EFFECT OF GEOMETRIC PARAMETERS ON DROP-POSITIONING FORCES FOR
A LEVITATED DROP REACTOR

BY

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THESIS

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Abstract

A boundary-element method (BEM) is used to compute axisymmetric acoustic fields in the gap between the radiating plate and reflector of a single-axis acoustic drop levitator, in order to better understand the dependence of levitation capability on geometric parameters. Modifications to the drop levitator radiating plate are investigated using the BEM. Gor’kov’s theory is used to calculate the acoustic forces acting on a drop in the levitation field. The levitation capability of a modified geometry is compared to that of a current experimental geometry. Indenting, and increasing the area of the radiating plate significantly enhances levitation capability.
I would like to thank my advisor, Professor Arne J. Pearlstein for giving me the opportunity to do research under his direction. The time I have spent working on this computational problem has been the most challenging and rewarding period in my academic career and has inspired me to continue with research in graduate school. I believe that this would not have been possible without the patience and understanding of Professor Pearlstein. I would also like to thank our collaborator from the Department of Chemistry at the University of Illinois at Urbana-Champaign, Professor Alexander Scheeline, for his contribution to the research.

I also wish to express my sincere appreciation to Mr. Dirk van Aarde and Mr. Thys Louwrens for their financial support during my Master’s Degree. Additionally, thanks to my loving fiancé and family for their help and patience throughout the process of earning a graduate degree.
# Table of Contents

Chapter 1 Introduction .................................................. 1  
Chapter 2 Formulation and Numerical Approach ................... 4  
Chapter 3 Code Validation ............................................... 13  
Chapter 4 Computational Results for the Experimental Geometry . 16  
Chapter 5 Effects of Geometry on the Computed Field ............. 20  
Chapter 6 Spherically-Indented Radiating Plate .................... 22  
Chapter 7 Approximate Treatment of Elasticity of the Radiating Plate 29  
Chapter 8 Radially-Localised Emission from a Planar Radiating Plate 32  
Chapter 9 Effects of Volumetric Absorption .......................... 37  
Chapter 10 Absorption Investigation Using a Complex Wavenumber . 40  
Chapter 11 Conclusions ................................................... 46  
Nomenclature ..................................................................... 47  
References ......................................................................... 52  
Appendix A Xie and Wei Test Case ....................................... 54  
Appendix B Grid Refinement ............................................... 57
Chapter 1

Introduction

Understanding the kinetics of enzymatic reactions is critical in many life-science contexts, including drug discovery, metabolic engineering, and defense against chemical and biological warfare agents. In most cases, however, enzyme is available at reasonable cost or effort only in minute quantities, so that kinetic studies in macroscopic reactors are possible only at enzyme concentrations much lower than those of physiological interest. Kinetic measurements at more realistic concentrations thus require the use of microscopic amounts of solution, and have been typically conducted in conventional microfluidic geometries\(^1\)-\(^2\).

Unfortunately, at the high surface-to-volume ratios typical of conventional microfluidic geometries, significant adsorption of enzyme to the quartz or polydimethylsiloxane (PDMS) surface can occur, as recently discussed by Pierre et al.\(^3\). If the adsorbed enzyme catalyzes reaction, then one is faced with the problem of distinguishing contributions of such heterogeneous (surface) reaction from the homogeneous (bulk) reaction of interest. Even if the adsorbed enzyme does not catalyze reaction, the fraction of enzyme adsorbed on the surface must still be known, in order that one can subtract it from the initial amount to determine the concentration of enzyme in solution.

The need to avoid adsorption on solid surfaces in kinetic studies involving minute quantities of enzyme has led to the development by Scheeline and co-workers of the “levitated drop reactor” (LDR)\(^3\)-\(^4\) in which an acoustic field is used to levitate small liquid drops (on the order of 1 nL) in air, with the progress of reaction monitored by noncontact spectroscopic techniques, or by an electrochemical probe. The system has been used to study the kinetics of both luminol chemiluminescence, as well as the reaction of pyruvate with nicotinamide.
adenine dinucleotide, catalyzed by lactate dehydrogenase.

Levitation of small particles (including liquid drops) has for some time been recognized as having several potential advantages when the particle must be thermally or chemically processed. Chief among these advantages is that chemical and mechanical interaction with solid boundaries can be avoided. While there are many ways to levitate small particles, including use of magnetic and electrostatic forces, and radiation pressure, the most attractive approach for aqueous drops uses an acoustic field. Although the possibility of positioning particles with an acoustic field was first demonstrated by Kundt in 1866, acoustic levitation in the Earth’s gravitational field was developed much later by Apfel and co-workers (Apfel and Trinh and Apfel). The approach is quite general, in that it does not rely on special properties of the drop (e.g., conductivity), and dissipates only modest levels of power within the drop (primarily through viscous stresses), allowing for good thermal control.

Typically, acoustic levitation of liquid drops is accomplished in the axisymmetric gap between a piezoelectric radiating plate driven sinusoidally in time, and a reflector, with the symmetry axis aligned with the vertical. Drops position themselves along the symmetry axis, at nodes of the standing acoustic wave field.

In kinetic measurements, where concentrations are to be monitored spectroscopically or electrochemically, a key design issue is maintenance of drop positional stability. In the LDR, several types of disturbances (in addition to thermodynamic fluctuations that occur in any fluid at a nonzero temperature) can contribute to drop wander, including air currents within the enclosure surrounding the radiating plate and reflector, impact of very small “ballistic” droplets used to supply substrates and cofactors, and fluctuating acoustic forces on the drop associated with oscillations in drop shape driven by the ultrasonic field.

It has been known experimentally for some time that changes in the shape of the radiating plate and reflector can significantly affect the positional stability of levitated drops in the face of disturbances. Approaches include fabricating these components with surfaces corresponding to portions of a sphere, or with surfaces with grooves. To date, however,
there has been no systematic study of the effect of radiating plate or reflector shape on the positional stability of drops, or even on the acoustic field.

In this work, we use a boundary-element method (BEM) to reduce the computational task for a given axisymmetric geometry to a one-dimensional computation, and systematically investigate the effect of geometry on the acoustic field as well as on several scalar measures that attempt to qualitatively relate the acoustic field to positional stability of the levitated drop.

The work is organized as follows. In §2, we present the mathematical formulation of the acoustic problem and briefly describe the BEM used to approximately compute the acoustic field. In §3, we discuss Code Validation by means of comparison with previous results. In §4, we present results for the particular geometry employed in the experimental work of Scheeline and co-workers\textsuperscript{3-4}. In §5, we present results showing how the acoustic field depends on geometric parameters for several classes of radiating plate and reflector geometries. In §6, we present results for a geometry employing a spherically-indented radiating plate. In §7, we consider an approximate treatment of radiating plate elasticity, and show that for material properties typical in experiments, the effects are likely to be very small. In §8, we consider the situation in which the “radiating plate” is divided into two parts: an inner, vibrating, portion coupled to the piezoelectric driver, and an outer stationary portion that essentially serves as a reflector. In §9, we consider the effects of volumetric absorption of ultrasound by air (and moist air) within the cavity, and in §10 present computations that validate the use of a complex wavenumber when absorption occurs. Some conclusions are offered in §11.
Chapter 2
Formulation and Numerical Approach

The acoustic field in the air surrounding the drop is, to an excellent approximation, governed by the classical wave equation

\[
\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = \nabla^2 p, \tag{2.1}
\]

where we assume constant sound speed \( c \). Equation 2.1 does not account for nonlinear effects or absorption within the volume. The cycle period \( T \), wavelength \( \lambda \), and excitation frequency \( f \) are related to the sound speed \( c \) according to

\[
c = \frac{\lambda}{T} = f\lambda. \tag{2.2}
\]

Introducing an acoustic potential \( \Phi \), we have \( p = -\rho_o \frac{\partial \Phi}{\partial t} \), where \( \rho_o \) is the nominal density. In the case where the excitation, and hence the response, are harmonic, we can write \( \Phi(x,t) = \phi(x)e^{-i\omega t} \) and the wave equation (2.1) reduces to the Helmholtz equation

\[
\nabla^2 \Phi + k^2 \Phi = 0, \tag{2.3}
\]

where we have defined the wavenumber \( k = \omega/c \).

For a field generated by a unit concentrated harmonic source, the fundamental solution of the Helmholtz equation in two dimensions can be written as

\[
G(r) = -\frac{i}{4} H_0^{(1)}(kr), \tag{2.4}
\]
where \( r \) is the distance between the field point \( x_f \) and the source point \( x_s \), and \( H_0^{(1)} \) is the Hankel function of the first kind of order zero. The fundamental solution has the property

\[
\nabla^2 G(x_s, x_f) + k^2 G(x_s, x_f) = \begin{cases} 
-1 & x_s = x_f \\
0 & x_s \neq x_f 
\end{cases}.
\tag{2.5}
\]

To formulate the boundary integral equation representation for the Helmholtz problem, Green’s second identity is used together with the fundamental solution, to give

\[
\int_V (\Phi \nabla^2 G - G \nabla^2 \Phi) dV = \int_S (\Phi \frac{\partial G}{\partial n} - G \frac{\partial \Phi}{\partial n}) dS. \tag{2.6}
\]

Substitution of (2.3) into (2.6) yields

\[
\int_V (\nabla^2 G + k^2 G) \Phi dV = \int_S (\Phi \frac{\partial G}{\partial n} - G \frac{\partial \Phi}{\partial n}) dS. \tag{2.7}
\]

The boundary integral representation of the problem is obtained by substituting equation (2.5) into equation (2.7) to get

\[
\Phi(x_s) = \int_S \frac{\partial \Phi}{\partial n} G(x_s, x_f) dS - \int_S \Phi \frac{\partial G(x_s, x_f)}{\partial n} dS. \tag{2.8}
\]

In the limit as the source point \( x_s \) approaches a boundary point \( x_s' \), (2.8) becomes

\[
c_s(x_s') \Phi(x_s') = \int_S \frac{\partial \Phi}{\partial n} G(x_s', x_f) dS - \int_S \Phi \frac{\partial G(x_s', x_f)}{\partial n} dS, \tag{2.9}
\]

where \( 0 \leq c_s(x_s') \leq 1 \) is given by

\[
c_s(x_s') = \frac{\gamma_s}{2\pi}, \tag{2.10}
\]

and the subtended angle \( \gamma_s \) associated with any source point at the intersection of two straight-line segments on the surface \( S \) is shown in Figure 2.5 of Wrobel\textsuperscript{12}.  

5
To solve (2.9) for arbitrary geometries, we need to proceed numerically. Key to the boundary element method is the concept of solving the boundary integral equations using a spatial discretization of the boundary $S$ into $N_e$ elements, $S_j$ ($1 \leq j \leq N_e$) so that

$$\bigcup_{j=1}^{N_e} S_j = S,$$

thus yielding the discretized form of (2.9)

$$c_s(x'_s)\Phi(x'_s) = \sum_{j=1}^{N_e} \left( \int_S \frac{\partial \Phi}{\partial n} G(x'_s, x_f) \, dS - \int_S \Phi \frac{\partial G(x'_s, x_f)}{\partial n} \, dS \right). \quad (2.11)$$

There are many ways to approximate the variation of $\Phi$ and $\partial \Phi / \partial n$ within each element. If one takes both quantities to be piecewise constant (i.e., each assumes different constant values over each element of the boundary), one obtains

$$c_s(x'_s)\Phi(x'_s) = \sum_{j=1}^{N_e} \frac{\partial \Phi_j}{\partial n} \int_{S_j} G(x'_s, x_f) \, dS - \sum_{j=1}^{N_e} \Phi_j \int_{S_j} \frac{\partial G(x'_s, x_f)}{\partial n} \, dS, \quad (2.12)$$

where $\Phi_j$ and $\partial \Phi_j / \partial n$ are the constant values of $\Phi$ and its normal derivative, respectively, evaluated at the center of each element $j$. The discretized continuum problem can then be restated as

$$(c_s \Phi)_i = \sum_{j=1}^{N_e} \frac{\partial \Phi_j}{\partial n} \int_{S_j} G(x'_s, x_i) \, dS - \sum_{j=1}^{N_e} \Phi_j \int_{S_j} \frac{\partial G(x'_s, x_i)}{\partial n} \, dS \quad (2.13)$$

where $x_i$ corresponds to $x_f$ for any nodal point $i$, $1 \leq i \leq N_e$. Defining

$$V_{ij} = \int_{S_j} G_i \, dS_j, \quad (2.14)$$

$$\hat{H}_{ij} = \int_{S_j} \frac{\partial G_i}{\partial n} \, dS_j, \quad (2.15)$$

6
and

$$H = \hat{H} + C_s,$$

(2.16)

where $H$ and $\hat{H}$ are $N_e \times N_e$ arrays, $C_s$ is a $N_e \times N_e$ diagonal array with the $i$-th diagonal element being $(c_s)_i$ (see (2.13)) and $G_i$ refers to $G(x'_s, x_i)$, so that (2.13) can be written as

$$\sum_{j=1}^{N_e} H_{ij} \Phi_j = \sum_{j=1}^{N_e} V_{ij} \frac{\partial G}{\partial n_j}.$$

(2.17)

Selecting nodal points on the bounding surfaces, as shown in Figure 2.1, and evaluating the potential at each of them, we obtain the matrix-vector equation

$$H \Phi = V Q,$$

(2.18)

where $V$ is a $N_e \times N_e$ array and $\Phi$ and $Q$ are vectors representing the discrete values of the potential and its normal derivative on the boundary. Boundary conditions are inserted to $\Phi$ and $Q$ to obtain a system of equations with $N_e$ number of unknowns on the boundary $S$. There are $N_u$ number of unknown values of the potential in $\Phi$, not determined by the boundary conditions and $N_e - N_u$ number of unknown values of the normal derivative of the potential in $Q$. The system (2.18) can be reordered as

$$A u = l,$$

(2.19)

where $u$ is a vector of length $N_e$ containing all unknown boundary values, $l$ is a 'load' vector and $A$ is a full non-symmetric matrix. The system (2.19) may be solved by iterative or direct schemes to obtain the values of the potential on the boundary.
The acoustic field was computed using the exterior version of the BEMHELM BEM code developed by Kirkup\textsuperscript{13}. Computations were performed for LDR geometries having an indented reflector, qualitatively similar to that in Figure 2.1. Using the output from this code, we can approximate the velocity potential anywhere on the exterior domain. We denote this continuous function of $r$ and $z$ by $\phi_a(r, z)$. Assuming time-harmonic motion, the time-dependent velocity potential calculated from $\phi_a(r, z)$, is expressed as

$$\Phi_{BEM}(r, z, t) = \phi_a(r, z)e^{-i\omega t} = [\phi_{a,r}(r, z) + i\phi_{a,i}(r, z)]e^{-i\omega t}, \quad (2.20)$$

where both the real and imaginary part of $\Phi_{BEM}(r, z, t)$ approximately satisfy the Helmholtz
equation. Using only the imaginary part of \( \phi_a(r, z) \), we write the potential as

\[
\Phi(r, z, t) = i \phi_{a,i}(r, z) [\cos \omega t - i \sin \omega t]
= \phi_{a,i}(r, z) \sin \omega t + i \phi_{a,i}(r, z) \cos \omega t
= \phi_{a,i}(r, z) e^{-i(\omega t - \pi/2)}.
\] (2.21)

The complex acoustic pressure is calculated as

\[
\hat{p}_c(r, z, t) = \rho_0 \frac{\partial \Phi(r, z, t)}{\partial t}
= \rho_0 \omega \phi_{a,i}(r, z) \cos \omega t - i \rho_0 \omega \phi_{a,i}(r, z) \sin \omega t
= -i \rho_0 \omega \phi_{a,i}(r, z) e^{-i(\omega t - \pi/2)}
= -i \rho_0 \omega \Phi(r, z, t),
\] (2.22)

where \( \rho_0 \) is the nominal air density. Since we are only interested in the real part of the complex pressure \( \hat{p}_c \), we have

\[
\hat{p}_r(r, z, t) = \text{Re}(\hat{p}_c)
= \rho_0 \omega \phi_{a,i}(r, z) \cos \omega t
= -\rho_0 \omega \phi_{a,i}(r, z) \sin(\omega t - \pi/2),
\] (2.23)

where \( \hat{p}_r(r, z, t) \) is the time-dependent real acoustic pressure and the complex acoustic velocity is calculated as

\[
\hat{v}_c(r, z, t) = - \left\{ \frac{\partial \phi_{a,i}(r, z)}{\partial r} e_r + \frac{\partial \phi_{a,i}(r, z)}{\partial z} e_z \right\} e^{-i(\omega t - \pi/2)}
= -\nabla \phi_{a,i}(r, z) (\sin \omega t + i \cos \omega t).
\] (2.24)
Since we are only interested in the real part of the solution, we have

\[ \hat{v}_r(r, z, t) = \text{Re}(\hat{v}_c) = -\nabla \phi_{a,i}(r, z) \sin \omega t, \]  

(2.25)

where \( \hat{v}_r(r, z, t) \) is the time-dependent real acoustic velocity. The acoustic velocity leads the acoustic pressure by 90°. Calculation of the acoustic pressure and acoustic velocity from the complex part of \( \phi_a \) is performed as

\[ p(r, z) = -\rho_0 \omega \phi_{a,i}(r, z) \]  

(2.26)

and

\[ v(r, z) = -\nabla \phi_{a,i}(r, z). \]  

(2.27)

For any specific LDR geometry and input frequency and amplitude, there are several modes of acoustic resonance. In order to determine the resonant modes of different geometries, we calculate the emitted power

\[ P = \int_{\Gamma_{\text{emitter}}} < pv_n > d\Gamma = \int_0^{2\pi} \int_0^{R_E} < pv_n > r dr d\theta, \]  

(2.28)

where integration is performed over the emitting surface \( \Gamma_{\text{emitter}} \), \( p \) is the acoustic pressure on the emitter surface, \( v_n \) is the amplitude of the normal acoustic velocity on the emitter surface, \(< >\) denotes a time-average, and \( \theta \) denotes the azimuthal coordinate.

We normalize \( P \) as

\[ \tilde{P} = \frac{Pk^2}{\rho_0 c^2 v_0^2}. \]  

(2.29)

For purposes of calculating the emitted power, it is only necessary to determine the velocity potential on the surface of the emitter. The normal velocity is simply equal to the boundary value specified on the emitting surface. A sample emitted power calculation for the experimental geometry in the Scheeline laboratory is shown in Figure 2.2. In the
range shown, four distinct power maxima are apparent. Since the first two (near $H/\lambda$ values of 0.58 and 1.15) correspond to emitter/reflector gaps too small for adequate optical access, and the fourth (near $H/\lambda = 2.25$) corresponds to insufficient emitted power for the experiment (A. Scheeline, private communication), we focus on the third resonant mode (i.e., the third-smallest value of $H/\lambda$ that produces resonance).

![Figure 2.2. Emitted power vs. distance $H$ from emitter to reflector for a uniform velocity distribution on the radiating plate boundary.](image)

When a water drop is introduced into the standing-wave acoustic field, it will migrate to a pressure node (where the wave amplitude in a standing acoustic wave is zero for all time), where it will experience surface forces (due to pressure) in opposition to gravity\textsuperscript{14}. Barmatz and Collas\textsuperscript{15} developed a method to determine the acoustic pressure force potential $U$, for a sphere in an arbitrary sound field, based on Gor’kov’s theory. This potential is defined as

$$U = 2\pi R_s^3 \left[ \frac{\langle p^2 \rangle}{3\rho_0 c^2} - \rho_0 \frac{\langle v^2 \rangle}{2} \right]$$  \hspace{1cm} (2.30)
and is related to the force by
\[ \mathbf{F} = -\nabla U, \]  
(2.31)

where \( R_s \) is the drop radius, and \( \mathbf{F} \) is the acoustic radiation force. Although the force potential distribution over the acoustic field is of importance, the value of the force potential is of less concern and it is thus useful to work with nondimensional quantities, as defined by

\[ \tilde{p} = \frac{p}{\rho_0 c v_0}, \]  
(2.32)

\[ \tilde{v} = \frac{v}{v_0}, \]  
(2.33)

\[ \tilde{U} = \frac{U}{2\pi R_s^3 \rho_0 v_0^2}, \]  
(2.34)

\[ \tilde{F}_r = \frac{\partial \tilde{U}}{\partial \tilde{r}}, \]  
(2.35a)

\[ \tilde{F}_z = \frac{\partial \tilde{U}}{\partial \tilde{z}}, \]  
(2.35b)

where \( \tilde{F}_r \) and \( \tilde{F}_z \) are the nondimensional versions of the dimensional axial and radial force components

\[ F_z = \frac{\partial U}{\partial z}, \]  
(2.36a)

\[ F_r = \frac{\partial U}{\partial r}, \]  
(2.36b)

respectively, and \( \tilde{z} = z/\lambda \), and \( \tilde{r} = r/\lambda \). Note that \( v_0 \) is the velocity amplitude on the emitter surface, whereas this symbol denotes the maximum velocity amplitude in the field in Ref. 15.
Chapter 3

Code Validation

Initial validation studies were performed using an interior version of the code for a rectangular domain, and results were in excellent agreement with the closed-form analytical solution.

To validate the exterior code, we computed the acoustic field between an emitting disk (transducer) and a coaxial reflector disk, corresponding to the geometry of Andrade et al.\textsuperscript{10}, shown in Figure 3.1a. The calculation in Ref. 10 is based upon a coupled finite-element method (FEM) treatment of the acoustic and solid, utilizing interfacial fluid-structure elements at the gas/solid interface and a no-reflection boundary condition on the nonsolid part of the boundary, to approximate radiation to infinity. The acoustic radiation potential field, defined in Ref. 10, is

\[ \tilde{U}_R = \frac{< p^2 >}{3 \rho_0 c^2} - \frac{\rho_0 < v^2 >}{2}. \]  

(3.1)

The present computations, shown in Figure 3.1b, are in good qualitative agreement with the finite-element results of Ref. 10, supporting the conclusion of Andrade et al.\textsuperscript{10} that a nonuniform normal velocity at the emitter/gas boundary (due to accounting for elastic deformation in the solid) has no significant influence on the acoustic problem. The maximum and minimum values of the acoustic radiation potential calculated from our results are approximately 30\% smaller and approximately 25\% larger than the maximum and minimum potential of Ref. 10, respectively. It is unclear whether the quantitative differences between our results and those of Andrade result from their modeling of deformation in the solid, or from differences in the no-reflection boundary condition. In Figure 3.1a two distinct, off-axis,
separated pressure minima lobes exist close to the corners of the transducer, whereas in Figure 3.1b corresponding lobes are visible, but seem to coalesce.

A second validation, in which our BEM computations are compared to the results of Xie and Wei, is provided in Appendix A. The Xie and Wei geometry used for comparison corresponds to a circular spherically concave reflector and a coaxial emitting disk. Contour plots of the nondimensional force potential $\tilde{U}$ are compared and excellent agreement between current results and that of Xie and Wei is shown.
Figure 3.1. Acoustic radiation potential field for a radiating plate-to-reflector spacing of 25 mm and an excitation frequency of 19.9 kHz. a) finite-element results of Andrade et al.\textsuperscript{10}; b) present BEM results. Note that in Ref. 10, the radial coordinate in Figure 5 was labelled as if it was Cartesian, and the size of the reflector (which in the computation is identical to ours), was truncated.
Chapter 4
Computational Results for the Experimental Geometry

Figure 2.1 is a schematic geometry broadly representative of those considered in this work. The emitter (radiating plate) radius is $R_E$ and the reflector radius is $R_R$. The maximum thickness of the reflector is $H_R$ and the emitter thickness is $H_E$. The difference between the on-axis thickness of the reflector and its thickness at $r = R_R$ is denoted by $D_R$, and the on-axis spacing between the radiating plate and reflector is denoted by $H$. The radius of the concave section on the reflector is $R_C$, and

$$R = \frac{D_R^2 + R_C^2}{2D_R},$$

(4.1)

is the radius of curvature of the spherical indentation. Using the parameters defined for the experimental geometry in Table 4.1 and an air density of 1.225 kg/m$^3$, sound speed of 340 m/s, and emitter-plate driving frequency of 20.7 kHz, the acoustic pressure field was calculated at acoustic resonance.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_E$</td>
<td>3.18</td>
</tr>
<tr>
<td>$H_R$</td>
<td>20.0</td>
</tr>
<tr>
<td>$R_E$</td>
<td>12.7</td>
</tr>
<tr>
<td>$R_R$</td>
<td>34.925</td>
</tr>
<tr>
<td>$R_C$</td>
<td>23.813</td>
</tr>
<tr>
<td>$R$</td>
<td>33.454</td>
</tr>
<tr>
<td>$H$</td>
<td>27.594</td>
</tr>
<tr>
<td>$D_R$</td>
<td>9.9568</td>
</tr>
</tbody>
</table>
The emitted power calculated according to (2.28) is shown in Figure 2.2, from which it is seen that resonance is achieved at $H/\lambda \approx 1.671$. The computed acoustic pressure field is shown in Figure 4.2.

On-axis levitation positions are located where the axial acoustic force $F_z = 0$ and the force potential $U$ reaches a minimum value to form a potential “well” or levitation “node”. Due to nonzero drop weight, the drop will shift slightly downwards to a position where $F_z$ balances drop weight, but that shift is typically very small for small drops. In order to calculate the node locations, the force potential $U$ and its axial derivative $F_z$ are computed on-axis. Figure 4.1 is a plot of $\tilde{U}$ and $\tilde{F}_z$ calculated on-axis for a range of $z$ that includes the first two levitation nodes, as measured from the radiating plate surface.

![Figure 4.1. Experimental geometry on-axis dimensionless force potential and axial force. $\bigcirc$ $\tilde{U}$; $\ast$ $\tilde{F}_z$; $\bullet$ levitation node location.](image)

In Figure 4.2, three distinct pressure nodes are apparent, indicated by the white $+$ symbols. Additional insight into the mechanism of drop levitation is provided by Figure 4.3, which shows the radial-locating nondimensional force component $\tilde{F}_r$ along a traverse of the $\tilde{F}_z = 0$ contour (indicated by the thick black curve in Figure 4.2 and denoted by $z_0(r)$,
where $z_0(0)$ corresponds to the location of the second levitation node for the geometry in Figure 4.2). The dimensionless radial-locating force distribution varies strongly close to the drop levitation position. Beyond $r/\lambda \approx 0.43$, where it assumes a maximum value, $|\tilde{F}_r|$ decreases rapidly to zero. To counteract drop wander due to disturbances, it is important that the radial-locating force component be sufficiently large at radii as far away from the axis as expected excursions of the drops.

Figure 4.2. Nondimensional acoustic pressure field for the region between the flat emitter plate and the concave reflector for the experimental LDR geometry, at third-mode resonance. The curve on which $\tilde{F}_z = 0$ is denoted by $z_0(r)$.
Figure 4.3. Nondimensional radial-locating force component and force potential along a traverse of \( z_0(r) \), where \( z_0(0) \) corresponds to the second levitation node for the geometry in Figure 4.2. - - - \( \tilde{U} \); —– \( \tilde{F}_r \).
Chapter 5

Effects of Geometry on the Computed Field

An important geometric parameter of the LDR is the reflector depth $D_R$ (see Figure 2.1). With the values of all geometric parameters other than $D_R$ and $H$ held fixed, the emitted power was calculated for $0.2 \leq D_R/\lambda \leq 0.9$ and $1.6 \leq H/\lambda \leq 1.75$, with the latter values being chosen to nearly correspond to the value of $H/\lambda$ at the third resonance for the experimental geometry in §4. The third resonant value was selected on the same basis as discussed in the case of the experimental geometry in §4. A plot of the nondimensional emitted power as a function of the geometric parameters $H/\lambda$ and $D_R/\lambda$ is shown in Figure 5.1.

Clearly, for each value of $D_R/\lambda$, there is a value of $H/\lambda$ for which the nondimensional emitted power assumes a local maximum. At higher $D_R/\lambda$, the emitted power at resonance becomes very high. This is characteristic of the increased resonance and the field’s increased sensitivity to $H/\lambda$ for higher values of $D_R/\lambda$, and it is advantageous to focus the emitted power so as to achieve high acoustic power at resonance.

Resonant modes are identified at values of $H/\lambda$ and $D_R/\lambda$ for which the emitted power assumes a local maximum. Resonant values of $H/\lambda$ as a function of reflector depth $D_R/\lambda$ are shown in Figure 5.2, in which we see that, for the range of $D_R/\lambda$ shown, the resonant value of $H/\lambda$ depends approximately linearly on $D_R/\lambda$. 

20
Figure 5.1. Emitted power as a function of $H/\lambda$ and $D_R/\lambda$, retaining the values for the experimental geometry of all other geometric parameters.

Figure 5.2. Reflector-radiating plate spacing at third-mode resonance as a function of reflector depth.
Chapter 6

Spherically-Indented Radiating Plate

As a modification of the experimental LDR geometry, we considered a spherically-Indented radiating plate. Figure 6.1 shows the discretized boundary geometry used in the calculation for geometric parameters given in Table 6.1. Here, $R_1$ and $R_2$ are the radii of curvature of the spherical indentation in the reflector and emitter, respectively, and are given by

$$R_1 = \frac{D_R^2 + R_C^2}{2D_R}$$

(6.1)

and

$$R_2 = \frac{D_E^2 + R_E^2}{2D_E}.$$  

(6.2)

The difference between the on-axis thickness of the emitter and its thickness at $r = R_E$ is denoted by $D_E$. The power as a function of $H/\lambda$ is shown in Figure 6.2. Comparing the power curves in Figures 2.2 and 6.2, it is clear that, for fixed values of the input vertical velocity, the spherically-Indented plate radiates more power at resonance. Figure 6.3 shows a plot of the acoustic pressure field for the spherically-Indented radiating plate.
Table 6.1. Geometric parameters for the case of a spherically-indented radiating plate, used in the present calculations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_E$</td>
<td>20.0</td>
</tr>
<tr>
<td>$H_R$</td>
<td>20.0</td>
</tr>
<tr>
<td>$R_E$</td>
<td>23.813</td>
</tr>
<tr>
<td>$R_R$</td>
<td>34.925</td>
</tr>
<tr>
<td>$R_C$</td>
<td>23.813</td>
</tr>
<tr>
<td>$R_1$</td>
<td>33.454</td>
</tr>
<tr>
<td>$R_2$</td>
<td>33.454</td>
</tr>
<tr>
<td>$H$</td>
<td>27.995</td>
</tr>
<tr>
<td>$D_R$</td>
<td>9.9553</td>
</tr>
<tr>
<td>$D_E$</td>
<td>9.9553</td>
</tr>
</tbody>
</table>
Figure 6.1. Geometry and discretization for the case of a spherically-indented radiating plate. ○ boundary-element nodes.
Figure 6.2. Third resonant mode calculated for the spherically-indented radiating plate levitator with parameters given in Table 6.1.
Figure 6.3. Nondimensional acoustic pressure field calculated at the third resonant mode spacing for the region between the spherically-indented radiating plate and the reflector.

Figure 6.4 shows the radial variation of the radial-locating force $\tilde{F}_r$, along a traverse of the $\tilde{F}_z = 0$ contour (indicated by the thick black curve in Figure 6.3 and denoted by $z_0(r)$, where $z_0(0)$ corresponds to the levitation position of experimental interest, that is the second levitation node for the geometry in Figure 6.3). The advantage of using a spherically-indented radiating plate is evident from Figure 6.5, where it is evident that the indented plate produces a much larger on-axis radial-locating force at elevations near the levitation node than does the flat plate.
The acoustic pressure “lobes” of the standing wave field, shown in Figure 6.3, contribute significantly to vertical positioning of the drop, and the lobe shape should thus correlate directly to drop stability. When the calculated fields in Figures 4.2 and 6.3 are compared, it is clear that the spherically-indented radiating plate produces “flatter” pressure lobes just above and below the second levitation node, which should provide better drop stability than those of Figure 4.2.

Figure 6.4. Nondimensional radial-locating force component and force potential on the $F_z = 0$ contour ($z_0(r)$ in Figure 6.3, where $z_0(0)$ corresponds to the second levitation node for the geometry in Figure 6.3). - - - $\tilde{U}$; —– $\tilde{F}_{r}$. 

Figure 6.5. Radial force on a 1 mm diameter sphere (or spherical drop), calculated for the spherically concave radiating plate and flat radiating plate. a) concave; b) flat.
Calculations were completed to simulate finite stiffness of the radiating plate, compared to infinite stiffness or piston-like behavior, used in the simulation of §4. The finite stiffness was modeled assuming a Bessel-function radial dependence of the axial velocity (see Rayleigh\textsuperscript{16}) on the radiating plate surface, compared to the uniform velocity distribution assumed in the case of infinite stiffness. The calculations were performed using parameters as shown in Figure 2.1 and Table 4.1 for a concave reflector and flat radiating plate.

For the nonuniform surface velocity distribution, a Bessel function of the first kind of order zero was evaluated at the centers of the discretized boundary elements. The comparison is made on the basis of determining the displacement boundary condition by equating the emitted power produced by the nonuniform and uniform surface velocity distributions. From the linear dependence of the acoustic pressure on the excitation amplitude, it follows that the emitted power calculated from (2.28) depends quadratically on the excitation amplitude. For a given uniform surface velocity amplitude $v_0$, the nonuniform or Bessel surface velocity amplitude $v_{B,0}$ is calculated from

$$v_{B,0} = v_{B,0}^{(1)} \sqrt{P_0/P_{B,0}^{(1)}},$$

(7.1)

where $v_{B,0}^{(1)}$ is an arbitrary nonzero amplitude of the nonuniform surface velocity distribution, $P_{B,0}^{(1)}$ is the emitted power calculated using $v_{B,0}^{(1)}$, and $P_0$ is the emitted power calculated using the uniform velocity distribution $v_0$. 
The nonuniform surface velocity distribution at the boundary is given by

\[ v_B(r) = v_{B,0} J_0(\chi r/R_E), \quad (7.2) \]

where \( \chi = 2.4048 \ldots \) is the smallest zero of \( J_0 \), corresponding to the vanishing of \( v_B(r) \) at \( r = R_E \). Figure 7.1 shows the acoustic pressure field obtained for this nonuniform boundary condition at third mode resonance. In the current calculation, the radiating plate is excited at the same frequency as in the uniform case. It was found that third-mode resonance is also achieved at \( H/\lambda = 1.671 \), as in the case of a uniform boundary condition. The acoustic pressure for the current calculation, \( p_B \), is nondimensionalized using the uniform surface velocity that gives the same power, according to

\[ \tilde{p}_B = \frac{p_B}{\rho_0 c v_0}. \quad (7.3) \]

Comparing Figure 4.2 and Figure 7.1, a good quantitative comparison is observed between the rigid and elastic treatments, with a maximum relative error of 4.45% in the nondimensional acoustic pressure. For calculation purposes, a uniform boundary velocity was assumed.
Figure 7.1. Nondimensional acoustic pressure field for the experimental geometry at third resonant mode spacing with the nonuniform velocity boundary condition, approximating elastic behavior, applied to the radiating plate surface.
Chapter 8

Radially-Localized Emission from a Planar Radiating Plate

Using the experimental geometry, computations were performed to determine the effect of radially localizing the power emission on a planar radiating plate. Practically, this velocity distribution is equivalent to having a vibrating sound transducer in the central region of the vibrating plate. The boundary surface velocity distribution is given by

\[
v_{\text{conc}}(r) = \begin{cases} 
    v_{\text{conc},0} & 0 \leq r \leq \beta R_E \\
    0 & \beta R_E < r \leq R_E 
\end{cases},
\]

(8.1)

where the fraction of the radius of the plate over which emission occurs is denoted by $\beta$. Following the same procedure adopted in §7, the boundary surface velocity amplitude is determined by

\[
v_{\text{conc},0} = v^{(1)}_{\text{conc},0} \sqrt{\frac{P_0}{P^{(1)}_{\text{conc},0}}},
\]

(8.2)

where $v^{(1)}_{\text{conc},0}$ is an arbitrary velocity amplitude, $P^{(1)}_{\text{conc},0}$ is the emitted power calculated using $v^{(1)}_{\text{conc},0}$, and $P_0$ is the emitted power calculated using the uniform distribution $v_0$. The emitter excitation frequency is 20.7 kHz, and resonance corresponding to three levitation nodes occurs for $H/\lambda = 1.680$. Nondimensionalization of the acoustic pressure, $\tilde{p}_{\text{conc}}$, for the radially-localized case is performed with the uniform surface velocity that gives the same power

\[
\tilde{p}_{\text{conc}} = \frac{p_{\text{conc}}}{\rho_0 cv_0}.
\]

(8.3)

Investigating Figure 8.1 to Figure 8.3, it is observed that there exists little qualitative difference in results when compared to Figure 4.2. Although there is a qualitatively good
comparison, the figures show reduced amplitudes in the nondimensional pressure. The reduction is approximately 20% for $\beta = 1/2$ and $1/4$, and significantly higher at approximately 50% for $\beta = 1/8$. 
Figure 8.1. Nondimensional acoustic pressure field calculated using a concentrated emission with $\beta = 1/2$. 
Figure 8.2. Nondimensional acoustic pressure field calculated using a concentrated emission with $\beta = 1/4$. 

\[ \tilde{p}_{\text{conc}} \]
Figure 8.3. Nondimensional acoustic pressure field calculated using a concentrated emission with $\beta = 1/8$. 
Chapter 9

Effects of Volumetric Absorption

A sound wave propagating in the atmosphere will experience energy loss due to shear viscosity and molecular relaxation. Atmospheric absorption reduces the pressure amplitude of a propagating plane wave approximately exponentially according to

\[ p_d = p_i e^{-0.1151 \alpha s} \tag{9.1} \]

where \( \alpha \) is the attenuation coefficient, \( p_i \) is the "upfield" amplitude, and \( s \) is the propagation distance. The attenuation coefficient is dependent on the laboratory atmospheric conditions, and can be approximated using the procedure outlined in Ref. 17, giving

\[
\alpha = 8.686 f_{\text{ANSI}}^2 \left\{ 1.84 \times 10^{-11} \frac{b}{a} + b^{-5/2} \times \left( \frac{0.01275 f_{\text{ro}} e^{-2230.1/T_{\text{ANSI}}}}{f_{\text{ro}}^2 + f_{\text{ANSI}}^2} + \frac{0.1068 f_{\text{rN}} e^{-3352/T_{\text{ANSI}}}}{f_{\text{rN}}^2 + f_{\text{ANSI}}^2} \right) \right\}, \tag{9.2}
\]

where

\[
f_{\text{ANSI}} = \frac{f}{\text{Hz}} \tag{9.3a}
\]

\[
T_{\text{ANSI}} = \frac{T}{\text{K}} \tag{9.3b}
\]

\[
a = \frac{p_a}{p_r} = 10^C \tag{9.3c}
\]

\[
C = 4.6151 - 6.8346 \left( \frac{T}{T_{01}} \right)^{1.261} \tag{9.3d}
\]
\[ b = \frac{T}{T_r}. \]  

(9.3e)

The oxygen and nitrogen relaxation frequencies \( f_{rO} \) and \( f_{rN} \) are functions of pressure and temperature, and are approximated by

\[ f_{rO} = a \left[ 24 + \frac{(4.04 \times 10^4)(0.02 + h_{\text{ANSI}})}{0.391 + h_{\text{ANSI}}} \right] \]  

(9.4a)

\[ f_{rN} = ab^{-1/2} \left[ 9 + 280h_{\text{ANSI}}e^{-4.17(b^{-1/2} - 1)} \right], \]  

(9.4b)

where the nondimensional molar concentration of water vapor, \( h_{\text{ANSI}} = h \text{ M}^{-1} \) is related to the relative humidity \( h_{\text{rel}} \) by

\[ h_{\text{ANSI}} = h_{\text{rel}} \frac{p_{\text{sat}}}{p_r} a^{-1}. \]  

(9.5)

The attenuation coefficient was calculated at the laboratory conditions with the values in Table 9.1, yielding a value of \( \alpha = 0.2418 \text{ dB/m} \). The complex wavenumber used in the BEM calculation is defined as

\[ k_c = \frac{\omega}{c} + i\alpha. \]  

(9.6)

Figure 9.1 is a plot of the attenuation coefficient calculated at 1 atm for a range of temperature and relative humidity. Due to the negligibly small attenuation that will occur over length scales of interest for the UIUC experimental geometry (\( \alpha R_R \) and \( \alpha H \) are both \( \sim 10^{-2} \text{ dB} \)), we neglect absorption, and use a purely real wavenumber. This assumption is further reinforced by the conclusions in §10 to follow.
Table 9.1. Values of the variables used in the attenuation coefficient calculation.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value and Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{rel}$</td>
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</tr>
<tr>
<td>$f$</td>
<td>20.7 kHz</td>
</tr>
<tr>
<td>$p_a$</td>
<td>99 kPa</td>
</tr>
<tr>
<td>$p_r$</td>
<td>101.325 kPa</td>
</tr>
<tr>
<td>$T$</td>
<td>298.16 K</td>
</tr>
<tr>
<td>$T_r$</td>
<td>293.15 K</td>
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<tr>
<td>$T_0$</td>
<td>273.16 K</td>
</tr>
</tbody>
</table>

Figure 9.1. Attenuation coefficient calculated at 1 atm as a function of temperature and relative humidity

39
Chapter 10
Absorption Investigation Using a Complex Wavenumber

To validate the BEM code’s treatment of cases in which absorption occurs (i.e., cases in which the wavenumber is complex), a pulsating sphere, for which an analytic solution is available, was considered. The spherical geometry is defined by

\[ r^2 + (z - R)^2 = R^2 \]  \hspace{1cm} (10.1a)

\[ z = \sqrt{R^2 - r^2} + R, \]  \hspace{1cm} (10.1b)

where \( r \) is the radial coordinate, \( z \) the axial coordinate and \( R \) is the radius of the sphere. A range of attenuation values was evaluated by varying the imaginary part of the wavenumber according to

\[ k_c = 382.54(1 + \sigma i), \quad \sigma = 0, \ 0.01, \ 0.1, \ 1. \]  \hspace{1cm} (10.2)

Where \( \text{Re}(k_c) = 382.54 \) is the wavenumber calculated at laboratory conditions (see §2 and §4). A discretization of 500 boundary elements on the surface of the sphere was used in the BEM calculations. On the spherical boundary, \( \Gamma_{sphere} \), the boundary condition was taken as

\[ \frac{\partial \phi}{\partial n} \bigg|_{\Gamma_{sphere}} = -v_0. \]  \hspace{1cm} (10.3)
It can easily be verified that the acoustic potential in an attenuating medium, driven by a harmonically pulsating sphere, is given by

\[ \phi(r) = \frac{4\pi R^2 v_0}{4\pi r \sqrt{1 + (\omega R/c)^2}} e^{i[k_c(r-R)-\tan^{-1}(\omega R/c)]}. \]  

(10.4)

Figures 10.1, 10.2, 10.3, 10.4 and 10.5 show the relative error in the absolute difference in nondimensional pressure, \( E_{rel} = |\tilde{p}_{BEM} - \tilde{p}_{anal}|/\max|\tilde{p}_{anal}| \), where \( \max|\tilde{p}_{anal}| \) is the maximum value of \( |\tilde{p}_{anal}| \) over \( 1.2329 \leq r/\lambda \leq 3.0593 \), \( \tilde{p}_{BEM} \) corresponds to the nondimensional pressure calculated using the BEM and the nondimensional pressure \( \tilde{p}_{anal} \), was calculated using the analytic solution (10.4) for \( \sigma = 0, 0.01, 0.1, 1 \) and at experimental conditions, for which \( \sigma = 6.323 \times 10^{-4} \). It is clear that a sufficiently large imaginary part of the wavenumber, corresponding to significant absorption, produces large error in the BEM calculation.

Figure 10.6 is a plot of the nondimensional acoustic pressure at a radial distance \( r \) for \( z = R \) and \( \sigma = 6.323 \times 10^{-4} \) using both the analytic solution and the BEM. Figure 10.7 is a plot of the nondimensional acoustic pressure \( \tilde{p} \), at a radial distance \( r \) for \( z = R \), using the attenuation coefficient calculated for the laboratory conditions \( \sigma = 6.323 \times 10^{-4} \) and for \( \sigma = 0 \). This confirms that at the laboratory conditions, attenuation was negligible.
Figure 10.1. For $\sigma = 0$ (no absorption), relative error $E_{rel} = |\tilde{p}_{BEM} - \tilde{p}_{anal}|/\max|\tilde{p}_{anal}|$, where $\max|\tilde{p}_{anal}|$ is the maximum value of $|\tilde{p}_{anal}|$ over $1.2329 \leq r/\lambda \leq 3.0593$.

Figure 10.2. For $\sigma = 0.01$, relative error $E_{rel} = |\tilde{p}_{BEM} - \tilde{p}_{anal}|/\max|\tilde{p}_{anal}|$, where $\max|\tilde{p}_{anal}|$ is the maximum value of $|\tilde{p}_{anal}|$ over $1.2329 \leq r/\lambda \leq 3.0593$. 
Figure 10.3. For $\sigma = 0.1$, relative error $E_{\text{rel}} = |\tilde{p}_{\text{BEM}} - \tilde{p}_{\text{anal}}|/\max|\tilde{p}_{\text{anal}}|$, where $\max|\tilde{p}_{\text{anal}}|$ is the maximum value of $|\tilde{p}_{\text{anal}}|$ over $1.2329 \leq r/\lambda \leq 3.0593$.

Figure 10.4. For $\sigma = 1$, relative error $E_{\text{rel}} = |\tilde{p}_{\text{BEM}} - \tilde{p}_{\text{anal}}|/\max|\tilde{p}_{\text{anal}}|$, where $\max|\tilde{p}_{\text{anal}}|$ is the maximum value of $|\tilde{p}_{\text{anal}}|$ over $1.2329 \leq r/\lambda \leq 3.0593$. 

43
Figure 10.5. For $\sigma = 6.323 \times 10^{-4}$, relative error $E_{rel} = |\tilde{p}_{BEM} - \tilde{p}_{anal}| / \max|\tilde{p}_{anal}|$, where $\max|\tilde{p}_{anal}|$ is the maximum value of $|\tilde{p}_{anal}|$ over $1.2329 \leq r/\lambda \leq 3.0593$.

Figure 10.6. For $\sigma = 6.323 \times 10^{-4}$, nondimensional acoustic pressure field plotted along the radial line centered at $z = R$. $\circ$ BEM; —— analytic.
Figure 10.7. Nondimensional acoustic pressure field calculated using the BEM, plotted along the radial line centered at $z = R$. $\sigma = 0$; $\sigma = 6.323 \times 10^{-4}$. 
Chapter 11
Conclusions

In this work a boundary-element method is used to analyze the properties of two types of axisymmetric geometries for drop levitation. To study the dependence of drop levitation capability on geometric parameters, the acoustic field between the radiating plate and a reflector is calculated. The acoustic levitation force acting on a drop in the field is calculated by Gor’kov’s theory\textsuperscript{14} and investigated in the region of drop levitation.

Using this model, the levitation capability of a current experimental geometry, utilizing a spherically-indented reflector and flat emitter was compared to that of a geometry utilizing a spherically-indented reflector and emitter. Calculated results indicate that the levitation field is strongly dependent on reflector curvature and that, up to an optimum reflector depth, increasing the reflector curvature produces higher emitted acoustic power compared to a flat reflector. Beyond that optimum depth, emitted acoustic power decreases.

The radially-locating acoustic force acting on a levitated drop is significantly enhanced in geometries in which the radius of the radiating plate is increased and the radiating plate surface is indented. It is also observed that the radial location of the maximum amplitude of the radially-locating force is little affected by indenting the radiating plate surface. Drop vertical positioning capability is also significantly improved due to the higher amplitude of the acoustic pressure field produced by the spherically-indented radiating plate model.

Although a relatively simple geometry was investigated, this method can be used in an optimization study of more complicated drop levitation geometries.
Nomenclature

\( \mathbf{A} \)          full non-symmetric matrix, see Equation 2.19
\( c \)              sound speed
\( c_s \)        coefficient used in BEM discretization, see Equation 2.10
\( C_s \)     square array of influence coefficients, see Equation 2.18
\( D_E \)      radiating plate depth, see Figure 6.1
\( D_R \)      reflector depth, see Figure 2.1
\( E_{ \text{rel}} \)   relative error in the absolute difference in nondimensional pressure, see §10
\( \mathbf{e}_r \)    unit vector in radial direction
\( \mathbf{e}_z \)   unit vector in axial direction
\( f \)            excitation frequency
\( f_{ \text{ANSI}} \)  \( f \ \text{Hz}^{-1} \), see Equation 9.2
\( f_{ \text{rO}} \)  oxygen relaxation frequency
\( f_{ \text{rN}} \)  nitrogen relaxation frequency
\( \mathbf{F} \)    acoustic radiation force vector
\( F_r \)        radial component of \( \mathbf{F} \)
\( \tilde{F}_r \)  dimensionless radial component of \( \mathbf{F} \)
\( F_z \)        axial component of \( \mathbf{F} \)
\( \tilde{F}_z \)  dimensionless axial component of \( \mathbf{F} \)
\( G \)            fundamental solution of the Helmholtz equation
\( h \)            molar concentration of water vapor
\( h_{ \text{ANSI}} \)  \( h \ \text{M}^{-1} \), where M = moles/liter
$h_{rel}$ relative humidity

$H$ square array of influence coefficients, see Equation 2.18

$H$ distance from radiating plate to reflector, see Figure 2.1

$H_E$ radiating plate height, see Figure 2.1

$H_R$ reflector height, see Figure 2.1

$H_0^{(1)}$ Hankel function of the first kind of order zero

$i \sqrt{-1}$

$J_0$ Bessel function of the first kind of order zero

$k$ wavenumber

$k_c$ complex wavenumber

$l$ load vector, see Equation 2.19

$N$ number of boundary elements used in BEM calculation

$N_e$ number of boundary elements

$N_u$ number of unknown elements of $\phi$, not determined by the boundary conditions

$p$ acoustic pressure

$\tilde{p}$ dimensionless acoustic pressure

$\tilde{p}_{anal}$ dimensionless acoustic pressure calculated using the analytic solution, see §10

$\tilde{p}_{BEM}$ dimensionless acoustic pressure calculated using the BEM, see §10

$p_a$ ambient atmospheric pressure, as used in Equation 9.3b

$p_B$ acoustic pressure for the elastic radiating plate calculation, see Equation 7.3

$\hat{p}_c$ complex acoustic pressure

$p_{conc}$ acoustic pressure for the radially localized surface velocity calculation, see Equation 8.3

$p_d$ propagating sound wave pressure amplitude, calculated in Equation 9.1
$p_i$ propagating sound wave initial pressure amplitude, as used in Equation 9.1

$p_r$ reference atmospheric pressure, as used in Equation 9.3b

$\dot{p}_r$ time-dependent real acoustic pressure

$p_{sat}$ saturation pressure of water vapor, as used in Equation 9.5

$P$ emitted power

$P_0$ emitted power calculated using $v_0$

$P_{B,0}$ emitted power calculated using $v_{B,0}$

$P_{conc,0}$ emitted power calculated using $v_{conc,0}$

$\tilde{P}$ dimensionless emitted power

$Q$ array of values of the velocity potential at BEM points

$r$ radial coordinate

$\tilde{r}$ dimensionless radial coordinate, $\tilde{r} = r/\lambda$

$r_0$ location of maximum radial restoring force

$R$ radius

$R_C$ concave section radius, see Figure 2.1 and Figure 6.1

$R_E$ radiating plate radius, see Figure 2.1

$R_R$ reflector radius, see Figure 2.1

$R_s$ drop radius

$s$ distance, as used in Equation 9.1

$S$ boundary surface

$t$ time

$T$ temperature

$T_{ANSI}$ $T$ K$^{-1}$

$T_r$ reference atmospheric temperature, as used in Equation 9.3d

$T_{01}$ triple-point isotherm temperature, as used in Equation 9.3c

$u$ vector containing all unknown boundary values, see Equation 2.19

$U$ acoustic force potential
$\tilde{U}$ dimensionless acoustic force potential, see Equation 2.34

$\tilde{U}_R$ acoustic radiation potential

$v_n$ amplitude of $v_n$

$v$ acoustic velocity

$\hat{v}$ dimensionless acoustic velocity

$\hat{v}_r$ time-dependent real acoustic velocity

$\hat{v}_c$ complex acoustic velocity

$v_B$ nonuniform surface velocity

$v_{B,0}$ amplitude of $v_B$

$v_0$ uniform surface velocity amplitude

$v_{conc}$ radially-confined surface velocity

$v_{conc,0}$ amplitude of $v_{conc}$

$V$ matrix of influence coefficients

$x_f$ general field point

$x_i$ $i$-th field point

$x_s$ source point

$x'_s$ source point on the boundary

$z$ axial coordinate

$z_0(r)$ radially dependent elevation where $F_z = 0$

$\tilde{z}$ dimensionless axial coordinate, $\tilde{z} = z/r$

$\alpha$ attenuation coefficient

$\beta$ radially-localized emission parameter

$\chi$ zeros of $J_0$

$\gamma_s$ subtended angle

$\Gamma_{emitter}$ emitting surface boundary on the radiating plate

$\Gamma_{sphere}$ sphere emitting surface boundary

$\rho_0$ nominal air density
\( \theta \) azimuthal coordinate
\( \phi \) velocity potential
\( \phi_a \) continuous velocity potential as defined in §2
\( \Phi \) vector of velocity potential values on the boundary
\( \Phi \) time-dependent velocity potential
\( \omega \) angular frequency
\( \lambda \) wavelength
\( \nabla \) gradient operator
References


Appendix A

Xie and Wei Test Case

As a part of the validation study, we computed the acoustic field between a circular planar emitter and a circular, spherically-indented reflector. The boundary discretization of this geometry used in the current work is shown in Figure A.1. This geometry corresponds to the geometry of Xie and Wei\textsuperscript{9}, who performed a BEM calculation for the exterior problem, using the reflector and emitter geometry shown in Figure A.2. The nondimensional force potential, \( \tilde{U} \), defined in (2.35), was calculated and plotted and compared to the results of Ref. 9. The potential field calculated by Ref. 9 is shown on the left in Figure A.2 and the present results are shown on the right in Figure A.2. Good comparison between the present results and those of Ref. 9 is found.
Figure A.1. Discretization of the boundary for the geometry used for the Xie and Wei geometry. Note that in keeping with the computational procedure of the current work, the reflector is located above the emitter, but in Figure A.2, the orientation is inverted.
Figure A.2. Geometry used in Ref. 9. Contours of the nondimensional force potential as calculated according to (2.35) is shown. Note that in Ref. 9, the radial coordinate was labelled as if it was Cartesian.
Appendix B

Grid Refinement

A well-refined grid is essential to capture the physics of the acoustic problem. The experimental geometry was used in the grid refinement investigation. The study was performed using the increasingly refined boundary discretizations in Table B.1, where the boundary segment notation is shown in Figure B.1. For each of four cases, the total number of boundary elements is increased.

Table B.2 shows the values of the nondimensional pressure $\tilde{p}$, at select locations in the region between the emitter and reflector for the four cases. Figures B.2 - B.5 are plots of the calculated nondimensional acoustic pressure field for the four grids whose discretizations are given by the parameters in Table B.1. It is observed that excellent agreement is achieved for case 2 and higher.

Table B.1. Grid refinement parameters.

<table>
<thead>
<tr>
<th>Case</th>
<th>$N_{AB}$</th>
<th>$N_{BC}$</th>
<th>$N_{CD}$</th>
<th>$N_{DE}$</th>
<th>$N_{FG}$</th>
<th>$N_{GH}$</th>
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<td>1</td>
<td>5</td>
<td>5</td>
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<td>60</td>
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</tr>
</tbody>
</table>
Figure B.1. LDR boundary segment notation.
Table B.2. Nondimensional pressure $\tilde{p}$, at select positions for grids characterized in Table B.1.

<table>
<thead>
<tr>
<th>Position</th>
<th>Case</th>
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</thead>
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<tr>
<td>$r/\lambda$</td>
<td>$z/\lambda$</td>
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<tr>
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<td>1.7008</td>
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<tr>
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<td>0.4000</td>
</tr>
<tr>
<td>0.3600</td>
<td>0.4000</td>
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<tr>
<td>0.5400</td>
<td>0.4000</td>
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<tr>
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<tr>
<td>0.9000</td>
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<tr>
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<tr>
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<td>1.2000</td>
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<tr>
<td>0.6000</td>
<td>1.2000</td>
</tr>
</tbody>
</table>
Figure B.2. Nondimensional acoustic pressure field for grid refinement case 1.
Figure B.3. Nondimensional acoustic pressure field for grid refinement case 2.
Figure B.4. Nondimensional acoustic pressure field for grid refinement case 3.
Figure B.5. Nondimensional acoustic pressure field for grid refinement case 4.