EFFECT OF NOISE ON THE PULL-IN DYNAMICS OF AN ELECTROSTATIC MICROSWITCH

BY

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THESIS

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The goal of this research is to investigate the effects of noise on the pull-in dynamics of a micro-electromechanical switch. We take into account thermal noise and noise present in the voltage source. Thermal noise is modeled as white noise. First, we study its effect on the linear response of the system. Then, we study the uncertainty in pull-in time caused by thermal noise. We show that thermal noise does not have a significant effect on the pull-in dynamics. For noise in voltage we study the models of Johnson and flicker noise. We see that, as these noise sources are coupled with non-linear electrostatic force, they enhance the stability of the system and thus delay the occurrence of pull-in. We further see that this noise enhanced stability is aided by the damping forces.
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CHAPTER 1

INTRODUCTION

Micro-Electro Mechanical Systems (MEMS) are increasingly being used as sensors and actuators in many engineering systems. In addition to the integrated circuit, all these systems have a small number of mechanical components like various kinds of cantilevers and diaphragms. As these MEMS transducers reduce in size, they achieve higher resolution and better sensitivity. Several studies have focused on identifying the limiting factors on their performance [1, 2, 3]. The sensitivity of micro-systems is limited by the random fluctuations or noise present in the system or the surrounding environment.

In both electrical and mechanical domains, fluctuations in signals arise due to random motion of discrete physical entities such as electrons, atoms and molecules. For example, the Brownian motion of electrons in a resistor gives rise to electrical noise or that of molecules in surrounding fluidic medium causes randomness in mechanical forces. Fluctuations may arise either due to internal dynamics of the system or because of coupling with the external environment. In mechanical domain any form of dissipative mechanism, whether it is internal dissipation or due to surrounding fluid, will give rise to a noise force. The different sources of mechanical noise include thermo-mechanical noise, noise induced due to the internal motion of defects, and adsorption - desorption noise. The well known mechanisms of electrical noise include the Johnson-Nyquist noise, generation-recombination noise and Flicker noise [4].

Though noise forces are small in magnitude when compared to the deterministic forces, they can interact with systems having low inertia and influence their sensitivity. For example, in a sensor, noise governs the minimum level of signal that can be detected, while in an actuator, noise in the system affects the minimum resolvable displacement. Design considerations for such systems therefore requires accounting for the various noise sources present and the effect that these random forces have on the dynamics of the system. Noise
does not always have a degrading effect. As MEMS systems are often subjected to deformations in the non-linear regime, noise forces can lead to new and interesting dynamics in such regimes of operation. In many cases, noise force is introduced deliberately in the system to modify its dynamics in a desired way. For example, in a stochastic strain sensor[5] white noise voltage is applied to a microelectromechanical switch to sense strain by modifying the pull-in dynamics. Noise does not always degrade the signal of a device, on the contrary it can amplify the signal as seen in the case of a bistable nano-mechanical oscillators[6]. The study of MEMS under the influence of random forces is therefore an important problem. It has different practical considerations as well as it offers a tool for exploring new dynamics.

The dynamic pull-in of an electrostatic actuator is a widely investigated phenomenon in MEMS. It is an instability phenomenon that takes place when the elastic structure collapses under the influence of electrostatic force. It is undesired in most cases, such as the case of micro-resonator, where it leads to the failure of the device. The dynamic pull-in behavior has the presence of an inherent non-linearity because of the electrostatic forces. The random forces can therefore significantly alter the dynamics of such a non-linear system. With this motivation in mind, in this work, we study the pull-in dynamics under the influence of fluctuating forces.

This thesis is organized as follows: In Chapter 2 we discuss the mathematical formulation and numerical simulation of white noise and 1/f noise, as these models provide a basis for modeling noise in physical sources. In Chapter 3 we talk about thermo-mechanical noise. First, we study the effect of noise on the linear response of an electrostatic switch using a spring mass damper model. Then, we simulate its effect on pull-in dynamics using the Monte-Carlo method. In Chapter 4 we discuss two major sources of noise in DC voltage: Johnson noise and flicker noise. We simulate their effect on the pull-in dynamics. The results and observations are then summarized.
CHAPTER 2
CHARACTERIZATION OF NOISE

Noise is a process which varies randomly in time. This means that every realization of a noisy signal in time could be entirely different from all the others. However it has some well-defined statistical properties, which help in studying the effect of such processes on a physical system. For example, in an electrostatic MEMS switch this randomness could be present in fluidic forces which act on the mechanical components or in the fluctuations of electrostatic voltage. Noise in time domain is a continuous signal, which can be viewed as a set of infinite number of random variables described as

\[ \xi(t) = [\xi(t_1), \xi(t_2) \ldots \xi(t_n) \ldots], \quad (2.1) \]

where \( \xi(t_i) \) is the value of the process at time \( t_i \).

We assume that \( \xi(t) \) is a Markov process; this is true for noise models considered in this work. This implies that future evolution of the process depends only on the present state and is independent of the preceding states. Then, these random variables are defined by their probability density functions (PDFs): \( p(\xi_i; t_i) \) is the probability that \( \xi(t_i) = \xi_i \) and \( p(\xi_{i+1}; t_{i+1}|\xi_i; t_i) \) is the conditional probability that \( \xi(t_{i+1}) = \xi_{i+1} \) given \( \xi(t_i) = \xi_i \).

In time domain these random variables are described by the mean, \( \mu(t) \) of \( \xi(t) \) and autocorrelation function \( R(t, \tau) \) between \( \xi(t) \) and \( \xi(t + \tau) \):

\[ \mu(t) = E[\xi(t)] = \int_\Omega \xi(t)p(\xi; t)d\xi, \quad (2.2) \]

where \( \Omega \) is the domain for \( \xi(t) \).
for $\tau > 0$,

$$R(t, \tau) = E[\xi(t)\xi(t + \tau)] = \int_{\Omega} \int_{\Omega} \xi_{t+\tau}\xi_t p(\xi_{t+\tau}; t + \tau | \xi_t; t) p(\xi_t; t) d\xi_t d\xi_{t+\tau}. \quad (2.3)$$

for $\tau = 0$,

$$R(t, 0) = E[\xi(t)^2] = \int_{\Omega} \xi_t^2 p(\xi_t; t) d\xi_t. \quad (2.4)$$

Here we will consider only wide sense stationary (WSS) processes, for which $\mu$ is constant over time and $R(t, \tau)$ is dependent only on the time lag $\tau$. In the frequency domain, such a WSS process is described by its power spectral density (PSD). This is the Fourier transform of its autocorrelation function as given by Wiener-Khintchine theorem:

$$S(\omega) = \int_{-\infty}^{+\infty} R(\tau) e^{-j\omega \tau} d\tau. \quad (2.5)$$

$S(\omega)$ is double sided PSD defined for $\omega \in (-\infty, +\infty)$. We will now discuss two major kinds of noise phenomena found in the MEMS domain.

### 2.1 Gaussian White Noise

White noise is completely an uncorrelated process. Each $\xi(t_i)$ is independent of the values at past times as well as in the future. At each time instant, the random variable has a Gaussian distribution with zero mean and variance $\sigma^2$. Thus it has the following probability distribution functions:

$$p(\xi; t_i) = \frac{1}{\sqrt{2\pi\sigma^2}} exp \left( -\frac{\xi^2}{2\sigma^2} \right), \quad (2.6)$$

$$p(\xi_{i+1}; t_{i+1} | \xi_i; t_i) = p(\xi_i; t_i). \quad (2.7)$$

Using Eqn.(2.3) and Eqn.(2.4) we now derive the autocorrelation function:

$$R(\tau) = \begin{cases} 
0 & : \tau > 0 \\
\sigma^2 & : \tau = 0 
\end{cases} \quad (2.8)$$

From Eqn.(2.8) we see that this process has zero correlation time. Correlation time is
defined as the maximum possible time lag between two random variables $\xi(t)$ and $\xi(t+\tau)$ which have non-zero correlation. This autocorrelation function can be expressed in terms of a Dirac Delta function, $\delta(\tau)$

$$R(\tau) = D\delta(\tau),$$  \hspace{1cm} (2.9)

where $D$ is the noise strength. Its relationship with variance $\sigma^2$ will be derived later in the section. Power spectral density is given by Eqn.(2.5):

$$S(\omega) = \int_{-\infty}^{+\infty} D\delta(\tau)e^{-j\omega\tau}d\tau = D.$$  \hspace{1cm} (2.10)

This explains why the process is called 'White', since its spectral density is similar to that of white light with all frequencies having equal intensity. In general, the physical models relate the noise strength $D$ with process parameters. Therefore we need to define a relationship between $\sigma^2$ and $D$ as follows:

$$\sigma^2 = R(0) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} D\delta d\omega.$$  \hspace{1cm} (2.11)

From Eqn.(2.11) we see that white noise is just an idealization as infinite bandwidth implies infinite variance. Also if infinite frequency is present in the signal, it will be continuously fluctuating. Whereas all physical processes will have a finite response time due to inherent inertial forces, which means that they can exhibit only discrete fluctuations in time. Therefore during the numerical simulation of white noise, we assume some cut-off bandwidth $\omega_o$ for the PSD. For such bandlimited processes, the autocorrelation function gets modified into a $\text{sinc}$ function with a finite correlation time, as shown below:

$$S(\omega) = D \quad \forall -\omega_o < \omega < \omega_o,$$  \hspace{1cm} (2.12)

$$R(\tau) = \frac{1}{2\pi} \int_{-\omega_o}^{+\omega_o} D e^{j\omega\tau} d\omega = \frac{D\omega_o}{\pi} \text{sinc} \left( \frac{\omega_o\tau}{\pi} \right).$$  \hspace{1cm} (2.13)

Figure 2.1 shows $R(\tau)$ for a band-limited white noise process. Since $R(\tau)$ goes to zero at time lags in the multiples of $\frac{\pi}{\omega_o}$, we can generate an uncorrelated process by sampling it at time steps of $\Delta t = \frac{\pi}{\omega_o}$. These samples are drawn from a Gaussian distribution with mean
zero and variance $\sigma^2 = \frac{2D\omega_0}{2\pi}$.

![Figure 2.1: Autocorrelation function for band-limited white noise.](image)

2.2 1/f noise

1/f noise is another major form of noise found in various domains of a MEMS device. In different systems it may have different physical origins but in general has a similar mathematical description. Its autocorrelation function is an exponential function and the power spectral density scales with frequency $f$ as $\frac{1}{f^\gamma}$ where $0.5 < \gamma < 2$. In our discussion, we have taken $\gamma = 2$. This noise can be described by a Ornstein-Uhlenbeck(O-U) process. It has the following probability density functions:[7, 8]:

$$p(\xi_t; t) = \frac{1}{\sqrt{2\pi \sigma^2}} exp \left( -\frac{\xi_t^2}{2\sigma^2} \right)$$

(2.14)

$$p(\xi_{t+\tau}; t + \tau | \xi_t; t) = \frac{1}{\sqrt{2\pi S}} exp \left( -\frac{(\xi_{t+\tau} - \xi_t e^{-\frac{\tau}{\alpha}})^2}{2S} \right)$$

(2.15)

where $S = 2\sigma^2\left(1 - e^{-\frac{2\tau}{\alpha}}\right)$
The autocorrelation function is given by Eqn.(2.3) and Eqn.(2.4):

\[
R(\tau) = \sigma^2 \exp\left(\frac{-|\tau|}{\alpha}\right),
\]

(2.16)

where \(\alpha\) is the relaxation time of the process. From the Wiener-Khintchine theorem, we get the PSD as:

\[
S(\omega) = \frac{2\alpha\sigma^2}{1 + (\omega\alpha)^2} \quad \forall -\infty < \omega < \infty.
\]

(2.17)

Figure 2.2: (a) A sample of 1/f noise in time. (b) Autocorrelation function for the generated 1/f noise.

We can numerically simulate O-U noise by passing Gaussian white noise through a low pass filter. This is given by the first order ordinary differential equation (ODE)[7]:

\[
x + \alpha \frac{dx}{dt} = \eta(t),
\]

(2.18)

where \(\eta(t)\) is Gaussian white noise with double-sided power spectral density \(S_n(\omega) = 2\alpha\sigma^2\).

Figure 2.2 shows a sample of the generated noise realization along with the computed autocorrelation function.
CHAPTER 3
THERMO-MECHANICAL NOISE

Thermo-mechanical noise is evident in the motion of a micro-beam immersed in a fluid medium in absence of an external driving force. The beam exhibits random movements due to the impulses exerted by molecules of the fluidic medium when they collide with the beam. When an external force is applied, the beam experiences a frictional force proportional to its velocity. This dissipative force also originates from the molecular motion of the fluid. These two forces are the consequences of maintaining thermal equilibrium. The amount of energy dissipated by the frictional force must equal the amount of energy imparted by the random collisions in order to maintain the system at a constant temperature.

Since this systematic dissipative force and random force have the same origin, they must be related. This relation is given by the Fluctuation Dissipation Theorem[9, 10]. Analogous to Nyquist’s relation for noise in the voltage across a resistor, the power spectral density (PSD) of the thermal noise force, $S_n$, is given as [11]

$$S_n = 2K_bTc \frac{N^2}{Hz}, \quad (3.1)$$

where $K_b$ is the Boltzmann’s constant, $T$ is the temperature of the system and $c$ is damping coefficient. It is a WSS process with auto-correlation function, $R(\tau)$ given by inverse Fourier transform of power spectral density:

$$R(\tau) = 2K_bTc \delta(\tau), \quad (3.2)$$

where $\tau$ is the time lag.

$R(\tau)$ describes how the random variables (which describe a random process in time) at two different instances are related. A complete description of the force requires knowledge
of the PDF at any given instant of time. At each time instant this noise force is assumed to have a Gaussian distribution. This assumption is physically valid as there will large number of molecules hitting the beam at any instant of time. The resultant force which is the sum of these individual impacts will approach a Gaussian distribution as given by the central limit theorem.

![Figure 3.1: Schematic of the fixed-fixed beam actuator.](image)

An electrostatic fixed-fixed beam actuator is shown in Figure 3.1. It consists of a fixed-fixed beam electrode suspended over a parallel fixed electrode. The dielectric medium between the two electrodes offers fluid resistance to the motion of beam. As potential difference in form of a DC voltage $V_a$ is applied across the electrodes, the fixed-fixed beam is attracted towards the bottom electrode and if the electrostatic force exceeds a threshold, it snaps to ground electrode, which is called pull-in and the corresponding voltage is called pull-in voltage. The response of this fixed-fixed beam to the electrostatic and thermal noise forces can be studied through its fundamental mode. This assumption is a valid one for the cases when the force is uniformly distributed on the beam. We assume that both the electrostatic as well as the noise force have a uniform spatial distribution along the length of the beam. The beam dynamics can then be captured, with negligible error, through the fundamental mode. The equation of motion for displacement $x$ of the equivalent spring-mass-damper
system under the influence of electrostatic force is given as

\[ m\ddot{x} + c\dot{x} + kx = \frac{\epsilon AV^2}{2(d_0 - x)^2} + \chi(t), \quad (3.3) \]

where \( m \) is the mass, \( c \) is the damping coefficient, \( k \) is the stiffness, \( \epsilon \) is the dielectric permittivity constant, \( d_0 \) is the initial gap distance and \( \chi(t) \) is the thermal noise. The initial conditions are \( x(0) = 0 \) and \( \dot{x}(0) = 0 \). With the known PDF for \( \chi(t) \) and given that it is \( \delta \) correlated, the noise values can be generated using the method described in Section(2.1). Simulations are done using Monte-Carlo approach where different ensembles of \( \chi(t) \) were generated and the stochastic differential equation(3.3) was integrated using the Newmark scheme. The mean and the standard deviation of displacement was computed. The simulations are done for a gold fixed-fixed beam with Young’s modulus \( E = 79GPa \), density \( \rho = 19300Kg/m^3 \), length \( L = 6.6\mu m \), width \( b = 0.5\mu m \), height \( h = 0.05\mu m \) and initial gap \( d_0 = 0.5\mu m \). The corresponding parameters for first mode of the beam are mass \( m = 3.1845X10^{-6}\mu g \), stiffness \( k = 0.1099N/m \) and damping coefficient \( c = 5.25X10^{-9}Ns/m \).

For voltages lower than the critical pull-in voltage, the system attains a steady state in the absence of noise. For small steady state displacements \( x \ll d_0 \), the system behaves as a low pass filter for white noise. For this linear response the PSD of displacement will be the superposition of responses due to electrostatic force and the thermal noise. This displacement spectrum is related to force spectrum as

\[ S_x(\omega) = (|H(\omega)|)^2(S_f(\omega) + S_n(\omega)), \quad (3.4) \]

where the transfer function, \( H(\omega) \), is given as \( H(\omega) = \frac{1}{k - mw^2 + jcw} \). The PSD of electrostatic force , \( S_f(\omega) \) is obtained as \( S_f(\omega) = \left( \frac{\epsilon AV^2}{2(d_0)^2} \right)^2 \delta(\omega) \)

The PSD of displacement in shown in Figure 3.2. For \( \omega < \omega_n \) (natural frequency) the output displays a flat power spectrum and for \( \omega > \omega_n \) it has a roll-off factor of \( \frac{1}{\omega^2} \). This shows that for a linear system frequencies of thermal noise above a certain cut-off have negligible effect on the response of the system.

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After sufficient time, the mass fluctuates about the deterministic steady state value. The standard deviation of these displacements can be computed from the fact that the mean energy of the system, $E[U(x)]$ remains constant and is equal to $\frac{k_bT}{2}$. Using this relation and the expression for $E[U(x)]$ we get [11]

$$\frac{K_{eff}E[x^2]}{2} = \frac{K_bT}{2}, \quad (3.5)$$

where $K_{eff}$ is the effective spring constant. This gives the the root mean square (RMS) of displacement, $x_{rms} = \sqrt{E[x^2]}$, as

$$x_{rms} = \sqrt{\frac{k_bT}{2K_{eff}}} \quad (3.6)$$

As DC voltage is increased, due to electrostatic spring softening, the effective spring constant of the system reduces to $K_{eff} = k - \frac{\epsilon AV^2}{(d_0-x)^4}$. Figure 3.3 shows that the relation for RMS of displacement, given by Eqn(3.6), is satisfied even for high voltages where the system vibrates in a nonlinear region.
3.1 Pull-in Dynamics

Dynamic pull-in is a characteristic instability phenomenon in electrostatic switches. This phenomenon can be understood in terms of movement of a particle with mass $m$ along the potential energy curve of the equivalent spring-mass-damper system. The total potential energy $U(x)$ is the sum of elastic and electrostatic energy given as:

$$U(x) = \frac{1}{2}kx^2 - \frac{\epsilon AV^2_a}{2(d_0 - x)} \quad (3.7)$$

Figure 3.4 shows the effective potential energy. The curve is characterized by a stable state and a metastable state, with a potential energy barrier. When the applied DC voltage is higher than a threshold value, the mass gains a high velocity by the time it reaches the stable state. It has sufficient kinetic energy to overcome the mechanical restoring force. Beyond the metastable state the electrostatic force dominates, hence the mass crosses the metastable state. This escaping of mass from the potential barrier is equivalent to the fixed-fixed beam
pulling in to the ground electrode. The time taken for beam to pull-in starting from initial state is the pull-in time. This pull-in time is a fixed value for a particular system in absence of noise.

![Figure 3.4: The effective potential profile for a particle under the influence of elastic and electrostatic force.](image)

Noise force causes random displacements in both positive and negative directions, it is therefore expected to introduce a degree of uncertainty in this pull-in time. The thermo-mechanical noise strength is related to damping constant by the fluctuation dissipation theorem. Hence the mean pull-in time depends on the damping constant as well. Figure 3.5 shows the mean pull-in time for different values of the noise strength $D = 2K_bTcN/sqrtHz$. We see that with increasing noise strength, and hence damping, the pull-in time decreases. This is expected as higher dissipation makes the response sluggish. The standard deviation in pull-in time, shows no discernible scaling with the noise strength.
Figure 3.5: The variation of the mean pull-in time with noise strength. The inset shows the standard deviation in pull-in time.
CHAPTER 4

NOISE IN DC VOLTAGE

Electrical domain is the one of the primary sources of noise in MEMS. Since the power supply circuit on a microelectronics chip consists of several semiconductor devices and metal-oxide transistors (MOSFETs), the discrete nature of free charge carriers in these systems gives rise to stochastic phenomenon. This results in fluctuations in the output voltage. Two major sources of noise are the free electron movement in resistors (Johnson noise) \[4, 12\] which has a white spectrum and variable trapping of free carriers in MOSFETs (flicker noise) which has a 1/f spectrum \[13\]. We study the effect of white noise and 1/f noise in DC bias separately.

4.1 Johnson Noise

Johnson noise was first predicted by Einstein \[14\] and was then observed by Johnson \[12\]. The power spectrum of Johnson noise was calculated by Nyquist \[15\]. This noise can be observed in a resistor in absence of any externally applied voltage. The resistor will exhibit a small fluctuating current due to the net movement of free electrons in one direction at any particular time. The magnitude of this effective voltage bias is proportional to the resistance value. It is similar to thermal noise in mathematical treatment, with power spectral density and autocorrelation function given as \[15\]

\[
S_v(\omega) = 2K_bT R \frac{V^2}{Hz}; \quad (4.1)
\]

\[
R(\tau) = 2K_bT R \delta(\omega), \quad (4.2)
\]

where \( R \) is the resistance value. We will again study the dynamics of an electrostatic switch in presence of voltage noise using a spring-mass-damper model. The equation of motion for
the displacement of mass is now given as

\[ m\ddot{x} + c\dot{x} + kx = \frac{\epsilon A(V_a + V_n(t))^2}{2(d_0 - x)^2}, \]  

(4.3)

where \( V_n(t) \) is Gaussian white noise in voltage with PSD given in Eqn(4.1). The system parameters used for this study are the same as those used for the case of thermal noise. For generating \( V_n(t) \), we use the method described in Chapter(2). Different ensembles of \( V_n(t) \) in time domain were simulated, while \( V_a \) was kept constant at a value of 11.4 V which is the dynamic pull-in voltage for the system under consideration. Eqn(4.2) was integrated for each ensemble of \( V_n(t) \) and the pull-in dynamics was studied.

4.1.1 Pull-in Dynamics

We discussed in Section(3.1) that dynamic pull-in is described by the escape of a particle of mass \( m \) over a potential barrier. The small positive velocity that the mass has at the metastable state results in its crossing the barrier. Due to noise in the voltage, this potential energy barrier will no longer be constant, but will be fluctuating with time. This can significantly alter the pull-in dynamics. These fluctuations are about a mean potential, which is given by taking mean of potential over the domain of randomly varying voltage. The mean potential, \( E[U(x)] \), is therefore obtained as

\[ E[U(x)] = \frac{1}{2}kx^2 - \frac{\epsilon AE[V^2]}{(d_0 - x)^3}, \]  

(4.4)

where \( V(t) = V_a + V_n(t) \). In Eqn.(4.3) \( E[V^2] \) is computed by averaging over time. \( V_n(t) \) and \( V \) is a ergodic process which means its time average will be equal to the ensemble average. At each time instant \( V \) will have a Gaussian distribution with mean \( V_a \) and variance \( \sigma^2 \) which is the variance of PDF of \( V_n(t) \). For Gaussian PDF, by ensemble average \( E[V^2] = V_a^2 + \sigma^2 \). Variance \( \sigma^2 \) depends on \( S_v(\omega) \) as shown in Section(2.1). Substituting the value of \( E[V^2] \) we get

\[ E[U(x)] = \frac{1}{2}kx^2 - \frac{\epsilon A(V_a^2 + \sigma^2)}{(d_0 - x)^3}. \]  

(4.5)
We now study the pull-in time distribution for different noise strength values $D = 2K_bTR$. Figure 4.1(a) shows the PDF for the pull-in time for a low noise strength value of $D = 0.006 \frac{mV^2}{Hz}$. We see that, for the case of small noise strength, the pull-in time has a single peak in the distribution. The distribution has a maximum at a value which is roughly equal to the deterministic pull-in time (which is $2 \mu s$ for the system under consideration). These cases, wherein the pull-in behavior is similar to the deterministic behavior but with some spread, are referred to as direct pull-in. The Monte-Carlo simulations were done with 50000 time domain realizations of $V_n(t)$.

The area under the PDF gives the fraction of ensembles that have pulled in. This was estimated to be 62%, for the case considered in Figure 4.1(a). This shows that for a significant fraction of the ensembles pull-in does not occur. This stabilizing effect of the noise can be understood with the help of phase plot of velocity vs. displacement of the mass. Figure 4.2(a) shows the phase plot for one of the ensembles for which no pull-in was observed. The plot shows that the mass comes to the metastable state with small positive velocity. It then retracts back towards the stable state and keeps on oscillating about the stable state. The positions of the stable and the metastable state are around $0.12\mu m$ and $0.2\mu m$ respectively.

The return of the system from the metastable state takes place due to the sufficient negative acceleration imparted by the noise force at that point. As the mass returns back towards the stable state it is further aided by the restoring force. The noise therefore needs to bring the mass only a relatively small distance back from the metastable state.

After the mass enters the potential well it continuously oscillates about the local minimum energy point. This is because damping continuously dissipates its energy and the noise strength is not sufficient to impart it enough energy to cross the barrier. We studied the distribution of displacement for oscillations in the energy well. Figure 4.3(a) shows the distribution obtained for the case of low noise strength. The plot shows a Gaussian distribution concentrated around the stable state. From such a distribution, we conclude that the probability of the mass reaching the metastable state is very low.

We then studied the pull-in time distribution for a larger noise strength. Figure 4.1(d) shows the PDF for the pull-time for a noise strength value of $D = 0.08 \frac{mV^2}{Hz}$. From the plot we see that the PDF shows a bi-modal distribution. The center of the first mode is roughly
Figure 4.1: (a-d) The pull-in time distribution for different values of the noise strength. Figures a, b, c and d correspond to the noise strength values of 0.006, 0.02, 0.026 and 0.08 mV$^2$/Hz, respectively.
Figure 4.2: (a-b) The displacement vs. velocity phase plot for one of the ensemble. Plot a and b are for the small and large noise value.

Figure 4.3: (a-b) The distribution of particle inside the well after it returns from the metastable state. Plot a and b are for the small and large noise value.
equal to the deterministic pull-in time. We refer to this mode, as done before, as the case of direct pull-in. The center of the second mode has a value three orders of magnitude higher than the deterministic pull-in time. We refer to the second mode as the case of delayed pull-in.

Figure 4.4: The variation of the mean pull-in time with the voltage noise strength. The figure shows the mean values for direct and delayed pull-in.

The direct pull-in cases, observed for the case of large strength values, have the same dynamics as described before for the case of low noise strength values. The new phenomenon observed in the case of larger noise strength value is the case of delayed pull-in. We now analyze the dynamics of these delayed pull-in cases. Figure 4.2(b) shows the displacement vs. velocity phase plot for one ensemble for which delayed pull-in was observed. From the phase
plot we can see that the mass reaches the metastable region, re-enters the well and again escapes from it. As the noise strength is of significant amount, it gives sufficient energy to the system in the well and helps it escape the potential barrier. In such cases a large amount of time is spent oscillating in the well. This explains why the pull-in is delayed for a very long time. We also studied the distribution of the displacement in the well for samples which did not pull-in. Figure 4.3(b) shows the distribution of displacements for $D = 0.08 \frac{mV^2}{Hz}$, we see that the distribution has a large variance. This means that the samples have a larger probability to reach the metastable state and hence have a larger probability for pull-in.

Figure 4.4 shows the mean of the direct and delayed pull-in time for different noise strength values. The delayed pull-in is observed only for a noise strength larger than some threshold value. Also we see that with the increasing noise strength, the two values come closer and closer. This is because after the particle comes back to the stable state, the time spent in oscillations is reduced with increasing noise strength. Such noise induced stable dynamics has been studied previously in electrical, chemical and biological systems operating under similar metastable potentials[16, 17]. We can see from Eqn(4.5) that mean barrier height decreases with increasing noise variance. This means that the stable and metastable points come close until they merge to become a point of inflection at some noise strength. Therefore the mean pull-in time decreases with increasing noise strength even though the mean voltage is kept constant at the deterministic dynamic pull-in threshold.

The interplay of damping and noise leads to the stabilizing effect as was observed in the case of low noise strength value. The noise force imparts a negative acceleration to the particle at the metastable state and the dissipative force extracts energy, preventing it from escaping the well. The relative magnitude of the two forces will therefore play an important role in deciding whether the delayed pull-in behavior is observed or not. We studied the pull-in dynamics by varying the damping parameter while keeping the noise strength constant. Figure 4.5 shows the mean of the direct and delayed pull-in time for different values of the damping coefficient. The noise strength is kept constant at a value of $D = 0.026 \frac{mV^2}{Hz}$. We see that for very high damping only the direct pull-in behavior is observed. For such cases, the high dissipative force stabilizes the particle once it re-enters the well from the metastable state. As damping coefficient $c$ is reduced, the second case emerges. For such cases, the noise
Figure 4.5: The variation of the mean pull-in time for direct and delayed pull-in with the damping constant.
force dominates over the relatively weak damping force.

4.2 Flicker Noise

Micro-power supply circuit consist of many semiconductor resistances, MOSFETs, piezoelectric conductors which have free charge carriers. Due to their random excitation and movement to conduction band from valence band or variable trapping and release by the electron holes, they cause fluctuations in the output voltage. These fluctuations result in flicker noise which has a characteristic 1/f spectrum. There are various models by Hooge, Handel, Voss and Clark, McWhorter[1] describing the 1/f noise spectrum. Here we take the model for exponentially correlated noise which is similar to McWhorter’s model for noise due to carrier density fluctuations. The auto-correlation function for this noise model is given by Eqn(2.18). Numerical treatment of this model has been discussed in Section(2.2).

1/f noise is characterized by two parameters, $\sigma$ and $\alpha$, which can be varied independently. $\sigma$ is a measure of the noise strength and $\alpha$ corresponds to the relaxation time of the process generating the noise. Figure 4.6 shows the variation of the pull-in time with the noise strength. The direct and the delayed pull-in time shows similar variation with the noise strength as was observed in the case of white noise. The trend observed in Figure 4.6 can be explained in a similar manner as was done in the case of white noise, and is omitted here for the sake of repetition.

We then studied the effect of $\alpha$ on the pull-in dynamics. Figure 4.7 shows the mean of the direct and delayed pull-in time with $\alpha$ for flicker noise. We see that the direct pull-in time almost remains constant with $\alpha$, while the delayed pull-in time shows non-monotonic behavior. The delayed pull-in time is determined by the time the particle stays in the energy well. The minimum in delayed pull-in time at some characteristic $\alpha$ value may correspond to the synchronization with the internal dynamics of the system having nearly equal relaxation time.
Figure 4.6: The variation of mean pull-in time with noise voltage strength for flicker noise. The inset shows the probability of delayed pull-in.

Figure 4.7: The variation of mean pull-in time with relaxation time for flicker noise. The inset shows the probability of delayed pull-in.
In this work we examined the major sources of noise present in electrical and mechanical domain in a MEMS switch. We described the mathematical model and numerical simulation of thermo-mechanical noise, Johnson Noise and flicker Noise. We studied their effect on the pull-in instability of the fixed-fixed beam switch using a mass spring damper model.

We observed that thermo-mechanical noise introduces a small degree of uncertainty in the pull-in time. It was shown that Johnson noise modulates the potential barrier of the system, thus affecting the pull-in dynamics. It forces the system to go back towards the stable state, after which the system oscillates in the well for a long time. This phenomenon is dependent on the strength of noise as well as on the degree of damping. Thus the mean pull-in time, in presence of noise, was observed to be significantly higher than the pull-in time in the absence of noise. This delay in pull-in behavior is also observed in presence of flicker noise. It was shown that relaxation time of flicker noise affects the pull-in dynamics.
REFERENCES


