINR can be formulated in terms of the covariance matrices. The covariance matrix given in Eqn. 5.8 is the total covariance matrix composed of the summation of separate desired, interference, and noise co-
variance matrices as shown below,

\[ \Phi = \Phi_n + \Phi_d + \Phi_i \]  

(5.26)

where

\[ \Phi_d = A_d^2 U_d^* U_d^T \]  

(5.27)

\[ \Phi_i = A_i^2 U_i^* U_i^T \]  

(5.28)

\[ \Phi_n = \sigma^2 I \]  

(5.29)

Eqns. 5.19 through 5.21 can be reformulated in terms of the separate covariance matrices given in Eqns. 5.27 through 5.29. The SINR can then be written as

\[ SINR = \frac{P_d}{P_i + P_n} = \frac{\frac{1}{2} W^\dagger \Phi_d W}{\frac{1}{2} W^\dagger \Phi_i W + \frac{1}{2} W^\dagger \Phi_n W}. \]  

(5.30)

5.1.6 SINR Calculation for Subarray Model Elements

Eqn. 5.30 is used in the adaptive algorithm to calculate the SINR for elements represented by subarray models. The optimal weight vector is produced by the LMS algorithm and is given for an array composed of two elements as

\[ W = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}. \]  

(5.31)

Four weights are present because each element is modeled as a subarray composed of two isotropic elements, and the algorithm generates a weight for each. Additionally, the covariance matrices are given by Eqns. 5.27 through 5.29, where \( f_n(\theta) = 1 \) for isotropic elements in \( U_d \) and \( U_i \). \( U_d \) and \( U_i \) then become

\[ U_d = \begin{bmatrix} 1 \\ e^{j \frac{2\pi}{\lambda} d_{\text{sub}} \cos(\theta_d)} \\ e^{j \frac{2\pi}{\lambda} d_{\text{main}} \cos(\theta_d)} \\ e^{j \frac{2\pi}{\lambda} (d_{\text{main}} + d_{\text{sub}}) \cos(\theta_d)} \end{bmatrix} \]  

(5.32)
and

\[
U_i = \begin{bmatrix}
\frac{1}{e^{j\frac{2\pi}{\lambda}d_{\text{sub}} \cos(\theta_i)}} \\
e^{j\frac{2\pi}{\lambda}d_{\text{main}} \cos(\theta_i)} \\
e^{j\frac{2\pi}{\lambda}(d_{\text{main}}+d_{\text{sub}}) \cos(\theta_i)}
\end{bmatrix},
\]  

(5.33)

where the subelements are spaced by distance \(d_{\text{sub}}\) and the elements are spaced by distance \(d_{\text{main}}\).

5.1.7 SINR Calculation Procedure for the Designed Element

The SINR of the subarray model was found using the optimal weights generated for each subelement in the subarray model. However, practical elements are driven by only a single weight. Therefore, the SINR has to be determined from practical weights, given in Eqn. 5.17, which are extracted from the subarray weights produced by the algorithm. Additionally, the normalized complex radiation pattern of the subarray model given in Eqn 5.16 must be replaced by the HFSS simulated normalized complex radiation pattern for the designed element whose SINR is desired. The SINR calculation is performed using Eqns. 5.19 through 5.21 shown earlier. The weight vector now consists of two practical weights and is given by

\[
W = \begin{bmatrix}
W_{\text{ele}1} \\
W_{\text{ele}2}
\end{bmatrix}.
\]  

(5.34)

The complex radiation pattern \(f_{n}(\theta)\) in \(U_d\) and \(U_i\) is replaced by the simulated HFSS normalized complex patterns. \(U_d\) and \(U_i\) are now given by

\[
U_d = \begin{bmatrix}
f_{1d}(\theta) \\
f_{2d}(\theta)e^{j\frac{2\pi}{\lambda}d_{\text{main}} \cos(\theta_d)}
\end{bmatrix},
\]  

(5.35)

and

\[
U_i = \begin{bmatrix}
f_{1i}(\theta) \\
f_{2i}(\theta)e^{j\frac{2\pi}{\lambda}d_{\text{main}} \cos(\theta_i)}
\end{bmatrix}.
\]  

(5.36)

where \(f_{n_d}(\theta)\) and \(f_{ni}(\theta)\) are the complex pattern values in the direction of the desired and interference signals, respectively.

Using this formulation, the SINR that was previously calculated using four subarray weights is now calculated using only two practical weights along
with simulated antenna patterns. As a result, the SINR is now expressed in terms of real, physical quantities, and the SINR of an array composed of practical designed elements can be determined. Using this method, the SINR was first calculated for an isolated, noncoupled null reconfigurable element. Using the same physical weights, the SINR of the null reconfigurable element in the presence of coupling was then determined. When coupling effects are included, the active element patterns (AEPs) of the elements in the array configurations selected by the algorithm are used as the normalized element patterns. The AEPs were described in Chapter 4 for four array configurations. Depending on the signal environment, the algorithm can also select array configurations consisting of elements with a $-14$ degree null tilt. Therefore, all nine possible array configurations were simulated and the 18 resulting patterns were recorded for use in the practical SINR calculations.

5.1.8 SINR Thermal Noise Power Issues

When relating the subarray model used in the algorithm to a practical element, the thermal noise power becomes an issue. The subarray model should represent the real, physical element implementation as closely as possible. This means that the array output desired, interference, and noise powers should be equivalent between the subarray model and the physical element implementations. The subarray model noise signal environment will have to be updated to model the physical situation. Consider a single subarray in the algorithm’s subarray model implementation. A noise power equal to $\sigma^2_{\text{sub}}$ is present for each of the subelements. The output noise power from the subelements, $P_{n\text{sub}}$, is found using Eqn. 5.21 as

$$P_{n\text{sub}} = \frac{\sigma^2_{\text{sub}}}{2} \mathbf{W}^\dagger \mathbf{W}$$

(5.37)

$$= \frac{\sigma^2_{\text{sub}}}{2} [w_1, w_2, w_3, w_4]^*$$

(5.38)

$$= \sigma^2_{\text{sub}}(|w_1|^2 + |w_3|^2),$$

(5.39)
where \( w_2 = w_1^* \) and \( w_4 = w_3^* \) from the complex conjugate constraint.

However, using the practical weights given in Eqn. 5.34 and the physical noise power, \( \sigma_{phys}^2 \), for a real two-element array, the output physical noise power, \( P_{nphys} \), becomes

\[
P_{nphys} = \frac{\sigma_{phys}^2}{2} \mathbf{W}^\dagger \mathbf{W}
\]

\[
= \frac{\sigma_{phys}^2}{2} [W_{ele1}, W_{ele2}]^* \begin{bmatrix} W_{ele1} \\ W_{ele2} \end{bmatrix}
\]

\[
= 2\sigma_{phys}^2 (|w_1|^2 + |w_3|^2),
\]

where \( W_{ele1} = 2|w_1|e^{jw_1} \) and \( W_{ele2} = 2|w_3|e^{jw_3} \). It is apparent that when the practical weights are used, the physical implementation output noise power differs from that of the subarray model. The subarray model produces half the noise power of the physical implementation. To account for this, the noise power in the algorithm using the subarray model is doubled from that desired in the physical implementation (\( \sigma_{sub}^2 = 2\sigma_{phys}^2 \)).

It is important to note that the magnitude of the algorithm thermal noise compensation is dependent on the formulation of the physical weights. In the current weight formulation given in Eqn. 5.17, the magnitude of the physical weight was chosen to be double the subarray weight magnitude to allow for a normalized radiation pattern. However, alternative physical weight formulations are possible, but may require a different noise power compensation in the algorithm. Changes to the algorithm desired signal power and interference power may even be necessary to ensure the array output power of each component is identical between arrays composed of models and the physical elements.

### 5.2 Case Studies

In this section SINR case studies are examined to investigate the limitations of the subarray model when representing the physical null reconfigurable design. The mutual coupling effects on the SINR performance are also examined. Four different element implementations are explored for each case. First, the SINR will be shown using the subarray model with the complex conjugate and switched null constraints. Next, the SINR for the isolated
switched pattern null reconfigurable antenna design without mutual coupling will be generated. This will show how well the subarray model matches the practical design. The SINR for the switched pattern null reconfigurable antenna with mutual coupling included will then be shown. This result will be compared against the design’s SINR without coupling included to show the effect of mutual coupling on adaptive array performance. Finally, the elements will be replaced by two isotropic radiators in order to see how the reconfigurable antennas compare to a traditional nonreconfigurable adaptive array of isotropic elements.

\[ \theta_d, \theta_i \]

**Figure 5.2:** Array configuration used in the case studies.

The signal environment is characterized by the number, the angular position, and the strength of the incident signals. The signal environments considered in this study will consist of one desired signal and one interference signal incident on the array from angles \( \theta_d \) and \( \theta_i \) as shown in Fig. 5.2. For each case the desired signal will be fixed to a specific angle while the interference signal’s angular position is updated. The SINR will be calculated and recorded for each interference signal angle for a two-element adaptive array composed of the four elements described above. A strong interference environment will be considered in which the signal-to-noise (SNR) ratio, \( \xi_d = \frac{A_d^2}{\sigma^2} \), is equal to 0 dB and the interference-to-noise (INR) ratio, \( \xi_i = \frac{A_i^2}{\sigma^2} \), is equal to 40 dB. These signal strengths were used by both Compton and Roach [6, 7] and will allow for a comparison between results. A spacing of 1.23\( \lambda \) between actual reconfigurable elements was selected for the comparison. This will show the performance of the array, and particularly the mutual coupling
effects, for an array with relatively close element spacing. Typically, $1.23\lambda$ is considered a large array spacing. However, the reconfigurable element is about one freespace wavelength in length, so the physical patches will be very closely positioned at $1.23\lambda$. An array with close element spacing is interesting to examine since an array with large element spacing will converge to the isolated element pattern SINR results.

5.2.1 Case 1: $\theta_d = 120^\circ$

The first signal environment that is considered is a fixed desired signal incident from 120 degrees while an interference signal is swept from 0 to 180 degrees. The SINR results are shown in Fig. 5.3. First inspection of the graph shows a very low SINR when the interference signal is from 120 degrees. This is when the interference and desired signal are incident from the same direction, and as a result, the interference cannot be eliminated without rejecting the desired signal too. This results in very low SINR.

For most interference angles, the SINR for the isolated elements matches closely with the coupled element results. This shows promise that even for relatively small element spacing, the coupling does not substantially impact the SINR performance for most interference angles. Instead, most of the SINR performance degradation from that predicted by the subarray model, which uses four isotropic radiators to represent two physical elements, is a result of the element design limitations and not the mutual coupling.

There are some instances when the coupled elements produce even better SINR performance than the isolated elements. Consider the peak in the coupled element SINR when the interference is incident from 98 degrees. The algorithm selects array configuration 1, which has both element nulls set for 90 degrees, to provide the best SINR performance. It is clear from the pattern plot of element 2 shown in Fig. 5.4 for each element implementation why the coupled element has substantially better SINR performance. The coupling causes the pattern to have its deepest null, of magnitude 0.13, at 98 degrees while its pattern maximum is located at 120 degrees. Although the subarray model has a similar magnitude at the interference location, the magnitude of the pattern at the desired location is only 0.7 which is lower than the coupled element’s magnitude. The isolated element’s pattern
Figure 5.3: SINR results for a desired signal incident from 120°.

has the lowest SINR even though its pattern magnitude is higher than the subarray model’s at 120 degrees. This is because its pattern magnitude at the interference angle of 98 degrees is 0.37 which is much larger than the subarray’s pattern magnitude of 0.2.

Figure 5.4: Element implementation patterns for array configuration 1, element 2.

A very high SINR result is present for interference angles near both 35 and 145 degrees. Upon first inspection, these peaks in the SINR may seem unwarranted because the elements cannot place a null anywhere near these angles. However, these high SINRs can be attributed to the weight angles
applied to the elements, not the element patterns. Consider when the interferer is at 35 degrees. The algorithm chooses array configuration 3 which produces particular practical element weight angles as described in Section 5.1.4. These element weight angles are such that when they combine with the phase of the subarray model pattern, an array pattern null is present at 35 degrees. This leads to the high SINR. However, the phase of the isolated and coupled element patterns differ from that of the subarray model phase at 35 degrees. The different element phases cause the array pattern null to move away from the 35 degree interference direction and lead to a substantial dip in the SINR seen in the graph at 35 degrees. This shows that not only the magnitude of the element pattern, but also the element pattern phases, are important and have a significant impact on SINR performance. Element pattern phases are particularly important when the best SINR is obtained from the element weight phases instead of the element pattern magnitude as is the case at 35 and 145 degrees. Additionally, the angular positions of these SINR peaks are dependent on the phase associated with the distance between elements. As a result, these peaks will change positions for different element spacings.

Next, consider the interference angles between 70 and 110 degrees. The SINR produced by the subarray model is, for the most part, much higher than that of both the isolated and coupled elements. Although small pattern phase difference effects are present, the lower SINR can be mostly attributed to the substantial difference in pattern shapes between the designed elements and the subarray model. When the subarray model pattern magnitudes are substituted for the isolated element pattern magnitudes, the isolated element SINR increases to levels very close to those seen for the subarray model. Because the isolated element’s pattern phases were left unaltered, this shows that the magnitude differences between the subarray model and isolated element pattern are mostly responsible for the low SINR over this range.

5.2.2 Case 2: \( \theta_d = 76^\circ \)

The next signal environment that is considered consists of a fixed desired signal incident from 76 degrees while an interference signal is swept from 0
to 180 degrees. In this case, the desired signal is incident from a direction in which an element null can be placed. The SINR results for these signal environments are shown in Fig. 5.5. As expected, a very low SINR is seen at 76 degrees when the interferer is incident from the same direction as the desired signal. Once again, the SINRs for the coupled patterns match very closely with those of the isolated. This shows that mutual coupling even for relatively close spacings does not affect the SINR results as much as the design limitations of the practical element.

However, when the interference angle is incident from 90 degrees, the coupled elements produce significantly worse performance than both the subarray model and the isolated designed elements. At this interference angle, the algorithm selects each element to have a null at 104 degrees. The magnitude and phase for the isolated and coupled elements in this array configuration are shown in Fig. 5.6. The graphs show that both the magnitude and phase differ between the isolated and coupled elements at the interference and desired signal angles. In this case, both the magnitude and phase differences are responsible for the decrease in the coupled SINR results. This shows that the coupling’s effect on the element pattern phase is also critical to SINR performance.
5.2.3 Case 3: $\theta_d = 55^\circ$

Case 3 will investigate the SINR performance when the desired signal is incident from 55 degrees. The coupled element SINR performance matches very well with the isolated reconfigurable element over all interference angle directions. In this case and the previous cases, the isolated and coupled reconfigurable elements outperform the subarray model for both small, below 30 degrees, and large, above 160 degrees, interference angle directions. This is a direct result of the designed element’s pattern shape. For the low and high angles, the algorithm selects a configuration with both element nulls tilted in the same direction. Consider the large interference incidence angles, above 155 degrees, in Fig. 5.7. The algorithm selects both element nulls to be directed toward 104 degrees. The subarray model pattern has a deep null at 104 degrees but maintains high magnitudes away from 104 degrees. The designed element pattern magnitudes shown in Fig. 5.6, however, taper off substantially toward 0 and 180 degrees. Therefore, when the interference is incident from low and high angles, the designed pattern can reduce its effects better than the subarray model pattern can. We see that the isolated element has the lowest magnitude at low and high angles resulting in the highest SINR. The coupled element has a slightly higher pattern magnitude at these angles resulting in a small decrease in SINR performance compared to the isolated element.

At 35 and 145 degrees, peaks in the SINR performance are seen as described for Case 1. Furthermore, well defined SINR peaks are also seen at
65 and 115 degrees as well in this case. These peaks are a result of the same effect seen at 35 and 145 degrees. The algorithm selects configuration 4, which has the element nulls positioned at 76 degrees, when the incident interference angle is 65 degrees. Configuration 4 results in particular element weights consistent with Section 5.1.4. These element weight angles are such that when they combine with the phase of the subarray model pattern an array pattern null is present at 65 degrees. This leads to peaks in the SINR performance, since the interference signals at these angles can be nulled out. These peaks like those at 35 and 145 degrees will change angular position if the distance between elements is altered.

5.3 Case Study Discussion

The case study results provide insight into the effect of mutual coupling on the SINR performance of the adaptive array. There are some situations shown in the case studies where the coupling significantly degrades the array performance. Other situations occur where the coupling significantly improves the SINR performance over that of the isolated element. However, the results show that for most signal environment situations, the mutual coupling does not significantly change the SINR performance of the array.
Instead, the results indicate that the coupling is less important than the subarray model limitations in representing the practical design. There are considerable SINR result differences between the subarray model and the practical element without coupling. As a result, a subarray model that more accurately represents the designed element pattern's magnitude and phase characteristics will provide more benefits than reducing the mutual coupling. This would ensure the algorithm selects the best patterns for a given signal environment.

Furthermore, the case study analysis compared the practical isolated and coupled elements to the subarray model in terms of SINR performance. The two-element isotropic reference gives an indication of the performance of a comparable traditional nonreconfigurable adaptive array. Throughout this study, the performance of the reconfigurable array was expected to increase compared to the traditional adaptive array. However, the results show that the adaptive array performance does not exceed that of the two-element nonreconfigurable reference array. The cause of this behavior originates in the LMS adaptive algorithm and subarray model technique developed by Roach [7] on which the current work is based. This requires some discussion.

In the adaptive algorithm using the subarray model, a reconfigurable element is modeled by constraining the weights that can be applied to the subarray elements. The result of this formulation is that the element patterns are not independent of the element weights. In this work, Eqn. 5.16 showed that the normalized pattern depends on the angle of the element weight. The subarray achieves a null in a particular direction by choosing a particular element weight. Therefore, when the pattern null or pattern maximum is constrained to certain angles, the angle of the weights generated by the algorithm are inherently limited. In reality, there is no relationship between the weight applied to the element and the radiation pattern of the element. They are independent. As a result, the LMS algorithm implementation is overconstrained, and the result is lower SINR performance than what would be expected from a real practical reconfigurable adaptive array.

The overconstrained behavior is also present in Roach’s results. However, the scenarios investigated in his work consisted of five array elements and each had a continuous beam steer range. Therefore, the algorithm is given much more flexibility in terms of weight angles, and the overconstrained nature of the algorithm is not obvious in the SINR results. In the current
work, however, the element null is constrained to three different angles to model the switched null element design. This significantly limits the element weight angles allowed, and a drastic decrease in SINR performance is seen. This means that, in reality, the SINR performance will be much better than that shown in this study and should exceed the isotropic traditional adaptive array reference SINR. In order to demonstrate the improvement, work is needed to develop a method to alleviate the overconstraints present in Roach’s algorithm.
CHAPTER 6

CONCLUSIONS

6.1 Summary

This thesis addressed the mutual coupling considerations for integrating pattern null reconfigurable elements into adaptive arrays. It is well known that the effects of coupling can be reduced by spacing the array elements far apart. However, modern wireless applications are requiring much smaller antennas, which makes widely spaced elements prohibitive. The material presented in this thesis will aid the design of pattern null reconfigurable adaptive arrays with relatively small element spacings. This study provides the framework to combine the benefits of reconfigurable elements and adaptive array technology to reduce the size of a practical array without sacrificing performance. Although previous work had analyzed widely spaced elements using theoretical models, this study analyzed the practical mutual coupling effects that are essential to understand in order to make the transition to small element spacing.

The thesis began by providing an overview of the state of the art in element pattern reconfigurability in adaptive arrays. This included introducing a subarray model developed to represent a practical reconfigurable element. A case study using the subarray model was highlighted to show the benefits of null reconfiguration in small adaptive arrays. Finally, a pattern null reconfigurable element that was specifically designed for use in adaptive arrays was introduced. This overview provided the framework necessary for a mutual coupling study.

After the state of the art in reconfigurable adaptive arrays was described, the work transitioned to characterizing the mutual coupling between pattern null reconfigurable elements in a two-element array configuration. The four array configurations studied throughout this work were shown. Each config-
uration was composed of two-elements set up with various null tilt combinations. Using the array configurations, the S-parameters, which characterize the coupling between elements and each element’s impedance match, were analyzed through simulation. The simulation results were then verified by measuring an array with an element spacing of $1.23\lambda$ that was constructed.

After the coupling between elements was characterized, the effects of coupling on the radiation pattern of each element in the four array configurations were analyzed. The coupling effects on element radiation patterns were analyzed as a function of element spacing using the active element pattern (AEP) concept. The null location and magnitude in each of the active element patterns for each spacing were plotted. Measurements of the AEPs for elements in configurations 2 and 4 with a spacing of $1.23\lambda$ verified the HFSS simulation results.

Finally, the effects of coupling on the adaptive array performance were investigated. Details of the integration of the subarray model into the least mean square (LMS) algorithm by Roach were presented. Modifications made in this work to Roach’s algorithm in order to accept the switched null element used in this study were described. Finally, case studies comparing the signal-to-interference-plus-noise ratio (SINR) of an array in the presence of coupling to that predicted by the subarray model were presented. The results provide insight into the effect of coupling on adaptive array performance.

6.2 Research Contributions

This study quantified the coupling between pattern null reconfigurable elements in an array. This is the first time coupling between reconfigurable elements has been explored for small adaptive arrays. The results show that the coupling between elements is relatively low for close element spacings. The results also show that changes in the spacing between elements and the resulting coupling does not have a sizeable impact on the element input impedances. Although the coupling between elements was shown to be relatively low, coupling does have a sizeable impact on the radiation pattern of each element. For some element spacings, coupling even eliminated the well-defined null expected. Additionally, for almost all spacings, the element null location in the presence of coupling is different from that of the isolated
element. These results provide insight into the coupling behavior not only for the configurations shown but also for elements of this design topology with continuous null tilts.

Additionally, this study investigated the SINR performance obtained for the practical pattern null reconfigurable element in an adaptive array. The results quantified the effect of mutual coupling on adaptive array performance. The results indicate that coupling severely degrades the array performance for some signal environments, yet substantially improves the SINR performance for others. In general, however, coupling was shown to not have a significant impact on the SINR performance of the array. Instead, the limitations of the subarray model in representing the practical null reconfigurable design were shown to be substantially more critical. The subarray model indicates much better performance for many signal environments than the practical element can achieve. This work shows that additional research is needed to further refine the subarray model to better represent the practical reconfigurable design. This will ensure the algorithm selects the best patterns for a given signal environment. Finally, this work revealed some key limitations to the LMS algorithm and subarray model developed for use with reconfigurable elements. The work shows that the algorithm, in its present form, is overconstrained and does not allow a practical adaptive array to reach its full SINR performance potential.

6.3 Future Work

This work revealed many important avenues for additional research. A large difference was seen between the subarray model and the isolated reconfigurable element SINR results. This indicates that the subarray model can better represent the designed pattern null reconfigurable element. More work is needed to refine the subarray model to better represent the practical element in terms of pattern magnitude and phase. Alternatively, the practical element design can be updated to obtain a deeper pattern null and a pattern phase that agrees better with the subarray model. Additionally, further investigation into the pattern phase and the effect of mutual coupling on pattern phase would provide more insight into the differences between the SINR results.
In addition to refining the subarray model, research is needed to address the algorithm constraints and their effects on the generation of the optimal element weights and patterns. The algorithm-generated weights and patterns in this study produced SINR results that were far below those that can be obtained from a traditional two-element isotropic array. This effect was shown to be a result of overconstraints present in Roach’s algorithm and subarray model. In the algorithm formulation, the subarray patterns are dependent on the array element weights. When the element patterns are constrained, the element weights are constrained as well. In reality, the element pattern and weight are independent. Further research is needed to remove the dependence of the subarray pattern on the element weights in the LMS algorithm implementation. This will allow the algorithm to select true optimal weights and patterns that will produce SINR performance results that exceed traditional, nonreconfigurable adaptive arrays.

After the algorithm constraints are modified to produce true optimal weights and patterns, the next step in this research will be to introduce the elements into a real adaptive array that can be controlled by the adaptive algorithm. Allowing the algorithm to adapt in real-time to a changing signal environment will provide additional insight into the benefits and limitations of reconfigurable elements in adaptive arrays. In order to successfully introduce the element into an experimental adaptive array, the null position must be electronically controlled by the algorithm. Therefore, an element that can switch its null without rewiring copper tape connections is essential. Other versions based on the topology used in this work have an electronically controllable null position and are available in [14]. These advanced designs include elements with continuous 2D and 3D null tilts which will be highly effective for eliminating interference signals. These advanced designs and the issues arising from the use of solid state components in the elements need to be characterized for array applications.

Before an experimental adaptive array can be operated, the outputs of the adaptive algorithm that select the desired element pattern must be investigated in more depth. Currently, the LMS algorithm developed by Roach outputs the phase and magnitude that is applied to each subarray element. However, a physical reconfigurable element’s pattern is not controlled by a magnitude and phase. Instead, the desired pattern is selected by applying a DC bias to solid state components that control the null tilt, such as PIN
diodes or varactors. As a result, the subelement magnitude and phases must be translated in order to drive a DC bias network that can change the reconfigurable element’s pattern to that selected by the algorithm.
REFERENCES


