MODEL-BASED FEEDFORWARD-FEEDBACK CONTROL
FOR REAL-TIME HYBRID SIMULATION OF LARGE-SCALE STRUCTURES

BY

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DISSERTATION
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ABSTRACT

Extreme dynamic events continue to demonstrate the fragility of civil infrastructure worldwide. Over the years, design codes and computational tools have come to reflect an improved understanding of dynamic loads and effects. However, experimental testing often drives these changes. Experimental testing is vital to understanding the behavior of structures subjected to these dynamic loads and evaluating new solutions for hazard mitigation. Common experimental frameworks include quasi-static testing, shake table testing, and hybrid simulation. The tradeoffs in loading protocol make each experimental framework attractive in different situations.

Hybrid simulation is a powerful, cost-effective framework for testing structural systems, closely coupling numerical simulation and experimental testing to obtain the complete response of a structure. Through substructuring, the well-understood components of the structure are modeled numerically, while the components of interest are tested physically. Generally, an arbitrary amount of time may be used to calculate and apply displacements at each step of the hybrid simulation. However, when the rate-dependent behavior of the physical specimen is important, real-time hybrid simulation (RTHS) must be employed.

In RTHS, computation, communication, and actuator limitations cause delays and lags which lead to inaccuracies and potential instabilities. At the same time, the phenomenon of control-structure interaction (CSI) leads to a coupling of the dynamic behavior of the actuators and the structure. Traditional actuator control approaches for RTHS compensate for an apparent time delay or time lag rather than address the actuator dynamics directly. Furthermore, most actuator control approaches focus on single-actuator systems. The model-based actuator control approach proposed herein directly addresses actuator dynamics including CSI and actuator coupling through model-based feedforward-feedback control. The feedback controller is flexible
to include multi-metric measurements for improved tracking of higher-order derivatives, moving beyond the traditional focus solely on displacement tracking. The proposed approach is illustrated for predefined trajectories as well as RTHS of both single and multi-actuator systems.

The similarities between actuator control for RTHS and shake tables are leveraged to apply the proposed model-based control approach to acceleration tracking. Shake tables provide a direct means by which to evaluate structural performance under earthquake excitation. Essentially, an actuator excites a base plate on which a structural model is mounted with a predefined acceleration record. Improvements to acceleration tracking is explored in the presence of large nonlinearities in shake table behavior as well as changes in shake table dynamics through CSI.

The research presented in this dissertation provides an advanced framework for the dynamic performance evaluation of structural systems. A broad class of structures is considered for RTHS, including multi-degree-of-freedom (MDOF) structures through accurate control across a broad frequency range, multi-actuator systems through the modeling of actuator coupling, and improved tracking of higher-order derivatives through multi-metric feedback control. Application to shake table control demonstrates the versatility of the proposed actuator control scheme for general real-time actuator control.
To my mother and father.
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CHAPTER 1     INTRODUCTION

1.1 Motivation

Extreme dynamic events continue to demonstrate the fragility of civil infrastructure. Unlike static loads, for which humans have had an intuitive understanding for many millennia, these dynamic loads cause unpredictable and widespread damage worldwide. Earthquakes are one of the most destructive natural hazards; recent examples include a magnitude 8.0 earthquake in Wenchuan, China in 2008, a magnitude 7.0 earthquake in Haiti in 2010, and a magnitude 9.0 earthquake off the coast of Tōhoku, Japan in 2011. Every new catastrophic event provides the influential combination of public interest and a rich set data that is used to develop seismic resistant devices and systems, improve design codes, and change public policy.

To find an event comparable in magnitude and destruction in the US, one must look back to the magnitude 7.9 earthquake in San Francisco, California in 1906. Since 1906, California has been hit by earthquakes in San Fernando in 1971, Loma Prieta in 1989, and Northridge in 1994. However, in each case, the earthquakes were less than 7.0 in magnitude, the impacted areas were limited in size, and the communities rebounded within weeks (NAS, 2011). Looking back even further, a series of three earthquakes, each with an estimated magnitude of 8.0 followed by several hundreds of aftershocks, hit the New Madrid seismic zone in the Midwest of the US from 1811 to 1812. At the time, the Midwest was sparsely populated and damage to civil infrastructure was very limited. If a similar event were to strike today, the cities of Memphis and St. Louis are expected to sustain severe damage. It is estimated that a magnitude 7.7 earthquake in the New Madrid seismic zone would result in 86,000 casualties and $300 billion dollars in direct economic losses (Elnashai et al., 2009). Despite a lack of recent catastrophic seismic
events within the US, the inevitability of future events should keep US from a sense of complacency.

Natural hazards are an unfortunate reactive means by which to affect changes in understanding and policy. Considering human and economic losses, it is the downtime between events when engineers should be most vigilant, acting in a preventative manner through extensive research programs. Over the years, design codes have seen considerable improvement and computational methods for the analysis of dynamic structure response have become more advanced. However, experimental testing is often driving these changes. When the response of a structural component or system is unknown or difficult to model numerically, experimental testing provides the only way to accurately assess the dynamic response.

1.2 Experimental Testing Frameworks

Engineers have a number of experimental testing frameworks with which to assess the behavior of structural systems under dynamic loads. The tradeoffs in loading protocol make each framework attractive in different situations. The most straightforward method of experimental testing is quasi-static testing, whereby a structure or structural component is loaded in a predefined manner at a slow rate. Because of the slow rate of loading, very large forces can be generated by hydraulic actuators, and thus full-scale specimens are easily accommodated. Typically quasi-static testing is used to investigate the capacity or hysteretic behavior of a material, structural component, or structural system. This information is particularly useful for calibrating numerical models that can be used extend the results of the quasi-static tests. Also, quasi-static testing is appropriate for parametric studies where the influence of incremental changes in structural design or material can be objectively assessed. Such parametric studies are essential in developing and improving design code provisions. However, quasi-static tests are
limited by their predefined loading protocol in that the behavior of the specimen will not affect
the future load steps. In short, quasi-static tests provide capacity information rather than
performance information.

Another well-established experimental testing framework uses shaking tables. In shake
table testing, the entire structural system is modeled physically and ground motion is applied in
real-time. Naturally, this type of testing imposes the limitation that dynamic loads such as wind
or blast cannot be considered directly. For ground motion simulation, the dynamic effects of the
structure are completely captured, making this testing method attractive for earthquake studies.
Considering the payload of most shake tables, reduced-scale structural models are typically
required. Maintaining similitude relationships is essential to assure that the scaled models are
providing accurate representation of their full-scale counterparts. However, there are many
phenomenon, such as fatigue, fracture, friction, local buckling in steel, and crack propagation
and shear in concrete that have size effects limiting the accuracy of scale models. Also, it is
difficult to capture soil-structure interaction unless the foundation system is included in the
physical model. Some large-scale shaking tables exist that are capable of testing full-scale
structural models including the foundation system; however, the cost of their use and
maintenance is restrictive.

Hybrid simulation combines experimental testing and numerical simulation to provide an
efficient and cost-effective framework to test large, complex structures (e.g., Hakuno et al.,
1969; Takanashi et al., 1975; Mahin and Shing, 1985; Takanashi and Nakashima, 1987; Mahin et
al., 1989; Shing et al., 1996). In hybrid simulation, the dynamic effects from loads such as
earthquakes, strong winds, or tsunami are calculated through numerical integration and used for
determining the loading protocol of the experimental component. The response of the
experimental component is measured and used to update the numerical integration in a loop of action and reaction to simulate the dynamic response of the total structure. Because the structural dynamics are expressed numerically and the rate-dependent effects of the experimental component are assumed negligible, hybrid simulation typically employs an extended time scale. This “slowing” of the experiment places less stringent restrictions on experimental component. For example, because large velocities are not required, the response lags of actuators are not a concern. Therefore, actuators are not limited by their flow rating, and large, precise forces can be easily imposed on the physical specimens. Likewise, the extended time scale allows for more flexibility during testing. The specimen may be under continual observation and any problems addressed immediately without advancing to the next load step. The experiment may be paused and restarted at any time with negligible effect on the results. Thus, hybrid simulations may last hours or even days. A direct result of the extended time scale is an inability to apply a specified velocity time history, which can be problematic when testing rate-dependent specimens (e.g., supplemental energy dissipation devices) or simply when rate-dependent behavior is not negligible.

When substructuring is used, structural components for which the response is well understood can be modeled numerically, greatly reducing the required laboratory space and equipment. Because only the critical structural components are physically tested, they can be large or even full-scale representations of the actual component, reducing size effects. In this way, even small laboratories can create and conduct accurate testing of complex structures. Several factors must be considered in choosing the critical structural component. Because damage is intrinsically a local phenomenon, the component may be selected based on a failure mechanism; failure is highly nonlinear and inherently difficult to model numerically. Also, when
testing a structural system with a new device or material for which numerical models are inadequate or simply do not exist, such components should be tested experimentally. Hybrid simulation is nearly identical to quasi-static testing in regard to the types of specimens that can be tested and the equipment required.

Quasi-static testing, shake table testing, and hybrid simulation provide different approaches for assessing the dynamic performance of structures. Hybrid simulation, often considered the most versatile of the three, has evolved into a family of distinct testing methods to serve specific needs of the engineering community. Of the many variations of hybrid simulation, real-time hybrid simulation will be the focus of this research as the rate-dependent behavior of the physical specimen can be captured directly.

1.3 Real-time Hybrid Simulation

Advances in supplemental energy dissipation devices, such as base isolation, fluid dampers, and friction devices, provide promising solutions for mitigating damage resulting from dynamic loads (Soong and Spencer, 2002). The responses of these devices are rate-dependent, requiring real-time experimental evaluation. When these devices are used as part of a hybrid simulation, real-time execution of the experiment is necessary to obtain accurate and stable results (i.e., real-time hybrid simulation (RTHS)). As real-time hybrid simulation is a challenging experimental framework due to interdisciplinary requirements including high-speed actuator control, a number of unexplored areas still remain before the framework can reach its full potential. A detailed understanding of the RTHS dynamics and interaction of the experimental equipment is required.

1.3.1 Real-Time Actuator Control

RTHS requires accurate tracking of a desired trajectory in real-time, typically using servo-hydraulic actuators. Close examination of the system response shows that experimental
equipment introduces both a time delay and frequency-dependent time lag into the RTHS loop. Time delays are not a function of frequency, generally being caused by the communication of data, analog to digital (A/D) and digital to analog (D/A) data conversion, and computation time. These delays can be reduced by using faster hardware, smaller numerical integration time steps, and more efficient software. In contrast, time lags are a result of the physical dynamics and limitations of the servo-hydraulic actuators and vary with both the frequency of excitation and specimen conditions (Dyke et al., 1995). Time delays and lags are an intrinsic part of experimental testing, and mitigation of their effects is an essential part of RTHS.

Horiuchi et al. (1996) demonstrated that for a linear-elastic, single-degree-of-freedom (SDOF) system, the effect of the energy introduced by a time delay is equivalent to negative damping. The negative damping was shown to be large for experiments with large stiffness or a large time delay. If the negative damping exceeds the inherent structural damping in the system, the experiment will become unstable. Moreover, even if the system remains stable, the results will be inaccurate.

A single apparent time delay, lumping together all of the actual time delays and lags present in the RTHS loop, is the basis for early efforts at actuator control for RTHS. For this reason, early approaches are referred to simply as delay compensation. Note that a pure time delay has a constant, unit gain; thus, these approaches also ignored the frequency-dependent amplitude variation of the servo-hydraulic actuator response. One of the most widely used approaches for delay compensation is the polynomial extrapolation method (Horiuchi et al., 1996). In this approach, known displacements are fit in time with a polynomial, and the displacement after a constant time delay is extrapolated in time (predicted). The extrapolated displacement is sent to the servo-hydraulic system as the commanded displacement.
Extrapolation methods have been improved by adding adaptive online estimation of the time delay. One of the first attempts to measure the time delay online was proposed by Darby et al. (2002). This research also demonstrated that the apparent time delay depends on the stiffness of the physical specimen, which can change as the specimen undergoes damage.

Because time lags are not constant, but rather frequency and specimen dependent, assuming a single time delay is not adequate to characterize the dynamic behavior of servo-hydraulic actuators. At the same time, extrapolation approaches have limited frequency bandwidth for accurate compensation, which diminishes as the apparent time delay increases. This problem is particularly acute when structural response is significant at multiple frequencies (e.g., multi-degree-of-freedom (MDOF) structures).

Recently, researchers have begun to address the servo-hydraulic system as a dynamic system, creating low-order transfer functions to represent the dynamics (Jung et al., 2007; Wallace et al., 2007; Chen and Ricles, 2009a). Inverses of these models can provide accurate compensation over the frequency range for which the model is accurate. With stiff or MDOF structures, there is a potential for instabilities to manifest due to unmodeled high frequency servo-hydraulic dynamics. These approaches are generally heuristic, designed to compensate for an observed time delay or time lag in the system.

1.3.2 Actuator Coupling

As real-time hybrid simulation gains traction as an acceptable testing method for civil structures, pushes are being made toward testing increasingly complex physical specimens. Multiple actuators may be required to excite the specimen and measure corresponding restoring forces. A coupling of the dynamics between actuator and specimen was observed and explained by Dyke et al., (1995) and identified as the phenomenon of control-structure interaction (CSI). CSI has
been well studied for single-actuator systems, and RTHS actuator control approaches considering specimen dependency through CSI have been proposed (Carrion and Spencer, 2007; Carrion et al., 2009; Phillips and Spencer, 2011). For multi-actuator systems, CSI leads to a complex actuator control challenge. Because the dynamics of a single actuator are coupled to a specimen, when multiple actuators are connected to the same specimen, the dynamics of all of the actuators become coupled to each other through the specimen. Studies involving multiple actuators for real-time control have been limited.

1.4 Overview of Dissertation

This dissertation develops new model-based actuator control strategies that provide high-fidelity tracking of a desired trajectory necessary for accurate and stable RTHS. Tracking over a broad frequency range is considered to accommodate stiff experimental structures where much of the mass may be simulated numerically, MDOF systems with lightly damped higher modes, and specimens that may add high-frequency dynamics to the test through change or damage.

With the proposed model-based approach, rather than compensating for an apparent time delay or time lag in the system response, focus will be placed on system modeling and control theory based trajectory tracking. The term model-based refers to the model of the servo-hydraulic system. To accommodate both single and multi-actuator systems, the dynamic behavior of the actuators and the dynamic coupling between actuators are considered, assuring accurate control in the presence of CSI. At the same time, model-based multi-metric feedback control algorithms are proposed that represent a new paradigm in RTHS, providing more accurate tracking of higher-order derivatives including velocity and acceleration. This research has five major components: (a) characterization and modeling of a rate-dependent semi-active control device (b) development of model-based actuator control, (c) application to single-
actuator systems, (d) application to multi-actuator systems, and (e) application to shake table control. A description of the contents of each chapter is provided below.

Chapter 2 contains a detailed review of previous studies in hybrid simulation and real-time hybrid simulation with a focus on numerical integration schemes and actuator control strategies. A review of shake table control strategies is also presented to juxtapose the extension of proposed model-based control strategy to state-of-the-art shake table control.

Chapter 3 provides technical background necessary for this dissertation that is outside of traditional civil engineering research. The basics of both classical and modern control are presented, both of which form the basis of the proposed actuator control strategy. Models of the servo-hydraulic system are discussed for which the actuator control strategy will be derived.

Chapter 4 investigates the behavior of a large-scale magnetorheological (MR) damper device. Characterization tests of the MR damper are provided, followed by the development of a high-fidelity numerical model. Semi-active control strategies are developed to improve the response time of the MR damper. The unique behavior of the MR damper is also discussed, including the effects of idle time on damper performance.

Chapter 5 details the formulation of the model-based actuator controller for a general multi-actuator system, starting with a separation into feedforward and feedback control links. Implementation of the feedforward controller is discussed, including the realization of improper systems and their discrete time equivalents for use in digital signal processors. The feedback controller is based on the servo-hydraulic system model with a shaping filter to restrict the frequency range the feedback control effort. Multi-metric feedback control is also proposed, recognizing that rate-dependent specimens are sensitive to velocities and accelerations.
Chapter 6 applies the model-based controller to a single-actuator system. An advanced hardware and software system is assembled for large-scale RTHS and used as the primary means to evaluate the proposed actuator control strategies. The effects of control-structure interaction are demonstrated using a MR damper specimen, whose properties depend on an applied electric current. The proposed model-based controller, including multi-metric feedback, is used to control this highly nonlinear specimen, which can undergo significant changes during testing in RTHS.

The successes of Chapter 6 for a single-actuator system are extended in Chapter 7 to multi-actuator systems. A simple numerical model is used to demonstrate multi-actuator control which is followed by a proof-of-concept test on a physical three-story steel frame structure.

Chapter 8 extends the model-based control framework to shake table testing. Acceleration and multi-metric feedback is proposed as part of the outer-loop controller to ensure accurate tracking of the desired acceleration. Acceleration tracking is explored in the presence of strong shake table nonlinearities as well as changes in shake table dynamics through CSI for a specimen undergoing damage.

Chapter 9 summarizes the research presented in this dissertation. Recommendations for future work are proposed in regard to actuator control as well as RTHS and shake table testing.
CHAPTER 2 LITERATURE REVIEW

This chapter provides a review of the literature on hybrid simulation, with a focus on the numerical integration schemes and actuator control strategies that have successfully been used in RTHS. A brief review of shake table control strategies is also included.

2.1 Hybrid Simulation Framework

Hybrid simulation is the currently favored term for what is also known as pseudodynamic testing (Shing and Mahin, 1984), hardware-in-the-loop simulation (Hanselmann, 1993), and virtual prototyping (Wang, 2002). The concept of hybrid simulation was first proposed by Hakuno et al. (1969) to test a single degree of freedom system under seismic loads. The equations of motion were solved using an analog computer while an electromagnetic actuator was used to excite the physical specimen in real-time. Dynamic response was obtained for the first time without the use of a shaking table, although the analog computer limited the accuracy of the results. Hybrid simulation was established in its current recognizable form through the introduction of discrete time systems and digital controllers (Takanashi et al., 1974, 1975). Using a digital controller to solve the equations of motion, the real-time loading constraint could be relaxed to a ramp and hold procedure over an extended time scale. Typical quasi-static testing equipment could be used while numerical integration could be performed at a slower rate appropriate for the computers at the time.

Efforts to validate and expand the hybrid simulation framework were pursued in parallel both in Japan and the US (Takanashi and Nakashima, 1987; Mahin and Shing, 1985; Mahin et al., 1989; and Shing et al., 1996). Researchers have since extended the original hybrid simulation framework into a versatile family of techniques available today. These include (a) substructure hybrid simulation (Dermitzakis and Mahin, 1985), (b) continuous hybrid simulation (Takanashi
and Ohi, 1983), (c) real-time hybrid simulation (Nakashima et al., 1992), (d) effective force testing (Dimig et al., 1999), (e) distributed substructure hybrid simulation (Watanabe et al., 2001), (f) distributed continuous hybrid simulation (Mosqueda et al., 2004), and (g) distributed real-time hybrid simulation (Kim et al., 2011). Note that when some of the earlier techniques were published, the term pseudodynamic testing was more widely accepted. A detailed review of these test methods can be found in Carrion and Spencer (2007). Figure 2.1 illustrates the evolution of the family of hybrid simulation techniques, adapted from Carrion and Spencer (2007).

**Figure 2.1. Family of hybrid simulation techniques**

When substructuring is employed to hybrid simulation, the components of interest can be modeled experimentally while the rest of the structure can be modeled numerically. Throughout a hybrid test, communication between the experimental and numerical components is maintained in a loop of action and reaction as presented in Fig. 2.2.

The equations of motion governing the dynamic response of the system are solved by a time-stepping numerical integration scheme using the numerically imposed excitation $F^N$, measured displacements $y$, and measured restoring forces $R^E$. Based on the solution of the
numerical integration, the command displacement $u$ is sent to the servo-hydraulic system to impose on the physical specimen. Through displacement feedback, the servo-controller ensures that the commanded displacement is realized by the specimen. Then, the new measured displacement $y$ and measured restoring force $R^E$ are sent back to the computer to be used in the next step of numerical integration. The process is repeated until the complete response of the structure is calculated.

**Figure 2.2. Hybrid simulation loop with substructuring**

### 2.2 Real-Time Hybrid Simulation

The first hybrid simulation, due to the limitations of using an analog computer, was naturally conducted in real-time (Hakuno et al., 1969). Hardware limitations also compromised the accuracy of the experiment by adding a phase lag that was recognized but uncompensated. The development of rate-dependent structural control devices such as base isolation bearings and fluid dampers has spurred interest in expanding hybrid simulation to include a more rigorously
verified real-time framework. The first modern real-time hybrid simulation using digital computers was conducted by Nakashima et al. (1992) on a SDOF system. In this system, accurate velocity control was sought by introducing a digital servo-mechanism between the computer performing the numerical integration and the servo-controller. This digital servo-mechanism acted as a ramp generator between numerical integration time steps and also included a feedback loop to improve the displacement performance at substeps of the numerical integration.

Horiuchi et al. (1996) studied the effect of time delay on RTHS in detail and proposed the polynomial extrapolation delay compensation scheme. In this system, a super real-time controller (Umekita et al., 1995) using parallel computing and a special programming language was used to perform all calculations within the required time step. Nakashima and Masaoka (1999) proposed separating the tasks of signal generation and response analysis to allow RTHS to be performed on commercially available processors. The response analysis task, including numerical integration, could be performed at a slow rate (e.g., $\Delta t = 10$ msec). In order to assure accurate velocity tracking, the signal generation task could be performed at a faster rate (e.g., $\delta t = 1$ msec). Many RTHS studies have been conducted since these pioneering studies. Focus is typically placed on developing or applying new numerical integration schemes, achieving more accurate real-time actuator control, developing new hardware and software for improved computational power, or combining RTHS with other hybrid simulation techniques such as geographically distributed testing.
2.3 Equations of Motion

For hybrid simulation, the equations of motion, which are second-order ordinary differential equations, must be expressed in discrete time form (Eqns. 2.1 and 2.2). During numerical integration, equilibrium must be satisfied at each time step.

\[ t_{i+1} = t_j + \Delta t \]  

\[ \mathbf{M}^N \ddot{\mathbf{x}}_{i+1} + \mathbf{C}^N \dot{\mathbf{x}}_{i+1} + \mathbf{R}^N_{i+1} + \mathbf{R}^E_{i+1} (\mathbf{x}_{i+1}, \dot{\mathbf{x}}_{i+1}, \ddot{\mathbf{x}}_{i+1}) = \mathbf{F}^N_{i+1} \]

where \( \mathbf{M}^N \) is the mass matrix of the numerical substructure, \( \mathbf{C}^N \) is the linear damping matrix of the numerical substructure, \( \mathbf{R}^N \) is the restoring force vector of the numerical substructure, \( \mathbf{R}^E \) is the restoring force vector of the experimental substructure, \( \mathbf{F}^N \) is the vector of excitation forces, \( \mathbf{x}_{i+1}, \dot{\mathbf{x}}_{i+1}, \text{ and } \ddot{\mathbf{x}}_{i+1} \) vectors of displacement, velocity, and acceleration at time \( t_{i+1} \), and \( \Delta t \) is the numerical integration time step. Note that the restoring force of the experimental substructure naturally includes contributions from static, damping, and inertial forces. When damping and inertial forces in the experimental substructure are negligible, then the physical specimen can be loaded at a slow rate without compromising accuracy.

2.4 Numerical Integration

Numerical integration is required to solve the equations of motion representing the complete behavior of the structure in Eqn. (2.2). Numerical integration schemes typically fall into two broad categories: explicit and implicit. Schemes solely based on previous and current time steps to determine future responses are referred to as explicit. Explicit schemes are preferred for real-time hybrid testing as they are generally less computationally intensive and require no iteration on the solution. The main drawback is that explicit schemes are often conditionally stable, placing limits on the maximum natural frequency of the structure or the maximum integration time step size. Schemes that make use of future time steps to determine future responses are
classified as implicit. These schemes are generally unconditionally stable, regardless of the time step chosen. However, iteration is required to converge on the solution which leads to large computational demands and potential spurious excitation of the physical substructure.

Numerical integration schemes can also be divided into the class of problem that they are designed to solve. Equations of motion for civil engineering applications are second-order ordinary differential equations and schemes have been designed specifically for such problems. These second-order ordinary differential equations could also be expanded into a system of first-order ordinary differential equations, opening the door to even more schemes. First-order approaches are attractive in that they can be directly applied to state-space formulations which are popular in modern dynamics and control theory.

The numerical integration schemes presented in the remainder of this section include algorithms either designed for or successfully used in RTHS.

2.4.1 Central Difference Method

The central difference method (CDM) is one of the most popular numerical integration schemes for RTHS (Nakashima et al., 1992; Shing et al., 1996; Darby 1999; Horiuchi et. al., 1999; Nakashima and Masaoka 1999; Horiuchi and Konno 2001; Wu et al., 2005; Carrion and Spencer, 2007; Phillips and Spencer, 2011). With this method, the velocity and acceleration are calculated using the following central difference equations:

\[
\dot{x}_i = \frac{x_{i+1} + x_{i-1}}{2\Delta t} \hspace{1cm} (2.3)
\]

\[
\ddot{x}_i = \frac{x_{i+1} - 2x_i + x_{i-1}}{\Delta t^2} \hspace{1cm} (2.4)
\]

The CDM is computationally efficient, extensively used in research, and easy to implement. The main benefit of the CDM is that it is an explicit scheme in displacement. The
displacement at time step \( t_{i+1} \) (to be imposed on the physical specimen) can be directly calculated by substituting Eqns. (2.3) and (2.4) into (2.2).

\[
x_{i+1} = \left[ \frac{1}{\Delta t^2} M^N + \frac{1}{2 \Delta t} C^N \right]^{-1} \left[ \frac{2}{\Delta t^2} \left( M^N \dot{x}_i - \left( \frac{1}{\Delta t^2} M^N - \frac{1}{2 \Delta t} C^N \right) x_{i-1} - R_i^N - R_i^E + F_i^N \right) \right]
\] (2.5)

One drawback of the CDM is that it is conditionally stable, with the stability criterion is given as \( \omega \Delta t \leq 2 \), where \( \omega \) is the highest natural frequency of the system (in radians/sec). Additionally, the velocity cannot be explicitly calculated (although it is not generally needed).

### 2.4.2 Central Difference Method – Real-Time Substructure Testing

Wu et al. (2005) proposed a modification to the CDM such that it is explicit in velocity, identified as the central difference method – real-time substructure testing (CDM–RST). The velocity is calculated using a backward difference as in Nakashima et al. (1992):

\[
\dot{x}_{i+1} = \frac{x_{i+1} - x_i}{\Delta t}
\] (2.6)

With this constraint on the velocity, the stability of the CDM becomes dependent upon the damping of the physical substructure. Generally an explicit formulation of the velocity is not required for RTHS; if the displacement is generated quickly and accurately tracked, then the correct velocity trajectory will also be achieved.

### 2.4.3 Newmark-Beta Method

The Newmark-Beta method (Newmark, 1959) comprises a family of numerical integration schemes for second-order ordinary differential equations. The Newmark-Beta method can be either explicit or implicit, depending on the selection of parameters \( \beta \) and \( \gamma \). These parameters determine how the acceleration will vary over the time step \( \Delta t \).

\[
x_{i+1} = x_i + \Delta t \dot{x}_i + \Delta t^2 \left( \frac{1}{2} - \beta \right) \ddot{x}_i + \beta \ddot{x}_{i+1}
\] (2.7)
\[ \mathbf{x}_{i+1} = \mathbf{x}_i + \Delta t \left[ (1 - \gamma) \mathbf{\ddot{x}}_i + \gamma \mathbf{\dddot{x}}_{i+1} \right] \quad (2.8) \]

With \( \gamma = \frac{1}{2} \), the Newmark-Beta method is second order accurate and exhibits no numerical damping. If \( \gamma < \frac{1}{2} \), negative numerical damping is introduced while if \( \gamma > \frac{1}{2} \), positive numerical damping is introduced. In these cases, the accuracy drops to first order. Adding numerical damping to the structure offers some advantages. Typically, accurately representing the higher natural frequencies of a structure is difficult. At the same time, these higher natural frequencies are sensitive to experimental errors (Mahin and Shing, 1985). In such cases, adding numerical damping can reduce spurious excitation of higher modes. This section discusses some of the RTHS integration schemes that can be created using the Newmark-Beta method.

**Newmark Explicit Scheme**

By selecting \( \beta = 0 \) from the Newmark-Beta method, the method becomes explicit in displacement and is thus referred to as the Newmark explicit scheme. When further selecting \( \gamma = \frac{1}{2} \), the Newmark explicit scheme becomes numerically equivalent to the CDM and thus possesses the same order of accuracy and stability criteria. However, the Newmark explicit scheme exhibits some advantages over the CDM. Shing and Mahin (1983) demonstrate the Newmark explicit scheme to have better error-propagation than the CDM. Also, the initialization procedure is more straightforward than the CDM since no information is required before the first time step at \( t = 0 \). Applications of the Newmark explicit scheme for RTHS include Blakeborough et al. (2001) and Bonnet et al. (2008).

**Constant Average Acceleration Method**

The constant average acceleration (trapezoidal) method (CAAM) is achieved from the Newmark-Beta method by selecting \( \beta = \frac{1}{4} \) and \( \gamma = \frac{1}{2} \). This method is second order accurate and adds no numerical damping, similar to the CDM, while exhibiting unconditional stability.
However, this method is implicit, limiting its application to RTHS. Horiuchi et al. (2000) applied the constant average acceleration method to RTHS. In this research, iteration on the command was eliminated by using the signals at time $t_i$ rather than $t_{i+1}$ to generate the actuator commands. The added delay of $\Delta t$ was lumped into the overall delay of the system for delay compensation.

**Chang Algorithm**

Chang (2002) modified the Newmark-Beta method to assume the following relationships for displacement and velocity:

$$\dot{x}_{i+1} = \dot{x}_i + \frac{1}{2} \Delta t \ddot{x}_i + \frac{1}{2} \Delta t \dddot{x}_{i+1}$$  \hspace{1cm} (2.9)

$$x_{i+1} = x_i + \dot{x}_i \Delta t + \dot{x}_i \Delta t^2$$  \hspace{1cm} (2.10)

Similar to the CDM, the Chang algorithm is explicit in displacement. The parameters $\beta_1$ and $\beta_2$ are defined for a linear SDOF system as:

$$\beta_1 = \frac{4(1 + \xi \omega_n \Delta t)}{4 + 4 \xi \omega_n \Delta t + \omega_n^2 \Delta t^2}$$  \hspace{1cm} (2.11)

$$\beta_2 = \frac{2}{4 + 4 \xi \omega_n \Delta t + \omega_n^2 \Delta t^2}$$  \hspace{1cm} (2.12)

where $\xi$ is the damping ratio and $\omega_n$ is the natural frequency of the structure. The following parameters were proposed for linear MDOF structures:

$$\beta_1 = \left( I + \frac{1}{2} \Delta t CM^{-1} + \frac{1}{4} \Delta t^2 KM^{-1} \right)^{-1} \left( I + \frac{1}{2} \Delta t CM^{-1} \right)$$  \hspace{1cm} (2.13)

$$\beta_2 = \frac{1}{2} \left( I + \frac{1}{2} \Delta t CM^{-1} + \frac{1}{4} \Delta t^2 KM^{-1} \right)^{-1}$$  \hspace{1cm} (2.14)

where $M$ is the mass matrix of the structure, $C$ is the linear damping matrix of the structure, and $K$ is the linear stiffness of the structure. The Chang algorithm exhibits the same numerical
properties for a linear system as the CAAM. Thus, for a linear system the Chang algorithm does not introduce any numerical damping and is unconditionally stable. For a nonlinear system, the initial linear stiffness was proposed for the calculation of $\beta_1$ and $\beta_2$, although a complete study of accuracy and stability for nonlinear systems was not performed.

**CR Algorithm**

Cheng and Ricles (2008) proposed the CR algorithm based on a frequency domain analysis of numerical integration schemes, formulating an explicit scheme for both displacement and velocity.

$$\dot{x}_{i+1} = \dot{x}_i + \Delta t \alpha_1 \ddot{x}_i$$  \hspace{1cm} (2.15)

$$x_{i+1} = x_i + \Delta t \dot{x}_i + \Delta t^2 \alpha_2 \ddot{x}_i$$  \hspace{1cm} (2.16)

The parameters $\alpha_1$ and $\alpha_2$ are derived by examining the discrete time transfer function of a linear SDOF oscillator. By equating the SDOF oscillator transfer function to the discrete time representation of the CR algorithm in Eqns. (2.15) and (2.16), the parameters are determined to be the following:

$$\alpha_1 = \alpha_2 = \frac{4}{4 + 4 \xi \omega_n \Delta t + \omega_n^2 \Delta t^2}$$  \hspace{1cm} (2.17)

Similar results are obtained for linear MDOF structures:

$$\alpha_1 = \alpha_2 = 4\left(4M + 2\Delta t C + \Delta t^2 K\right)^{-1} M$$  \hspace{1cm} (2.18)

The CR Algorithm does not introduce any numerical damping and exhibits the same period elongation as the CAAM and the Chang algorithm. Modifications to Eqns. (2.17) and (2.18) have been proposed for nonlinear structures (Chen and Ricles, 2009b). With these modifications, the algorithm was shown to be unconditionally stable for softening behavior.
A modification to the Newmark method was proposed by Hibler et al. (1977) to allow for the introduction of numerical damping without reducing the order of accuracy. The parameter $\alpha$ is added to the equations of motion to control the numerical damping:

$$M^N \ddot{x}_{i+1} + (1 + \alpha)C^N \dot{x}_{i+1} - \alpha C^N \dot{x}_i + (1 + \alpha)\left[R^N_{i+1} + R^E_{i+1}\right] - \alpha \left[R^N_i + R^E_i\right] = (1 + \alpha)F^N_{i+1} - \alpha F^N_i$$

(2.19)

The equations for the Newmark-Beta method still apply, with parameters determined using the following equations:

$$\beta = \frac{1}{4}(1 - \alpha^2)$$

(2.20)

$$\gamma = \frac{1}{2}(1 - 2\alpha)$$

(2.21)

$$-\frac{1}{3} \leq \alpha \leq 0$$

(2.22)

With $\alpha = 0$, the HHT-$\alpha$ method introduces no numerical damping and is equivalent to the CAAM. The HHT-$\alpha$ method is implicit, unconditionally stable, and introduces numerical damping in the high modes while not affecting lower modes appreciably. The numerical damping increases with the square of the frequency. Jung et al. (2007) implemented this iterative numerical integration scheme in RTHS for a nonlinear structure using a modified Newton approach.

### 2.4.4 Equivalent Force Control

Equivalent force control (EFC) is an approach to implicit numerical integration where the iteration procedure is replaced by a tunable feedback loop (Wu et al., 2007). Using the CAAM, the equations of motion are written in the following form:
\[
K_{PD} x_{i+1} + R^N(x_{i+1}) + R^E(\dot{x}_{i+1}, \ddot{x}_{i+1}, x_{i+1}) = F_{EQ,i+1}
\]

(2.23)

where \(K_{PD}\) is the pseudodynamic stiffness and \(F_{EQ}\) is an equivalent applied force that contains the externally applied force and a pseudodynamic effect depending only on the previous time step. \(K_{PD}\) and \(F_{EQ}\) are given below:

\[
K_{PD} = \frac{4M^N}{\Delta t^2} + \frac{2C^N}{\Delta t}
\]

(2.24)

\[
F_{EQ,i+1} = F_{i+1}^N + M^N \ddot{x}_i + \left(\frac{4M^N}{\Delta t} + C^N\right) \dot{x}_i + \left(\frac{4M^N}{\Delta t^2} + \frac{2C^N}{\Delta t}\right) x_i
\]

(2.25)

Essentially, \(F_{EQ,i+1}\) can be computed explicitly at \(t_i\) using Eqn. (2.25). Then, over the time step from \(t_i\) to \(t_{i+1}\), \(F_{EQ,i+1}\) will be computed at a faster rate using Eqn. (2.23) while exciting the experimental substructure as to reduce the error between Eqn. (2.25) and (2.23). The calculation of Eqn. (2.23) is illustrated in Fig. 2.3.

A PID loop is used to reduce the error in the desired equivalent force \(F_{EQ,i+1}\), and the measured equivalent force \(F'_{EQ,i+1}\). This loop runs at a faster rate than \(\Delta t\) such that the error is small by the end of each time step. A transformation matrix is necessary to convert the PID control effort \(e_{EQ}\) to a displacement command \(u\). Wu et al. (2007) proposes the following matrix to convert from force to displacement:

\[
C_F = \left(K^N + K_{PD} + K^E\right)^{-1}
\]

(2.26)

where \(K^N\) and \(K^E\) are the initial linear stiffness matrices associated with the numerical and experimental substructures, respectively. In the case of a nonlinear test structure, the above approximation may be inaccurate. Therefore, nonlinear control algorithms may be developed in
place of the PID loop and $C_p$ approximation. Suggested approaches in Wu et al. (2007) include adaptive or sliding mode control; however neither approach were investigated in detail.

![Figure 2.3. Equivalent force control experimental loop](image)

At the end of each time step $t_{i+1}$, the displacement $x_{i+1}$ is needed to calculate $\dot{x}_{i+1}$ and $\ddot{x}_{i+1}$ using the CAAM. Rather than using the commanded displacement $u$ or measured displacement $y$ from Fig. 2.3, Wu et al. (2007) recommends to calculate $x_{i+1}$ from Eqn. (2.23) such that equilibrium is satisfied.

### 2.4.5 Integral Form of Equations of Motion

Chang et al. (1998) proposed using an integral form of the equations of motion for hybrid simulation. The study considered an integral form of the Newmark explicit method. The stability, accuracy, period elongation, and other basic properties were found identical between the original equations of motion and integral form. However, using the integral of externally measured forces adds a smoothing effect to the numerical integration scheme, reducing noise and allowing large time steps to be accommodated. Darby et al. (1999) and Blakeborough et al. (2001) make use of integral forms of the equations of motion for RTHS using first-order numerical integration schemes.
2.4.6 Runge-Kutta

One of the most widely used numerical integration schemes outside of hybrid simulation is the Runge-Kutta scheme, developed to solve first-order ordinary differential equations. Runge-Kutta schemes can be either explicit or implicit and can have a high order of accuracy. The first-order equations of motion are written for an initial-value problem as below:

\[ \dot{x} = f(t, x) \quad (2.27) \]
\[ x(t_0) = x_0 \quad (2.28) \]

where \( x \) is a vector of the states of the system, \( \dot{x} \) is the time derivative of the states, and \( x_0 \) is a vector of initial conditions. The solution at time \( t_{i+1} \) is given as the solution at time \( t_i \) plus a linear combination of intermediate stages \( k_s \) that approximate Eqn. (2.27).

\[ x_{i+1} = x_i + \Delta t \sum_{n=1}^{\#s} b_n k_n \quad (2.29) \]

One of the most common Runge-Kutta schemes is a fourth-order explicit scheme. Equation (2.29) can be rewritten as Eqn. (2.30) which contains the weighted average of four slopes calculated at subintervals of \( \Delta t \) in Eqns. (2.31) through (2.34). This method is fourth-order accurate.

\[ x_{i+1} = x_i + \frac{1}{6} \Delta t \left( k_1 + 2 k_2 + 2 k_3 + k_4 \right) \quad (2.30) \]
\[ k_1 = f(t_i, x_i) \quad (2.31) \]
\[ k_2 = f\left(t_{i+\frac{1}{2}}, x_i + \frac{1}{2} \Delta t k_1\right) \quad (2.32) \]
\[ k_3 = f\left(t_{i+\frac{1}{2}}, x_i + \frac{1}{2} \Delta t k_2\right) \quad (2.33) \]
\[ k_4 = f\left(t_{i+1}, x_i + \Delta t k_3\right) \quad (2.34) \]
The fourth-order Runge-Kutta scheme has been used in RTHS by Carrion and Spencer (2007), Carrion et al. (2009), Lin and Christenson (2009), Jiang and Christenson (2010), Friedman et al. (2010), Phillips and Spencer (2010), Phillips et al. (2010), and Kim et al. (2011).

2.4.7 First-Order Hold Discretization

Darby et al. (2001) proposed using a first-order hold approximation of the continuous equations of motion in state-space form for RTHS. In order to step the method in time, the states at time \( t_{i+1} \) were calculated using a first-order extrapolation. Between numerical integration time steps, a quadratic interpolation scheme was proposed to generate smooth displacement commands. In this research, the equations of motion were expressed in modal coordinates, restricting nonlinearities to the physical specimen.

2.4.8 Tustin’s Method

Blakeborough et al. (2001) proposed a similar method to Darby et al. (2001) using Tustin’s method to develop the discrete time equations of motion. Tustin’s method is achieved by equating integration in the \( s \)-plane (continuous) to the trapezoidal rule between consecutive samples in the \( z \)-plane (discrete). Tustin’s method was used to create a discrete time-stepping form of the equations of motion; therefore, no extrapolation was needed to generate the states at time \( t_{i+1} \). Because modal coordinates were used to express the equations of motion, it was proposed that higher frequency modes could be directly reduced or eliminated if necessary.

2.5 Experimental Errors

Hybrid simulation is relatively sensitive to experimental error because the measured restoring force influences future control efforts. Experimental error can be introduced from flexibility in the reaction frame, misalignment of the specimen, force relaxation, ambient noise, instrumentation calibration issues, instrumentation precision limitations, instrumentation noise,
A/D and D/A conversion, and errors in actuator control. The net result of these errors is that displacement and force may be incorrectly applied and measured.

In hybrid simulation, experimental feedback errors accumulate through the numerical integration scheme due to displacement control errors and force measurement errors. These errors consist of both systematic and random components (Shing and Mahin, 1983). Among the systematic feedback errors, both overshooting the displacement or a lead in displacement can increase the apparent damping in the system (removing energy), while undershooting the displacement or a lag in the displacement will decrease the apparent damping (adding energy) (Shing and Mahin, 1983; Shing and Mahin, 1987). For MDOF structures, higher modes have relatively little contribution to the dynamic response of a structure; however, they are more sensitive to systematic errors than lower modes. If energy is added to these higher modes, the cumulative errors can grow indefinitely in a resonance-like fashion (Shing and Mahin, 1987). Random errors are typically low amplitude, high frequency, and zero mean. These errors, which are fed back to the numerical integration, can excite higher frequency modes, especially if they are underdamped. Researchers commonly add numerical damping at higher modes to reduce high frequency excitation (Hilbert et al., 1977; Shing and Mahin, 1984). The equations of motion can also be converted to integral form as in Chang (1998) to reduce noise.

The most significant experimental error in RTHS is poor phase tracking of the desired displacement. For a simple illustration, the total effect of both time delays and time lags will be approximated as a single time delay $T_d$. Horiuchi et al. (1996) demonstrated that for a linear-elastic SDOF system, a time delay $T_d$ introduces negative damping into the system equal to $-k^E T_d$, where $k^E$ is the stiffness of the specimen. Figure 2.4a illustrates the time delay between the desired displacement $r$ and the measured response $y$. Figure 2.4b shows that although the
specimen response will track the linear-elastic black line, if the force $R^E$ is consistently measured when $y$ is achieved yet associated with $r$, then a counter-clockwise hysteresis loop is perceived. This loop is equivalent to negative damping, which will introduce inaccuracies into the RTHS. Furthermore, in cases when the negative damping exceeds the structural damping, the experiment can become unstable. Negative damping can be especially problematic for steel frames which exhibit high stiffness and low structural damping.

![Figure 2.4. (a) Time delay (b) Effect of time delay on hysteresis](image)

2.6 Real-Time Actuator Control Strategies

Accurate actuator control is essential to the stability and accuracy of RTHS and has been the focus of numerous studies. Early approaches amount to time delay compensation, assuming a constant apparent time delay in the servo-hydraulic system and extrapolating the displacement after the time delay. Improvements were added to the time delay approach by including online prediction of the time delay. More recently, the servo-hydraulic system has been modeled as a dynamic system and low-order inverse models or lead compensators have been proposed for actuator control. Some actuator control strategies include feedback controllers to minimize the tracking error and add robustness. Alternatively, some approaches accept a small time delay/lag.
and instead adjustments make adjustments to the force measurements. Specific examples of these approaches to actuator control will be examined in this section.

The controllers presented are outer-loop controllers, built around the inner-loop control of a servo-controller which provides stability to the servo-hydraulic system and some degree of tracking control. Thus, the goal of the outer-loop controller is to improve upon the performance of the inner-loop control. Alternatively, the inner-loop could be removed and a single controller designed for high-fidelity control of the system, however this approach has not been the focus of research in RTHS to date.

### 2.6.1 Polynomial Extrapolation

Prior to the work of Horiuchi et al. (1996), RTHS did not consider delay compensation. These early studies were conducted on structures with low natural frequencies or large structural damping, and therefore the negative effects of delay were considered negligible. To make RTHS available to a wider range of structural systems, Horiuchi et al. (1996) proposed the polynomial extrapolation method for delay compensation. In this approach, previous and current desired displacements are fit with a polynomial, and the displacement after a constant estimated time delay $T_d$ is extrapolated. This extrapolated displacement is then sent to the servo-hydraulic system as the commanded displacement. A polynomial of any order may be employed, with higher-order polynomials leading to higher accuracy coupled with more computational effort and sensitivity to noise. The accuracy and stability of this method become an issue when the time delay $T_d$ is large compared to the smallest period of the structure $\omega$. For a SDOF system, the stability is related to the nondimensional parameter $\omega T_d$. This constraint can be problematic for lightly-damped, stiff, or MDOF structures, all of which would exhibit higher natural frequencies and thus lower periods.
2.6.2 Linear Acceleration

Horiuchi and Konno (2001) proposed another approach to delay compensation which makes use of higher-order derivatives for extrapolation. With the CDM, which was used for this study, only the desired displacement \( x_{i+1} \) is known explicitly. To circumvent this issue, rather than extrapolating over the interval \( T_d \) from the desired displacement, extrapolation is performed over the interval \( T_d + \Delta t \) using the known displacement, velocity, and acceleration of the current time step. In this method, the acceleration after \( T_d + \Delta t \) is linearly extrapolated. Then, assuming that the acceleration is linearly varying between the current and extrapolated accelerations, the predicted displacement after the delay \( T_d + \Delta t \) is calculated. This method was shown to exhibit improved stability over the polynomial extrapolation method.

2.6.3 Least Squares Extrapolation

With the polynomial extrapolation method, the extrapolation is based on \( N + 1 \) points for an \( N^{th} \) order polynomial. Wallace et al. (2005) relaxes these restrictions by fitting an \( N^{th} \) order polynomial to a greater number of data points using a least-squares best fit. Increasing the number of data points reduces the influence of noise on the extrapolation while making the compensation more computationally intensive (i.e., more data must be stored). With this method, the time delay \( T_d \) does not have to be an integer multiple of the numerical integration sampling period \( \Delta t \), relaxing another restriction of the polynomial extrapolation method.

2.6.4 Derivative Feedforward

Jung et al. (2007) proposed a modification to the servo-hydraulic inner-loop control for RTHS. A feedforward term proportional to the derivative of the commanded displacement is added to the existing PID loop. The feedforward term is tuned to an anticipated apparent time delay in the RTHS loop.
2.6.5 Discrete Feedforward Compensation

Jung et al. (2007) also proposed modifying the displacement command based on the anticipated tracking error. The tracking error for the next time step is assumed to be approximately the same as the tracking error for the current time step. The current error is multiplied by a proportional gain and added to the command for the next time step to correct for the anticipated error.

2.6.6 Lead Compensator

Zhao et al. (2003) proposed using a phase-lead compensator network to eliminate the effects of amplitude and phase error. A single-pole single-zero lead compensator is added to the command signal to compensate for the frequency-dependent amplitude response of the servo-hydraulic system. This compensator also reduces some of the time delay in the RTHS loop. A second single-pole single-zero lead compensator is added to the measured restoring force. The goal of this second compensator is to remove any remaining time delay in the loop. Jung et al. (2007) implemented this idea in RTHS using a single phase-lead compensator for feedforward compensation.

2.6.7 Force Correction Methods

In RTHS, focus can also be placed on adjusting the force measurement to compensate for any time delays and time lags, in place of or in addition to displacement compensation schemes. Extrapolation techniques are difficult to employ on force measurements since they are generally too noisy for accurate extrapolation over a large time delay. Ahmadizadeh et al. (2008) proposed fitting the latest force and displacement measurements each to their own second order polynomials. The time when the displacement polynomial will cross the desired displacement is calculated using the quadratic formula. After choosing the appropriate root of the quadratic formula (the closest root to the current time), the force at that same time is calculate using the
force polynomial. The measured force and the desired displacement are thus aligned when they are expected to occur without having to explicitly calculate a time delay. With this approach, the quadratic formula could result in a complex time root, leading to a complex force. If the imaginary component is small, then the absolute value of the complex force was shown to provide a good estimate.

2.6.8 CR Inverse

Chen and Ricles (2008) represented the actuator time delay $T_d$ at a multiple $\alpha$ of the time step $\Delta t$. With this simplification, a discrete time relationship was established using the $\alpha$ parameter to relate the commanded displacement to the measured displacement. By taking the $z$-transform, a discrete time transfer function was created, the inverse of which was used to provide feedforward compensation. The transfer function contains one pole and one zero, both stable, therefore an inverse is easily obtained.

2.6.9 Model-Based Inverse

Carrion and Spencer (2007) proposed a model-based control method to account directly for the frequency-dependent dynamics (both amplitude and phase) of the servo-hydraulic system over a broad frequency range. Accurate transfer function models of the servo-hydraulic system were developed based on experimentally collected transfer functions and inverses of these models were used for feedforward control. A low-pass filter was added to the model inverse to create a proper system for implementation.

2.6.10 Scheduling Control

Carrion and Spencer (2007) also proposed a scheduling control method to account for changes in the specimen conditions affecting the dynamics of the servo-hydraulic system. Feedforward controllers were developed for the two extremes of expected specimen conditions. The transition
between the two feedforward controllers was determined using a bumpless transfer. The bumpless transfer method has merits for a specimen with behavior that is controlled by the user (e.g., the input current to MR dampers), however is not generally applicable (e.g., degrading structures).

### 2.6.11 Adaptive Schemes

In the case that a single time delay $T_d$ is used to approximate the delay across all frequencies, adaptive approaches that estimate the delay online can be used. These approaches require learning gains which must be tuned to achieve good convergence and avoid instabilities.

One of the first attempts to assess the time delay online was proposed by Darby et al. (2002). Darby demonstrated that the delay depends on the stiffness of the specimen. Thus, specimens that undergo damage during an experiment may require an adaptive compensation scheme. In this approach, a best guess of the initial time delay is used for compensation. Then, based on whether or not measured signal is leading or lagging the desired signal, the delay estimate is updated. This delay estimation technique involves determining two learning gains before the test begins. These gains control a balance between the rate of convergence, overshoot, and oscillation of the delay estimation. Although a method for choosing these empirical parameters is presented, their calibration remains a drawback to the method.

Ahmadizadeh et al. (2008) proposed an improvement to online delay estimation using the slopes of the desired and measured signals, demonstrating faster convergence and reduction in oscillations in the estimated delay. However, in this approach, the presence of the division operator in the slope calculation is problematic. If the denominator quantity becomes very small, the time delay estimation can jump considerably. This method also contains a learning gain which must be calibrated prior to testing.
Cheng and Ricles (2010) proposed using the tracking indicator (Mercan, 2007) to adjust the initial estimate of actuator delay. The tracking indicator is a stable measure of whether or not one signal is leading or lagging another, making it suitable for RTHS applications. Proportional-integral (PI) control is used to translate the tracking indicator quantity into an adjustment of the estimated time delay. The PI gains must be calibrated prior to testing.

### 2.6.12 Feedback Control

Real-time actuator control schemes can be augmented by feedback control to further minimize tracking error between the desired and measured signals. Carrion and Spencer (2007) and Cheng and Ricles (2009c) added a proportional feedback gain to the main feedforward control schemes to increase robustness. However, caution must be exercised, as high feedback gains for outer-loop control can also lead to system instabilities in RTHS (Carrion and Spencer, 2007). Feedback controllers proposed thus far employ a proportional gain, but do not take advantage of the known dynamics of the system (i.e., are not model-based).

### 2.6.13 Minimal Controller Synthesis – Modified Demand (MCSmd) Outer-Loop Scheme

Lim et al. (2007) proposed a model-following compensation scheme for RTHS. In this approach, the servo-hydraulic controller and actuator are treated together as a transfer system. This transfer system is not determined directly; rather a reference system with desired closed-loop performance is defined by the user. The outer-loop controller is comprised of inverse of this reference system in conjunction with an adaptive control scheme to assure that the transfer system tracks the reference system. The performance was found extremely sensitive to the initial selection of the adaptive gain parameters. A fixed-gain study was also conducted to provide guidance on initial gain selection. Bonnet et al. (2007) used the MCSmd scheme in multiple RTHS studies. The reference model in this study was created by performing system
identification on the transfer system and fitting to a first order transfer function model. This study also adopted a multi-tasking strategy whereby the outer-loop compensation was performed at a faster rate than the numerical integration.

### 2.7 Control-Structure Interaction

When mechanical actuators are used to excite a specimen, a strong dynamic coupling is usually present between the actuator and specimen, identified as control-structure interaction (CSI). This phenomenon was observed and explained by Dyke et al. (1995). Prior to this study, many researchers neglected CSI in experimental testing. This oversight was acceptable for slow-speed tests including conventional hybrid simulation, but unacceptable for the emerging framework of RTHS.

When specimens undergo changes in behavior (e.g., through damage), the dynamics of the actuator will change through CSI. Actuator control schemes tuned to one specimen condition may no longer be effective. CSI has been well studied for single-actuator systems, and RTHS actuator control approaches considering specimen dependency through CSI have been proposed (Carrion and Spencer, 2007; Carrion et al., 2009; Phillips and Spencer, 2011). However, as RTHS is being used for more complicated tests, control of multiple actuators will be required. For multi-actuator systems, CSI leads to a complex actuator control challenge. When multiple actuators are connected to the same specimen, the dynamics of all of the actuators become coupled to each other through the specimen.

### 2.8 Multi-Actuator Systems

To date, most studies in RTHS have focused on single-actuator systems, overlooking the anticipated challenges of actuator coupling. When the multiple actuators are not physically coupled through the specimen, their dynamics can be independently compensated. Chen and
Ricles (2011) investigated a structural system with two physically isolated MR dampers connected through the numerical substructure as part of the same lateral resisting system. Independently developed actuator controllers proved adequate for RTHS. In Kim et al. (2012), actuator controllers for two geographically distributed MR dampers were independently developed and successfully applied. In these studies, although the dynamics of the isolated specimens are tied together through the numerical substructure, there is no physical coupling in place.

A few tests with physically coupled actuators have been investigated, however in each case the coupling of the actuators was noted but not accounted for in the actuator control. Jung et al. (2007) investigated both a SDOF and a two-degree-of-freedom (2DOF) shear frame with the stiffness modeled by a physical column in each case. The column was cantilevered with one actuator attached in the SDOF case and two actuators attached in the 2DOF case. Relative to the SDOF system, the 2DOF system exhibited larger experimental error when compared to numerical simulation. The authors attribute the difference to a synchronization error between the actuators due to differences in load and servo-valve capacities. Both discrete feedforward compensation and lead compensation were explored for actuator control in this study.

Bonnet et al. (2007) created a modular system of inline mass, damping, and stiffness elements. Two boundary conditions for the physical substructure were explored: (a) actuator at one end and fixed at the other and (b) identical actuators at each end. Noticeably larger control errors were observed for the two-actuator system when compared to the single-actuator system. To further investigate the phenomenon of actuator coupling, the stiffness of the physical specimen was increased. With the stiffer specimen, stable tests were only achievable when the actuators moved in synch. The MSCmd outer-loop control scheme was used in this study.
Early work in multi-actuator control illustrates the need for the consideration of physical actuator coupling in controller development. A more detailed investigation of actuator coupling is required such that RTHS can be applied to a broader class of structures.

2.9 Shake Table Control

Current advances in shake table control will be explored to investigate the potential for applying RTHS actuator control strategies to shake table testing. Shake table control has many of the same challenges of actuator control for RTHS including tracking of a desired trajectory in real-time, nonlinearities in actuator performance, and specimen dependent dynamics through CSI.

Shake tables provide a direct means by which to impart a desired ground motion on a physical specimen. Shake table testing is unique in that the desired trajectory is an acceleration signal; however, for stability, servo-hydraulic actuators still operate in displacement feedback. The acceleration signal is known a priori and is not affected by the response of the structure as in hybrid simulation. Because the entire structure is mounted on the table, dynamic effects can be directly captured. Shake tables are inherently nonlinear devices due to nonlinearities in actuator behavior, friction in the table, and CSI. Due to the nonlinearities, it is difficult to reproduce a desired earthquake record over a wide range of frequencies.

As with RTHS, many shake table controllers are outer-loop controllers built around an inner-loop displacement feedback controller. The most basic approach to achieving the desired acceleration record is to first integrate twice to determine a compatible displacement record. Simova (1980) presents this offline method, whereby the resulting displacement record is tracked by the shake table using displacement feedback. The following shake table control methods provide improvements upon this simple approach.
2.9.1 Transfer Function Iteration

Fletcher (1990) presents the transfer function iteration method used by many commercial shake tables, later applied to a small-scale shake table in Spencer and Yang (1998). This approach is based on a linearized model of the shake table commands to measured acceleration. An inverse of this model is used to generate a command signal history from the acceleration record, taking into account the modeled table behavior. However, nonlinearities lead to error between desired and measured accelerations. These errors are used offline to iteratively modify the command signal. A similar approach using iterative learning control is proposed by Daley et al. (2004) whereby the reference trajectory is iteratively modified to reduce the tracking error.

2.9.2 Feedforward-Feedback Approaches

Newell et al. (1995) proposed an offline optimization approach to develop the reference signal to track a desired acceleration based on a time-varying nonlinear model of the shake table dynamics. The nonlinear model is also linearized about the reference signal to create a Kalman filter for feedback control. Displacement and acceleration measurements are compared to reference displacements and accelerations to calculate a differential reference signal correction. The use of acceleration feedback can be problematic because acceleration measurements are sensitive at high frequencies; high frequency behavior is apparent in the measured accelerations. At the same time, linearization of the time-varying nonlinear model is computationally burdensome.

Another feedforward-feedback approach to shake table control was proposed by Kuehn et al. (1999) based on receding horizon control. In this approach, measurements of displacement and differential pressure were used to approximate the states of the system model. The approach was demonstrated to be superior to traditional LQR control in terms of phase performance.
Nakata (2010) proposed an acceleration trajectory tracking control in which a linearized model of the shake table is used as a feedforward controller, joined by a displacement feedback controller based on a compatible displacement record. In this approach commands are sent directly to the servo-valve (no inner-loop controller is used). Thus, the displacement feedback controller is necessary to provide stability to the shake table. An intentional delay is added to the feedback controller to ensure that the more aggressive control effort is provided by the feedforward controller and the feedback controller simply prevents excessive drift. This method requires the assembly of special equipment to bypass the inner-loop servo-controller (Nakata, 2011).

2.10 Summary

This chapter presented an overview of hybrid simulation focusing on extensions into real-time hybrid simulation. A review of numerical integration schemes and actuator control schemes are presented with a focus on real-time applications. Considerable room for improvement in the area of actuator control remains, starting with an accurate understanding of actuator dynamics. Moreover, most studies only consider single-actuator systems and those that do consider multi-actuator systems neglect actuator coupling. Also, actuator control strategies to date focus on displacement control and displacement tracking. Actuator control stands to benefit from multi-metric feedback to achieve accurate tracking of higher-order derivatives such as velocity and acceleration, which are important to capture rate-dependent behavior accurately. Advances made to actuator control for RTHS can be applied to other areas of experimental testing, including shake table control. Multi-metric feedback control, including acceleration feedback, has particular promise to improve the tracking of a desired acceleration.
CHAPTER 3  BACKGROUND

Some background in control theory is provided in this chapter to lay the groundwork for the development of advanced actuator control schemes for RTHS. A linearized model of the servo-hydraulic actuator system is also proposed as the plant to which the control theory will be applied.

3.1 Classic Control Theory

Classic control theory focuses on frequency domain approaches to examining stability, noise, bandwidth, and performance. Transfer functions are fundamental to classic control theory, providing a frequency domain representation of the input/output relationship of a linear time-invariant system. Transfer functions are typically expressed in the Laplace plane. The Laplace transform of a function $f(t)$ is defined as:

$$F(s) = L\{f(t)\} = \int_{-\infty}^{\infty} e^{-st} f(t) dt$$  \hspace{1cm} (3.1)

Where $L\{\}$ represents the Laplace transform and $s$ is a complex number in the $s$-plane.

Consider the dynamic single-input single-output (SISO) system in Fig. 3.1.

![Figure 3.1. Dynamic system](image)

The transfer function is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input as in:

$$G_{yu}(s) = \frac{Y(s)}{U(s)}$$  \hspace{1cm} (3.2)
where zero initial conditions are assumed. A transfer function written with a subscript \( yu \) indicates the input \( u \) to the output \( y \). This notation is helpful when there are multiple loops and multiple dynamic systems.

The transfer function can be expressed in pole-zero form or expanded to ratio of polynomials:

\[
G_{yu}(s) = K \prod_{i=1}^{m} \frac{(s-z_i)}{\prod_{i=1}^{n} (s-p_i)} = \frac{b_0 + b_1s + b_2s^2 \ldots + b_ms^m}{a_0 + a_1s + a_2s^2 \ldots + a_ns^n}
\]

(3.3)

where \( m \) is the number of zeros \( z \), \( n \) is the number of poles \( p \), \( K \) is the transfer function gain, and \( a_0 \) through \( a_n \) and \( b_0 \) through \( b_m \) are polynomial coefficients. The poles correspond to the locations on the \( s \)-plane where the transfer function becomes infinite while the zeros correspond to the locations on the \( s \)-plane where the transfer function is zero.

Controllers are designed to produce an input to a plant (dynamic process coupled with an actuator) that achieves a desired response from the plant. In the area of tracking control, this desired response is given by a reference signal. In the following discussion, \( P(s) \) represents the plant, \( C(s) \) represents the controller, \( r \) is the reference signal, \( u \) is the control input, \( d \) is an unknown disturbance, \( y \) is the output of the plant, and \( e \) is the error between the reference signal and the plant output. Both open-loop (Fig. 3.2) and closed-loop (Fig. 3.3) control will be examined using classic control theory for a SISO system.

The following criteria will be used to assess controller performance:

(a) Tracking: The error \( e \) between the reference signal \( r \) and measured response \( y \) should be minimized. The transfer function from the reference signal to the plant output \( G_{yr}(s) \) provides insight into tracking.
(b) Disturbance Rejection: The output of the plant $y$ should not be greatly affected by disturbances $d$ in the system. The transfer function from the disturbance to the plant output $G_{yd}(s)$ provides insight into disturbance rejection.

(c) Sensitivity: Good tracking of the reference signal should be achieved even if the plant model is not accurate or undergoes changes. The sensitivity of the transfer function $G_{yr}(s)$ to small variations in the plant is given by:

$$ S(s) = \frac{\delta G_{yr}(s)}{G_{yr}(s)} = \frac{\delta G_{yr}(s)P(s)}{\delta P(s)G_{yr}(s)} $$

(3.4)

Where $\delta G_{yr}(s)$ is the variation in $G_{yr}(s)$ caused by $\delta P(s)$, the variation in $P(s)$ (Phillips and Nagle, 1995).
3.1.1 Open-Loop Control

Open-loop control (Fig. 3.2) modifies the reference signal directly to produce a command to the plant. Such a controller can be designed with knowledge of the plant dynamics, such as through an identified plant model. The controller command does not depend on the response of the plant, which can lead to tracking problems.

(a) Tracking: In open-loop control, the open-loop transfer function from the reference signal to the plant output is given by the following:

\[ G_{yr}(s) = \frac{Y(s)}{R(s)} = C(s)P(s) \]  

(3.5)

Perfect tracking would be indicated by \( G_{yr}(s) = 1 \), meaning that the output is equal to the reference signal over all frequencies, which can be achieved by choosing \( C(s) = P(s)^{-1} \). However, plant inversion is not always straightforward or possible. At the same time, the modeling errors can lead to an inverse that will not cancel out the true plant dynamics. Thus, open-loop control provides the possibility for perfect tracking.

(b) Disturbance Rejection: The transfer function from the disturbance to the plant output is given by the following:

\[ G_{yd}(s) = \frac{Y(s)}{D(s)} = P(s) \]  

(3.6)

Open-loop control provides no means by which to reject disturbances, as \( C(s) \) does not appear in Eqn. (3.6).

(c) Robustness: The sensitivity function can be reduced to unity as in Eqn. (3.7), indicating that the open-loop system provides no robustness.

\[ S(s) = \frac{\delta G_{yr}(s)P(s)}{\delta P(s)G_{yr}(s)} = C(s) \frac{P(s)}{C(s)P(s)} = 1 \]  

(3.7)
3.1.2 Closed-Loop Control

Closed-loop control uses the output of the plant to adjust the command signal. In the closed-loop control representation in Fig. 3.3, the control effort depends on the error between the plant output and the reference signal.

(a) Tracking: In closed-loop control, the transfer function from the reference signal to the plant output is given by the following:

$$G_{yr}(s) = \frac{Y(s)}{R(s)} = \frac{C(s)P(s)}{1 + C(s)P(s)}$$

Since perfect tracking would be indicated by $G_{yr}(s) = 1$, one can choose $C(s) = K$ where $K$ is a very large constant. Such a controller would achieve good tracking across all frequencies. This controller is not always practical nor the best solution, however, the possibility for good tracking exists.

(b) Disturbance Rejection: The transfer function from the disturbance to the plant output is given by the following:

$$G_{yd}(s) = \frac{Y(s)}{D(s)} = \frac{P(s)}{1 + C(s)P(s)}$$

In this case, the transfer function $G_{yd}(s)$ should be as small as possible, indicating that the disturbance does not affect the output. If $C(s) = K$ where $K$ is a very large constant, then the transfer function becomes small. Thus, closed-loop control can provide good disturbance rejection.

(c) Robustness: The sensitivity function can be reduced to Eqn. (3.10). As with tracking control and disturbance rejection, if $C(s) = K$ where $K$ is a very large constant, a small sensitivity function is created. Therefore, closed-loop control can provide good robustness.
3.1.3 PID Control

Proportional-Integral-Derivative (PID) control is a very common approach to closed-loop control (Fig. 3.3) and generally favored when the dynamics of the plant are unknown. The PID controller contains three gains that can be tuned to achieve the desired closed-loop performance:

\[
S(s) = \frac{\delta G_{yr}(s)P(s)}{\delta P(s)G_{yr}(s)} = \frac{C(s)}{(1+C(s)P(s))^2} \cdot \frac{P(s)}{1+C(s)P(s)} = \frac{1}{1+C(s)P(s)}
\] (3.10)

or, in the Laplace domain:

\[
C(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_ds
\] (3.12)

Note that multiplying by \( s \) is equivalent to differentiation in the time domain while dividing by \( s \) is equivalent to integration in the time domain.

The proportional gain, \( K_p \), adds control effort proportional to the tracking error. This gain generally contributes most of the control effort. A high proportional gain improves the system response time, although if set too high the system may become unstable. The integral gain, \( K_i \), adds control effort proportional to the accumulated error. Integral control is most useful to eliminate steady-state errors; however, it can also introduce overshoot into the system. The derivative gain, \( K_d \), adds control effort proportional to the derivative of the error. Derivative control slows the transient response of the system, reduces overshoot, and improves system stability. However, derivative control is sensitive to noise and can lead to instabilities in the presence of such noise.
3.1.4 Lead and Lag Compensators

Many alternative compensator designs are available for closed-loop control. Two simple yet effective designs include lead and lag compensators, taking the form:

$$C(s) = K \frac{(s-z)}{(s-p)}$$  \hspace{1cm} (3.13)

These compensators are typically placed in the open-loop path of the closed-loop system as in Fig. 3.3. If $z < p$, then a lead compensator is created, adding positive phase to the open-loop system. In the closed loop system, lead compensators approximate the function of proportional-derivative (PD) control, decreasing rise time and overshoot. If $z > p$, then a lag compensator is created, adding negative phase to the open-loop system. In the closed-loop system, lag compensators approximate the function of proportional-integral (PI) control, reducing steady-state error (Franklin et al., 2006). In the case that the desired performance cannot be achieved with a single lead or lag compensator, multiple compensators can be cascaded. Alternatively, if the lead and lag compensators are placed in the feedback path, the compensators have the same effect on the closed-loop system poles, however the transient response to a reference input will change.

3.2 Modern Control Theory

Whereas classical control theory focuses on transfer function approaches, modern control theory is based on ordinary differential equations, creating a mathematical model of system dynamics in the time domain. Ordinary differential equations can be written in state-space form as a system of first-order differential equations. State-space models provide a convenient form to represent multi-input multi-output (MIMO) systems, non-zero initial conditions, as well as nonlinearities. If the dynamics are linear and time-invariant, the state-space model can be written as below for a SISO system.
\[
\dot{x} = Ax + Bu \\
y = Cx + Du
\]

where \(x\) is the state variable vector with initial condition \(x(0) = x_0\), \(\dot{x}\) is its time derivative vector of the states, \(u\) is the system input, \(y\) is the measured system output and \(A, B, C, \text{ and } D\) are the system, input, state output, and feedthrough matrices, respectively. For the following, it is assumed that the feedthrough matrix \(D = 0\), indicating that the input does not directly enter the output.

### 3.2.1 State Feedback

In state-space design, the closed-loop system will typically take the form of Fig. 3.4 assuming that all states are available. State feedback in this form will bring the derivatives of the states to zero, holding the system output steady against unknown disturbances. Such a controller is called a regulator.

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx \\
\end{align*}
\]

**Figure 3.4. State feedback**

By multiplying the states by a constant gain \(K\), the command to the plant \(u\) is determined:

\[
u = -Kx
\]

With Eqn. (3.16) as an input, the closed-loop state-space model can be written as:

\[
\dot{x} = (A - BK)x
\]
\[ y = Cx \]  \hspace{1cm} (3.18)

The poles of the closed loop system are the eigenvalues of the matrix \((A - BK)\). If and only if the closed-loop system is controllable, then the feedback gain \(K\) can be chosen to achieve any arbitrary poles in the closed-loop system and thus any arbitrary closed-loop response. A discussion on controllability can be found in Stengel (1986). The closed-loop poles can be selected to meet a design objective, such as rise time or transient response.

### 3.2.2 Linear Quadratic Regulator (LQR)

An effective and widely used method for determining the state feedback gain matrix \(K\) is to minimize an objective cost function. In the case of a linear system with a quadratic cost function, LQR design can be applied to create an optimal regulator design. When designed for an infinite time horizon (steady-state control), the quadratic function cost is given by:

\[
J_{\text{LQR}} = \int_{0}^{\infty} \left[ x^T Q_x x + u^T R_u u \right] dt
\]  \hspace{1cm} (3.19)

where \(J_{\text{LQR}}\) is the cost function, \(Q_x\) is the weighting matrix on the system outputs, and \(R_u\) is the weighting on the system input. The algebraic Riccati equation can be solved to obtain the symmetric, positive definite matrix \(P\):

\[
A^T P + PA - PB R_u^{-1} B^T P + Q_x = 0
\]  \hspace{1cm} (3.20)

The optimal feedback gain matrix \(K_{\text{LQR}}\) based on the control weights selected is given by:

\[
K_{\text{LQR}} = R_u^{-1} B^T P
\]  \hspace{1cm} (3.21)

### 3.2.3 Observers

State feedback is formulated assuming that all of the states are known. In practice, sensors are required to measure the states of a plant. Sensors can be expensive and sometimes difficult or impossible to install. At the same time, certain plant states might not have a physical
representation and are therefore unmeasurable. In order to place the closed-loop poles using state feedback, these states must be reconstructed from available data.

An observer (also called an estimator) can combine the measurable quantities $y$, known commands $u$, as well as the modeled plant dynamics to create estimates of the states. First, the observer dynamics are created to mimic the original plant, where $\hat{x}$ are the estimated states. If the initial states of the system are known and the plant is a perfect model of the physical system, then the states can be estimated exactly from the open-loop observer:

$$\dot{x} = Ax + Bu$$  \hspace{1cm} (3.22)

However, initial state conditions may not be known and the estimated states may begin to diverge from the actual states due to noise or disturbances in the system. Thus, a corrective term is added to the observer dynamics using the available measurements $y$:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$$  \hspace{1cm} (3.23)

where $L$ is the observer gain. This form of observer is called a Luenberger observer. The closed-loop observer and plant are shown together in Fig. 3.5.

![Figure 3.5. Observer](image)

The poles of the observer are the eigenvalues of the $(A - LC)$, which can be chosen arbitrarily for an observable system. A discussion on observability can be found in Stengel (1986).
3.2.4 Kalman Filter

The Kalman filter (Kalman, 1960; Kalman and Bucy, 1961) is an approach to optimal observer design. Aside from reconstructing the states of the system, a Kalman filter removes noise from the measurements based on the modeled plant dynamics and weighting parameters. Kalman filters do not add lag to the system as traditional noise filters do, making them attractive alternatives toward achieving clean measurement signals in real-time applications. The state-space model with noise can be expressed as:

\[
\dot{x} = Ax + Bu + Ew \tag{3.24}
\]

\[
y_v = Cx + v \tag{3.25}
\]

where \(w\) is the process noise (i.e., disturbance) assumed to be zero-mean Gaussian white noise with covariance \(Q_w(\tau) = E[w(t)w(t+\tau)]\) and \(v\) is the measurement noise assumed to be zero-mean Gaussian white noise with covariance \(R_v(\tau) = E[v(t)v(t+\tau)]\), \(E[\ ]\) is the expected value of the quantity in brackets, and \(y_v\) is the output measurement contaminated by noise. The noises \(w\) and \(v\) are considered independent such that \(E[v(t)w(t+\tau)] = 0\). Equations (3.24) and (3.25) are represented in Fig. 3.6.

![Plant with process and measurement noise](image)

**Figure 3.6. Plant with process and measurement noise**

The relative ratio between the noise covariances affect the performance of the Kalman filter. The error covariance of the estimated state is defined by:

\[
P_e = \lim_{t \to \infty} E\left[\{x(t) - \hat{x}(t)\}\{x(t) - \hat{x}(t)\}^T\right] \tag{3.26}
\]
The stationary error covariance matrix can be calculated from the algebraic Riccati equation:

$$AP_e + P_e A^T - P_e C^T R_v^{-1} C P_e - E Q_w E^T = 0$$  \hspace{1cm} (3.27)

The optimal observer gain $L_{Kal}$ that minimizes the error covariance based on the selected noise covariances can be calculated by:

$$L_{Kal} = P_e C^T R_v^{-1}$$  \hspace{1cm} (3.28)

Note that the Kalman filter design is independent of the input command $u$.

### 3.2.5 LQG Control

Feedback control, in the absence of complete state information or in the presence of uncertainty, may require a combination of controller and observer designs. In a general case, the designs of the two components cannot be separated. However, for a linear system with a quadratic cost function and certain assumptions on the process and measurement noise, the optimal controller and optimal observer can be designed independently.

The infinite-horizon Linear-Quadratic-Gaussian (LQG) control problem is defined for Eqns. (3.24) and (3.25) assuming the quadratic cost function given by:

$$J = \lim_{T \to \infty} \frac{1}{T} E \left[ \int_0^T \left[ x^T Q x + u R u \right] dt \right]$$  \hspace{1cm} (3.29)

The optimal controller can be designed as if there is no process or measurement noise, using the control law of Eqn. (3.19), or LQR control, in the case that the noise is a Gaussian white noise or Gaussian colored noise, as well as a few other specialized cases (Stengel, 1986). This property is called the certainty equivalence property, whereby the optimal control law can be derived without considering the uncertainty. Also, the Kalman filter design (optimal observer) of Eqn. (3.28) does not depend on the feedback control effort. Thus, LQG and Kalman filter designs can
be performed independently, illustrating the separation principle. The combined system is shown in Fig. 3.7.

The closed-loop system can be described by the states of the plant and observer as:

\[
\begin{bmatrix}
\dot{x} \\
\dot{\hat{x}}
\end{bmatrix} =
\begin{bmatrix}
A & -BK_{\text{LQG}} \\
L_{\text{Kal}}C & A - BK_{\text{LQG}} - L_{\text{Kal}}C
\end{bmatrix}
\begin{bmatrix}
x \\
\hat{x}
\end{bmatrix}
\]

(3.30)

The eigenvalues of a matrix remain unchanged if a similarity transform is performed.

\[
\text{eig}\left(\begin{bmatrix}
A & -BK_{\text{LQG}} \\
L_{\text{Kal}}C & A - BK_{\text{LQG}} - L_{\text{Kal}}C
\end{bmatrix}\right) = \text{eig}\left(\begin{bmatrix}
I & 0 \\
I & -I
\end{bmatrix}\begin{bmatrix}
A & -BK_{\text{LQG}} \\
L_{\text{Kal}}C & A - BK_{\text{LQG}} - L_{\text{Kal}}C
\end{bmatrix}\begin{bmatrix}
I & 0 \\
I & -I
\end{bmatrix}^{-1}\right) =
\text{eig}\left(\begin{bmatrix}
A - BK_{\text{LQG}} & BK_{\text{LQG}} \\
0 & A - L_{\text{Kal}}C
\end{bmatrix}\right)
\]

(3.31)

In the form of Eqn. (3.30) on the right-hand side, it is apparent that the eigenvalues of the complete closed loop system are the eigenvalues of \((A - BK_{\text{LQG}})\) and \((A - L_{\text{Kal}}C)\), the same eigenvalues obtained by independent design regulator and observer design. The separation principle guarantees that design of the state feedback gain and observer gain can be performed independently.

Figure 3.7. Combined controller and observer
3.2.6 Tracking Control using State-Space Design

The controller obtained through combined state feedback design and observer design is a regulator, maintaining a steady-state while achieving good disturbance rejection. A complete tracking controller requires good tracking of a reference input as well as disturbance rejection. To formulate a state-space tracking controller, a reference input must be introduced. For a simple example, the reference signal \( r \), control effort \( u \), and measurement \( y \) are assumed to be scalars. Terms proportional to the reference signal \( r \) will be added to the estimated states and control effort.

\[
\dot{x} = (A - BK - LC)\dot{x} + Ly + Mr \tag{3.32}
\]

\[
u = -K\dot{x} + nr \tag{3.33}
\]

where \( M \) is a vector, \( n \) is a scalar, and the process and measurement noise are not considered. This formulation is illustrated in Fig. 3.8.

![Figure 3.8. Combined controller and observer with reference input](image)

Because \( r \) is an external signal, the selections of \( M \) and \( n \) do not influence the characteristic equation of the combined controller and observer system in Eqn. (3.30). Their
selection only influences the transient response and thus tracking performance of the system. Three simple designs strategies are presented in Franklin et al. (2006) as follows:

(a) Zero-assignment estimator: \( M \) and \( n \) can be selected to place the zeros of the combined system at desired locations to achieve the desired transient response. The ratio of \( M/n \) influences the location of the zeros. The following two design strategies (b) and (c) are special cases of (a).

(b) Autonomous estimator: \( M \) and \( n \) can be selected such that the error in the state estimation is independent of \( r \). This is achieved by making sure that \( r \) does not appear in \( \dot{x} - \dot{\hat{x}} \), assuring good estimator performance in the presence of a reference signal. The result of such a design is that \( M = Bn \).

(c) Tracking-error estimator: \( M \) and \( n \) can be selected such that only the tracking error \( e = r - y \) is used for control. This approach is useful if a sensor can only measure error, as in some thermostats and some radar tracking systems. The result is that \( M = -L \) and \( n = 0 \).

Many other approaches to tracking control in state-space are available. In particular, an approach using regulator redesign is presented in Chapter 5 as part of the proposed model-based actuator control scheme. By transforming the tracking problem into a regulator problem, optimal regulator design can be applied.

### 3.3 Discrete Time

Discrete time equivalents, expressed in \( z \)-plane, can be expressed for all of the previously discussed control theory. In such cases, the transfer function is written as:

\[
G_{ju}(z) = \frac{Y(z)}{U(z)} = K \prod_{i=1}^{m} \frac{(z - z_i)}{(z - p_i)} = \frac{b_0 + b_1 z + b_2 z^2 + \ldots + b_m z^m}{a_0 + a_1 z + a_2 z^2 + \ldots + a_n z^n}
\]
where \( z \) is a complex number in the \( z \)-plane, not to be confused with the \( z \) commonly used to represent zeros. Or, in the time domain, a discrete state-space representation can be created:

\[
\begin{align*}
x_{i+1} &= Ax_i + Bu_i \quad (3.35) \\
y_i &= Cx_i + Du_i \quad (3.36)
\end{align*}
\]

where the discrete time state-space matrices are different from the continuous time state-space matrices. Continuous controller designs, including actuator controllers, must be expressed in discrete time for use in a digital signal processor (DSP) for RTHS. Discrete controller designs can be created either directly or by converting a continuous controller designs to discrete time. A few alternatives for creating discrete time representations of continuous systems are presented herein. MATLAB contains built-in algorithms to translate between continuous and discrete systems using most of the methods presented.

### 3.3.1 Zero-Order Hold

A zero-order hold (ZOH) approximation creates a discrete model by sampling and holding each input for one sample period. ZOH approximations are exact when the input to the system is a staircase and can be used for MIMO systems.

\[
u(t) = u_i \text{ for } i\Delta t \leq t < (i+1)\Delta t \quad (3.37)
\]

### 3.3.2 First-Order Hold

A first-order hold (FOH) approximation creates a discrete model through a similar sample and hold procedure as the ZOH conversion. In this case, the hold is a linear interpolation between samples. Either acausal (Eqn. 3.38) or causal (Eqn. 3.39) interpolation may be used.

\[
\begin{align*}
u(t) &= u_i + \frac{t - i \Delta t}{\Delta t} (u_{i+1} - u_i) \text{ for } i\Delta t \leq t < (i+1)\Delta t \quad (3.38) \\
u(t) &= u_i + \frac{t - i \Delta t}{\Delta t} (u_i - u_{i-1}) \text{ for } i\Delta t \leq t < (i+1)\Delta t \quad (3.39)
\end{align*}
\]
FOH approximations are exact when the input to the system is a piecewise-linear and can be used for MIMO systems. The discrete approximation $G(z)$ of a continuous system $G(s)$ is represented in Fig. 3.9 for both ZOH and FOH.

$$G(z)$$

**Figure 3.9. ZOH and FOH**

### 3.3.3 Pole-Zero Matching

A pole or zero in the $s$-plane at $\alpha$ is equivalent to a pole or zero in the $z$-plane at $e^{\alpha \Delta t}$. The following relationship can be established between the $s$-plane and the $z$-plane:

$$z = e^{\alpha \Delta t}$$

(3.40)

Pole-zero matching achieves a good match in the frequency domain between continuous and discrete systems, however it is only applicable to SISO systems.

### 3.3.4 Numerical Integration Equivalence

A family of discrete time equivalents is achieved by equating integration in both the $s$-plane and the $z$-plane and is applicable to MIMO systems. In the $s$-plane, integration is given by $1/s$. In the $z$-plane, three discrete time integration equivalents are presented in Fig. 3.10, including the forward rectangular rule, backward rectangular rule, and trapezoidal rule.

With the forward rectangular rule between consecutive samples, the following relationships between continuous and discrete time can be derived:

$$z = 1 + s \Delta t$$

(3.41)
With the backward rectangular rule between consecutive samples, the following relationships between continuous and discrete time can be derived:

\[
zs = \frac{z - 1}{\Delta t}
\]  
(3.42)

Tustin’s method, also known as the bilinear approximation, is achieved by using the trapezoidal rule between consecutive samples. With this method, the following relationships can be derived:

\[
z = \frac{1 + s\Delta t / 2}{1 - s\Delta t / 2}
\]  
(3.45)

\[
s = \frac{2}{\Delta t} \left( \frac{1 - z^{-1}}{1 + z^{-1}} \right)
\]  
(3.46)

Tustin’s method achieves a good match in the frequency domain between continuous and discrete systems, mapping the entire stable region of the s-plane to the stable region of the z-plane, although some frequency distortion occurs. Techniques are available to compensate for this frequency distortion through prewarping (Franklin et al., 2006).
3.4 **Servo-Hydraulic System Modeling**

The servo-hydraulic system is an assemblage of mechanical and electrical components used to excite a specimen, typically to a prescribed displacement (see Fig. 3.11). Individual component models can be assembled to create a dynamic model for the complete servo-hydraulic system. Components with nonlinear behavior will be represented by linear models with respect to an operating point such that the complete system model is also linear. The linear model will facilitate the use of frequency domain techniques including the Laplace transform as well as frequency domain based system identification. The transfer function model can also be expressed in state-space form to apply modern control theory approaches.

![Servo-Hydraulic System Diagram](image)

**Figure 3.11. Components of the servo-hydraulic system**

3.4.1 **Valve Flow**

The flow of oil through the actuator chambers can be approximated by the following linearization (Meritt, 1967):

\[ Q_L = K_q x_v - K_c p_L \]  \hspace{1cm} (3.47)
where $Q_L$ is the oil flow through the load, $K_q$ is the valve flow gain, $x_v$ is the position of the valve spool, $K_c$ is the valve flow-pressure gain, and $p_L$ is pressure drop across the load. The system in Eqn. (3.47) has been linearized about the origin where $Q_L = 0$, $x_v = 0$, and $p_L = 0$.

### 3.4.2 Actuator

The behavior of the actuator can be described by force equilibrium of the flow rate (Meritt, 1967):

$$Q_L = A\dot{x} + C_1 p_L + \frac{V_t}{4\beta_c} \dot{p}_L$$

(3.48)

where $C_1$ is the total leakage coefficient of the actuator piston, $V_t$ is the total volume of fluid under compression in both actuator chambers, $\beta_c$ is the effective bulk modulus of the system, and $A$ is the area of the actuator piston. Equation (3.48) can be rewritten in the Laplace domain as:

$$\frac{p_L(s)}{Q_L(s) - Ax(s)} = \frac{1}{C_1 + \frac{V_t}{4\beta_c} s}$$

(3.49)

The force generated by the actuator $f^E$ and thus imparted on the specimen is given by:

$$f^E = Ap_L$$

(3.50)

### 3.4.3 Specimen

The specimen is excited actuator, moving due to the applied force. The equation of motion of the specimen (SDOF) is given by:

$$m^E \ddot{x}^E + c^E \dot{x}^E + k^E x^E + F_s = f^E$$

(3.51)

where $m^E$, $c^E$, and $k^E$ represent the mass, damping, and stiffness values of the specimen and attachments (which may include the piston rod, load cell, clevis, etc.), $F_s$ represents the friction
in the piston rod, \( x^E \) represents the displacement of the specimen, and dots indicate differentiation with respect to time. Modern actuators often use low-friction seals such that the friction force can be assumed negligible. The equation of motion can be rewritten without friction as the following transfer function:

\[
G_{sf}(s) = \frac{x^E(s)}{f^E(s)} = \frac{1}{m^E s^2 + c^E s + k^E}.
\]  
(3.52)

An extension to MDOF specimens will be explored in Chapter 5.

### 3.4.4 Servo-Controller

The servo-controller is used to stabilize the inherently unstable hydraulic actuator. With displacement feedback, the error signal \( e_c \) is equal to the difference between the command \( u \) and measured displacement \( x^E \).

\[
e_c = u - x^E.
\]  
(3.53)

Servo-controllers often use PID control to eliminate the error. For real-time applications, proportional gain alone is generally adequate, avoiding the lag introduced by integral control and sensitivity to noise of derivative control. With a proportional controller, the servo-controller dynamics can be expressed as:

\[
i_c = K_p e_c.
\]  
(3.54)

where \( K_p \) is the proportional feedback gain of the servo-controller and \( i_c \) is the command signal to the servo-valve.

### 3.4.5 Servo-Valve

The servo-valve provides an interface between the electrical and mechanical components of the system. The servo-valve receives an electrical signal from the servo-controller which moves the position of the valve spool, controlling the flow of oil into the actuator. A constant gain may be
used to approximate the servo-valve dynamics over low-frequency ranges (Meritt, 1967; Dyke et al., 1995, Carrion 2007).

\[ x_v = k_v i_c \]  

where \( k_v \) is the valve gain. In the Laplace domain, Eqn. (3.55) can be written as:

\[ G_v(s) = \frac{x_v(s)}{i_c(s)} = k_v \]  

If a constant gain is inadequate over the frequency range of interest, a first order model may be used. This model includes a time lag, presented below:

\[ G_v(s) = \frac{k_v}{s + \tau_v} \]  

where \( \tau_v \) is the servo-valve time constant.

### 3.4.6 Complete Model

The components of the servo-hydraulic system can be combined into the block diagram model of Fig. 3.12.

**Figure 3.12. Block diagram model of the servo-hydraulic system**

With a constant gain representing the servo-valve dynamics, the servo-hydraulic system model can be represented by the following three-pole transfer function:
\[
G_{su}(s) = \frac{K_p \frac{K_q A}{K_c}}{\left( \frac{V_t}{4 \beta_c K_c} m^E \right) s^3 + \left( m^E + \frac{V_t}{4 \beta_c K_c} c^E \right) s^2 + \left( c^E + \frac{V_t}{4 \beta_c K_c} k^E + \frac{A^2}{K_c} \right) s + \left( k^E + K_p \frac{K_q A}{K_c} \right)}
\]

(3.58)

where \( K_q = K_q k_v \) is the servo-valve gain and \( K_c = K_c C_1 \) is the total flow-pressure coefficient.

With a first-order model for the servo-valve dynamics, the transfer function would contain four poles as in:

\[
G_{su}(s) = \frac{K_p \frac{K_q A}{K_c}}{D_4 s^4 + D_3 s^3 + D_2 s^2 + D_1 s + D_0}
\]

(3.59)

where

\[
D_4 = \left( \frac{V_t}{4 \beta_c K_c} m^E \tau_v \right)
\]

(3.60)

\[
D_3 = \left( \frac{V_t}{4 \beta_c K_c} m^E + m^E \tau_v + \frac{V_t}{4 \beta_c K_c} c^E \tau_v \right)
\]

(3.61)

\[
D_2 = \left( m^E + \frac{V_t}{4 \beta_c K_c} c^E + \frac{A^2}{K_c} \tau_v + c^E \tau_v + \frac{V_t}{4 \beta_c K_c} k^E \tau_v \right)
\]

(3.62)

\[
D_1 = \left( c^E + \frac{V_t}{4 \beta_c K_c} k^E + \frac{A^2}{K_c} + k^E \tau_v \right)
\]

(3.63)

\[
D_0 = \left( k^E + K_p \frac{K_q A}{K_c} \right)
\]

(3.64)

### 3.4.7 Equivalent Simplified Model

For simplicity, the model of Fig. 3.12 is equivalently rearranged as shown in Fig. 3.13. This simplification will facilitate the presentation of MIMO system control in Chapter 5. In this
figure, \( G_s, G_a, \) and \( G_{sf} \), represent the transfer functions of the servo-controller and servo-valve, actuator, and specimen, respectively.

![Servo-Hydraulic System Diagram](image)

**Figure 3.13. Servo-hydraulic system with CSI**

The individual block models are presented as:

\[
G_s(s) = \frac{k_s}{s - p_a} \quad (3.65)
\]

\[
G_{sf}(s) = \frac{1}{mE s^2 + cE s + kE} \quad (3.66)
\]

where

\[
k_s = \frac{4\beta c A}{V_t} \quad (3.67)
\]

\[
p_a = \frac{4\beta c}{V_t K_c} \quad (3.68)
\]

Furthermore, for a constant gain servo-valve model:

\[
G_s(s) = k_s \quad (3.69)
\]

where

\[
k_s = K_p K_q \quad (3.70)
\]
And for a first-order servo-valve model:

$$G_s(s) = \frac{k_s}{s - p_s} \quad (3.71)$$

where

$$k_s = \frac{K_p K_q}{\tau_v} \quad (3.72)$$

$$p_s = -\frac{1}{\tau_v} \quad (3.73)$$

The resulting simplified third-order model for the servo-hydraulic system is:

$$G_{xu}(s) = \frac{k_a k_s}{m^E s^3 + (-p_a m^E + c^E) s^2 + (-p_a c^E + A k_a + k^E) s + (-p_a k^E + k_a k_s)} \quad (3.74)$$

And the simplified fourth-order model for the servo-hydraulic system is:

$$G_{xu}(s) = \frac{k_s k_a}{D_4 s^4 + D_3 s^3 + D_2 s^2 + D_1 s + D_0} \quad (3.75)$$

where

$$D_4 = m^E \quad (3.76)$$

$$D_3 = \left(c^E - (p_s + p_a) m^E \right) \quad (3.77)$$

$$D_2 = \left(p_s p_a m^E - (p_s + p_a) c^E - k_a A + k^E \right) \quad (3.78)$$

$$D_1 = \left(p_s p_a c^E - (p_s + p_a) k^E - p_s k_a A \right) \quad (3.79)$$

$$D_0 = k_s k_a + p_s p_a k^E \quad (3.80)$$

### 3.5 Summary

This chapter provided background on control theory that is essential to the development of the proposed research. Both classic control theory and modern control theory are explored. In
addition, a total model of the servo-hydraulic system is derived from component models. This model will form the basis of the proposed model-based actuator control.
Semi-active devices such as MR dampers combine the desirable properties of both passive and active control systems. They have the ability to adapt to loading demands on the structure, as with active control systems; however, as with passive systems, they cannot inject energy into the structure, eliminating stability concerns. Also, in the event of power loss or controller damage, the devices function as passive energy dissipaters. With an MR damper, changes in the input current can be used to achieve forces predictably in semi-active control algorithms (Spencer et al., 1997).

RTHS allows for the dynamic performance evaluation of MR damper devices in a cost-effective and repeatable framework. An isolated large-scale MR damper can be considered as the experimental substructure while the rest of the structure is simulated numerically, reducing demands on laboratory space and equipment while still fully capturing the true nonlinear and current-dependent dynamics.

This chapter investigates the modeling and control of an MR damper, the specimen used throughout this research for RTHS. First, the dynamic performance of a large-scale 200 kN MR damper specimen is identified through a series of characterization tests. A numerical model of the MR damper is then developed from the experimental data to aid in offline simulation studies on semi-active control and to verify RTHS results. The observed dynamic behavior of the MR damper is used to develop advanced semi-active control strategies. Both structural control (e.g., semi-active control algorithms) and the accuracy of the RTHS framework (e.g., actuator control strategies) can be investigated using this specimen. The focus of this dissertation is placed on the RTHS framework with the MR damper seen as an example application which necessitates real-time experimental evaluation.
4.1 MR Damper Specimen

The specimen is a second-generation, large-scale 200 kN MR damper manufactured by the Lord Corporation (see Fig. 4.1) on loan from Professor Richard E. Christenson. The damper has a stroke of ±292 mm (±13 in) and can generate forces slightly higher than the nominal 200 kN. The damper has an accumulator charged to 5.17 MPa (750 psi) to compensate for the thermal expansion of the MR fluid (Christenson et al., 2008). The unique properties of MR dampers are derived from the internal MR fluid. In the presence of a magnetic field, the fluid changes from a linear viscous fluid to a semi-solid with controllable yield strength (Carlson and Jolly, 2000). This yield strength is dependent upon the strength of the magnetic field, while the maximum yield strength is determined by the composition of the MR fluid. The source of the magnetic field is an electromagnet located in the piston head, excited by an external current which can vary as required by a structural control algorithm.

Figure 4.1. Cross-section of the 200 kN MR damper
The current to the MR damper is controlled using a pulse-width modulator (PWM), which consists of an Advanced Motion Controls model PS2x300W unregulated power supply providing 80 VDC to an Advanced Motion Controls model 20a8 analog servo-drive, shown in Fig. 4.2.

![Figure 4.2. PWM for MR damper excitation current](image)

The analog servo-drive can measure the current in the closed-loop circuit for current feedback control, which is suitable for MR damper applications. The benefit of using a PWM is power efficiency and quick response time. An AC line filter is added to prevent noise from the PWM from leaking into the AC supply and contaminating nearby equipment. A ferrite suppression core is added to attenuate noise from the switching of the PWM.

4.2 Characterization Tests

4.2.1 Sine Wave Tests

A series of sine wave tests are used to characterize the frequency, amplitude, and current dependency of the MR damper dynamics. Each test is run for a displacement sine wave of fixed
frequency and amplitude with a fixed current to the MR damper circuit. The results are presented in Fig. 4.3 through Fig. 4.10, with each figure presenting multiple current levels for the same displacement sine wave. The MR damper responds to increases in current with corresponding increases in the restoring force during dynamic events. The magnitude of the restoring force changes dramatically, yet predictably with the input current. This characteristic makes MR dampers ideal for semi-active structural control. At the same time, the rate-dependent behavior requires real-time experimental evaluation, making MR dampers candidates for RTHS.

Figure 4.3. MR damper with 25.4 mm, 1.0 Hz sine wave

Figure 4.4. MR damper with 25.4 mm, 0.5 Hz sine wave
Figure 4.5. MR damper with 25.4 mm, 0.2 Hz sine wave

Figure 4.6. MR damper with 25.4 mm, 0.1 Hz sine wave

Figure 4.7. MR damper with 25.4 mm, 0.05 Hz sine wave
Figure 4.8. MR damper with 2.54 mm, 5.0 Hz sine wave

Figure 4.9. MR damper with 2.54 mm, 2.0 Hz sine wave

Figure 4.10. MR damper with 2.54 mm, 0.5 Hz sine wave
4.2.2 Force Rise Time Tests

The MR damper does not respond instantaneously to an input current. To investigate this dynamic current behavior, steps in input current are commanded over a constant velocity. The experimental results during the constant velocity (50 mm/sec) tests for steps in current from 0.0 Amps to higher current levels are shown in Fig. 4.11.

![Graph showing force rise time](image)

**Figure 4.11. Over-driven current during constant 50 mm/sec**

Results show that commanding a higher current to the MR damper results in a faster force rise time. This improved rise time can be included in semi-active controller designs (Yang et al., 2002). For example, to quickly achieve a force of 200 kN, a relatively high current (such as 8 Amps) could be applied until 200 kN is achieved (i.e., over-driving the circuit), then dropped to a lower current (such as 2.5 Amps) to maintain this force. Leaving a high current on for a long period of time has the potential of both overshooting the desired force and overheating the MR damper coils, so it is important to only use a high current level for overdriving the
circuit, not maintaining a force level. Note that there are two lags present in the system. The first lag is the amount of time it takes to realize the desired current in the MR damper circuit. The second lag, which is much larger, is due to the amount of time required to realize the corresponding restoring force. Both lags can be seen in Fig. 4.11.

4.2.3 Force Decay Time

If a high level of force exists in the MR damper as a result of an input current, the force can be is reduced by setting the current to zero. This approach is analogous to driving a boat at a high velocity and stopping the engine to reach zero velocity. However, a boat engine can also be set in reverse, and doing so would bring the velocity to zero much more quickly. Likewise, the current in the MR damper could be commanded in the opposite direction rather than set to zero (i.e. back-driving the circuit), which aids in dissipating any residual magnetic field that would cause a slow force decay time. The concept of back-driving the current is a complement to over-driving the current, which can also be included in semi-active controller designs (Yang et al., 2002). Deleteriously, if the back-driven current were left on for too long, the force would eventually start to rise again as a magnetic field is generated in the opposite direction. Therefore, some intelligence must be added to control the duration of the back-driven current. Figure 4.12 shows experimental results for applying back-driven current during a constant velocity (50 mm/sec) after a large force is achieved (170 kN) using 2.5 Amps.

A back-driven current of -7.5 Amps is applied beginning at 0.0 seconds for various durations (as identified in the figure’s legend), after which the current is returned to 0.0 Amps. Results show that using back-driven current reduces the force decay time considerably. However, as noted previously, leaving the back-driven current on for too long causes the force to rise again. Through trial and error, an optimal current command to bring the MR damper force to
zero as quickly as possible is determined, presented in Fig. 4.12. In this optimal case, the current is back-driven for 0.1 seconds and thereafter the current is set to decay exponentially to zero.

![Graph showing force and current over time](image)

**Figure 4.12. Back-driven current during constant 50 mm/sec**

### 4.3 High-Fidelity MR Damper Model

To assist in developing semi-active control algorithms, a high-fidelity MR damper model is identified. This model can also be used to assess the stability and feasibility of RTHS offline; however, the results of such simulations are restricted by the accuracy of the model.

The comprehensive characterization of the MR damper behavior is used to develop parameters for the phenomenological model originally proposed by Spencer *et al.* (1997) which is based on a Bouc-Wen hysteretic model. Other MR damper models have been proposed based on a Dahl friction model (Ikhouane and Dyke, 2007) and a hyperbolic tangent function (Kwok *et al.*, 2006; Bass and Christenson, 2007; Jiang and Christenson, 2011). These models boast fewer parameters with comparable results to the phenomenological model (Kwok *et al.*, 2006;
Ikhouane and Dyke, 2007). A comprehensive evaluation of MR damper models can be found in Jiang and Christenson (2011). Little effort was required to fit the parameters of the phenomenological model to the 200 kN MR damper characterization tests, therefore this well-established model was employed without difficulty.

Figure 4.13 illustrates the underlying mechanics of the model. Essentially the model outputs the restoring force $F$ for a given input displacement $x$ and velocity $\dot{x}$. Equating the forces on either side of the center rigid bar in Fig. 4.13 leads to the following relationship:

$$c_1 \ddot{y} = \alpha z + k_0 (x - y) + c_0 (\dot{x} - \dot{y}) \quad (4.1)$$

The force $\alpha z$ is determined by the evolutionary variable $z$ modeled by a Bouc-Wen hysteretic element (Baber and Wen, 1981).

$$\dot{z} = -\gamma |\dot{x} - \dot{y}|^n \dot{z}^{n-1} - \beta (\dot{x} - \dot{y})|z|^n + A(\dot{x} - \dot{y}) \quad (4.2)$$

The restoring force $F$ can be described by equating the forces on either side of the right-hand-side rigid bar in Fig. 4.13.

$$F = \alpha z + c_0 (\dot{x} - \dot{y}) + k_0 (x - y) + k_1 (x - x_0) \quad (4.3)$$
Because the MR damper piston rod is double-ended, no force offset is present under zero displacement; thus, the stiffness term $k_1$ can be set to zero. The other model parameters are fit using Simulink’s parameter estimation tool within MATLAB. To model the current-dependent behavior of the MR damper, Eqns. (4.4) through (4.8) are incorporated into the model, where $i_c$ is the input current. Parameters with the subscript “a” were fit to 0.0 Amp data while parameters with subscript “b” were fit to 2.5 Amp data. An exponential relationship between the extremes was found best to match the behavior intermediate levels of current, with the rate of change described by the parameters with subscript “c”. The optimized parameters are presented in Table 4.1.

\[
\alpha = \alpha_b + (\alpha_a - \alpha_b) \times \exp(-\alpha_c i_c)
\]

(4.4)

\[
c_0 = c_{0,b} + (c_{0,a} - c_{0,b}) \times \exp(-c_{0,c} i_c)
\]

(4.5)

\[
c_1 = c_{1,b} + (c_{1,a} - c_{1,b}) \times \exp(-c_{1,c} i_c)
\]

(4.6)

\[
\beta = \beta_b + (\beta_a - \beta_b) \times \exp(-\beta_c i_c)
\]

(4.7)

\[
\gamma = \gamma_b + (\gamma_a - \gamma_b) \times \exp(-\gamma_c i_c)
\]

(4.8)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{0,a}$</td>
<td>0.080 kN·sec/mm</td>
<td>$c_{1,a}$</td>
<td>3.0 kN·sec/mm</td>
<td>$\gamma_a$, $\beta_a$</td>
<td>0.050 mm$^2$</td>
</tr>
<tr>
<td>$c_{0,b}$</td>
<td>0.32 kN·sec/mm</td>
<td>$c_{1,b}$</td>
<td>15.0 kN·sec/mm</td>
<td>$\gamma_b$, $\beta_b$</td>
<td>0.0020 mm$^2$</td>
</tr>
<tr>
<td>$c_{0,c}$</td>
<td>1.5 A$^{-1}$</td>
<td>$c_{1,c}$</td>
<td>2.0 kN·sec/mm</td>
<td>$\gamma_c$, $\beta_c$</td>
<td>5.2 A$^{-1}$</td>
</tr>
<tr>
<td>$k_0$</td>
<td>0.0 kN/mm</td>
<td>$\alpha_a$</td>
<td>0.11 kN/mm</td>
<td>$A$</td>
<td>300</td>
</tr>
<tr>
<td>$k_1$</td>
<td>0.0 kN/mm</td>
<td>$\alpha_b$</td>
<td>0.55 kN/mm</td>
<td>$n$</td>
<td>2.0</td>
</tr>
<tr>
<td>$x_0$</td>
<td>0.0 mm</td>
<td>$\alpha_c$</td>
<td>1.0 A$^{-1}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In addition to the current-dependent behavior of the MR damper at static levels of current, changes in current introduce dynamics that must be modeled. These dynamics can be described as a time lag consisting of two components: (a) the lag between when a current is commanded to the PWM device and it is realized in the MR damper circuit, and (b) the lag between when the current is realized in the MR damper circuit and the corresponding restoring force is achieved in the MR damper. The aggregate effects of both lags are modeled by a first order transfer function (Eqn. 4.9) curve fit to match experimental data. The desired current $i_d$ is input to the transfer function and the effective resulting current $i_c$ is then input to the MR damper model. Note that a second-order low pass filter with cutoff frequency of 75 Hz is added in series with Eqn. (4.9) to avoid numerical stability issues sometimes found for quickly changing current in numerical simulation.

\[
i_c = \frac{(s + 9\pi)}{9(s + \pi)} i_d \tag{4.9}
\]

To verify the proposed MR damper model under semi-active conditions (varying current), a band-limited white noise (BLWN) with a 0 to 5 Hz bandwidth and a 2.78 mm RMS was input to the physical MR damper. At the same time, a current pulse between 0.0 and 2.5 Amps was input at a frequency of 2 Hz (50% duty cycle). The resulting displacement, velocity, and current histories from the experiment were then input into the numerical model. A comparison between the restoring force of the physical MR damper and model MR damper is presented in Fig. 4.14. Force time histories, as well as force-displacement and force-velocity hysteresis loops, are shown. The model is seen to work well even under varying specimen conditions, although some inaccuracies are apparent, especially as the current decreases. Differences between the model and physical specimen highlight the need for RTHS in that the
highly nonlinear behavior of the MR damper cannot be completely captured by current modeling approaches.

\[ \text{Figure 4.14. Performance of MR damper model under semi-active conditions} \]

### 4.4 Advanced Semi-Active Control Algorithms

Semi-active control algorithms for the MR damper use external sensors to determine the optimal current command to the MR damper. One of the most straightforward approaches to semi-active control is to create a two-stage controller. The first stage is used to determine the optimal control force through feedback control design. If an LQG controller is chosen for the first stage, then the optimal control force depends upon weighting of the structural response with the control effort. The second stage is used to turn the optimal control force into the control effort (current command) of the semi-active device.
4.4.1 Clipped-Optimal (CO) Control

A commonly referenced second-stage controller for MR damper devices is the clipped-optimal controller (Dyke et al., 1996). This semi-active controller is based on a clipped-optimal (CO) control algorithm, i.e.,

\[ i_d = i_{\text{max}} H \left\{ (f_d - f_m) f_m \right\} \]  

(4.10)

where \( i_d \) is the desired current (sent to MR damper), \( i_{\text{max}} \) is the maximum current (2.5 Amps in this case), \( f_d \) is the desired force, \( f_m \) is the measured force, and \( H \) is the Heaviside function. In short, when the desired force \( f_d \) is greater in magnitude than the measured force \( f_m \) and of the same sign, the maximum current \( i_{\text{max}} \) is sent to the damper. Thus, the magnitude of the force \( f_m \) will increase in an attempt to reach \( f_d \). In all other cases, the current is set to 0.0 Amps. The desired force \( f_d \) is determined using an LQG controller. The clipped-optimal controller logic is illustrated in Fig. 4.15.

![Figure 4.15. Graphical representation of clipped-optimal control algorithm](#)
4.4.2 Over-Driven Back-Driven Clipped-Optimal (ODBDCO) Control

In order to achieve quicker response from the clipped-optimal control algorithm, over-driven and back-driven concepts are incorporated. When the current is switched on by the clipped-optimal control algorithm, instead of jumping to the maximum current (2.5 Amps), a PI feedback loop is used. Through the feedback loop, more current is applied when the force error is greater (over a range of 0 to 7.5 Amps). To prevent the MR damper coils from overheating, the maximum allowable current is decreased (to 2.5 Amps) after a few seconds.

To achieve quicker force decay time, when the current is switched off (from a previous on-state), a negative current is applied. Three conditions are set, and if any of them are met then the back-driven current would be disengaged. (a) When, after engaged for a minimum amount of time to allow for the force to begin to decrease in magnitude, the force begins to increase in magnitude. This condition indicates that a magnetic field is being generated in the opposite direction, an undesirable outcome. (b) When the preceding commanded current is 0.0 Amps. Because back-driving the current can only reduce MR damper forces caused by current as opposed to velocity, it is important to ensure that such a current-induced force exists. (c) When the measured current is less than 25 kN in magnitude. Little benefit is gained by back-driving the current at these low levels of force. This condition would also turn the back-driven current off as the measured force enters this threshold from a higher magnitude, complimenting condition (a) without the time restriction. As with the CO controller, the desired force \( f_a \) is determined using an LQG controller.

4.5 Force Tracking Exercise

To evaluate the ability of the second-stage controllers to track a desired force, an arbitrary displacement and force history were generated numerically. Then, the displacement history was
imposed on the physical MR damper specimen. Assuming that the force history is the desired force, second-stage controllers were used to track the force history. Both the CO and ODBDCO controllers were evaluated. As can be seen in Fig. 4.16, the ODBDCO controller offers some slight improvement in force tracking, especially on the force decay at 4.0 seconds. Results for passive-off and passive-on are also presented as a rough envelope of the range of possible MR damper performance. The velocity is also shown as an indication of the direction of the MR damper motion. Only when the velocity and desired force are in the same direction can the desired force actually be achieved. For example, from 4.1 to 4.25 seconds, the desired force and velocity are in opposite directions and thus no current is applied.

Figure 4.16. Force tracking exercise
4.6 MR Damper Fluid Settling

Over time, the iron particles in the MR fluid may settle and cause inconsistent behavior. With an isolated MR damper in a laboratory setting, the MR fluid can be cycled repeatedly before testing. However, inside a lateral load resisting frame, it may be difficult to cycle the MR damper and maintain fluid consistency. The behavior of the MR damper after remaining idle is of interest for practical application, where the MR damper may sit idle for years before a significant dynamic event. The cyclic behavior of the MR damper after short idle times and under large displacements is shown in Fig. 4.17. The input displacement is a 25.4 mm, 1 Hz sine wave with a 1 cycle ramp time under an MR damper current of 2.5 Amps switched on at zero seconds. In the measured force, there is a small irregularity for the first full cycle of the 1.5 week idle test; however the MR damper behavior is relatively stable.

![Figure 4.17. MR damper behavior after short idle time](image)

Assessing the MR damper behavior for longer idle periods is difficult in that it ties up experimental resources. Over the course of this research, one window of six months idle time
was available, after which a test was conducted for a 7.62 mm, 1 Hz sine wave with for a 2.5 Amp current switched on at zero seconds. The lower displacement was to prevent the first cycle from mixing the fluid considerably. After the prescribed five cycle test (including the one cycle ramp), the original restoring force was not reached. Therefore, the cyclic test was repeated five times until consistent behavior was observed.

Longer idle times followed by smaller excitations have a noticeable effect on the performance of the MR damper. For seismic applications, a large impulse may mix up the fluid adequately, however further study of this phenomenon is required. Similarly, if a larger stroke sine wave was used, the mixing might have occurred sooner, thus reaching stable performance in a more reasonable time frame. The phenomenon is also dependent upon the MR fluid, as different additives influence setting time.

![Figure 4.18. MR damper behavior after six months idle time](image-url)
4.7 Summary

Characterization of the MR damper behavior led to the development of a numerical model to facilitate studies on semi-active control. The numerical model also serves to confirm the feasibility of RTHS before physical experiments are run as well as verify the results. However, the model cannot perfectly capture the true dynamics of the device and therefore RTHS is required to confirm any controller designs or insight gained from numerical simulation.

The characterization tests have also demonstrated the benefit of over-driving and back-driving the MR damper circuit to improve response time. Semi-active controllers have been presented to incorporate these ideas. Future studies on semi-active control utilizing the proposed RTHS framework are planned. Also, the observed phenomenon of MR damper fluid settling requires further investigation.
CHAPTER 5  MODEL-BASED ACTUATOR CONTROL FOR RTHS

In this chapter, a model-based actuator control strategy for RTHS is proposed. The equations of motion describing the structural system are partitioned for hybrid simulation and presented alongside the dynamics of the servo-hydraulic system. The goal of the proposed actuator control strategy is to eliminate the dynamics of the servo-hydraulic system such that compatibility is achieved between numerical and experimental components. Model-based control is developed around a linearized model of the servo-hydraulic system including feedforward and feedback links. The formulation is flexible to accommodate multi-actuator systems considering actuator coupling as well as multi-metric feedback control.

5.1 Problem Formulation

The equations of motion governing the dynamic response of a linear structure subjected to an input ground motion can be represented as follows:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\Gamma\ddot{\mathbf{x}}_{g}$$

(5.1)

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is stiffness matrix, $\Gamma$ is the mass influence matrix, $\ddot{\mathbf{x}}_{g}$ is the ground acceleration, $\mathbf{x}$ is the vector of displacements relative to the ground, and the dots represent differentiation with respect to time. A linear system is presented here for clarity of presentation, although the formulation can be adapted to include nonlinear systems. In hybrid simulation, the equations of motion can be separated into numerical and experimental components as indicated by superscripts “N” and “E”, respectively:

$$\left(\mathbf{M}^{N} + \mathbf{M}^{E}\right)\ddot{\mathbf{x}} + \left(\mathbf{C}^{N} + \mathbf{C}^{E}\right)\dot{\mathbf{x}} + \left(\mathbf{K}^{N} + \mathbf{K}^{E}\right)\mathbf{x} = -\left(\mathbf{M}^{N} + \mathbf{M}^{E}\right)\Gamma\ddot{\mathbf{x}}_{g}$$

(5.2)

Traditionally, numerical integration is performed on all DOFs using a single time integration scheme (Shing, 2008). The responses can thus be partitioned into DOFs that are purely numerical and DOFs that are at the interface between numerical and experimental
components, represented by superscripts “N” and “I”, respectively. With the total displacement vector arranged as \( \mathbf{x} = \{ \mathbf{x}^N \quad \mathbf{x}^I \}^T \), the following partitions of the mass, damping, and stiffness matrices are created:

\[
\begin{align*}
\mathbf{M}^N &= \begin{bmatrix} \mathbf{M}_{NN}^N & \mathbf{M}_{NI}^N \\ \mathbf{M}_{IN}^N & \mathbf{M}_{II}^N \end{bmatrix}, & \mathbf{C}^N &= \begin{bmatrix} \mathbf{C}_{NN}^N & \mathbf{C}_{NI}^N \\ \mathbf{C}_{IN}^N & \mathbf{C}_{II}^N \end{bmatrix}, & \mathbf{K}^N &= \begin{bmatrix} \mathbf{K}_{NN}^N & \mathbf{K}_{NI}^N \\ \mathbf{K}_{IN}^N & \mathbf{K}_{II}^N \end{bmatrix} \\
\mathbf{M}^E &= \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{M}_{II}^E \end{bmatrix}, & \mathbf{C}^E &= \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{C}_{II}^E \end{bmatrix}, & \mathbf{K}^E &= \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{K}_{II}^E \end{bmatrix}
\end{align*}
\] (5.3)

The restoring forces of the experimental component can be lumped into the vector \( \mathbf{R}^E \), which contains contributions from static, damping, and inertial forces:

\[
\mathbf{M}^N \ddot{\mathbf{x}}^N + \mathbf{C}^N \dot{\mathbf{x}}^N + \mathbf{K}^N \mathbf{x}^N + \mathbf{R}^E (\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}) = -\left( \mathbf{M}^N + \mathbf{M}^E \right) \Gamma \ddot{\mathbf{x}}^g 
\] (5.5)

The loop of action and reaction between numerical and experimental components during RTHS is illustrated in Fig. 5.1. From numerical integration of Eqn. (5.5), the structure is excited and displacements \( \mathbf{x} \) are calculated. To achieve compatibility between numerical and experimental components, the subset of \( \mathbf{x} \) corresponding to the interface DOFs \( \mathbf{x}^I \) are commanded to the experimental component using servo-hydraulic actuators. Inner-loop actuator control provides nominal tracking of the command vector \( \mathbf{u} \) to the servo-hydraulic system as measured by \( \mathbf{x}^E \), the vector of interface DOFs physically realized by the experimental component. Outer-loop actuator control is typically added to determine \( \mathbf{u} \) such that \( \mathbf{x}^E \) tracks \( \mathbf{x}^I \) very accurately and in real-time. The equations of motion of the experimental component can be represented as below, responding to an input force \( \mathbf{f}^E \) from the servo-hydraulic actuators.

\[
\mathbf{M}_{II}^E \ddot{\mathbf{x}}^E + \mathbf{C}_{II}^E \dot{\mathbf{x}}^E + \mathbf{K}_{II}^E \mathbf{x}^E = \mathbf{f}^E
\] (5.6)

Further compatibility of \( \dot{\mathbf{x}}^E = \dot{\mathbf{x}}^I \) and \( \ddot{\mathbf{x}}^E = \ddot{\mathbf{x}}^I \) is desired, but typically it is assumed that realizing the displacements \( \mathbf{x}^I \) accurately in real-time will ensure compatibility of higher-order
derivatives. The restoring forces of the specimen, as measured by the actuator load cells or external load cells, are returned to the numerical integration scheme as $R^E$. Lastly, through the natural velocity feedback loop, the dynamics of the specimen directly influence the dynamics of the servo-hydraulic system (Dyke et al., 1995).

![Figure 5.1. Multiple feedback loops in real-time hybrid simulation](image)

In model-based control, the outer-loop controller is created to cancel out the dynamics of the servo-hydraulic system, including specimen dependency through the natural velocity feedback loop (Carrion and Spencer, 2007). Consider the input-output transfer function model $G_{sx}(s)$ of the linearized servo-hydraulic system, including the actuator, servo-valve, servo-controller, and specimen (experimental component), as represented in Fig. 5.2. The dynamics of the servo-controller and servo-valve, actuator, and specimen have been condensed into transfer functions $G_s(s)$, $G_a(s)$, and $G_{sf}(s)$, respectively. The parameter $A$ represents the effective cross-sectional area of the actuator piston. The input-output transfer function can be written in the Laplace domain as:

$$G_{sx}(s) = \frac{X^E(s)}{U(s)} = \frac{G_s(s)G_a(s)G_{sf}(s)}{1 + G_s(s)G_a(s)G_{sf}(s) + AsG_a(s)G_{sf}(s)}$$

(5.7)
Figure 5.2 can represent both SISO and MIMO systems for single and multi-actuator systems, respectively.

![Servo-Hydraulic System Block Diagram](image)

**Figure 5.2. Servo-hydraulic system with CSI**

### 5.2 Regulator Redesign

The model-based control approach proposed herein is based on a linearized model of the servo-hydraulic system, as in Eqn. (5.7), which is represented in state-space form to facilitate modern control theory design (Phillips and Spencer, 2011):

\[
\dot{z} = Az + Bu 
\]

(5.8)

\[
x^E = Cz 
\]

(5.9)

where \( z \) is the state vector and \( A, B, \) and \( C \) are the system, input, and output matrices, respectively. The tracking error between the desired and measured displacement (or \( x^1 \) and \( x^E \), respectively) is given by:

\[
e = x^1 - x^E 
\]

(5.10)

The command \( u \) should be chosen such that the tracking error is minimized. If perfect tracking is achieved, an ideal state \( \bar{z} \) and an ideal input \( \bar{u} \) leading to an output \( \bar{x}^E \) must exist such that \( \bar{x}^E = x^1 \). The ideal system is described as:
Deviations of the state, control, and output from this ideal system with respect to the original system are defined as:

\[ \ddot{z} = A\tilde{z} + B\tilde{u} \quad (5.11) \]
\[ \dot{x}^E = C\tilde{z} = x^i \quad (5.12) \]

The tracking problem has now been redefined as a regulator problem about a setpoint (Lewis and Syrmos, 1995). The control law in Eqn. (5.14) can be rewritten in terms of the original system, which consists of a feedforward component \( \bar{u} = u_{FF} \) determined from the ideal system and a feedback component \( \tilde{u} = u_{FB} \) determined from the deviation system, i.e.,

\[ u = \bar{u} + \tilde{u} = u_{FF} + u_{FB} \quad (5.18) \]

The model-based controller incorporating both feedforward and feedback links is represented schematically in Fig. 5.3. The servo-hydraulic system of Fig. 5.2 has been condensed to show the details of the model-based controller, which acts as an outer-loop controller around the system.
5.3 Model-Based Controller

The development of the feedforward and feedback links for model-based control will be presented for the general case followed by examples for single and multi-actuator systems.

5.3.1 Feedforward Controller

The feedforward controller is designed to cancel the modeled dynamics of the servo-hydraulic system. Placed in series with the servo-hydraulic system, the inverse of the servo-hydraulic system model will serve as the feedforward controller.

\[
G_{FF}(s) = G_{su}(s)^{-1} = \frac{U_{FF}(s)}{X^I(s)} = I + G_{su}(s)^{-1} G_a(s)^{-1} G_s(s)^{-1} + A_s G_s(s)^{-1}
\] (5.19)

To illustrate implementation issues associated with model inversion, a SISO system will be examined. As shown in Chapter 3, the servo-hydraulic system can be represented by the following transfer function model:

\[
G_{su}(s) = \frac{b_0 + b_1 s + \cdots + b_m s^m}{a_0 + a_1 s + \cdots + a_n s^n}
\] (5.20)

For an accurate model of a servo-hydraulic system, the number of poles is generally larger than the number of zeros, as was the case for the model in Chapter 3. For the model in Eqn. (5.20), the feedforward controller can be expressed as the inverse, or:
\[ G_{FF}(s) = \frac{a_0 + a_1 s + \cdots + a_n s^n}{b_0 + b_1 s + \cdots + b_m s^m} \] (5.21)

### 5.3.2 Proper versus Improper Inverses

If Eqn. (5.21) is both proper and stable, meaning \( m \geq n \) and all poles are stable, then the feedforward controller can be implemented without modification. For use with a digital controller, a discrete time approximation, such as pole-zero matching or Tustin’s method may be used.

If \( m < n \), the feedforward controller is improper and requires modification. A low-pass filter could be added to Eqn. (5.21) to reduce the degree to which the inverse is improper. With enough poles, the low-pass filter could even create a proper system (Carrion and Spencer, 2007):

\[ G_{FF}(s) = \frac{a_0 + a_1 s + \cdots + a_n s^n}{b_0 + b_1 s + \cdots + b_m s^m} \cdot \frac{1}{c_0 + c_1 s + \cdots + c_{n-m} s^{n-m}} \] (5.22)

where \( c_0 \) through \( c_{n-m} \) are the coefficients of the low-pass filter. However, low-pass filters typically introduce unwanted dynamics into the feedforward controller. The approach proposed herein for accommodating the improper transfer function is to make use of higher-order derivatives which are available from numerical integration during RTHS. The improper feedforward model of Eqn. (5.21) can be separated into proper and improper terms:

\[ G_{FF}(s) = \frac{a_0 + a_1 s + \cdots + a_m s^m}{b_0 + b_1 s + \cdots + b_m s^m} + \frac{a_{n-m} s^m}{b_0 + b_1 s + \cdots + b_m s^m} s + \cdots + \frac{a_n s^m}{b_0 + b_1 s + \cdots + b_m s^m} s^{n-m} \] (5.23)

Equation (5.23) can be expressed in the time domain as:

\[ u_{FF}(t) = \frac{a_0 + a_1 s + \cdots + a_m s^m}{b_0 + b_1 s + \cdots + b_m s^m} r(t) + \frac{a_{n-m} s^m}{b_0 + b_1 s + \cdots + b_m s^m} r(t) + \cdots + \frac{a_n s^m}{b_0 + b_1 s + \cdots + b_m s^m} r(t)^{(n-m)} \] (5.24)

For example, if \( m = 0 \) and \( n = 3 \), the feedforward controller could be written as:
\[ u_{ff}(t) = \frac{a_0}{b_0} r(t) + \frac{a_1}{b_0} \dot{r}(t) + \frac{a_2}{b_0} \ddot{r}(t) + \frac{a_3}{b_0} \dddot{r}(t) \]  (5.25)

Or if \( m = 2 \) and \( n = 5 \),

\[ u_{ff}(t) = \frac{a_0 + a_1 s + a_2 s^2}{b_0 + b_1 s + b_2 s^2} r(t) + \frac{a_3 s^2}{b_0 + b_1 s + b_2 s^2} \dot{r}(t) + \frac{a_4 s^2}{b_0 + b_1 s + b_2 s^2} \ddot{r}(t) + \frac{a_5 s^2}{b_0 + b_1 s + b_2 s^2} \dddot{r}(t) \]  (5.26)

For implementation with a digital controller, the proper components can be discretized as before and the higher-order derivatives can be calculated at each time step via numerical integration. Also, in some other applications, the desired trajectory is known a priori (e.g., earthquake motion reproduction on shaking tables); for such cases, smooth derivatives can be created offline.

### 5.3.3 Positive Zeros

If the servo-hydraulic system contains zeros with a positive real component, the resulting inverse (i.e., feedforward controller) would be unstable due to positive poles. The most straightforward solution is to create the best possible model without the use of positive zeros, avoiding stability concerns in the inverse. In the case that positive model zeros cannot be avoided, a zero phase error tracking controller (ZPETC) can be applied to the feedforward controller (Tomizuka, 1987).

First, the system model is described by numerator and denominator polynomials:

\[
G_{su}(s) = \frac{n(s)}{d(s)}
\]  (5.27)

with inverse (feedforward controller) given by:

\[
G_{ff}(s) = \frac{d(s)}{n(s)}
\]  (5.28)

The feedforward controller poles should be separated into acceptable (negative) and unacceptable (positive) poles, indicated by subscripts “a” and “u”, respectively.
The unacceptable poles are then removed and the DC gain adjusted for their absence:

\[ G_{\text{ff}}(s) = \frac{d(s)}{n_a(s)n_u(0)} \]  

(5.30)

To maintain the same phase as the original unstable controller, the following adjustment is made:

\[ G_{\text{ff}}(s) = \frac{d(s)n_u^*(s)}{n_a(s)[n_u(0)]^2} \]  

(5.31)

where * indicates the complex conjugate. From Eqn. (5.31), it can be seen that an unacceptable inverse pole will lead to an additional zero in the feedforward controller. The ZPETC approach matches the phase of the unstable inverse, but distorts magnitude at higher frequencies. The magnitude distortion may negate the benefit of increased model accuracy through inclusion of positive zeros.

### 5.3.4 Feedback Controller

The feedback controller is added to complement the feedforward controller, providing robustness in the presence of changing specimen conditions, modeling errors, and disturbances. For the proposed model-based feedback controller, LQG control is applied to bring the deviation states to zero and thus reduce the tracking error. The deviation system of Eqn. (5.16) and (5.17) is rewritten as:

\[ \dot{\tilde{z}} = A\tilde{z} + Bu_{\text{FB}} + Ew_f \]  

(5.32)

\[ \tilde{x}^E = C\tilde{z} + v_f \]  

(5.33)

where \( w_f \) is the disturbance to the system, \( E \) is a matrix that describes how the disturbance enters the system, and \( v_f \) is the vector of measurement noise. It is assumed that \( E = B \), such that
the disturbance enters the servo-hydraulic system in the same way as the command. Only the output of the deviation system (i.e., \( \ddot{x}^E = x^E - x^l \)) is measurable. Thus, an observer is needed to estimate the unknown states of the deviation system. Evoking the separation principle, an LQG controller can be designed from independent LQR (optimal state feedback control) and Kalman filter (optimal observer) designs (Stengel, 1986).

To improve the LQG controller’s performance and robustness in the frequency range of interest, the disturbance \( w_d \) is assumed to be Gaussian white-noise \( w \) passed through a second-order shaping filter, i.e.,

\[
\dot{z}_d = A_d z_d + E_d w \\
w_d = C_d z_d
\]

where

\[
A_d = \begin{bmatrix} 0 & 1 \\ -\omega_d^2 & -2\xi_d \omega_d \end{bmatrix}
\]

\[
E_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

\[
C_d = \begin{bmatrix} \omega_d^2 & 2\xi_d \eta_d \omega_d \end{bmatrix}
\]

and \( z_d \) is the state vector the shaping filter, \( \dot{z}_d \) is its time derivative, and the parameters \( \xi_d, \omega_d \), and \( \eta_d \) control the peak, bandwidth, and roll-off of the disturbance, respectively. The deviation system can be rewritten as an augmented system that includes the dynamics of the shaping filter. This augmented system, denoted by the subscript “a”, is given by:

\[
\dot{z}_a = A_a z_a + B_a u_{FB} + E_a w
\]
\[ \tilde{x}^E = C_a z_a + v_f \]  

(5.41)

where

\[ A_a = \begin{bmatrix} A_f & 0 \\ E C_f & A \end{bmatrix} \]  

(5.42)

\[ B_a = \begin{bmatrix} 0 \\ B \end{bmatrix} \]  

(5.43)

\[ E_a = \begin{bmatrix} E_f \\ 0 \end{bmatrix} \]  

(5.44)

\[ C_a = [0 \ C] \]  

(5.45)

and the measurement noise vector \( v_f \) is assumed to be comprised of independent Gaussian white noises.

The control \( u_{FB} \) can be obtained using LQR design assuming full state feedback and output weighting as follows:

\[ J_{LQR} = \int_0^T \left[ (\tilde{x}^E)^T Q_{LQR} \tilde{x}^E + u_{FB}^T R_{LQR} u_{FB} \right] dt \]  

(5.46)

\[ u_{FB} = -K_{LQR} z_a \]  

(5.47)

where \( K_{LQR} \) is the optimal state feedback gain matrix, \( J_{LQR} \) is the cost function minimized by LQR design, \( Q_{LQR} \) is the weighting matrix on the system outputs, and \( R_{LQR} \) is the weighting matrix on the system inputs. With the certainty equivalence property, LQR design can be performed without regard to the process and measurement noise.

The augmented system states \( z_a \) can be estimated using a Kalman filter:

\[ \dot{\hat{z}}_a = A \hat{z} + B u_{FB} + L_{Kal} \left( \tilde{x}^E - C \hat{z}_a \right) \]  

(5.48)
where \( \hat{z}_a \) represents the estimated states and \( L_{\text{Kal}} \) is the optimal observer gain matrix.

The control law in Eqn. (5.47) is then written in terms of the estimated states and included in the estimator:

\[
\mathbf{u}_{\text{FB}} = -K_{LQR} \hat{z}_a 
\]

\[
\dot{\hat{z}}_a = (A - L_{\text{Kal}} C - B K_{LQR}) \hat{z}_a + L_{\text{Kal}} \hat{x}^E 
\]

The control systems toolbox in MATLAB is used for both LQR and Kalman filter designs.

### 5.4 SISO Example

The proposed model-based actuator control strategy will be examined for a single-actuator system. A parameterized servo-hydraulic system model as presented in Chapter 3 will be used as an example to develop both feedforward and feedback controllers. The model contains three poles and no zeros:

\[
G_m(s) = \frac{X^E(s)}{U(s)} = \frac{k_a k_s}{m^E s^3 + (-p_a m^E + c^E) s^2 + (-p_a c^E + A k_a + k^E) s + (-p_a k^E + k_a k_s)}
\]

#### 5.4.1 Feedforward Controller

The feedforward controller is taken as the inverse of servo-hydraulic system:

\[
G_{\text{FF}}(s) = \frac{U_{\text{FF}}(s)}{X^I(s)} = \frac{s - p_a}{k_a k_s + sA + 1}
\]

The feedforward controller can be rewritten as:

\[
G_{\text{FF}}(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3
\]

where the coefficients \( a_0 \) through \( a_3 \) can be determined by expanding Eqn. (5.52). The feedforward controller is improper by three degrees. In the time domain, Eqn. (5.53) becomes:

\[
u_{\text{FF}}(t) = a_0 x^1(t) + a_1 \dot{x}^1(t) + a_2 \ddot{x}^1(t) + a_3 \dddot{x}^1(t)
\]
where dots denote differentiation with respect to time and, as before, “I” refers to the interface DOF. In general, the equations of motion are solved at time step $i-1$ for the displacements at time step $i$ (i.e., time-stepping numerical integration) and the displacements are imposed on the physical specimen. In discrete time, Equation (5.54) can be written as:

$$u_{FF,i} = a_0 x_i^I + a_1 x_i^I + a_2 x_i^I + a_3 x_i^I$$  \hspace{1cm} (5.55)

Thus, the feedforward controller for the example actuator system requires the calculation of displacement, velocity, acceleration, and jerk (derivative of the acceleration) at time step $i$; however, most numerical integration schemes are only explicit in displacement. Two methods for calculating the necessary higher-order derivatives are proposed, including the CDM with linear acceleration extrapolation and the backward difference method (BDM). Note that these methods are proposed simply to estimate the higher-order derivatives at the required time step and can be selected independently from the numerical integration scheme. In addition a discrete model fitting approach is proposed to avoid the need for estimating higher-order derivatives.

**Central Difference Method with Linear Acceleration Extrapolation**

With most explicit numerical integration schemes, only the desired displacement $x_i^I$ is known. The desired acceleration can be linearly extrapolated over one time step:

$$\ddot{x}_i^I = 2\ddot{x}_{i-1}^I - \ddot{x}_{i-2}^I$$  \hspace{1cm} (5.56)

Note that the accelerations (and all other signals) must be in relative coordinates such that they describe the desired trajectory of the physical specimen. The desired velocity can be computed using Eqn. (5.57), which can be derived from Eqns. (2.3) and (2.4).

$$\dot{x}_i^I = \dot{x}_{i-1}^I + \frac{\Delta t}{2} \left( \ddot{x}_{i-1}^I + \ddot{x}_i^I \right)$$  \hspace{1cm} (5.57)
Finally, the desired jerk can be calculated directly from the acceleration. Since a linear extrapolation of the acceleration is chosen, the jerk can be calculated as the slope of the extrapolation:

$$\ddot{x}_i = \frac{1}{\Delta t} \left( \ddot{x}_{i-1} - \ddot{x}_{i-2} \right)$$  \hspace{1cm} (5.58)

The proposed feedforward controller coupled with the CDM for numerical integration is illustrated in Fig. 5.4 as it would be implemented by a digital controller for RTHS.

![Figure 5.4. Implementation of proposed feedforward controller in discrete time](image)

When the CDM is used for numerical integration, both velocity and acceleration can be expressed in terms of displacement. Thus, the feedforward controller using the CDM and a linear acceleration extrapolation with the CDM for numerical integration can be expressed explicitly as:

$$u_{FF,j} = \left[ a_0 + \frac{2a_1}{\Delta t} + \frac{2a_2}{\Delta t^2} + \frac{a_3}{\Delta t^3} \right] x_i + \left[ -\frac{7a_1}{2\Delta t} + \frac{5a_2}{\Delta t^2} + \frac{-3a_3}{\Delta t^3} \right] x_{i-1} + \left[ \frac{2a_1}{\Delta t} + \frac{4a_2}{\Delta t^2} + \frac{3a_3}{\Delta t^3} \right] x_{i-2} + \left[ \frac{-a_1}{2\Delta t} + \frac{-a_2}{\Delta t^2} + \frac{-a_3}{\Delta t^3} \right] x_{i-3}$$  \hspace{1cm} (5.59)

Or, as a discrete time transfer function:
\[ G_{\text{FF}}(z) = \left[ a_0 + \frac{2a_1 + 2a_2 + a_3}{\Delta t} \right] + \left[ -7a_1 + \frac{-5a_2 + -3a_3}{\Delta t^2} \right] z^{-1} + \left[ \frac{2a_1 + 4a_2 + 3a_3}{\Delta t^3} \right] z^{-2} + \left[ -\frac{a_1 + -a_2 + -a_3}{\Delta t^4} \right] z^{-3} \]  

(5.60)

**Backward Difference Method**

The BDM provides an alternative to discretize an improper continuous time system. Derivatives up to the third order calculated using the BDM are given by:

\[ \ddot{x}_i = \frac{1}{2\Delta t} \left( 3x_i - 4x_{i-1} + x_{i-2} \right) \]  

(5.61)

\[ \dddot{x}_i = \frac{1}{\Delta t^2} \left( 2x_i - 5x_{i-1} + 4x_{i-2} - x_{i-3} \right) \]  

(5.62)

\[ \ddddot{x}_i = \frac{1}{2\Delta t^3} \left( 5x_i - 18x_{i-1} + 24x_{i-2} - 14x_{i-3} + 3x_{i-4} \right) \]  

(5.63)

where the derivatives are second order accurate. Since the derivatives are calculated from available displacements, the feedforward controller in Eqn. (5.55) can be expressed as:

\[ u_{\text{ff},i} = \left[ a_0 + \frac{3a_1 + 2a_2 + 5a_3}{2\Delta t} \right] x_i + \left[ -2a_1 + \frac{-5a_2 + -9a_3}{2\Delta t^2} \right] x_{i-1} + \left[ \frac{a_1 + 4a_2 + 12a_3}{2\Delta t^3} \right] x_{i-2} + \left[ -\frac{a_2 + -7a_3}{2\Delta t^4} \right] x_{i-3} + \left[ \frac{3a_3}{2\Delta t^5} \right] x_{i-4} \]  

(5.64)

Or, as a discrete time transfer function:

\[ G_{\text{ff}}(z) = \left[ a_0 + \frac{3a_1 + 2a_2 + 5a_3}{2\Delta t} \right] + \left[ -2a_1 + \frac{-5a_2 + -9a_3}{2\Delta t^2} \right] z^{-1} + \left[ \frac{a_1 + 4a_2 + 12a_3}{2\Delta t^3} \right] z^{-2} + \left[ -\frac{a_2 + -7a_3}{2\Delta t^4} \right] z^{-3} + \left[ \frac{3a_3}{2\Delta t^5} \right] z^{-4} \]  

(5.65)

The CDM with linear acceleration extrapolation and BDM methods to creating a discrete time transfer function are compared to the continuous time transfer function in Fig. 5.5 for multiple sampling rates. The parameters for the example feedforward controller are taken as \( a_0 = 1.000, \) \( a_1 = 8.950 \times 10^{-3}, \) \( a_2 = 2.497 \times 10^{-5}, \) and \( a_3 = 6.210 \times 10^{-8}. \) For both methods, the discrete time approximation approaches the continuous time model as the sampling rate increases. The
BDM is more accurate over the frequency range of interest, however requires one more data point in the calculation.

Figure 5.5. Discrete time approximations of feedforward controller

**Discrete Model Fitting**

Examining Eqn. (5.59) and (5.64), the discrete time improper feedforward controller can be seen as a model-based extrapolation. That is, a series of previous displacement commands are used to extrapolate future displacements with coefficients determined by the servo-hydraulic system model. Therefore, rather than discretizing a continuous time model, the transfer function model could be directly fit in discrete time by adjusting the parameters $a_0$ through $a_n$ in:

$$G_{FF}(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}$$  \hspace{1cm} (5.66)

More generally, the transfer function could be directly fit as a discrete time system with both poles and zeros in the case that the servo-hydraulic system model possesses zeros:

$$G_{FF}(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}}{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m}}$$  \hspace{1cm} (5.67)
5.4.2 Feedback Controller

A state-space model of the servo-hydraulic system in Eqn. (5.51) is used to develop the model-based feedback controller. A parametric state-space model for the single-actuator system can be created by examining the dynamics of the force applied by the actuator in Fig. 3.13:

\[
\begin{align*}
\dot{f}^E &= k_a k_s u - k_a k_s x^E - k_a A x^E + f^E p_a \\
(5.68)
\end{align*}
\]

The state-space representation of the system is given by:

\[
\begin{bmatrix}
\dot{x}^E \\
\dot{\dot{x}}^E \\
\dot{f}^E
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
-\frac{k_a}{m} & -\frac{c_a}{m} & \frac{1}{m} \\
-k_a k_s & -k_a A & p_a
\end{bmatrix}
\begin{bmatrix}
x^E \\
\dot{x}^E \\
f^E
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
k_a k_s
\end{bmatrix} u \\
(5.69)
\]

With a physical servo-hydraulic system, it may be difficult to fit a parameterized model. Therefore, a nonparametric state-space model (fit using frequency domain system identification) can also be used for feedback design; in such cases, Eqn. (5.69) provides insight into the meaning of the nonparametric realization.

5.5 MIMO System with Actuator Coupling

When multiple actuators are connected to the same specimen, the dynamics of the actuators become coupled through the specimen (i.e., when an actuator applies a force to the structure, the other actuators will also experience this force). Actuator coupling will be demonstrated using the 3DOF linear building structure shown in Fig. 5.6.
This specimen employs three servo-hydraulic systems, each comprised of a servo-valve, a servo-controller, and an actuator which are commanded independently and can be represented in Fig. 5.2 by the following diagonal matrices:

\[ G_s(s) = \begin{bmatrix} k_{s1} & 0 & 0 \\ 0 & k_{s2} & 0 \\ 0 & 0 & k_{s3} \end{bmatrix} \] \hspace{1cm} (5.70)

\[ G_a(s) = \begin{bmatrix} \frac{k_{a1}}{(s-p_{a1})} & 0 & 0 \\ 0 & \frac{k_{a2}}{(s-p_{a2})} & 0 \\ 0 & 0 & \frac{k_{a3}}{(s-p_{a3})} \end{bmatrix} \] \hspace{1cm} (5.71)

\[ A = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix} \] \hspace{1cm} (5.72)

where the parameter subscripts identify the DOF to which the actuator is attached for the general case where the servo-hydraulic hardware is not identical. Likewise, the commanded displacements, measured forces, and measured displacements can be written as
hydraulic systems. The off-diagonal terms in Eqn. (5.74) are the source of the interaction between the three servo-valves. Fully coupled; in such a case, Eqn. (5.6) can be written as:

\[
\begin{bmatrix}
m_{11}^E & m_{12}^E & m_{13}^E \\
m_{21}^E & m_{22}^E & m_{23}^E \\
m_{31}^E & m_{32}^E & m_{33}^E
\end{bmatrix} \begin{bmatrix}
\ddot{x}_1^E \\
\ddot{x}_2^E \\
\ddot{x}_3^E
\end{bmatrix} + \begin{bmatrix}
c_{11}^E & c_{12}^E & c_{13}^E \\
c_{21}^E & c_{22}^E & c_{23}^E \\
c_{31}^E & c_{32}^E & c_{33}^E
\end{bmatrix} \begin{bmatrix}
\dot{x}_1^E \\
\dot{x}_2^E \\
\dot{x}_3^E
\end{bmatrix} + \begin{bmatrix}
k_{11}^E & k_{12}^E & k_{13}^E \\
k_{21}^E & k_{22}^E & k_{23}^E \\
k_{31}^E & k_{32}^E & k_{33}^E
\end{bmatrix} \begin{bmatrix}
x_1^E \\
x_2^E \\
x_3^E
\end{bmatrix} = \begin{bmatrix}
\dot{f}_1^E \\
\dot{f}_2^E \\
\dot{f}_3^E
\end{bmatrix}
\] (5.73)

where \( m \), \( c \), and \( k \) represent entries in the mass, damping, and stiffness matrices, respectively, with their position indicated by the subscripts. Taking the Laplace transform of Eqn. (5.73) yields the transfer function relating the input force from the actuators to the output displacement:

\[
G_{sf}(s) = \frac{X^E(s)}{F(s)} = \begin{bmatrix}
m_{11}^E s^2 + c_{11}^E s + k_{11}^E \\
m_{22}^E s^2 + c_{22}^E s + k_{22}^E \\
m_{33}^E s^2 + c_{33}^E s + k_{33}^E
\end{bmatrix}^{-1}
\] (5.74)

The off-diagonal terms in Eqn. (5.74) are the source of the interaction between the three servo-valves.

Substituting Eqns. (5.70) through (5.72) and Eqn. (5.74) into Eqn. (5.7), the MIMO servo-hydraulic system transfer function model is obtained:

\[
G_{su}(s) = \frac{X^E(s)}{U(s)} = \begin{bmatrix}
\prod_{i=1}^{6} (s - z_{11,i}) \\
\prod_{i=1}^{6} (s - z_{12,i}) \\
\prod_{i=1}^{6} (s - z_{13,i})
\end{bmatrix}
\begin{bmatrix}
\prod_{i=1}^{6} (s - z_{21,i}) \\
\prod_{i=1}^{6} (s - z_{22,i}) \\
\prod_{i=1}^{6} (s - z_{23,i})
\end{bmatrix}
\begin{bmatrix}
\prod_{i=1}^{6} (s - z_{31,i}) \\
\prod_{i=1}^{6} (s - z_{32,i}) \\
\prod_{i=1}^{6} (s - z_{33,i})
\end{bmatrix}
\prod_{i=1}^{9} (s - p_i)
\] (5.75)
where \( z \) and \( p \) represent the model zeros and poles, respectively. Note that the model is a \( 3 \times 3 \) transfer function, with each input-output pair possessing six zeros and nine poles. The actual poles and zeros can be obtained in closed-form, although they are too complicated for concise presentation. In Eqn. (5.75), the off-diagonal terms describe the interaction between the three servo-hydraulic systems.

### 5.5.1 Feedforward Controller

The feedforward controller for the multi-actuator system considering actuator coupling can be created from Eqn. (5.19). Substituting Eqns. (5.70) through (5.72) and Eqn. (5.74) into Eqn. (5.19) yields:

\[
G_{vi}(s) = \begin{bmatrix}
\frac{m_{i1}^v s^2 + c_{i1}^v s + k_{i1}}{k_{i1} k_{i1}} \frac{(s - p_{a1})}{k_{a1}} + \frac{s a_1}{k_{a1}} + 1 \\
\frac{m_{i2}^v s^2 + c_{i2}^v s + k_{i2}}{k_{i2} k_{i2}} \frac{(s - p_{a2})}{k_{a2}} + \frac{s a_2}{k_{a2}} + 1 \\
\frac{m_{i3}^v s^2 + c_{i3}^v s + k_{i3}}{k_{i3} k_{i3}} \frac{(s - p_{a3})}{k_{a3}} + \frac{s a_3}{k_{a3}} + 1 \\
\end{bmatrix}
\]

Even with the complexity of the servo-hydraulic system transfer function model of Eqn. (5.75), Eqn. (5.76) is relatively simple (three zeros for each input-output pair). For each input-output pair of Eqn. (5.76), there are two zeros that appear as a result of the second-order specimen dynamics. If the mass, damping, or stiffness matrices are not fully populated, as in a lumped mass system or a shear building, then Eqn. (5.76) could be further simplified. Thus, an understanding of the behavior of the physical specimen can aid in determining the number of zeros (and poles) to use in the feedforward controller. The third zero in each input-output pair arises from the first-order actuator model. Note that each column has the same actuator parameters (as seen by the subscripts), since each column is associated with the input to one actuator.
The feedforward controller \( \mathbf{U}_{\text{FF}}(s) = \mathbf{G}_{\text{FF}}(s) \mathbf{X}(s) \) can be rewritten as:

\[
\begin{align*}
\mathbf{U}_{\text{FF},1}(s) &= \begin{bmatrix} a_{11} + b_{11} s + c_{11} s^2 + d_{11} s^3 \quad a_{12} + b_{12} s + c_{12} s^2 + d_{12} s^3 \quad a_{13} + b_{13} s + c_{13} s^2 + d_{13} s^3 \end{bmatrix} \begin{bmatrix} x_1^1 \\ \dot{x}_1^1 \\ \ddot{x}_1^1 \\ \dddot{x}_1^1 \end{bmatrix} \\
\mathbf{U}_{\text{FF},2}(s) &= \begin{bmatrix} a_{21} + b_{21} s + c_{21} s^2 + d_{21} s^3 \quad a_{22} + b_{22} s + c_{22} s^2 + d_{22} s^3 \quad a_{23} + b_{23} s + c_{23} s^2 + d_{23} s^3 \end{bmatrix} \begin{bmatrix} x_2^1 \\ \dot{x}_2^1 \\ \ddot{x}_2^1 \\ \dddot{x}_2^1 \end{bmatrix} \\
\mathbf{U}_{\text{FF},3}(s) &= \begin{bmatrix} a_{31} + b_{31} s + c_{31} s^2 + d_{31} s^3 \quad a_{32} + b_{32} s + c_{32} s^2 + d_{32} s^3 \quad a_{33} + b_{33} s + c_{33} s^2 + d_{33} s^3 \end{bmatrix} \begin{bmatrix} x_3^1 \\ \dot{x}_3^1 \\ \ddot{x}_3^1 \\ \dddot{x}_3^1 \end{bmatrix}
\end{align*}
\]

(5.77)

where \( a \) though \( d \) are coefficients with their position indicated by the subscripts and the feedforward controller is seen to be improper. The proposed approach for accommodating the improper transfer function is to make use of higher-order derivatives which are available from numerical integration during RTHS. In such a case, the feedforward controller can be rewritten in the time domain as:

\[
\begin{align*}
\{ \mathbf{u}_{\text{FF},1} \} &= \begin{bmatrix} a_{11} & b_{11} & c_{11} & d_{11} & a_{12} & b_{12} & c_{12} & d_{12} & a_{13} & b_{13} & c_{13} & d_{13} \end{bmatrix} \begin{bmatrix} x_1^1 \\ \dot{x}_1^1 \\ \ddot{x}_1^1 \\ \dddot{x}_1^1 \end{bmatrix} \\
\{ \mathbf{u}_{\text{FF},2} \} &= \begin{bmatrix} a_{21} & b_{21} & c_{21} & d_{21} & a_{22} & b_{22} & c_{22} & d_{22} & a_{23} & b_{23} & c_{23} & d_{23} \end{bmatrix} \begin{bmatrix} x_2^1 \\ \dot{x}_2^1 \\ \ddot{x}_2^1 \\ \dddot{x}_2^1 \end{bmatrix} \\
\{ \mathbf{u}_{\text{FF},3} \} &= \begin{bmatrix} a_{31} & b_{31} & c_{31} & d_{31} & a_{32} & b_{32} & c_{32} & d_{32} & a_{33} & b_{33} & c_{33} & d_{33} \end{bmatrix} \begin{bmatrix} x_3^1 \\ \dot{x}_3^1 \\ \ddot{x}_3^1 \\ \dddot{x}_3^1 \end{bmatrix}
\end{align*}
\]

(5.78)

Thus, with the knowledge of the desired displacement, velocity, acceleration, and jerk at the interface DOFs, the feedforward control is reduced to matrix multiplication. Methods for accurately estimating the higher-order derivatives during RTHS are discussed previously for the single-actuator system.
5.5.2 Feedback Controller

Model-based feedback control can be derived from a state-space representation of the transfer function given in Eqn. (5.75). A state-space representation can also be created by examining the dynamics of the force in each of the actuators. From Fig. 5.2, Eqns. (5.70) through (5.72), and Eqn. (5.74), the equations governing the actuator responses are:

\[
\begin{align*}
\dot{f}_1^E &= k_{a1}k_{s1}u_1 - k_{a1}k_{s1}x_1^E - k_{a1}A_1\dot{x}_1^E + f_1^E p_{a1} \\
\dot{f}_2^E &= k_{a2}k_{s2}u_2 - k_{a2}k_{s2}x_2^E - k_{a2}A_2\dot{x}_2^E + f_2^E p_{a2} \\
\dot{f}_3^E &= k_{a3}k_{s3}u_3 - k_{a3}k_{s3}x_3^E - k_{a3}A_3\dot{x}_3^E + f_3^E p_{a3}
\end{align*}
\]

(5.79)  (5.80)  (5.81)

The state-space representation of the system is thus:

\[
\begin{align*}
\begin{bmatrix}
\dot{x}_1^E \\
\dot{x}_2^E \\
\dot{x}_3^E \\
\dot{f}_1^E \\
\dot{f}_2^E \\
\dot{f}_3^E
\end{bmatrix} &= \begin{bmatrix}
0_{3\times3} & I_{3\times3} & 0_{3\times3} \\
-M^E)^{-1}K^E & -(M^E)^{-1}C^E & (M^E)^{-1}
\end{bmatrix}
\begin{bmatrix}
x_1^E \\
x_2^E \\
x_3^E \\
f_1^E \\
f_2^E \\
f_3^E
\end{bmatrix} + \\
\begin{bmatrix}
-p_{a1} & 0 & 0 \\
0 & p_{a2} & 0 \\
0 & 0 & p_{a3}
\end{bmatrix}
\begin{bmatrix}
0_{3\times3} \\
0_{3\times3} \\
-k_{a1}k_{s1} & 0 & 0 \\
0 & -k_{a2}k_{s2} & 0 \\
0 & 0 & -k_{a3}k_{s3}
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2 \\
u_3
\end{bmatrix}
\end{align*}
\]

(5.82)

As expected, the state-space model contains nine poles. In a practical situation where the specimen dynamics must be determined from system identification, identifying the inverse model first (Eqn. 5.76) and then calculating the servo-hydraulic system model (Eqn. 5.75) from...
the inverse may be easier. Such an approach is discussed and applied in Chapter 7. As with the single-actuator system, it may be easier to identify a nonparametric model for control design.

The preceding model-based multi-actuator control scheme was presented for a three-actuator system. The same approach can be applied to an arbitrary number of actuators. Likewise, the approach can be easily adapted to higher-order servo-hydraulic system models.

5.6 Multi-Metric Feedback Control

RTHS is used to test the rate-dependent behavior of structural components. Actuator control schemes to date focus on accurate displacement tracking without concern for the higher-order derivatives. The proposed model-based strategy has the flexibility to include feedback from additional measurement devices to get better estimates of the states of the system through the Kalman filter. At the same time, feedback controller weight can be placed on the additional measurements through LQR control. By incorporating higher-order derivatives in the model-based controller, more accurate tracking of higher-order derivatives can be achieved toward a better representation of rate-dependent specimen behavior.

For example, accelerometers can be added to the actuators in line with each displacement transducer. In this case, the servo-hydraulic system model will have twice as many outputs as inputs.

$$G_{yu}(s) = \left[ \begin{array}{c} X^E(s) \\ \dot{X}^E(s) \end{array} \right][U(s)]^{-1} = \frac{Y^E(s)}{U(s)}$$

(5.83)

The state-space realization would contain this additional output.

$$\dot{z} = Az + Bu$$

(5.84)

$$y^E = Cz$$

(5.85)
where \( y^E = \{ x^E, \dot{x}^E \}^T \). With desired displacement and acceleration signal given by \( y^I = \{ x^I, \dot{x}^I \}^T \), the tracking error between the desired and measured signals is:

\[
e = y^I - y^E
\]  

(5.86)

Through regulator redesign, the deviation system outputs are:

\[
\tilde{y}^E = \begin{bmatrix} x^E - x^I \\ \dot{x}^E - \dot{x}^I \end{bmatrix}
\]  

(5.87)

where the additional output is equal to the difference between the measured and desired acceleration. The dynamics of the augmented deviation system including the process noise shaping filter and measurement noise are given by:

\[
\dot{z}_a = A_a z_a + B_a u_{FB} + E_a w_f
\]  

(5.88)

\[
\tilde{y}^E = C_a z_a + v_f
\]  

(5.89)

where the measurement noise vector includes both displacement and acceleration noise. The LQR controller can be designed based on output weighting including both the displacement and acceleration outputs:

\[
J_{LQR} = \int_0^\infty \left[ (\tilde{y}^E)^T Q_{LQR} \tilde{y}^E + u_{FB}^T R_{LQR} u_{FB} \right] dt
\]  

(5.90)

Acceleration measurements increase in magnitude with the square of the frequency relative to displacement measurements. Thus, the shaping filter contained within the augmented system is especially important to attenuate the response of the feedback controller at high frequencies. The augmented system states \( z_a \) can be estimated using a Kalman filter designed for an assumed process noise and displacement and acceleration measurement noise:

\[
\dot{\hat{z}}_a = A \hat{z}_a + B u_{FB} + L_{Kal} (\tilde{y}^E - C \hat{z}_a)
\]  

(5.91)
The total combined feedback controller would have the dynamics:

$$\dot{z}_a = (A - L_{Kal} C - BK_{LQR}) \dot{z}_a + L_{Kal} \tilde{y}^E$$  \hspace{1cm} (5.92)

A schematic of the proposed multi-metric feedback controller is presented in Fig. 5.7.

![Schematic of the proposed multi-metric feedback controller](image.png)

**Figure 5.7. Multi-metric feedback control**

Note that the feedforward controller used to achieve the ideal system in the regulator redesign has not been changed.

**5.7 Summary**

This chapter presented a framework for model-based actuator control including both feedforward and feedback links. Example designs are presented for both single and multi-actuator systems. In the single-actuator system example, focus is placed on the implementation of improper feedforward controllers, which are typical of servo-hydraulic systems. In the multi-actuator system example, focus is placed on illustrating the phenomenon of actuator coupling. A framework for incorporating multi-metric feedback is also presented toward better tracking of higher-order derivatives.
CHAPTER 6 SINGLE ACTUATOR CONTROL

Model-based actuator control is first applied to a large-scale, single-actuator system. An experimental framework combining advanced computational hardware and software with a high-performance servo-hydraulic system has been developed to evaluate the performance of rate-dependent structural components in RTHS. The physical specimen is chosen as a large-scale MR damper, which exhibits highly nonlinear, user controllable behavior. Through repeatable changes in the specimen behavior, the robustness of the proposed actuator control strategies can be investigated. Control performance is evaluated in the time domain and frequency domain for predefined commands as well as for both SDOF and MDOF structures in RTHS.

6.1 Experimental Setup

The RTHS testing framework at the University of Illinois is located in the Newmark Structural Engineering Laboratory (NSEL, http://nsel.cee.illinois.edu) and is a part of the Smart Structures Technology Laboratory (SSTL, http://sstl.cee.illinois.edu). The servo-hydraulic system hardware is shown in Fig. 6.1, accelerometer placement in Fig. 6.2, and the computational hardware and servo-controller in Fig. 6.3. The components depicted are described subsequently.

Figure 6.1. Servo-hydraulic hardware for RTHS
6.1.1 Test Frame

The actuator and specimen are both mounted on a 7.62 cm (3 in) thick steel plate using reaction angles. Steel blocks and wedges are used to prevent differential movement of the actuator and specimen. The steel plate is secured to the NSEL strong floor using threaded rods spaced 45.72 cm (1.5 ft) on center to prevent flexing of the plate and shear keys are used to prevent
longitudinal translation of the plate. The frame is designed to minimize backlash and elastic deformation under the high forces produced during testing. This setup (see Fig. 6.4) has proven successful for the dynamic testing of large-scale MR dampers (Yang et al., 2002; Phillips et al., 2010; Phillips and Spencer, 2011).

Figure 6.4. Test frame and fixturing (not to scale)

6.1.2 Servo-Hydraulic System

The actuator, manufactured by the Shore Western Corporation, is rated at 556 kN (125 kips) with a stroke of ±152.4 mm (±6 in), has an effective piston area of 271 cm² (42 in²), and is double ended to provide equal performance both extending and retracting. A Schenck-Pegasus model 1800 three-stage servo-valve rated at 300 lpm (80 gpm) is employed with a model 20B rated at 3.26 lpm (0.86 gpm) as the pilot servo-valve, shown in Fig. 6.5. Three-stage servo-valves are typically used in dynamic applications to achieve high flow rates required for the faster response of large actuators. Both flow ratings assume a 6.89 MPa (1000 psi) pressure drop.

The system is connected to the main hydraulic power supply of NSEL, providing 20.7 MPa (3000 psi) at 341 lpm (90 gpm). Hydraulic oil is routed through a Schenck-Pegasus model 3170804S hydraulic service manifold (HSM), which is rated at 300 lpm (80 gpm). The HSM has accumulators for the pilot pressure, main pressure, and return pressure of 10.3 MPa (1500 psi), 5.17 MPa (750 psi), and 0.345 MPa (50 psi) respectively, to provide additional flow for short durations. A Shore Western model 1104 digital servo-controller is used to control the actuator in
displacement feedback mode. This servo-controller contains an analog proportional feedback loop used for inner-loop control. Shore Western’s SC6000 software is used to interface with the servo-controller. The servo-controller is configured to accept external commands from the DSP board controlling the RTHS.

![Figure 6.5. Schenck-Pegasus model 1800 three-stage servo-valve with pilot valve](image)

### 6.1.3 Sensors

The displacement of the actuator is measured using an internal AC LVDT with a sensitivity of 46.3 mV/mm. A 445 kN (100 kip) Key Transducers, Inc. model 1411-114-02 load cell with a sensitivity of 12.4 mV/kN is mounted in line with the actuator, measuring the restoring force of the attached specimen. In addition, the temperature is monitored continuously during testing using three Omega Engineering model SA1XL-J thermocouples and model SMCJ-J analog converters with a sensitivity of 1mV/°F. Accelerations are measured using model 3701G3FA3G capacitive accelerometers manufactured by PCB Piezotronics. The accelerometers have a measurement range of ±3 g, a frequency range of 0-100 Hz, and a sensitivity of 1000 mV/g. Two accelerometers are placed on the actuator piston rod for comparison; however the accelerometer closest to the actuator housing in Fig. 6.2 is used for all reported acceleration measurements. The
current in the MR damper circuit is measured using a Tektronix current probe with a sensitivity of 100 mV/A.

6.1.4 Digital Signal Processor

The RTHS is controlled by a dSPACE model 1103 DSP board with a PPC 750GX processor. An I/O board CLP1103 is used to interface with the servo-controller. The DSP board has 20 A/D channels and 8 D/A channels, each with a resolution of 16-bits. The DSP board is mounted in an external chassis and connected to a host computer via fiber optic cable. This board is used to perform numerical integration of the equations of motion for the numerical substructure, apply the outer-loop actuator control schemes, and control the MR damper current based on semi-active control algorithms. These three numerical components are programmed on the host computer using Simulink, a block diagram programming tool within MATLAB. The Simulink model is converted to C language using MATLAB’s Real-Time Workshop and transferred to the DSP board. Real-time execution of the code is controlled and monitored from the host computer using dSPACE’s ControlDesk software. An example ControlDesk interface panel is presented in Fig. 6.6, allowing the user to start, pause, and stop the code execution, select the input ground motion, select the structural control strategy, select the actuator control strategy, and monitor the sensors readings in real-time. The host computer also acts as the DAQ, logging data from the DSP board.

The flow of information from the host computer to the experimental equipment is shown in Fig. 6.7. Solid arrows indicate communication in real-time (e.g., during RTHS).
Figure 6.6. Example ControlDesk software interface panel

Figure 6.7. Communication between software and hardware
6.2 Characterization of the Servo-Hydraulic System

The servo-hydraulic system has dynamic performance limitations which can be more restrictive than the load rating of the actuator. An understanding of these limitations will help to develop the dynamic operating range of the system. To explore the coupling between force and velocity, the dynamics of the servo-valve will be investigated. The output flow of a servo-valve (Eqn. 6.1) depends on the commanded valve current as well as the pressure drop across the valve (Meritt, 1967).

\[
Q_L = Q_R \left( \frac{i_c}{i_R} \right) \sqrt{\frac{P_S - P_L}{\Delta P}} \tag{6.1}
\]

where \(Q_L\) is the output flow, \(Q_R\) is the rated flow, \(i_c\) is the input current, \(i_R\) is the maximum input current, \(P_S\) is the supply pressure, \(P_L\) is the pressure drop across the load, and \(\Delta P\) is the rated valve pressure drop. For the servo-hydraulic system in this study, the maximum input current \(i_R\) is 50 mA and the supply pressure \(P_S\) is 3000 psi. The rated flow \(Q_R\) for this particular servo-valve is 80 gpm for a rated valve pressure drop \(\Delta P\) of 1000 psi. Servo-valves are rated at this valve pressure drop because it corresponds to the largest output power (force times velocity) for a 3000 psi supply pressure system.

The output flow is plotted against the ratio of pressure drop across the load to supply pressure for a variety of input current ratios in Fig. 6.8 based on Eqn. (6.1). When the load to supply pressure ratio is equal to 2/3, the rated flow of 80 gpm is achieved at the maximum input current value. Output flows greater than the rated flow can be achieved, although the output power will not be at maximum.
The force on the piston rod is equal to the pressure drop across the load multiplied by the piston cross-sectional area as in:

$$f^E = P_L A$$

(6.2)

where $f^E$ is the piston force and $A$ is the piston cross-sectional area. The piston force is comprised of the force imparted on the specimen, the inertial force of the piston rod, as well as the friction force. The inertial force is typically negligible. The friction force depends on the friction between the piston rod and the actuator housing. Actuators use low-friction seals to minimize this force, ensuring that most of the total force can be used to excite the specimen.

The velocity of the piston rod $v$ is equal to:

$$v = \frac{Q_L}{A}$$

(6.3)

By combining Eqns. (6.1) through (6.3) for the maximum current, the following relationship between force and velocity for the servo-valve and actuator pair can be derived:
At the same time, there is a flow limitation imposed by the supply oil pressure. The hydraulic pumps are limited to 90 gpm. Further downstream, the servo-hydraulic manifold is rated at 80 gpm. With the 80 gpm limit, a second velocity limit can be derived based on the oil supply $Q_s$ flow and the equation below.

$$v = \frac{Q_s}{A}$$

The operating range of the servo-hydraulic system is presented in Fig. 6.9 with limits based on the servo-valve and actuator force-velocity relationship, oil supply limit, and specimen rating. The supply limit is straightforward; based on the rated flow rate of the servo-hydraulic manifold, no more than 186 mm/sec can be achieved in the piston rod. This limit is conservative, because the accumulators in the servo-hydraulic manifold will allow for short bursts of higher velocities.

Figure 6.9. Hydraulic power curve
The force limit of the servo-valve and actuator follows a parabolic relationship with the piston velocity (Eqn. 6.4). For low piston force, the flow rate can exceed the rated flow; however the supply limit cannot be exceeded. The operating range of the servo-hydraulic system has been identified and experiments will be designed such that these limits, namely the maximum velocity, are not exceeded. At the same time, the MR damper is rated at a nominal 200 kN, so forces much higher than 200 kN are not expected.

6.2.1 System Identification

The transfer function from the input commanded displacement to the output measured displacement (and acceleration for multi-metric feedback) is used to characterize the dynamics of the servo-hydraulic system. This transfer function includes the dynamics of the actuator, servo-valve, servo-controller, specimen, and sensors. Note that some time delay (e.g. data communication, A/D and D/A conversion) will naturally be included in the characterization of the servo-hydraulic system dynamics. Unless otherwise mentioned, the input is selected as a BLWN from 0 to 50 Hz with a displacement RMS of 0.254 mm, providing insight into the servo-hydraulic dynamics over this range of frequencies. The dSPACE system is used to generate the commanded signal and measure the response at 2048 Hz. Data is down-sampled to 128 Hz and the transfer function is calculated using 2048 FFT points, a Hanning window with 50% overlap, and 10 averages. The time lag is calculated by dividing the phase by the frequency, which is noticeably sensitive to noise at the lower frequencies.

6.2.2 Actuator Tuning

The user has control over the inner-loop PID gains of the servo-controller. For RTHS applications, integral gain is typically not used because it adds delay to the response of the servo-hydraulic system and has little impact on the dynamic performance of the system. Also,
derivative gain is not used because it causes the servo-hydraulic system to be more sensitive to noise. Therefore, focus is placed on tuning the proportional gain. Figure 6.10 shows the servo-hydraulic system transfer function with the MR damper specimen attached (set at 0.0 Amps) for various proportional gain values. A proportional gain of 600 is selected for its superior performance before the onset of stability concerns seen by further increasing the gain. Note that when a specimen with higher restoring force is attached, such as the MR damper at 2.5 Amps, the servo-hydraulic system performance will degrade due to a larger restoring force resisting the motion.

![Figure 6.10. Tuning of the servo-hydraulic system at 0.0 Amps](image)

**6.2.3 Servo-Hydraulic System Linearization**

The experimental transfer function is a linear representation of the input-output dynamics of the servo-hydraulic system. The linearization occurs about the operating point of the system; therefore a different transfer function could be identified for different input amplitudes,
frequencies, or specimen conditions. To qualitatively investigate the sensitivity of the identified model to input conditions, the servo-hydraulic system transfer function is investigated at three different amplitude ranges. To safely accommodate higher amplitudes, the frequency ranges of the input BLWN are reduced. System identification is performed with RMS values of 0.254 mm, 0.400 mm, and 0.600 mm with BLWN frequency ranges of 0 to 50 Hz, 0 to 25 Hz, and 0 to 15 Hz, respectively. Sample time history signals are shown in Fig. 6.11.

![Figure 6.11. Time history of BLWN input signal](image)

The corresponding transfer functions are shown in Fig. 6.12, where both 0.0 Amp and 2.5 Amp conditions are investigated. The changes made to the input do not significantly affect transfer function of the system, showing that a linearized model is acceptable for a fixed specimen condition. On the other hand, changes to the specimen clearly lead to changes in the system dynamics, a phenomenon that will be addressed through the proposed controller.
6.2.4 Model Development

With the inner-loop PID gains set and confidence in the linear behavior of the servo-hydraulic system for a fixed specimen condition, experimentally derived transfer functions will be used to develop transfer function models. A nonparametric system identification technique, MFDID (Kim et al., 1995), is used to fit the experimental transfer function data to SISO or single-input multi-output (SIMO) model of poles and zeros.

Because the current to the MR damper can change during the RTHS, the servo-hydraulic dynamics must be investigated at multiple current levels. The measured displacement transfer function magnitude, phase, and time lag are presented in Fig. 6.13 for two conditions: 0.0 and 2.5 Amps. The results are also averaged to create a third transfer function appropriate for when the specimen conditions are unknown or changing.
Identified transfer function models are overlain on Fig. 6.13 in dashed black lines. Three pole models are found sufficient to accurately represent the dynamics over the frequency range of interest (up to 40 Hz). Models of the servo-hydraulic dynamics at 0.0 and 2.5 Amps, as well as the average of the two specimen conditions, are given by:

\[ G_{su,0.0A}(s) = \frac{X(s)}{U(s)} = \frac{1.730 \times 10^7}{(s + 182.7)(s^2 + 225.3s + 9.499 \times 10^4)} \]  
(6.6)

\[ G_{su,2.5A}(s) = \frac{1.613 \times 10^7}{(s + 134.2)(s^2 + 324.6s + 1.211 \times 10^7)} \]  
(6.7)

\[ G_{su,avg}(s) = \frac{1.600 \times 10^7}{(s + 151.7)(s^2 + 250.4s + 1.061 \times 10^5)} \]  
(6.8)

Figure 6.13 shows that the behavior of the servo-hydraulic system is frequency dependent, where the magnitude and phase (or equivalently, the time lag) varies with frequency. Traditional delay compensation approaches based on a single constant time delay would be
inadequate for systems that respond at multiple frequencies, such as MDOF structures. Likewise, traditional approaches do not address the decay in magnitude observed.

Typical time delays/lags reported in the literature range from 8 to 30 msec (Horiuchi et al., 1999; Nakashima and Masaoka, 1999; Darby et al., 2002; Carrion and Spencer, 2007; Wallace et al., 2007; Chen and Ricles, 2010). The time lag in this experimental setup was found to vary between 8 and 11 msec depending on the frequency of excitation and the specimen conditions, which is relatively small for such a large actuator. Subsequent tests comparing the lag between input sine waves and measured responses confirmed these results.

In addition to displacement based feedback, multi-metric feedback approaches will be considered. To implement multi-metric feedback control, the output accelerations are also measured, creating a SIMO system. The transfer function from input command to measured acceleration is identified, shown in Fig. 6.14.

Fitted transfer function models are overlain on Fig. 6.14 in dashed black lines. The combined SIMO system models (input command to output displacement and acceleration) are given by:

\[
G_{yu,0.0A}(s) = \frac{X(s)}{A(s)}[U(s)]^{-1} = \frac{1.730 \times 10^7}{(s + 182.7)(s^2 + 225.3s + 9.499 \times 10^4)} \quad (6.9)
\]

\[
G_{yu,2.5A}(s) = \frac{1.613 \times 10^7}{(s + 134.2)(s^2 + 324.6s + 1.211 \times 10^5)} \quad (6.10)
\]

\[
G_{yu,avgA}(s) = \frac{1.600 \times 10^7}{(s + 151.7)(s^2 + 250.4s + 1.061 \times 10^5)} \quad (6.11)
\]
Figure 6.14. Measured acceleration transfer functions at select current levels

When the restoring force is low in the MR damper (0.0 Amps), the measurements in the LVDT and accelerometer correlate well. As seen in Eqn. (6.9), the acceleration transfer function can be accurately represented by the second derivative of the displacement transfer function. However, when the restoring force in the MR damper is high (2.5 Amps), the elastic deformation of the testing frame will be larger. Since the accelerometer is mounted on the exposed actuator piston head and the LVDT is mounted internally in the rear of the actuator housing, the elastic deformation will lead to discrepancies in measurement. At low displacements, some of the measured displacement will go into the elastic deformation of the frame while the accelerometers are measuring absolute accelerations. This decrease in acceleration measurement relative to the displacement measurement can be seen by the gain adjustments in Eqns. (6.10) and (6.11). Future work will explore alternative sensor placement for better measurement correlation in the presence of elastic deformation.
6.3 Controller Designs

Actuator controllers are created based on (a) the proposed model-based strategy, (b) a previous model-based strategy (Carrion and Spencer, 2007), (c) the polynomial extrapolation method (Horiuchi et al., 1996), and (d) the lead compensator method (Zhao et al., 2003; Jung et al., 2007). Model-based controllers are designed based on the identified servo-hydraulic models while the polynomial extrapolation and lead compensator controllers were designed to compensate for the DC (i.e., zero frequency) time lag. The additional controllers will serve as a comparison for the proposed model-based controller among other widely-applied approaches to actuator control for RTHS.

6.3.1 Proposed Model-Based Controller

Based on the methods proposed in Chapter 5, three model-based feedforward controllers are created using the transfer function models in Eqns. (6.6), (6.7), and (6.8) and identified as $G_{FF,0.0A}$, $G_{FF,2.5A}$, and $G_{FF,avgA}$, respectively. The first two controllers are used when the specimen conditions are known and unchanging, while the third controller is used when the specimen conditions may be changing. With the proposed feedforward controllers, higher-order derivatives are required. For predefined displacements, the higher-order derivatives are calculated offline. In RTHS, higher-order derivatives are calculated in real-time using the CDM with linear acceleration extrapolation. To improve performance and compensate for system modeling errors and changes in specimen conditions, an LQG feedback controller is created using state-space representations of the models in Eqns. (6.8) and (6.11) for displacement-based and multi-metric feedback controllers, respectively. The complete controllers with feedback control are identified as $G_{FF,avgA} + xLQG$ and $G_{FF,avgA} + xaLQG$ for displacement and multi-metric feedback, respectively.
6.3.2 Previous Model-Based Controller

The previous model-based approach is based on a pole-only model of the servo-hydraulic system given by:

\[ G_{su}(s) = \frac{K}{\prod_{i=1}^{n}(s - p_i)} \]  

(6.12)

The feedforward controller is taken as the inverse in combination with a low-pass filter to create a proper system:

\[ G_{FF}(s) = \alpha^n \frac{\prod_{i=1}^{n}(s - p_i)}{\prod_{i=1}^{n}(s - \alpha p_i)} \]  

(6.13)

where the parameter \( \alpha \) controls the location of the poles of the feedforward controller. With \( \alpha = 1 \), the feedforward control effort is equal to unity (i.e., no compensation) while as \( \alpha \) approaches infinity, the feedforward controller approaches an improper inverse. Carrion and Spencer (2007) recommend using \( \alpha = 10 \) such that the poles of the feedforward controller do not significantly interfere with the inverse dynamics while the poles are small enough to be implemented with a reasonable sampling rate for RTHS. Two feedforward controllers are created based on the transfer function models in Eqns. (6.6) and (6.7), representing the extremes of the specimen conditions. The feedforward controllers with low-pass filters (\( \alpha =10 \)) are identified as \( G_{FF,0.0A} + LP \) and \( G_{FF,2.5A} + LP \). For cases when the specimen conditions may be changing, a bumpless transfer is created between the two feedforward controllers based on the input current \( 0.0 \leq i_d \leq 2.5 \) Amps. The bumpless transfer is illustrated in Fig. 6.15.
The transfer function \( G_t(s) \) provides a smooth transition between controllers based on the input current to the MR damper. By examining the response time of the MR damper, the following lag transfer function was selected:

\[
G_t(s) = \frac{10/2.5}{0.005s + 10}
\]  

(6.14)

### 6.3.3 Polynomial Extrapolation

The polynomial extrapolation technique fits the current desired displacement with previous displacements to a polynomial and extrapolates the displacement command after a fixed time step. The command to the servo-hydraulic system can be calculated by:

\[
u_{FF,i} = \sum_{j=0}^{n} a_j x_j^i
\]  

(6.15)

where \( n \) is the order of the polynomial, \( x_j^i \) is the displacement \( T_d \times j \) units of time ago, \( T_d \) is the estimated delay, and \( a_j \) are constants of extrapolation, depending on the order \( n \). For a third-order extrapolation,
\[ u_{FF,i} = 4x_i^1 - 6x_{i-T_d}^1 + 4x_{i-2T_d}^1 - x_{i-3T_d}^1 \]  \hspace{1cm} (6.16)

To accommodate different specimen conditions, three third-order polynomial extrapolation compensators are created, based on 8 msec delay for 0.0 Amps, 10 msec delay for 2.5 Amps, and 9 msec delay for changing specimen conditions.

### 6.3.4 Lead Compensator

The lead compensator is a pole-zero pair which is tuned to eliminate the low-frequency time lag:

\[ G_{FF}(s) = K \frac{(s - p)}{(s - z)} \]  \hspace{1cm} (6.17)

The DC time lag compensation provided can be calculated by Eqn. (6.18) to aid in determining \( p \) and \( z \) while \( K \) is chosen such that the DC gain of the controller is unity. The phase angle \( \theta \) can be calculated using Eqn. (6.19).

\[ \text{timelag (sec)} = \lim_{\omega \to 0} \left\{ \frac{\theta}{\omega} \right\} = \frac{1}{p} - \frac{1}{z} \]  \hspace{1cm} (6.18)

\[ \theta = \tan^{-1} \left( \frac{\text{Im}(G_{FF}(s))}{\text{Re}(G_{FF}(s))} \right) \]  \hspace{1cm} (6.19)

Three lead compensator designs are created, based on 8 msec delay for 0.0 Amps, 10 msec delay for 2.5 Amps, and 9 msec delay for changing specimen conditions.

Table 6.1 summarizes the controllers explored, identified by the controller type and specimen condition.
### Table 6.1. Real-Time Actuator Controllers

<table>
<thead>
<tr>
<th>Method</th>
<th>Specimen Conditions</th>
<th>Short Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Model-Based Tracking Control</td>
<td>0.0 Amps</td>
<td>$G_{FF,0.0A}$</td>
</tr>
<tr>
<td></td>
<td>2.5 Amps</td>
<td>$G_{FF,2.5A}$</td>
</tr>
<tr>
<td></td>
<td>Average / General</td>
<td>$G_{FF,avgA} + xLQG$</td>
</tr>
<tr>
<td></td>
<td>Average / General</td>
<td>$G_{FF,avgA} + xaLQG$</td>
</tr>
<tr>
<td>Model-Based Control with Low-Pass Filter</td>
<td>0.0 Amps</td>
<td>$G_{FF,0.0A} + LP$</td>
</tr>
<tr>
<td>(Carrion and Spencer, 2007)</td>
<td>2.5 Amps</td>
<td>$G_{FF,2.5A} + LP$</td>
</tr>
<tr>
<td></td>
<td>Average / General</td>
<td>Bumpless + LP</td>
</tr>
<tr>
<td>Third-Order Polynomial Extrapolation</td>
<td>0.0 Amps</td>
<td>$3^{rd}$ Poly 8ms</td>
</tr>
<tr>
<td>(Horiuchi et al., 1996)</td>
<td>2.5 Amps</td>
<td>$3^{rd}$ Poly 10ms</td>
</tr>
<tr>
<td></td>
<td>Average / General</td>
<td>$3^{rd}$ Poly 9ms</td>
</tr>
<tr>
<td>Lead Compensator (Zhao et al., 2003)</td>
<td>0.0 Amps</td>
<td>Lead Comp 8ms</td>
</tr>
<tr>
<td>(Jung et al., 2007)</td>
<td>2.5 Amps</td>
<td>Lead Comp 10ms</td>
</tr>
<tr>
<td></td>
<td>Average / General</td>
<td>Lead Comp 9ms</td>
</tr>
</tbody>
</table>

#### 6.4 Tracking Performance in the Frequency Domain

To evaluate performance in the frequency domain, the actuator controllers were implemented in dSPACE using a sampling rate of 2048 Hz. Then, a BLWN from 0 to 50 Hz with a displacement RMS of 0.254 mm was commanded to experimentally determine the servo-hydraulic system transfer function with outer-loop control.

**6.4.1 Constant Specimen Conditions**

Controllers were designed to match the specimen conditions, with results for the 0.0 Amp condition in Fig. 6.13 and the 2.5 Amp condition in Fig. 6.14. Perfect controller performance would be indicated by unit magnitude, zero phase, and zero time lag.

The polynomial extrapolation technique provides good compensation at low-frequencies. However, magnitude undershoot is found from 5 to 15 Hz, whereas above 15 Hz the magnitude begins to increase dramatically. Because of this amplification, the system was not excited above...
30 Hz for safety. At the same time, the polynomial extrapolation technique overcompensates for the time lag after 10 Hz. This overcompensation can add positive damping to the RTHS loop, adding stability while compromising accuracy. After about 25 Hz, the polynomial extrapolation technique begins to undercompensate.

The lead compensator also provides good compensation at low-frequencies. However, at about 10 Hz, the magnitude begins to increase and the time lag becomes undercompensated. A single pole and zero pair are not enough to provide adequate compensation over a broad frequency range, which can be problematic if high-frequency response is expected.

These results demonstrate the model-based approaches to have significantly better performance in terms of both magnitude and phase (or time lag). Excellent results can be seen in magnitude performance for model-based approaches up to 50 Hz. In terms of phase, the model-based approach using a low-pass filter has slightly poorer time lag compensation, which is due to
the dynamics of the low-pass filter adversely adding phase lag to the model-based inverse. The proposed model-based approach reduces this lag.

![Image of transfer functions for various control techniques with 2.5 Amps in damper](image)

**Figure 6.17. Transfer functions for various control techniques with 2.5 Amps in damper**

### 6.4.2 Time Varying Specimen Conditions

During RTHS with semi-active control, the current to the MR damper will be varying. Therefore, a representative semi-active command ranging from 0.0 to 2.5 Amps is created, with a one-second window shown in Fig. 6.18. The measured current in the MR damper circuit is also presented, illustrating the lag between command and measured current.

Three actuator control schemes are investigated for the semi-active MR damper case, with the frequency domain results shown in Fig. 6.19. The feedforward controller alone based on average specimen conditions provides good control. Adding model-based feedback significantly improves the magnitude response. Model-based multi-metric feedback, including acceleration
feedback measurements, further improves the range of excellent actuator control in both magnitude and phase.

Figure 6.18. Example semi-active current in MR damper

Figure 6.19. Transfer functions for various control techniques with semi-active conditions
6.5 Tracking Performance in the Time Domain

The controllers are also evaluated in the time domain using a predefined displacement and current command history. Two displacement histories were explored (a) BLWN with bandwidth of 0 to 5 Hz and an RMS of 2.78 mm and (b) BLWN with bandwidth of 0 to 15 Hz and an RMS of 0.595 mm. During this displacement, the current command to the MR damper was either maintained at 0.0 Amps (passive-off), 2.5 Amps (passive-on), or a pulse between 0.0 Amps and 2.5 Amps at 0.5 Hz (50% duty cycle, mimicking semi-active control conditions). Good tracking is indicated by a low RMS error (norm) between the desired and measured signal as calculated by:

\[
\text{RMS error (norm)} = \sqrt{\frac{\sum_{i} (r_i - y_i)^2}{\sum_{i} (r_i)^2}} \times 100\% \quad (6.20)
\]

where \( r_i \) is the desired signal and \( y_i \) is the measured signal at time step \( i \). Equation (6.20) is also used to calculate velocity and acceleration tracking errors.

6.5.1 Displacement Tracking

Results of the time domain displacement tracking tests are presented in Table 6.2. Results highlight that the proposed model-based control technique provide considerable improvement in system performance through reduction of the displacement RMS error for all specimen conditions. Model-based feedforward controllers designed to match the specimen conditions performed well while the model-based feedforward-feedback controller performed well under all specimen conditions. Adding multi-metric feedback slightly improves the tracking performance of the proposed model-based controller.
Table 6.2. Tracking Performance for Predefined Displacement Histories

<table>
<thead>
<tr>
<th>Specimen Condition</th>
<th>Controller</th>
<th>RMS Error (%)</th>
<th>RMS Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0 to 5 Hz BLWN</td>
<td>0 to 15 Hz BLWN</td>
</tr>
<tr>
<td>0.0 Amps</td>
<td>None</td>
<td>16.0</td>
<td>42.9</td>
</tr>
<tr>
<td></td>
<td>3rd Poly 8ms</td>
<td>1.22</td>
<td>12.8</td>
</tr>
<tr>
<td></td>
<td>Lead Comp 8ms</td>
<td>1.95</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,0.0A}$ + LP</td>
<td>1.01</td>
<td>4.27</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,0.0A}$</td>
<td>0.942</td>
<td>3.45</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,avgA} + xLQG$</td>
<td>1.16</td>
<td>3.89</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,avgA} + xaLQG$</td>
<td>0.924</td>
<td>3.98</td>
</tr>
<tr>
<td>2.5 Amps</td>
<td>None</td>
<td>20.1</td>
<td>51.7</td>
</tr>
<tr>
<td></td>
<td>3rd Poly 10ms</td>
<td>2.04</td>
<td>25.9</td>
</tr>
<tr>
<td></td>
<td>Lead Comp 10ms</td>
<td>3.34</td>
<td>15.1</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,2.5A} + LP$</td>
<td>2.55</td>
<td>9.40</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,2.5A}$</td>
<td>2.27</td>
<td>4.68</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,avgA} + xLQG$</td>
<td>1.41</td>
<td>5.57</td>
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<tr>
<td></td>
<td>$G_{FF,avgA} + xaLQG$</td>
<td>1.11</td>
<td>4.52</td>
</tr>
<tr>
<td>2.5 Amp Pulse</td>
<td>None</td>
<td>18.1</td>
<td>49.2</td>
</tr>
<tr>
<td></td>
<td>3rd Poly 9ms</td>
<td>1.80</td>
<td>18.3</td>
</tr>
<tr>
<td></td>
<td>Lead Comp 9ms</td>
<td>2.97</td>
<td>16.0</td>
</tr>
<tr>
<td></td>
<td>Bumpless + LP</td>
<td>2.04</td>
<td>8.45</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,avgA}$</td>
<td>1.93</td>
<td>6.35</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,avgA} + xLQG$</td>
<td>1.09</td>
<td>4.72</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,avgA} + xaLQG$</td>
<td>0.870</td>
<td>4.11</td>
</tr>
</tbody>
</table>

Figure 6.20 shows the time history results for the displacement tracking test corresponding to the 2.5 Amp pulse. Both 0 to 5 Hz BLWN and 0 to 15 Hz BLWN results are presented for a short window with identical scaling in both displacement and time. At 3 seconds, the current is switched from 0.0 Amps to 2.5 Amps, thus the results show a transition period in specimen conditions. Without compensation, the effect of the servo-hydraulic dynamics on magnitude and phase are apparent.
Figure 6.20. Displacement tracking during a pulse in current

The time history results reflect the observations made in the frequency domain study. The polynomial extrapolation technique shows slight undershoot at these frequencies. At the same time, the time lag is overcompensated, especially in the 0 to 15 Hz BLWN case. The lead compensator exhibits considerable overshoot, especially in the 0 to 15 Hz BLWN case. The time lag is slightly overcompensated in the 0 to 5 BLWN Hz case before 3 seconds because the lead compensator is designed for average conditions and the specimen is at 0.0 Amps. In the 0 to 15 Hz BLWN case, the time lag is undercompensated because the effectiveness of the lead compensator diminishes at high-frequencies. With the bumpless transfer approach, the time lag is well compensated under changing specimen conditions however there is a slight overshoot in magnitude. With the proposed model-based feedforward-feedback controller, accurate tracking of both magnitude and phase is achieved. Both displacement and multi-metric feedback controllers exhibit similar performance.
6.5.2 Velocity and Acceleration Tracking

The velocity and acceleration tracking performance of the proposed model-based controller is investigated for both displacement feedback and multi-metric feedback approaches. Rate-dependent devices are sensitive to higher-order derivatives, so accurate tracking of velocities and accelerations can lead to more accurate RTHS results. Table 6.3 shows the RMS error between desired and measured displacements, velocities, and accelerations for a 0 to 5 Hz BLWN predefined displacement. Table 6.4 shows the same quantities for a 0 to 15 Hz BLWN predefined displacement. Desired velocities and accelerations are calculated by differentiating the desired displacement. Velocity measurements are calculated by differentiating the displacement measurements while acceleration measurements are taken from the attached accelerometer. Since the acceleration readings enter the DSP unfiltered to avoid introducing time lag into the feedback loop, the acceleration measurements have been filtered in post-processing using a low-pass filter with a cutoff frequency of 50 Hz.

Table 6.3. Tracking Performance for Higher-Order Derivatives, 0 to 5 Hz BLWN

<table>
<thead>
<tr>
<th>Specimen Condition</th>
<th>Controller</th>
<th>RMS Error (%) 0 to 5 Hz BLWN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Displacement</td>
<td>Velocity</td>
</tr>
<tr>
<td>0.0 Amps</td>
<td>$G_{FF,avgA} + xLQG$</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,avgA} + xaLQG$</td>
<td>0.924</td>
</tr>
<tr>
<td>2.5 Amps</td>
<td>$G_{FF,avgA} + xLQG$</td>
<td>1.41</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,avgA} + xaLQG$</td>
<td>1.11</td>
</tr>
<tr>
<td>2.5 Amp Pulse</td>
<td>$G_{FF,avgA}$</td>
<td>1.93</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,avgA} + xLQG$</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,avgA} + xaLQG$</td>
<td>0.870</td>
</tr>
</tbody>
</table>
Table 6.4. Tracking Performance for Higher-Order Derivatives, 0 to 15 Hz BLWN

<table>
<thead>
<tr>
<th>Specimen Condition</th>
<th>Controller</th>
<th>RMS Error (%) 0 to 15 Hz BLWN</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Displacement</td>
<td>Velocity</td>
<td>Acceleration</td>
</tr>
<tr>
<td>0.0 Amps</td>
<td>$G_{FF,avgA} + xLQG$</td>
<td>3.89</td>
<td>7.97</td>
<td>23.3</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,avgA} + xaLQG</td>
<td>3.98</td>
<td>5.93</td>
<td>16.9</td>
</tr>
<tr>
<td>2.5 Amps</td>
<td>$G_{FF,avgA} + xLQG$</td>
<td>5.57</td>
<td>18.3</td>
<td>48.6</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,avgA} + xaLQG$</td>
<td>4.52</td>
<td>11.9</td>
<td>39.7</td>
</tr>
<tr>
<td>2.5 Amp Pulse</td>
<td>$G_{FF,avgA}$</td>
<td>6.35</td>
<td>11.2</td>
<td>36.9</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,avgA} + xLQG$</td>
<td>4.72</td>
<td>16.0</td>
<td>42.5</td>
</tr>
<tr>
<td></td>
<td>$G_{FF,avgA} + xaLQG$</td>
<td>4.11</td>
<td>9.24</td>
<td>32.4</td>
</tr>
</tbody>
</table>

The improvements in displacement tracking achieved by the displacement feedback controller also add higher frequency dynamics which manifests as degraded velocity and acceleration tracking. Through multi-metric feedback control, improved tracking is seen for displacement, velocity, and acceleration. In both cases, the LQG feedback controller weightings were tuned to get good performance; however different controller weighting can lead to slightly better or worse performance. Figure 6.21 presents the time history results for the 2.5 Amp pulse case, with the current switching from 0 Amps to 2.5 Amps at 3 seconds. The acceleration spikes are highest with displacement feedback control and reduced using multi-metric feedback control. Moreover, the noise in the acceleration readings demonstrates the importance of including a Kalman filter in feedback control design.
Figure 6.21. Acceleration tracking during a pulse in current

6.6 Preliminary Real-Time Hybrid Simulation Study

To illustrate the actuator controller performances in a closed-loop RTHS, a simple SDOF structure is selected. Mass, damping, and stiffness are simulated numerically while an MR damper at 0.0 Amps is used as the physical substructure. At this level of current, the MR damper can obtain approximately 20 kN restoring force, which would provide an appropriate level of control (approximately 10% of the mass) for a 20,000 kg structure. With the mass held constant, the stiffness is varied to achieve a set of structures with natural frequencies ranging from 0.5 Hz to 30 Hz. Although it is not likely that a civil engineering structure will have a single mode at such high frequencies, MDOF structures may possess modes in this range or beyond. For each structure, the damping coefficient is chosen to achieve 2% modal damping.

Each structure is excited with a BLWN ground acceleration from 0 to 50 Hz. The RMS values of the ground acceleration were chosen as 1500 mm/s$^2$ for the 0.5, 1, 5, and 10 Hz
structures and 2500 mm/s² for the 20 and 30 Hz structures. These RMS values are chosen to provide a safe level of excitation while achieving a response significantly above the noise floor of the measurement devices. Each structure is tested using no compensation, the polynomial extrapolation technique, the lead compensator, the model-based feedforward controller with a low-pass filter, the proposed model-based feedforward controller, the proposed model-based controller with displacement feedback, and the proposed model-based controller with multi-metric feedback. Since the specimen conditions are unchanging, all controllers are designed for 0.0 Amps in the MR damper. Numerical integration is performed using the CDM at 2000 Hz. Results are presented in Fig. 6.22 with each experiment summarized by the RMS error between the desired and measured displacement.

![Figure 6.22. RMS error in displacement for RTHS of SDOF structure](image)

All real-time actuator control schemes provide improved tracking when compared to the uncompensated case, except for the polynomial extrapolation for the 30 Hz structure. In this case, the response became amplified so greatly that the experiment was unsafe to continue. The polynomial extrapolation and lead compensators are not accurate in magnitude or phase at higher
frequencies, leading to poor performance in RTHS. The model-based feedforward controller with a low-pass filter works well, however the added filter dynamics detract from controller performance at high frequencies. The proposed model-based feedforward controller exhibits the best results over a wide range of frequencies. Thus, if a structure exhibits higher frequency responses, the proposed method would be able to provide the best tracking and avoid instability. Adding model-based displacement or multi-metric feedback control further improves displacement tracking performance at lower frequencies (as was designed using the LQG shaping filter). However, since the specimen conditions are unchanging, the proposed model-based feedforward controller alone performs very well.

Looking at the velocity instead (see Fig. 6.23), model-based approaches provide the best velocity tracking performance. Model-based displacement feedback control slightly detracts from velocity tracking compared to the feedforward controller alone, while multi-metric feedback control improves velocity tracking. As before, velocities are calculated by differentiating the displacements.

![Figure 6.23. RMS error in velocity for RTHS of SDOF structure](image)

Figure 6.23. RMS error in velocity for RTHS of SDOF structure
Looking at the acceleration tracking in Fig. 6.24, the model-based multi-metric feedback controller is seen to improve the acceleration tracking over the entire frequency range beyond the feedforward controller alone. On the other hand, displacement feedback control degrades acceleration tracking. With multi-metric feedback, the balance between good displacement and acceleration tracking can be adjusted through the LQG controller gains to suit the control objectives. The model-based feedforward controller with a low-pass filter performs the best, however has worse displacement and velocity tracking than the proposed model-based controllers. As before, the acceleration measurements have been post-processed using a low-pass filter with a 50 Hz cutoff frequency.

![Figure 6.24. RMS error in acceleration for RTHS of SDOF structure](image)

**Figure 6.24. RMS error in acceleration for RTHS of SDOF structure**

### 6.7 Real-Time Hybrid Simulation of a Semi-Actively Controlled Structure

To verify the model-based actuator control strategy for large-scale RTHS, a well-researched nine-story steel frame benchmark shear building is chosen (Ohtori *et al.*, 1994). This structure was designed to meet seismic code and represent a typical medium-rise building in Los Angeles, California. This structure has five bays in both the NS and EW directions. The NS lateral load
system consists of two identical moment resisting frames as shown in Fig. 6.25. For this study, a linear model of one of these NS moment resisting frames is used with half of the total seismic mass of the structure and excited in the NS direction.

The natural frequencies of the structure corresponding to the first five modes are 0.443, 1.18, 2.05, 3.09, and 4.27 Hz, respectively, with a maximum natural frequency of 63.6 Hz for the 29th mode. All modes are assumed to have 2% damping.

![Figure 6.25. Elevation view of nine-story structure (Ohtori et al., 1994)](image_url)

Structural control provided by MR dampers (added to the structure for this study) is assumed to keep response of the structure in the linear range for the earthquakes investigated. In this RTHS, the MR damper is represented by a physical specimen, while the rest of the structure is simulated numerically. The seismic mass that each NS moment frame must resist is $4.50 \times 10^6$ kg which is equivalent to 44,100 kN. A reasonable level of control can be achieved with about
10% of this force, or 4410 kN. Because a 200 kN MR damper is available as the physical specimen, 18 of these devices are assumed to be used in conjunction with the moment frame to resist lateral loads. MR dampers with higher capacities have been developed, so it is possible to reduce the number of dampers in a physical implementation of this study. All 18 devices will be placed between the ground and the first story. By doing so, the need to test multiple devices is eliminated as the force from one MR damper can be used to approximate all 18.

The structure is assumed to be equipped with sensors measuring the absolute story accelerations in the first, third, fifth, seventh, and ninth floor, the MR damper displacement, and the MR damper force. These measurements are available to the semi-active controller for use in determining the input current to the MR damper. Two passive controllers are considered in addition to one semi-active controller. In the passive controllers, the input current is maintained at 0.0 or 2.5 Amps for passive-off and passive-on, respectively. The semi-active control is based on the clipped-optimal control algorithm (Dyke et al., 1996) with equal acceleration weighting on all stories paired with very low weighting of the MR damper force. These weightings achieve good semi-active control results in simulation over wide range of earthquake records.

Reference earthquake ground motions from this benchmark study are used throughout this dissertation. These include: (a) the NS component of the Imperial Valley Irrigation District substation in El Centro, California during the El Centro earthquake of May 18th, 1940, (b) the NS component of the Hachinohe City record during the Tokachi-Oki earthquake of May 16th, 1968, (c) the NS component of the Sylmar County Hospital parking lot in Sylmar, California during the Northridge earthquake of January 17th, 1994, and (d) the NS component of the Japanese Meteorological Agency station during the Kobe earthquake of January 17th, 1995. The first 30 seconds of each record are shown in Fig. 6.26.
RTHS is used to evaluate the response of the nine-story structure subjected to the NS component of the 1940 El Centro earthquake with a scale factor of 0.5 (PGA 0.174 g). The numerical model, structural control algorithm, and real-time actuator control techniques are implemented in Simulink. A sampling rate of 2000 Hz is found adequate at achieving both numerical integration accuracy (using the CDM) and accuracy of the applied velocity to the MR damper.

Results from the RTHS are presented for the physical specimen in passive-off, passive-on and semi-active control modes in Fig. 6.27 through Fig. 6.29. These figures show the time histories of the displacement and force of the MR damper, the ninth-story acceleration, as well as the force-displacement hysteresis and the force-velocity hysteresis of the MR damper. Numerical
simulation results are also presented using the proposed phenomenological model of Chapter 4 to represent the physical MR damper.

The proposed model-based actuator control strategy for this application (three-pole transfer function model with the CDM and linear acceleration extrapolation) requires an extrapolation of the acceleration followed by a prediction of velocity. The RMS errors between the extrapolated acceleration and the actual acceleration one time step later are 1.33%, 2.23%, and 1.81% for the passive-off, passive-on, and semi-active control cases. The RMS errors between the predicted velocity and actual velocity one time step later are 0.0029%, 0.0119%, and 0.0061% for the passive-off, passive-on, and semi-active control cases. The low RMS error indicates that the extrapolated and predicted values provide accurate estimates toward implementing an improper inverse. In all cases, the same model-based controller is used (i.e., feedforward controller based on average specimen conditions with a feedback controller).

The results for passive-off control are presented in Fig. 6.27. While the RMS error is 3.05% without compensation and 0.381% with model-based feedforward-feedback control, the two results from the RTHS are quite similar. This close agreement is due to the fact that in passive-off control, the restoring force returned to the numerical substructure is relatively small. Thus, even if the restoring force has some time lag, it has little influence on the overall structural response. Also, the MR damper naturally adds some damping to the system which can counteract the negative damping included by the time lag. Simulations match the RTHS well, indicating that the MR damper model is doing a good job capturing the MR damper nonlinearities and providing confidence in the results.
Figure 6.27. MR Damper response using passive-off control

The results for passive-on control are presented in Fig. 6.28. Unlike passive-off control, the RTHS could not be completed in the absence of compensation due to large, unsafe oscillations in the servo-hydraulic actuator. In lieu of uncompensated results, results using polynomial extrapolation based on a 10 msec delay are presented. The force time history shows that for the polynomial extrapolation, some high-frequency oscillations are introduced because of the poor compensation provided at the higher frequencies. These oscillations are also present in the displacement time history, but much less apparent. Model-based feedforward-feedback control exhibits excellent performance and does not introduce high-frequency oscillations. The RMS errors are 1.22% and 0.571% for polynomial extrapolation and model-based cases,
respectively. The simulation matches the RTHS well, showing that the MR damper model also replicates the physical MR damper behavior for passive-on conditions.

Figure 6.28. MR Damper response using passive-on control

Accurate real-time actuator control is critical for passive-on control, which is counterintuitive, as passive-on control introduces more damping to the system than passive-off control. However, with the increase in damping also comes an increase in stiffness. At very small displacements (e.g. from 32 to 42 seconds in Fig. 6.28) the MR damper behaves more like a spring, because the MR fluid is not yielding and the frame is undergoing elastic deformation. Higher stiffness leads to more negative damping in the presence of time lag (Horiuchi et al., 1996). When the MR damper starts to move more significantly under the earthquake load, the additional damping provided by the MR damper helps to stabilize the oscillations. For the same
reason, the oscillations do not grow without bound. The more the MR damper oscillates, the more positive damping is added to the system, stabilizing it. However, these oscillations can be damaging to the servo-hydraulic equipment. Also if the oscillations occur at a lightly damped mode or a mode significant to the structural response, the RTHS accuracy would be greatly reduced.

In the final structural controller explored, the MR damper current was allowed to vary using the semi-active clipped-optimal control scheme, with results presented in Fig. 6.29. As with the passive-on case, the RTHS quickly became unstable in the absence of compensation. Semi-active control switches the specimen conditions between the extremes very quickly, adding high-frequency dynamics to the structure. The polynomial extrapolation technique handles these additional dynamics poorly, leading toward high amplitude oscillations in the force, most apparent in the hysteresis. On the other hand, results are similar for both the model-based bumpless transfer controller (Carrion and Spencer, 2007) and proposed model-based feedforward-feedback controller. Small oscillations in the force are apparent with the bumpless transfer controller, which are likely due to the low-pass filter adding phase lag to the controller at these frequencies. The RMS errors are 1.02%, 0.302% and 0.379% for polynomial extrapolation, bumpless transfer, and feedforward-feedback controller, respectively.

As with the other structural control cases (passive-off and passive-on), the numerical simulation matches the RTHS well. The differences can be attributed to the difficulty in modeling the behavior of the MR damper under changing current, as well as the semi-active control affecting future control efforts. These challenges aside, the model provides a good comparison even for the semi-active case and more importantly, a useful tool for semi-active controller design.
In regard to the performance of the MR damper as a semi-active device, a reduction in top story acceleration is seen when compared to the passive-on case. The maximum acceleration drops from 0.250 g to 0.203 g while the RMS acceleration drops from 0.0520 g to 0.0425 g. At the same time, the maximum control force decreases from 162 kN to 118 kN. Semi-active control is seen to be an effective means to balance good displacement and acceleration performance (of the structure) under a wide range of input loads.
To further investigate the proposed model-based controller, the semi-active case is repeated for feedforward control alone, feedforward with displacement feedback control, and feedforward with multi-metric feedback control. The results are shown in Fig. 6.30. The RMS errors in displacement are 0.514%, 0.404%, and 0.400% for each controller, respectively. Feedback control slightly improves the tracking performance; however, since the feedforward controller alone provides excellent control for this structure, the overall results are very similar.

Figure 6.30. Comparison of feedback controllers during RTHS
The RTHS conducted was an especially challenging case for actuator control. This difficulty arises from the fact that the building is lightly damped at high frequencies and the CDM adds no numerical damping. Light damping brings the structure closer to instability when coupled inadequately compensated time delay and time lag. At the same time, a single physical MR damper is used to represent 18 devices. Any measurement noise in the dampers is amplified and completely correlated. Also, the harmful effects of time delay and time lag, such as negative damping, are concentrated in one location and amplified. In spite of these challenges, model-based actuator control provided excellent results in RTHS.

Also, note that the responses of the overall structure do not change much with the compensation methods explored. The most significant effects are local to the MR damper, namely stability of the physical experiment and undesired oscillations at floors connected to the MR damper. However, these higher frequency oscillations are not significant to the overall response of the structure (in this case) and do not travel far from the source. It is worth mentioning that if the MR damper placement were different, the load cell measurement noise and any destabilizing negative damping would enter different stories of the structure and thus affect difference modes.

6.8 Summary

A framework for RTHS has been developed at the University of Illinois, including state-of-the-art software and hardware. The dynamic characterization shows that the system is capable of fast loading rates required for RTHS. The accuracy of actuator control, critical to RTHS, is improved with the proposed model-based controller. With predefined displacements, results showed near perfect tracking of the desired displacement signal. Multi-metric feedback control was demonstrated as a means to balance good displacement, velocity, and acceleration tracking. In
RTHS, the proposed model-based controller was proven successful for testing SDOF structures in a parametric study and a lightly damped MDOF structure, both using a 200 kN MR damper as the physical substructure. In the SDOF test, the proposed model-based controller provided the best tracking among the methods considered, especially when the natural frequency of the structure exceeded 5 Hz. In the MDOF test, the current in the MR damper was allowed to vary under semi-active control. Even under these changing specimen conditions, the proposed model-based controller showed excellent performance. Numerical simulation results compare well to RTHS, proving confidence in RTHS results.
CHAPTER 7     MULTI-ACTUATOR CONTROL

The proposed model-based actuator control strategy is flexible to accommodate multi-actuator systems. However, few experimental facilities are capable of multi-actuator RTHS. For this reason, focus is first placed on creating a simulated RTHS based on a well-researched three-story structure. The simulated RTHS has all of the components shown in Fig. 5.1, including the dynamics of the servo-hydraulic system. The multi-actuator control strategy is subsequently verified for a large-scale three-story steel frame specimen, which is part of a larger project on performance based design using semi-active control devices. For the study on multi-actuator control, feedback approaches are restricted to displacement-based feedback controllers for simplicity.

7.1     Multi-Actuator Nonlinear Numerical Study

To demonstrate the performance of the proposed model-based multi-actuator control strategy, a three-story semi-actively controlled building is considered. For simplicity, all DOF will be selected as interface DOF (having both numerical and experimental components). The experimental component is selected as the small-scale three-story building model from multiple studies on active and semi-active control (Dyke et al., 1995; Dyke et al., 1996). The simplified model parameters, as reported from system identification, are given by:

\[
\begin{align*}
\mathbf{M}^E &= \begin{bmatrix} 98.3 & 0 & 0 \\ 0 & 98.3 & 0 \\ 0 & 0 & 98.3 \end{bmatrix} \times 10^{-6} \text{ kN} \cdot \text{s}^2 / \text{mm} \\
\mathbf{C}^E &= \begin{bmatrix} 175 & -50 & 0 \\ -50 & 100 & -50 \\ 0 & -50 & 50 \end{bmatrix} \times 10^{-6} \text{ kN} \cdot \text{s} / \text{mm}
\end{align*}
\]
\[
\mathbf{K}^E = \begin{bmatrix}
1.20 & -0.684 & 0 \\
-0.684 & 1.37 & -0.684 \\
0 & -0.684 & 0.684
\end{bmatrix} \text{kN/mm} \tag{7.3}
\]

The corresponding natural frequencies are 5.46, 15.8, and 23.6 Hz, with damping ratios of 0.31, 0.62, and 0.63%. A numerical component is added with a mass matrix equal to 9 times the mass matrix of Eqn. (7.1), bringing the natural frequencies of the total structure (combining numerical and experimental components) to 1.73, 5.00, and 7.48 Hz. Rayleigh damping is added to the total structure to create damping ratios of 1.00, 1.00, and 1.57%. The additional damping required to achieve these damping ratios is added numerically. Finally, a small-scale MR damper is added between the ground and first story of the structure. This MR damper is considered part of the experimental component and modeled using the phenomenological model and parameters proposed by Spencer et al. (1997). The MR damper has a maximum force of approximately 1.5 kN, which is about 5% of the seismic mass of the total structure. The numerical and experimental components are illustrated in Fig. 7.1.

![Figure 7.1. Three-story nonlinear structure](image_url)
Servo-hydraulic actuators are connected to each of the three floors of the experimental structure to enforce compatibility with the numerical component and provide restoring force feedback from the load cells. The servo-hydraulic system parameters for all three actuators are based on the small-scale actuator model of Dyke et al. (1995). These parameters are \( p_a = -66.7 \text{ rad/sec} \), \( k_a = 50.1A \), and \( k_a = 83.3/A \). From the original parameters, \( k_a \) has been multiplied by 3 and \( k_a \) divided by 3 to scale the actuator model appropriately for the configuration of Fig. 7.1.

The experimental component is assumed to be equipped with sensors measuring the actuator displacements, the actuator restoring forces, the absolute story accelerations, the MR damper displacement, and the MR damper restoring force. Measurement noise is simulated with a 0 to 2000 Hz BLWN with RMS values of 180 mm/s\(^2\) for acceleration, 0.0254 mm for displacement, and 0.005 kN for force. These values are chosen by examining the noise floor for appropriate sensors found in the Smart Structures Technology Laboratory at the University of Illinois. The absolute story accelerations, the MR damper displacement, and the MR damper force are available to a semi-active controller for use in determining the input voltage to the MR damper. A semi-active controller is created based on a clipped-optimal control algorithm and controller weightings from Dyke et al. (1996). The components of the numerically simulated RTHS are presented in the Simulink block diagram of Fig. 7.2.

![Simulink diagram of simulated RTHS](image.png)
7.1.1 MIMO System Identification and Controller Design

In the likely case that the parameters of the specimen and servo-hydraulic system are unknown, nonparametric system identification can be used to obtain the servo-hydraulic system transfer function model (Kim et al., 2005). As indicated previously, the servo-hydraulic system transfer function model of the MIMO system has many poles and zeros to fit, whereas the inverse has relatively few poles and zeros to fit. The simplicity of the inverse is the basis for the proposed system identification method for model-based multi-actuator control.

Step 1: Determine the experimental MIMO transfer function. The first step is to conduct system identification on coupled actuator system attached to specimen. One actuator should excite the specimen with a BLWN (over the frequency range of interest) and the response be measured at all actuators. The process should be repeated for each actuator; the MIMO transfer function will thus be built one input at a time. During each test, the unexcited actuators should either be held at zero displacement or given a very low-amplitude BLWN to overcome static friction forces which can add damping to the system (Chang, 2011). In the case when a user-controllable device is attached to the specimen, such as an MR damper, the device should be acting as it would during RTHS to create the most accurate linearized model for RTHS.

Step 2: MIMO transfer function inversion. At this step, the experimental MIMO transfer function should be inverted. The operation will be a matrix inversion at each frequency.

Step 3: Fitting the inverse. Next, each input-output pair of the inverse MIMO transfer function should be fit with a SISO transfer function model. The SISO transfer function models can then be combined to create an inverse MIMO transfer function model, which can be used as the feedforward controller. Insight from Eqn. (5.76) can aid in the model fitting.
Step 4: Creating the servo-hydraulic system transfer function model. The inverse of the inverse MIMO transfer function model will be equal to the servo-hydraulic system transfer function model. This model, in state-space form, can be used for feedback control design. Note that when a MIMO transfer function model is converted into a state-space model, it will not necessarily be a minimal realization. A minimal state-space realization contains the minimal number of states necessary to represent the system dynamics. Such a realization is also necessarily both controllable and observable. Effort should be made to create a minimal realization; methods for creating minimal realizations are discussed in Chang (2011). Equation (5.82) demonstrates that a minimal realization is possible, whereby there are no duplicate or unnecessary states and all of the states are controllable through the actuators as well as observable using load cells and displacement transducers.

The four steps above are illustrated in Fig. 7.3 and Fig. 7.4 for the experimental component of Fig. 7.2 including the modeled actuator dynamics. Each actuator is excited one at a time using a 0 to 50 Hz BLWN for a total of three data sets. During this excitation, the other actuators are held at zero displacement (since the phenomenon of static friction is not included in the numerical actuator model). Also, the small-scale MR damper model is randomly switched from 0.0 to 2.0 Amps (0.0 V to 2.25 V) to simulate semi-active conditions during RTHS. The fitted feedforward model contains three zeros in each of the diagonals, one zero in each of the immediate off-diagonals, and no dynamics for the extreme off-diagonals. The resulting servo-hydraulic system model contains six zeros and nine poles in each of the diagonals, four zeros and nine poles in each of the immediate off-diagonals, and two zeros and nine poles in the extreme off-diagonals.
Figure 7.3. MIMO transfer function magnitude of the 3DOF experimental substructure

Figure 7.4. MIMO transfer function phase of the 3DOF experimental substructure
Figure 7.3 and Fig. 7.4 illustrate the accuracy of the models compared to the numerically simulated data. In the figures, columns one through three correspond to inputs to actuators one through three while rows one through three correspond to outputs from actuators one through three. Model-based control is developed using the inverse model and system model for the feedforward and feedback controller, respectively.

7.1.2 RTHS of MDOF Structure

RTHS is used to evaluate the response of the three-story nonlinear structure employing semi-active control subjected to 0.5x the NS component of the 1940 El Centro earthquake. The simulation is run at 2000 Hz using the fourth-order Runge-Kutta scheme for numerical integration. Both the numerical and experimental components are simulated numerically using MATLAB’s Simulink environment with the effects of actuator dynamics included (as in Fig. 5.1 and Fig. 7.2). Simulated measurement noise is included in all feedback loops (e.g., restoring force of experimental component, measured displacement for model-based feedback control, and input for the semi-active controller) with values as previously mentioned. This noise is not included in the sampled measurements used for post-processing, equivalent to perfect filtering of the noise.

Six cases are considered to evaluate the structural response: (a) idealized simulation (i.e., no actuator dynamics, substructuring, or measurement noise), (b) RTHS with actuator dynamics and no compensation, (c) RTHS with actuator dynamics and model-based feedforward control neglecting actuator coupling, (d) RTHS with actuator dynamics and model-based feedforward control considering actuator coupling, (e) RTHS with actuator dynamics and model-based feedforward-feedback control neglecting actuator coupling, and (f) RTHS with actuator dynamics and model-based feedforward-feedback control considering actuator coupling.
The simulation case (a) is considered the correct results from which a comparison of RTHS cases (b) through (f) will be made. For case (b), the RTHS immediately went unstable, illustrating the need for actuator control in the presence of actuator dynamics. As a representative case, (f) is presented alongside case (a) in Fig. 7.5 for displacement and absolute acceleration of the first story, as well as MR damper hysteresis loops. Excellent correlation between the two cases is observed for all quantities.

Figure 7.5. First story time histories and MR damper hysteresis

Graphically distinguishing cases (c) through (f) is difficult. Therefore, RMS error will be used as a quantitative measure of actuator controller performance:

\[
\text{RMS error} = \sqrt{\frac{\sum_{i}^{N} (r_i - y_i)^2}{\sum_{i}^{N} (y_i)^2}} \times 100 \%
\]

(7.4)
where \( i \) is the time step of numerical integration performed over \( N \) steps. Comparisons are made for cases (c) through (f) in Table 7.1 for both tracking error and response error. To calculate tracking error from Eqn. (7.4), \( r = x^1 \) and \( y = x^E \) (from the same case at the same story). Tracking error illustrates how well the actuator controller performs physically tracking the desired displacements. To calculate response error from Eqn. (7.4), \( r \) is the response quantity from case (a) and \( y \) is the response quantity from the other cases (at the same story). Response quantity errors illustrate how much the RTHS solution is diverging from the ideal simulation solution. With semi-active control, where future control efforts depend on past responses, solutions can diverge quickly due to small differences.

### Table 7.1. RMS Error of Tracking and Response for Actuator Control Strategies

<table>
<thead>
<tr>
<th>Actuator Control Strategy</th>
<th>Tracking Error (%)</th>
<th>Response Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( e_1 )</td>
<td>( e_2 )</td>
</tr>
<tr>
<td>(c) FF w/o Coupling</td>
<td>1.71</td>
<td>1.99</td>
</tr>
<tr>
<td>(d) FF w/ Coupling</td>
<td>0.133</td>
<td>0.009</td>
</tr>
<tr>
<td>(e) FF + FB w/o Coupling</td>
<td>0.470</td>
<td>0.547</td>
</tr>
<tr>
<td>(f) FF + FB w/ Coupling</td>
<td>0.045</td>
<td>0.003</td>
</tr>
</tbody>
</table>

In this study, the effect of actuator coupling is investigated along with the benefits of feedback control. In cases when actuator coupling is neglected (i.e., ignoring off-diagonal terms of Eqns. (5.76) and (5.75) for feedforward and feedback controller designs, respectively), appreciable tracking error is found, leading to large response error. On the other hand, considering actuator coupling when designing multi-actuator control improves the accuracy of the RTHS as measured by both tracking error and response error. As the amount of actuator coupling increases, for example due to a stiffer specimen relative to the actuator capacity, the benefits of considering the coupling for control design will also increase. In all cases, feedback
control improves the accuracy of the RTHS compared to feedforward control alone. Because the feedforward controller is based on a linear model of the servo-hydraulic system, the feedback controller will add robustness to changing specimen conditions, modeling errors, and nonlinearities (all of which are present in this numerical study).

### 7.2 Large-Scale Multi-Actuator Experimental Framework

An ongoing study on performance based design using semi-active control will be leveraged to evaluate the proposed model-based actuator control strategy for RTHS on a physical specimen. Advanced energy dissipation devices such as MR dampers have great potential toward mitigating damage as a part of seismic resistance system. However, a lack of procedures appropriate for incorporating MR dampers in design limits their acceptance and use in civil infrastructure. At the same time, large-scale experimental verification is required to confirm the performance structures designed with MR dampers. Testing of a lateral load resisting system with MR dampers requires real-time experimental evaluation, naturally lending itself to RTHS.

A three-story prototype building has been designed using performance based design incorporating MR dampers in the lateral load resisting system (Dong, 2013; see Fig. 7.6).

![Figure 7.6. Prototype structure for RTHS](image-url)
To evaluate the design and semi-active control algorithms, a moment resisting frame (MRF) and damped braced frame (DBF) will be physically constructed, representing the lateral system for a quarter of the total tributary seismic area in one direction. The remaining components within the tributary seismic area, namely the seismic mass, are simulated numerically as a lean-on column.

7.2.1 Equipment

An experimental framework for large-scale RTHS has been developed at the Real-Time Multi-Directional (RTMD) testing facility at Lehigh University. Each floor of frame is excited by servo-hydraulic actuator manufactured by Servotest Systems Ltd.; the first story uses a model 200-100-1700 with a 2300 kN (501 kips) capacity and ±500 mm (±19.7 in) stroke while the second and third story use a model 200-1000-1250 with a 1700 kN (382 kips) capacity and ±500 mm (±19.7 in) stroke. The actuators contain hydrostatic bearings to reduce friction and are configurable to support one to three servo-valves. Each actuator is powered by two three-stage model SV1200 servo-valves manufactured by Servotest Systems Ltd. The servo-valves have a maximum individual flow rate of 2082 lpm (550 gpm), a model G772-204 Moog pilot value, and their own model B550-3412 hydraulic service manifold manufactured by Servotest Systems Ltd. The hydraulic oil supply consists of five 450 lpm (119 gpm) pumps and 16 accumulators with 190 liters (50.2 gallons) capacity connected to 9 Nitrogen gas bottles of 1325 liters (850 gallons) capacity manufactured by Parker Hannifin Corporation.

The actuators are controlled in displacement feedback mode by a model DCS 2000 digital servo-controller manufactured by Servotest Systems Ltd. The controller runs with a clock speed of 1024 Hz and has a 16-bit resolution on the A/D and D/A cards. The servo-controller consists of a DSP real-time control card (Module 2201) connected to a computer identified as RTMDCtrl. The numerical component of the RTHS, outer-loop actuator control, and semi-active
control algorithms are programmed in MATLAB’s Simulink programming language on a computer identified as RTMDsim. The Simulink file is compiled and downloaded to an xPC computer that runs Mathwork’s real-time Target PC software (identified as RTMDxPC). The RTHS is controlled by the user on the RTMDsim computer through an RTMDxPC module. RTMDxPC communicates with RTMDctrl in real-time (at 1024 Hz) over SCRAMNet which is a proprietary shared memory bus that serves as the underlying communication mechanism between RTMD modules. Data is collected using a DAS 6000 DAQ system manufactured by Pacific Instruments Inc., operating as RTMDdaq.

### 7.2.2 Specimen

For a multi-actuator control proof-of-concept study, a simplified linear 3DOF model is created to capture the dynamics of the MRF, DBF, and lean-on column. The natural frequencies of this simplified model are 1.27, 4.04, and 8.28 Hz with assumed modal damping of 3%, 6%, and 6%, respectively. The corresponding mass, damping, and stiffness matrices are:

$$
M = \begin{bmatrix}
102.0 & 0 & 0 \\
0 & 102.0 & 0 \\
0 & 0 & 73.96 \\
\end{bmatrix} \text{kN} \cdot \text{s}^2/\text{m} \quad (7.5)
$$

$$
C = \begin{bmatrix}
461.6 & -190.2 & 8.151 \\
-190.2 & 353.1 & -148.1 \\
8.151 & -148.1 & 131.8 \\
\end{bmatrix} \text{kN} \cdot \text{s}/\text{m} \quad (7.6)
$$

$$
K = \begin{bmatrix}
1.720 & -1.065 & 0.2318 \\
-1.065 & 1.318 & -0.5193 \\
0.2318 & -0.5193 & 0.3216 \\
\end{bmatrix} \times 10^5 \text{kN}/\text{m} \quad (7.7)
$$

The DBF has been constructed first and will be taken as the experimental substructure while the remaining components (moment resisting frame and lean-on column) will be simulated numerically. A schematic of the DBF with member sizes is shown in Fig. 7.7.
The DBF is capable of housing one MR damper at each story; however for this study only one MR damper is installed at the bracing of the first story. The MR damper specimen is an identical model to the large-scale 200 kN MR damper identified in Chapter 4 and used in the single-actuator study of Chapter 6.

The DBF is constructed within an outer support frame designed to prevent out of plane deformation (see Fig. 7.8). Hydraulic actuators are connected to each of the stories using loading beams with multiple connections to distribute the applied force across the entire floor. The MR damper can be seen installed in the first story bracing.

The mass, damping, and stiffness matrices of the DBF will be estimated and subtracted from the total system to determine the remaining component to be simulated numerically in RTHS. The mass of the DBF, including loading beams and other fixturing, is calculated and lumped at each of the stories.

\[
M^E = \begin{bmatrix}
3.147 & 0 & 0 \\
0 & 3.147 & 0 \\
0 & 0 & 3.147 \\
\end{bmatrix} \text{ kN} \cdot \text{s}^2/\text{m} \quad (7.8)
\]

The static stiffness matrix is determined experimentally using the attached hydraulic actuators and load cells.

\[
K^E = \begin{bmatrix}
6.407 & -4.107 & 0.9937 \\
-4.107 & 5.034 & -2.119 \\
0.9937 & -2.119 & 1.306 \\
\end{bmatrix} \times 10^4 \text{ kN/m} \quad (7.9)
\]

Finally, a damping matrix is selected to match the transient results of preliminary dynamic testing.

\[
C^E = \begin{bmatrix}
83.13 & -33.84 & 3.225 \\
-33.84 & 67.16 & -26.10 \\
3.225 & -26.10 & 30.85 \\
\end{bmatrix} \text{ kN} \cdot \text{s}/\text{m} \quad (7.10)
\]
Figure 7.7. Three-story DBF with MR dampers
7.2.3 System Identification and Model-Based Control

System identification is performed on the multi-actuator system using a 0 to 25 Hz BLWN. Each actuator is excited one at a time, creating three SIMO systems that are assembled into a MIMO transfer function matrix. The actuators are excited to a maximum safe range, which is determined to be 0.8 mm, 0.6 mm, and 0.7 mm RMS in the first, second, and third actuator, respectively. During system identification, the current to the MR damper is switched from 0.0 to 2.5 Amps, creating a linearized model for semi-active conditions. The MIMO transfer function and its inverse are presented for magnitude in Fig. 7.9 and for phase in Fig. 7.10. System identification is performed using the same approach as the MIMO numerical example.
Figure 7.9. MIMO transfer function magnitude of the DBF

Figure 7.10. MIMO transfer function phase of the DBF
The inverse model contains three zeros in each of the diagonals and one zero in each of the off-diagonals. The feedforward controller is taken as the inverse model. The system model contains nine poles in each entry, six zeros in the diagonals, and four zeros in the off-diagonals. The feedback controller is designed based on the system model. Model-based controllers are created that both consider actuator coupling (i.e., including the off-diagonal terms) and neglect actuator coupling (i.e., ignoring the off-diagonal terms).

7.2.4 Tracking Performance

The tracking performance of the proposed model-based multi-actuator controller is evaluated by simultaneously exciting each actuator with a 0 to 5 Hz BLWN with an RMS of 1 mm. The low RMS value is chosen to avoid damage to the frame in the presence of asynchronous actuator motion. The tracking performance without compensation as well as for the proposed model-based feedforward-feedback approach with actuator coupling is shown in Fig. 7.11. The first, second, and third story actuators are considered actuators one, two, and three, respectively. In all tracking exercises, the current command is predefined, varying as it would during semi-active control. The proposed controller performs very well, matching almost exactly with the desired displacement signal in each actuators. The tracking exercise is repeated for a 0 to 15 Hz BLWN in each actuator with an RMS of 0.5 mm. Results are presented without compensation as well as for the proposed model-based feedforward-feedback approach with actuator coupling in Fig. 7.12. The controller has more difficulty tracking the higher frequency signal, but still performs well in both magnitude and phase.
Figure 7.11. Displacement tracking of 0 to 5 Hz BLWN

The tracking exercises are repeated for multiple controller designs, with a summary of the RMS errors in Table 7.2.

Table 7.2. Displacement Tracking Performance for Predefined Displacements

<table>
<thead>
<tr>
<th>Actuator Control Strategy</th>
<th>RMS Tracking Error (%)</th>
<th>0 to 5 Hz BLWN</th>
<th>0 to 15 Hz BLWN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Disp 1</td>
<td>Disp 2</td>
</tr>
<tr>
<td>No Compensation</td>
<td></td>
<td>44.8</td>
<td>47.8</td>
</tr>
<tr>
<td>FF w/o Coupling</td>
<td></td>
<td>4.13</td>
<td>5.09</td>
</tr>
<tr>
<td>FF w/ Coupling</td>
<td></td>
<td>3.85</td>
<td>4.41</td>
</tr>
<tr>
<td>FF + FB w/o Coupling</td>
<td></td>
<td>3.75</td>
<td>4.90</td>
</tr>
<tr>
<td>FF + FB w/ Coupling</td>
<td></td>
<td>3.75</td>
<td>4.43</td>
</tr>
</tbody>
</table>
From Table 7.2, the model-based feedforward-feedback control considering actuator coupling achieves the best tracking performance; however the benefits are not as noticeable as in the numerical simulation study. In numerical simulation, even for low levels of actuator coupling, improvement can clearly be seen when considering actuator coupling in controller design. However, at low levels of actuator coupling in the presence of experimental error, the benefits are reduced.

**Figure 7.12. Displacement tracking of 0 to 15 Hz BLWN**

The tracking performances of the actuator controllers are further investigated for actuator motion more representative of a RTHS. With the proposed large-scale phenomenological model of Chapter 4 representing the MR damper, the response of the total structure is simulated in RTHS offline. For this simulated RTHS, the structure is excited using the NS component of the
Hachinohe City record of the 1968 Tokachi-Oki earthquake (see Fig. 6.26) with a scale factor of 0.25 (PGA 0.057 g). The displacements determined from numerical simulation are imposed on DBF as a predefined input. For MR damper control, a semi-active control algorithm designed based on the clipped-optimal control algorithm (Dyke et al., 1996) is used with acceleration weighting on the top story and low weighting on the MR damper force. The MR damper current command from simulation is sent to the physical MR damper during the tracking exercise. Tracking of the predefined displacements are shown in Fig. 7.13.

Figure 7.13. Displacement tracking of predefined RTHS results

Tracking results for multiple controllers are presented in Table 7.3. In the previous tracking exercise, the actuator motions are asynchronous. In this case, the actuators move
together, mostly in the first mode of the structure. Thus, the actuators are not fighting each other as much and the benefits of considering coupling are further reduced. However, the tracking performance overall is excellent and consistently improved by adding model-based feedback control.

Table 7.3. Displacement Tracking Performance for Predefined RTHS

<table>
<thead>
<tr>
<th>Actuator Control Strategy</th>
<th>RMS Tracking Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Disp 1</td>
</tr>
<tr>
<td>No Compensation</td>
<td>21.8</td>
</tr>
<tr>
<td>FF w/o Coupling</td>
<td>5.69</td>
</tr>
<tr>
<td>FF w/ Coupling</td>
<td>5.59</td>
</tr>
<tr>
<td>FF + FB w/o Coupling</td>
<td>5.60</td>
</tr>
<tr>
<td>FF + FB w/ Coupling</td>
<td>5.48</td>
</tr>
</tbody>
</table>

7.2.5 Real-Time Hybrid Simulation

RTHS is chosen to evaluate the dynamic performance of the DBF under seismic loading. For a proof-of-concept study on actuator control, the DBF alone is taken as the experimental substructure while the rest of the structure is simulated numerically. Numerical integration is performed using the CDM at 1024 Hz. The semi-active control scheme described for the predefined RTHS tracking study is again used for RTHS. For comparison, numerical simulation of the RTHS is performed using the total structure along with the phenomenological model of the MR damper.

The structure is excited using the NS component of the 1940 El Centro earthquake with a scale factor of 0.2 (PGA 0.070 g). The excitation is chosen to avoid yielding the DBF such that multiple tests could be safely conducted. Although the DBF remains nominally linear, nonlinearity is introduced to the structure through the MR damper. Results are presented for the actuator displacements, MR damper current command, MR damper force-displacement
hysteresis and MR damper force-velocity hysteresis in Fig. 7.14 using the model-based feedforward-feedback controller with actuator coupling. The RTHS results compare very well with simulation although further improvement is expected with a more rigorous numerical model of the DBF including modeling of the bracing to which the MR damper is attached.

**Figure 7.14. RTHS of prototype structure for El Centro**

The tracking performances for multiple controllers in RTHS are presented in Table 7.4. Overall the controllers perform very well, achieving comparable results with the numerical
Model-based feedback control is seen to improve the tracking performance beyond feedforward control alone, while considering actuator coupling has limited benefits in this case. Note that unlike predefined displacements, RTHS is not performed in the absence of outer-loop actuator control to avoid instabilities.

### Table 7.4. Displacement Tracking Performance during RTHS for El Centro

<table>
<thead>
<tr>
<th>Actuator Control Strategy</th>
<th>RMS Tracking Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Disp 1</td>
</tr>
<tr>
<td>FF w/o Coupling</td>
<td>3.83</td>
</tr>
<tr>
<td>FF + FB w/o Coupling</td>
<td>3.48</td>
</tr>
<tr>
<td>FF w/ Coupling</td>
<td>3.54</td>
</tr>
<tr>
<td>FF + FB w/ Coupling</td>
<td>3.39</td>
</tr>
</tbody>
</table>

The structure is also evaluated for the NS component of the 1994 Northridge earthquake (see Fig. 6.26) with a scale factor of 0.12 (PGA 0.10 g). Results using the model-based feedforward-feedback controller with actuator coupling are presented in Fig. 7.15. Again, the results compare very well to numerical simulation.

The tracking performances of multiple controllers are presented in Table 7.5. As with the previous earthquake record, little benefit is seen considering actuator coupling while the feedback controller does improve tracking control in most cases. Most importantly excellent tracking is achieved with the proposed model-based controller, leading to accurate and stable results.
Table 7.5. Displacement Tracking Performance during RTHS for Northridge

<table>
<thead>
<tr>
<th>Actuator Control Strategy</th>
<th>RMS Tracking Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Disp 1</td>
</tr>
<tr>
<td>FF w/o Coupling</td>
<td>3.43</td>
</tr>
<tr>
<td>FF + FB w/o Coupling</td>
<td>3.31</td>
</tr>
<tr>
<td>FF w/ Coupling</td>
<td>3.64</td>
</tr>
<tr>
<td>FF + FB w/ Coupling</td>
<td>3.42</td>
</tr>
</tbody>
</table>
7.3 Summary

A framework for model-based multi-actuator control including both model-based feedforward and feedback links has been developed to directly address actuator dynamics including actuator coupling. A simple approach to identifying the servo-hydraulic transfer function model and its inverse for designing a model-based multiple-actuator controller was outlined. The controller performed very well during the simulated RTHS of a three-story nonlinear structure. Through this example, the benefits of considering actuator coupling were demonstrated. Feedback control was shown to further improve the performance of the feedforward controller in the presence of the nonlinear MR damper device. The proposed model-based multi-actuator controller was then applied to a three-story steel frame as the experimental substructure. Excellent control was demonstrated, although the low level of actuator coupling along with experimental error led to limited benefit of including the coupling in controller design. More importantly, the RTHS results compared very well to numerical simulation, demonstrating an accurate and stable framework for multi-actuator RTHS. Numerical simulation results could further be improved by modeling the bracing to which the MR damper is attached, since there may be considerable difference between the first story displacement and MR damper displacement for higher levels of MR damper restoring force (due to deformation in the bracing).
CHAPTER 8 ACTUATOR CONTROL FOR SHAKE TABLES

Shake tables provide a direct means by which to evaluate structural performance under earthquake excitation. Essentially, a base plate is excited by an actuator to replicate historic or analytically generated ground accelerations. Because the entire structure is mounted on the base plate and subjected to the ground motion in real-time, dynamic effects and rate-dependent behavior can be represented.

The payload (including table mass) is typically large relative to the capacity of the actuator, leading to pronounced control-structure interaction. Through this interaction, the dynamics of the specimen influence the dynamics of the shake table, which can be problematic when specimens change behavior due to damage or other nonlinearities. Moreover, shake tables are inherently nonlinear, making it difficult to accurately recreate a desired acceleration record accurately over a broad frequency range. The proposed model-based multi-metric feedback control strategy will be adapted to improve tracking of the desired acceleration, remaining robust to nonlinearities including changes in specimen conditions. The proposed strategy is verified for the shake table testing of both linear and nonlinear specimens.

Shake table control is not straightforward as the desired signal is an acceleration record. Since acceleration measurements (i.e. from accelerometers) cannot capture constant velocities or constant displacements, acceleration feedback alone cannot provide stable control of a shake table. For stability, even in shake table applications where the desired record to be reproduced is an acceleration, actuators operate in displacement feedback through an inner-loop PID controller. To ensure stability of the shake table when applying model-based control to shake table testing, the inner-loop displacement feedback control will not be modified. Rather, an outer-loop
controller with model-based feedforward and feedback links is proposed to improve acceleration tracking on the stable inner-loop system.

8.1 Model-Based Control for Shake Table Testing

Model-based control provides a promising alternative for shake table testing; however, it requires modification from RTHS applications. Most notably, the signal that the controller should track is an acceleration signal, a nontrivial task for displacement feedback systems. To assure accurate acceleration tracking, an accelerometer is attached to the table for both system identification and actuator control. In this section, first a feedforward controller will be developed for acceleration tracking followed by approaches for acceleration, displacement, and multi-metric feedback control.

8.1.1 Feedforward Control

To develop a model of the shake table for feedforward control, the relationship from the input command voltage $u$ to the measured output acceleration $a_m$ is examined, represented by the transfer function model:

$$G_{au}(s) = \frac{A(s)}{U(s)}$$  \hspace{1cm} (8.1)

The feedforward controller, designed to cancel the dynamics of the shake table, is selected as the inverse of the shake table model:

$$G_{au}^{-1}(s) = \frac{U(s)}{A(s)}$$  \hspace{1cm} (8.2)

The feedforward controller provides the ideal control effort about which regulator redesign is performed, described subsequently for different available feedback measurements. An accurate model of the shake table in Eqn. (8.1), which takes a displacement based command to an acceleration measurement, will consist of two zeros at the origin. Thus, a pure inverse as in
Eqn. (8.2) will have two poles at the origin, which can lead to low-frequency drift in the feedforward command. To avoid such drift, the reference acceleration record should be passed through a high-pass filter to unobtrusively remove very low-frequency behavior from the reference record. The filtered acceleration signal should be compared with the reference acceleration prior to testing to ensure that they match well. Moreover, the feedforward control effort can be calculated completely offline, allowing for a lot of flexibility including zero-phase digital filtering of the reference acceleration and the implementation of improper model inverses.

### 8.1.2 Acceleration Feedback Control

Acceleration feedback is based on the shake table model of Eqn. (8.1). A state-space representation of Eqn. (8.1) is given by:

\[
\dot{z} = Az + Bu \tag{8.3}
\]

\[
a_m = Cz \tag{8.4}
\]

As with actuator control for RTHS, regulator redesign can be used to create a model-based feedforward-feedback controller to minimize the acceleration tracking error given by:

\[
e = a_d - a_m \tag{8.5}
\]

where the desired acceleration is given by \(a_d\). For regulator redesign, the deviation system outputs are:

\[
\tilde{a} = a_m - a_d \tag{8.6}
\]

The dynamics of the augmented deviation system including the process noise (with shaping filter) and acceleration measurement noise are then:

\[
\dot{z}_a = A_z z_a + B_z u_{FB} + E_z w_f \tag{8.7}
\]

\[
\tilde{a} = C_z z_a + v_f \tag{8.8}
\]
The LQR control design is based on acceleration output weighting with the following cost function (using the certainty equivalence property):

\[
J_{LQR} = \int_0^\infty \left[ Q_{LQR} \ddot{a}^2 + R_{LQR} u_{FB}^2 \right] dt
\]  

(8.9)

The Kalman filter can be designed after selecting process and measurement noise covariances. The total combined feedback controller will have the dynamics:

\[
\dot{\hat{z}}_a = (A - L_{Kal} C - BK_{LQR}) \hat{z}_a + L_{Kal} \ddot{a}
\]  

(8.10)

Acceleration feedback control alone is unstable due to unobservable and thus uncontrollable poles at the origin. However, the inner-loop displacement based servo-controller provides stability to the shake table. As with RTHS applications, the LQG controller process noise is shaped by a second-order filter to attenuate the control effort at frequencies beyond the region of desired tracking performance. The combined feedforward controller with acceleration feedback is presented in Fig. 8.1.

---

**Figure 8.1. Model-based acceleration feedback control**
8.1.3 Displacement Feedback Control

Alternatively, the feedback controller could be designed based on displacement measurements. A displacement based feedback controller would require a model that describes the relationship between input command voltage $u$ and output measured displacement $x_m$:

$$G_{xm}(s) = \frac{X(s)}{U(s)}$$  \hspace{1cm} (8.11)

A state-space representation of Eqn. (8.11) is given by:

$$\dot{z} = Az + Bu$$  \hspace{1cm} (8.12)

$$x_m = Cz$$  \hspace{1cm} (8.13)

Regulator redesign can be used to create a model-based feedforward-feedback controller to minimize the tracking error given by:

$$e = x_d - x_m$$  \hspace{1cm} (8.14)

where the desired displacement is given by $x_d$. Although regulator redesign is performed using the displacement tracking error described in Eqn. (8.14), the feedforward controller used to achieve the ideal system will still be based on acceleration as in Eqn. (8.2) toward the overall goal of acceleration tracking. For regulator redesign, the deviation system outputs are:

$$\tilde{x} = x_m - x_d$$  \hspace{1cm} (8.15)

The dynamics of the augmented deviation system including the process noise shaping filter and measurement noise are then:

$$\dot{\tilde{z}} = A_a \tilde{z} + B_a u_{FB} + E_a w_f$$  \hspace{1cm} (8.16)

$$\tilde{x} = C_a \tilde{z} + v_f$$  \hspace{1cm} (8.17)

where the measurement noise is in the displacement measurements. The LQR controller can be based on displacement output weighting using the cost function:
After designing a Kalman filter based on covariances for process and measurement noise, the total combined feedback controller will have the following dynamics:

\[
\dot{\tilde{z}}_a = (A - L_{\text{Kal}} C - B K_{\text{LQR}}) \dot{\tilde{z}}_a + L_{\text{Kal}} \tilde{x}
\]  

(8.19)

To implement displacement feedback control, the desired displacement is required. This displacement can be calculated from the double integration of the desired acceleration signal. Alternatively, the desired displacement can be calculated using the feedforward controller (acceleration to command) and the shake table model (command to displacement) as in Eqn. (8.20), providing a better prediction of a compatible desired displacement (Nakata, 2010). Since the acceleration signal has been passed through a high-pass filter, drift in the desired displacement will be limited.

\[
G_{xx}(s)G_{au}^{-1}(s) = \frac{X(s)U(s)}{U(s)A(s)} = \frac{X(s)}{A(s)}
\]  

(8.20)

The combined feedforward controller with displacement feedback is shown in Fig. 8.2.

Figure 8.2. Model-based displacement feedback control
8.1.4 Multi-Metric Feedback Control

By combining displacement and acceleration measurements, a multi-metric feedback approach to shake table control can be realized. A greater number of measurements can be used for better estimates of the states of the shake table system model through the Kalman filter. More importantly, displacement measurements are more sensitive in the lower frequency range while acceleration measurements are more sensitive in the higher frequency range; thus, by combining the two measurements, accurate feedback control can be achieved over a broad frequency range. The shake table system will now be modeled as a SIMO system from input command voltage to both output measured displacement and output measured acceleration.

\[
G_y(s) = U(s)^{-1} \begin{bmatrix} X(s) \\ A(s) \end{bmatrix}
\]  

(8.21)

A state-space representation of Eqn. (8.21) is given by:

\[
\dot{z} = Az + Bu
\]  

(8.22)

\[
y_m = \begin{bmatrix} x_m \\ a_m \end{bmatrix} = Cz
\]  

(8.23)

Regulator redesign can be used to create a model-based feedforward-feedback to minimize the tracking error given by:

\[
e = \begin{bmatrix} e_s \\ e_a \end{bmatrix} = \begin{bmatrix} x_d \\ a_d \end{bmatrix} - \begin{bmatrix} x_m \\ a_m \end{bmatrix} = y_d - y_m
\]  

(8.24)

From regulator redesign, the deviation system outputs are:

\[
\tilde{y} = y_m - y_d
\]  

(8.25)

The dynamics of the augmented deviation system including the process noise shaping filter and measurement noise are then:

\[
\dot{z}_a = A_a z_a + B_a u_{FB} + E_a w_f
\]  

(8.26)
\[ \bar{y} = C_a z_a + v_f \]  
\((8.27)\)

The measurement noise includes both noises in the displacement and acceleration measurements. The LQR controller can be based on output weighting including both the displacement and acceleration outputs:

\[ J_{LQR} = \int_0^\infty [\bar{y}^T Q_{LQR} \bar{y} + R_{LQR} u_{FB}^2] dt \]  
\((8.28)\)

The Kalman filter design is based on assumed process and measurement noise covariances. The total combined feedback controller is given by:

\[ \dot{\hat{z}}_a = (A - L_{Kal} C - B K_{LQR}) \hat{z}_a + L_{Kal} \bar{y} \]  
\((8.29)\)

The combined feedforward controller with multi-metric feedback is presented in Fig. 8.3.

---

**Figure 8.3. Model-based multi-metric feedback control**

For shake table control studies, four model-based controllers are considered: (a) feedforward controller (i.e., Eqn. 8.2), (b) feedforward controller with displacement feedback (i.e., Fig. 8.2), (c) feedforward controller with acceleration feedback (i.e., Fig. 8.1), and (d)
feedforward controller with multi-metric feedback (i.e., Fig. 8.3). Note that throughout this chapter, once a controller is designed to a specific model of the shake table, no further modification is performed (i.e., controllers are designed independently of the ground motion, requiring no iteration).

Reference earthquake ground motions are selected from a benchmark study on structural control (Ohtori et al., 1994; see Fig. 6.26). Both reference and high-pass filtered ground motions are shown in Fig. 8.4 for the first 30 seconds of each record.

Figure 8.4. Filtered historic ground motions
8.2 Experimental Setup

The model-based control approach for shake table testing is verified using a small-scale single axis shake table as shown in Fig. 8.5. The shake table uses a custom built servo-motor manufactured by SMI Technology to move a 46 cm × 46 cm top plate with a stroke of ±5 cm. A Quanser Consulting MultiQ-3 Board and host PC are used to control the shake table in displacement control with a PD controller. The A/D and D/A of the board are both 12-bit. Displacement feedback is provided by a digital encoder. Accelerations are measured using model 3701G3FA3G capacitive accelerometers manufactured by PCB Piezotronics. The accelerometers have a measurement range of ±3 g, a frequency range of 0-100 Hz, and a sensitivity of 1000 mV/g. For system identification, input signals are generated using a Spectral Dynamic Siglab spectrum analyzer and responses measured using an m+p international VibPilot spectrum analyzer. Model-based outer-loop controllers are implemented using a dSPACE model 1103 DSP board, the details of which are provided in Chapter 6.
8.3 Bare Shake Table

Model-based control is first explored for the bare shake table. The shake table platform adds considerable mass, which influences the dynamics of the shake table. However, without an attached structure, the phenomenon of CSI will be minimized, leading to a simpler control problem.

8.3.1 System Identification

The input-output relationship of the shake table is determined experimentally using a 0 to 20 Hz BLWN command to the shake table. Measurements of the base plate are made in both displacement and acceleration. Four levels of excitation are considered, 0.1, 0.2, 0.4, and 0.6 V RMS, to investigate the influence of excitation amplitude on the shake table dynamics. Transfer function is calculated using 2048 FFT points, a Hanning window with 50% overlap, and 20 averages. Transfer functions are shown in Fig. 8.6 and Fig. 8.7, which display the input command to displacement and acceleration transfer functions, respectively. The units for command, measured displacement, and measured accelerations in the transfer function are Volts, cm, and m/s\(^2\). It is clear that the shake table exhibits highly nonlinear behavior due to the amplitude dependency of the transfer function. At low amplitudes, there is significant friction limiting the accurate tracking of the command signal. The transfer function at 0.4 V RMS excitation represents amplitude similar to that of the reference acceleration ground motions. Therefore, the model-based controller is based on a model of the transfer function at this level of excitation. Identified SISO models are presented in Fig. 8.6 and Fig. 8.7. By design, the feedback controller will make the system robust to nonlinearities, modeling inaccuracies, and changes in the system.
The identified model from command to displacement is given as:

\[ G_{ux}(s) = \frac{0.001669(s + 60.34)(s^2 - 1059s + 8.999 \times 10^5)}{(s + 32.69)(s^2 + 43.96s + 4614)} \]  

(8.30)

The identified model from command to acceleration is given as:

\[ G_{au}(s) = \frac{12.54s^2(s + 103.9)}{(s + 46.17)(s^2 + 46.67 + 4907)} \]  

(8.31)
For the bare shake table, the goal is to verify the feasibility of acceleration feedback control. Therefore, a SIMO system is not identified and multi-metric control is not explored (though it will be when the structure is attached).

8.3.2 Tracking Performance

For this preliminary study, the El Centro earthquake record is selected as the reference acceleration. In order to investigate nonlinearities associated with the amplitude of the input motion, both 0.2x and 0.4x of the record are investigated. Controllers include feedforward control (FF), feedforward control with displacement feedback (FF + xFB), and feedforward control with acceleration feedback (FF + aFB). Results are shown in Fig. 8.8 and Fig. 8.9 for 0.2x El Centro and 0.4x El Centro, respectively. Errors are reported as the RMS difference between the measured and desired accelerations (or measured and desired displacements) for the first 25 seconds of response. Note that the acceleration results have been filtered by a low-pass filter with a cutoff frequency of 15 Hz, which is well above the maximum natural frequency of the specimen that is to be studied. Filtering is necessary in post-processing because the acceleration measurements enter the DSP unfiltered to avoid introducing lag into the feedback loop. The time range for error calculations and filtering is consistent throughout this chapter.

Acceleration feedback control provides far superior acceleration tracking for both earthquake levels investigated. Most importantly, the peaks in acceleration which can cause the largest damage to a specimen are very accurately tracked. At higher levels of ground motion, the friction that plagues the low amplitude behavior of the shake table is reduced. Therefore, improved control is seen in all approaches for the larger earthquake. Improvements can be seen visually or by normalizing the RMS error by the maximum amplitude of acceleration as in Table 8.1.
Figure 8.8. Acceleration tracking of 0.2x El Centro with bare shake table

Figure 8.9. Acceleration tracking of 0.4x El Centro with bare shake table
Table 8.1. Normalized Acceleration Tracking Performance

<table>
<thead>
<tr>
<th>Earthquake Record</th>
<th>RMS Error (normalized by maximum acceleration)</th>
<th>FF</th>
<th>FF + xFB</th>
<th>FF + aFB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 x El Centro</td>
<td>0.266</td>
<td>0.304</td>
<td>0.067</td>
<td></td>
</tr>
<tr>
<td>0.4 x El Centro</td>
<td>0.170</td>
<td>0.179</td>
<td>0.043</td>
<td></td>
</tr>
</tbody>
</table>

The purpose of controlling the bare shake table is proof-of-concept. Therefore, only two amplitudes of one earthquake record are investigated and multi-metric control is not considered. A more extensive controller evaluation with multiple earthquakes and multi-metric control is performed for the shake table with payload.

The displacement tracking performance results are shown in Fig. 8.10 and Fig. 8.11 for 0.2x El Centro and 0.4x El Centro, respectively. Displacement feedback control provides the best displacement tracking in all cases, however is also shown to provide the worst acceleration tracking. Displacement feedback control slightly degrades acceleration tracking performance by adding high-frequency content to the acceleration measurements. With shake table control, the desired trajectory is an acceleration record, thus accurate acceleration tracking is a more desirable objective.

Normalized displacement tracking errors are summarized in Table 8.2. As with acceleration tracking, when the ground motion is larger and friction does not influence the shake table behavior as much, displacement tracking is more accurate.

Table 8.2. Normalized Displacement Tracking Performance

<table>
<thead>
<tr>
<th>Earthquake Record</th>
<th>RMS Error (normalized by maximum displacement)</th>
<th>FF</th>
<th>FF + xFB</th>
<th>FF + aFB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 x El Centro</td>
<td>0.054</td>
<td>0.028</td>
<td>0.076</td>
<td></td>
</tr>
<tr>
<td>0.4 x El Centro</td>
<td>0.036</td>
<td>0.014</td>
<td>0.047</td>
<td></td>
</tr>
</tbody>
</table>
Figure 8.10. Displacement tracking of 0.2x El Centro with bare shake table

Figure 8.11. Displacement tracking of 0.4x El Centro with bare shake table
8.4 Two-Story Linear Building

A two story steel frame structure is added to the shake table to study the performance of model-based control in the presence of strong CSI. The building is designed using 1.27 cm (½ inch) steel plates connected by spring-steel columns that nominally constrain the motion to a single axis, minimizing torsion. The spring-steel ensures that the building can undergo large deformation without yielding. The building is shown mounted to the shake table in Fig. 8.12.

![Shake table with linear structure](image)

**Figure 8.12. Shake table with linear structure**

The mass and stiffness matrices are by design:

\[
M = \begin{bmatrix}
24.30 & 0 \\
0 & 27.10 \\
\end{bmatrix} \text{kg} \quad (8.32)
\]

\[
K = \begin{bmatrix}
15.726 & -8.400 \\
-8.400 & 8.400 \\
\end{bmatrix} \text{N/m} \quad (8.33)
\]
The mass and stiffness matrices result in experimentally verified natural frequencies of 1.67 Hz and 4.63 Hz with corresponding experimentally determined damping ratios of 0.15% and 0.10%. The natural frequencies are designed to be similar to those of typical midrise steel structures (ASCE, 2010). In addition, the structure has a vibro-impact nonlinear energy sink attached to the top story, which is a remnant from a separate line of research. In its unlocked configuration, this device consists of a mass that moves on a round rail system with an impact stopper. In its current configuration the mass is locked into place; therefore, the only effect on the system is an increase in mass, which is included in the mass matrix.

8.4.1 System Identification

System identification is performed on the shake table with the same testing protocol as the bare shake table. Figure 8.13 shows the command to displacement transfer function while Fig. 8.14 shows the command to acceleration transfer function. These transfer function are very different from the bare shake table case, showing how influential the payload mass and dynamics are for this experimental setup. In fact, the two natural frequencies of the structure are apparent in the shake table transfer function. Due to the pronounced CSI, two modeling approaches are considered. The first modeling approach is a low-order model, where the pronounced peaks and valleys due to CSI are ignored. The second modeling approach is a high-order model, where a greater number of poles and zeros are added to model CSI accurately. Both model fits are illustrated in Fig. 8.13 and Fig. 8.14.

The low-order and high-order models are identified as SIMO systems in Eqns. (8.34) and (8.35), respectively.

\[
G_{yu}(s) = \frac{X(s)}{U(s)} = \left[ -\frac{0.003938(s - 506.6)(s + 478.1)(s + 76.53)}{9.814s^2(s + 75.00)} \frac{1}{(s + 32.48)(s^2 + 48.96s + 3833)} \right] \tag{8.34}
\]
In controller designs when only a SISO system is required (such as acceleration feedback and displacement feedback), the SISO system is extracted from the SIMO system.

\[
G_{yu}(s) = \frac{\left[949.4(s + 90.45)\left(s^2 + 0.1632s + 102.8\right)\left(s^2 + 0.2220s + 79.5\right)\right]}{\left[9.73s^2 \left(s^2 + 87.94\right)\left(s^2 + 0.1679s + 103.1\right)\left(s^2 + 0.2222s + 79.7\right)\right]} \\
\left(s + 37.71\right)\left(s^2 + 51.51s + 3906\right)\left(s^2 + 0.2882s + 99.26\right)\left(s^2 + 0.4706s + 79.09\right)
\]

(8.35)
8.4.2 Tracking Performance

The performance of the shake table with the building attached is investigated for model-based controllers based on both low and high-order models. The El Centro, Kobe, and Northridge earthquake records are selected as the reference accelerations. Controllers include feedforward control (FF), feedforward control with displacement feedback (FF + xFB), feedforward control with acceleration feedback (FF + aFB), and feedforward control with multi-metric feedback (FF + xaFB). The results for Kobe earthquake record using the low-order controller design are shown in Fig. 8.15 while the results using the high-order controller design are shown in Fig. 8.16.

Figure 8.15. Acceleration tracking of Kobe with linear structure and low-order controller

![Graphs showing acceleration tracking for different controllers with RMS errors for Kobe earthquake record.]
Figure 8.16. Acceleration tracking of Kobe with linear structure and high-order controller

Acceleration feedback improves the performance of acceleration tracking, especially in reducing higher frequency oscillations and matching the peak accelerations. The addition of multi-metric feedback control slightly improves tracking performance. The best multi-metric feedback controller designs are achieved by placing more importance on the acceleration measurement relative to the displacement measurement (through LQG design), illustrating the value of acceleration feedback. Furthermore, the controllers developed using the high-order model perform better than the lower-order model in most cases. If the frequency content of the input ground motion overlaps with the pronounced CSI effects observed in the transfer functions, the higher-order model will provide better control over this region.
The acceleration tracking results for all earthquake records, summarized by the RMS error, are presented for low-order controllers in Table 8.3 and high-order controllers in Table 8.4.

**Table 8.3. Acceleration Tracking Performance for Low-Order Controllers**

<table>
<thead>
<tr>
<th>Earthquake Record</th>
<th>FF</th>
<th>FF + xFB</th>
<th>FF + aFB</th>
<th>FF + xaFB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 x El Centro</td>
<td>0.177</td>
<td>0.192</td>
<td>0.065</td>
<td>0.049</td>
</tr>
<tr>
<td>0.4 x El Centro</td>
<td>0.259</td>
<td>0.283</td>
<td>0.085</td>
<td>0.065</td>
</tr>
<tr>
<td>0.15 Kobe</td>
<td>0.215</td>
<td>0.226</td>
<td>0.076</td>
<td>0.063</td>
</tr>
<tr>
<td>0.1 Northridge</td>
<td>0.153</td>
<td>0.203</td>
<td>0.052</td>
<td>0.057</td>
</tr>
</tbody>
</table>

**Table 8.4. Acceleration Tracking Performance for High-Order Controllers**

<table>
<thead>
<tr>
<th>Earthquake Record</th>
<th>FF</th>
<th>FF + xFB</th>
<th>FF + aFB</th>
<th>FF + xaFB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 x El Centro</td>
<td>0.157</td>
<td>0.198</td>
<td>0.057</td>
<td>0.049</td>
</tr>
<tr>
<td>0.4 x El Centro</td>
<td>0.220</td>
<td>0.245</td>
<td>0.074</td>
<td>0.058</td>
</tr>
<tr>
<td>0.15 Kobe</td>
<td>0.203</td>
<td>0.255</td>
<td>0.059</td>
<td>0.054</td>
</tr>
<tr>
<td>0.1 Northridge</td>
<td>0.126</td>
<td>0.160</td>
<td>0.038</td>
<td>0.031</td>
</tr>
</tbody>
</table>

Results from the other earthquake records confirm that the model-based acceleration feedback controller performs very well and, in most cases, is improved slightly by multi-metric feedback control. Again, the controllers based on higher-order models provide better control in most cases.

Displacement tracking results are presented for Kobe earthquake record using the low-order controller design in Fig. 8.17 and the high-order controller in Fig. 8.18. Displacement feedback control significantly improves the tracking of the desired displacement beyond other control schemes. With multi-metric feedback control, most control weighting is placed on acceleration; therefore the multi-metric feedback control results closely match the acceleration feedback control results. As with acceleration tracking, the high-order controllers reduce the displacement tracking error beyond the low-order controllers.
Figure 8.17. Displacement tracking of Kobe with linear structure and low-order controller

The displacement tracking results for all earthquake records investigated, summarized by the RMS error, are presented for low-order controllers in Table 8.5 and high-order controllers in Table 8.6. Overall, displacement feedback control provides the best displacement tracking. However, the control objective is acceleration tracking, where displacement feedback control provides the worst performance. Thus, displacement feedback alone may be inadequate toward achieving accurate ground motion replication. Multi-metric control allows for a balance between displacement and acceleration tracking; however focus is placed on the acceleration tracking.
Figure 8.18. Displacement tracking of Kobe with linear structure and high-order controller

Table 8.5. Displacement Tracking Performance for Low-Order Controllers

<table>
<thead>
<tr>
<th>Earthquake Record</th>
<th>RMS Error (cm)</th>
<th>FF</th>
<th>FF + xFB</th>
<th>FF + aFB</th>
<th>FF + xaFB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 x El Centro</td>
<td>0.067</td>
<td>0.023</td>
<td>0.097</td>
<td>0.101</td>
<td></td>
</tr>
<tr>
<td>0.4 x El Centro</td>
<td>0.076</td>
<td>0.034</td>
<td>0.067</td>
<td>0.105</td>
<td></td>
</tr>
<tr>
<td>0.15 Kobe</td>
<td>0.060</td>
<td>0.024</td>
<td>0.065</td>
<td>0.061</td>
<td></td>
</tr>
<tr>
<td>0.1 Northridge</td>
<td>0.091</td>
<td>0.044</td>
<td>0.102</td>
<td>0.112</td>
<td></td>
</tr>
</tbody>
</table>

Table 8.6. Displacement Tracking Performance for High-Order Controllers

<table>
<thead>
<tr>
<th>Earthquake Record</th>
<th>RMS Error (cm)</th>
<th>FF</th>
<th>FF + xFB</th>
<th>FF + aFB</th>
<th>FF + xaFB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 x El Centro</td>
<td>0.072</td>
<td>0.020</td>
<td>0.107</td>
<td>0.107</td>
<td></td>
</tr>
<tr>
<td>0.4 x El Centro</td>
<td>0.099</td>
<td>0.030</td>
<td>0.058</td>
<td>0.110</td>
<td></td>
</tr>
<tr>
<td>0.15 Kobe</td>
<td>0.048</td>
<td>0.019</td>
<td>0.054</td>
<td>0.053</td>
<td></td>
</tr>
<tr>
<td>0.1 Northridge</td>
<td>0.090</td>
<td>0.036</td>
<td>0.093</td>
<td>0.105</td>
<td></td>
</tr>
</tbody>
</table>
8.5 Nonlinear Structure

To verify the robustness of the model-based controller in the presence of nonlinearities, which can be caused by structural damage during experimentation, a nonlinear specimen will be investigated. Nonlinear structures provide a particular control challenge in the presence of strong CSI. For example, significantly pronounced structure dynamics through CSI are present in Fig. 8.13 and Fig. 8.14. If the structure was to become damaged and the natural frequencies shift, then the model-based controller would no longer be tuned to the structure. Robust controllers are needed to accommodate this change in shake table behavior and ensure accurate acceleration tracking even after damage. A shift in natural frequencies may affect the high-order model than the low-order model since the low-order model overshadows the peaks and valleys due to CSI; therefore both modeling approaches will be explored for the nonlinear specimen.

To consider nonlinear behavior, a modification is made to the existing two story frame structure. Two wooden columns constructed from basswood are added to each story, increasing the stiffness of the structure, as shown in Fig. 8.19. Each wooden column is 4.76 mm (3/16 inch) thick over the entire length, 12.7 mm (1/2 inch) wide over a 177.8 mm (7 inch) length, and 50.8 mm (2 inch) wide at the connections. System modeling and controller designs will be based on the modified structure with wooden columns. Under extreme loading, the columns are designed to fail, reducing the natural frequencies to approximately that of the linear steel-only structure. The robustness of the controller will be evaluated in the presence of this shift in structural dynamics.
8.5.1 System Identification

System identification is performed on the shake table with the same testing protocol as used in previous cases for 0.4 V RMS excitation. Figure 8.20 shows the command to displacement transfer function while Fig. 8.21 shows the command to acceleration transfer function. The original experimental transfer functions without wooden columns are shown to illustrate the change that the wooden columns bring to the dynamics of the shake table. Both low-order and high-order model fits are shown.

The low-order and high-order SIMO transfer function models are identified as Eqns. (8.36) and (8.37) respectively.

\[
G_{yu}(s) = \frac{\begin{bmatrix} X(s) \\ A(s) \end{bmatrix}}{U(s)}^{-1} = \frac{-0.0006289(s + 1356)(s - 1164)(s + 66.56)}{10.19s^2(s + 64.65)} \frac{(s + 29.81)(s^2 + 52.70s + 4082)}{(s + 29.81)(s^2 + 52.70s + 4082)}
\]

\[(8.36)\]
\[
G_{y u}(s) = \frac{-0.004537(s - 464.4)(s + 441.5)(s + 81.57)(s^2 + 0.2027s + 154.3)(s^2 + 0.3966s + 1185)}{9.915s^2(s + 76.57)(s^2 + 0.2008s + 154.5)(s^2 + 0.3944s + 1186)} \\
\left( s + 35.17 \right)(s^2 + 0.4220s + 147.7)(s^2 + 0.8011s + 1173)(s^2 + 54.14s + 4100)
\]

(8.37)

Figure 8.20. Shake table with nonlinear building displacement transfer function

Figure 8.21. Shake table with nonlinear building acceleration transfer function
8.5.2 Acceleration Tracking Performance

The El Centro earthquake record is selected as the reference acceleration for tracking control with maximum achievable amplitude of 0.4x. As the specimen will undergo damage, experiments are not easily repeated and directly compared; therefore, only the model-based multi-metric feedback controller will be explored for both low-order and high-order controller designs. To illustrate controller performance during specimen changes, the same earthquake record is run multiple times as the wooden columns receive damage and are eventually removed. During these tests, the model-based controller, which is based on the structure with intact wooden columns, is not adjusted.

The performance of the model-based multi-metric feedback controller based on the high-order model is shown in Fig. 8.22. The first row of the figure illustrates the performance of the controller with the column completely intact throughout the ground motion. At maximum amplitude, the shake table is unable to noticeably damage the columns. Therefore, prior to the next test, the columns are slightly notched. The notches are introduced at the location where the width of the column changes and the highest bending stresses are expected. The performance of the controller when the columns go from notched to damaged state is shown in the next row. Next, the damaged columns are again subjected to the same earthquake and damaged further, with results shown in the next row. Finally, performance of the controller with columns removed is shown in the last row. As the conditions of the specimen change, the high-order controller is no longer tuned, and performance degrades slightly.

The natural frequencies of the structure are evaluated before and after each test in Fig. 8.22 from the free response to an impulse load of low enough amplitude to prevent damage to the
column. The natural frequencies are reported in Table 8.7, providing quantitative values to characterize the change in dynamics.

**Figure 8.22. Acceleration tracking with nonlinear structure and high-order controller**

**Table 8.7. Natural Frequencies Before and After Testing Using High-Order Model**

<table>
<thead>
<tr>
<th>Natural Frequency (Hz)</th>
<th>Intact Columns</th>
<th>Notched Columns</th>
<th>Damaged Columns</th>
<th>Further Damaged Columns</th>
<th>Columns Removed</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Mode</td>
<td>1.97</td>
<td>1.94</td>
<td>1.69</td>
<td>1.67</td>
<td>1.67</td>
</tr>
<tr>
<td>Second Mode</td>
<td>5.49</td>
<td>5.38</td>
<td>5.10</td>
<td>5.08</td>
<td>4.63</td>
</tr>
</tbody>
</table>

The performance of the model-based acceleration feedback controller based on the low-order model controller is illustrated in Fig. 8.23. For this controller, experiments are run using
notched columns such that damage is introduced under accelerations achievable by the shake table. The first row shows the performance of the controller as the notched columns become damaged. In the second row, the damaged columns are again subjected to the same earthquake load and become further damaged. In the third row, the columns are removed. The low-order model controller is less sensitive to changes in the natural frequency, thus as the condition of the specimen does not affect the controller performance significantly. The best performance in terms of RMS error is actually seen in the case with the columns removed, perhaps due to the removal of the nonlinear behavior introduced by the wooden columns.

![Graph showing acceleration tracking with nonlinear structure and low-order controller](image)

**Figure 8.23. Acceleration tracking with nonlinear structure and low-order controller**

The natural frequencies of the structure are shown in Table 8.8 as evaluated before and after each test in Fig. 8.23. A typical damage case for the wooden columns is shown in Fig. 8.24. Damage occurred at two of the notch points, as expected.
Table 8.8. Natural Frequencies Before and After Testing using Low-Order Model.

<table>
<thead>
<tr>
<th></th>
<th>Natural Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Notched Columns</td>
</tr>
<tr>
<td>First Mode</td>
<td>1.97</td>
</tr>
<tr>
<td>Second Mode</td>
<td>5.47</td>
</tr>
</tbody>
</table>

Figure 8.24. Typical damage to wooden columns (base connection on right-hand side)

8.6 Summary

In this chapter, the proposed model-based controller was adapted to shake table testing. High-fidelity tracking of the desired accelerations was achieved through acceleration and multi-metric feedback approaches. Success was achieved by ensuring stability through inner-loop displacement feedback control while designing the model-based controller as an outer-loop controller. Also, the flexibility of LQG control for model-based feedback allows for shaping of the process and measurement noises. Through shaping filters, the feedback controller can be designed less sensitive to higher frequencies which may lead to high frequency oscillations and instabilities.

The model-based controller was demonstrated to be robust to changes in specimen conditions due to damage. In cases when damage is expected, the low-order controller less
precisely tuned to the peaks and valleys caused by CSI was shown to provide better control over all specimen conditions. For linear specimens, the high-order controller exactly tuned to the effects of CSI was shown to provide better control. In either case, model-based control provides an excellent alternative to existing control techniques used for shake table testing. Further improvement is expected when working with higher-quality hydraulic shake tables where the behavior will be less amplitude dependent.
CHAPTER 9  CONCLUSIONS AND FUTURE STUDIES

9.1 Conclusions

This research provides a rigorous framework for model-based actuator control including both model-based feedforward and feedback links to directly address added, unwanted dynamics in the RTHS loop. A simple approach to developing the model-based controller for a general servo-hydraulic system was proposed. With predefined displacements, results showed near perfect tracking of the desired displacement signal and good tracking of velocities and accelerations. In RTHS, the proposed model-based controller was proven successful for testing both single actuator and multi-actuator systems. The model-based control technique was successfully applied to the acceleration control of a shake table, extending the merits of the technique to other areas of experimental evaluation.

A review of literature in the area of hybrid simulation was first presented with a focus on RTHS. Both numerical integration schemes and actuator control techniques were explored alongside phenomena significant to RTHS including experimental error and CSI. A review of shake table control followed to juxtapose applications of model-based control for shake table testing. Basic control theory including transfer function and state-space approaches were also presented as necessary background for the development of the proposed actuator control algorithms.

RTHS allows for the experimental evaluation of the performance of rate-dependent components in a larger structural system. The application of the proposed framework has focused on MR dampers, providing the opportunity to explore the device’s behavior and semi-active control algorithms. A set of characterization tests were performed to develop a numerical model for verification of RTHS. Also an over-driven back-driven approach to semi-active control of the
MR damper was proposed to overcome observed response lag in restoring force after the input current changes. Although the specimen selected was an MR damper, the proposed strategies for RTHS are applicable to a much broader range of structures and structural components.

The framework for RTHS was illustrated with a focus on the dynamics of the servo-hydraulic system added to the RTHS loop. By examining the behavior of the servo-hydraulic system, a model-based multi-actuator control approach including both feedforward and feedback links was developed that directly addresses actuator dynamics including CSI. The feedforward link eliminated the modeled actuator dynamics as a model inverse. Accurate feedforward controllers were found to be improper systems, therefore approaches to implement the improper system in RTHS were proposed, taking higher-order derivatives from numerical integration. The feedback link provided robustness to modeling errors, nonlinearities, and changes in the specimen during testing. The feedback link was designed using model-based LQR control with output weighting to minimize displacement errors as well as acceleration errors through multi-metric feedback control. A Kalman filter was incorporated to estimate the unknown model states while providing filtering of the measurements without introducing time lag. A second-order shaping filter was added to the process noise, restricting the feedback control effort to the frequency range of interest. The source of actuator coupling for multi-actuator systems was illustrated by example. The model-based controller was by design flexible to accommodate multi-actuator systems including the effects of actuator coupling.

The large-scale experimental setup used to explore the proposed framework for RTHS on a single-actuator system was presented. The setup combined fast computational hardware and software with a high-performance servo-hydraulic system. As an application for RTHS, an MR damper was considered as the rate-dependent experimental specimen. The MR damper can
undergo significant and repeatable changes in behavior, making it an ideal specimen for exploring the robustness of actuator control to specimen-dependent actuator behavior. Using nonparametric system identification, the behavior of the servo-hydraulic system was characterized and used to develop model-based control approaches. The proposed model-based actuator controller was explored in parallel with other common actuator control approaches for RTHS. Superiority was demonstrated in both the frequency and time domain for predefined displacement tracking, especially at higher frequencies. To investigate actuator control for RTHS, a SDOF structure was created with an MR damper as the experimental specimen. The natural frequency of the structure was varied to explore the influence of accurate actuator control at higher frequencies. The proposed model-based controller provided the best tracking among the methods considered, especially when the natural frequency of the structure exceeded 5 Hz. A benchmark nine-story structure was then considered as a more realistic structure for RTHS where the current to the MR damper varied under semi-active structural control. Even under these changing specimen conditions which bring about changes to the servo-hydraulic system behavior and introduces higher frequency dynamics into the RTHS, the proposed model-based controller demonstrated excellent performance.

Multi-actuator systems were then explored through a numerically modeled three-story nonlinear structure. The model-based multiple-actuator controller performed very well during the simulated RTHS. This example demonstrated the benefits of considering actuator coupling in actuator control. Feedback control further improved the performance of the feedforward controller alone for the nonlinear structure. A physical three-story braced frame with an MR damper at the first story was then considered for RTHS with the remaining structural
components simulated numerically. Excellent results were obtained through RTTHS which compared well to numerical simulations.

Shake table testing presents a unique control challenge in that the desired trajectory is an acceleration record. The proposed model-based actuator control strategy was adapted to the tracking of an acceleration record with multi-metric feedback, including acceleration measurements, to improve the tracking performance. The small-scale shake table selected for the study had significant nonlinear behavior due to high levels of friction in the bearings. At the same time, significant CSI was observed such that the natural frequencies of the specimen, a two-story structure, were present in the shake table transfer function. Both low and high-order model-based controllers were developed, the former overshadowing the strong CSI while the later accurately matching the dynamics due to CSI. The robustness of the controllers to changes in specimen condition was demonstrated by creating a specimen that experienced damage during the prescribed ground motion. The high-order model was found more accurate for a linear specimen where the shake table dynamics would be unchanging. The low-order model was found more accurate for the nonlinear specimen in that the identified model is less sensitive to changes in shake table behavior. Overall, the model-based controller performed well, especially in matching the peak acceleration values of the prescribed ground motion.

The research has demonstrated RTTHS to be an effect means to test the behavior of rate-dependent structural components. Through RTTHS, advanced energy dissipation devices could be explored at large scale, which is important because the complex nonlinear behavior will likely not scale accurately to small scale models. The proposed model-based actuator control strategy addressed a number of gaps necessary for testing a broad class of structure including: (1) accurate actuator control over a broad frequency range, (2) multi-actuator control considering
actuator coupling, and (3) multi-metric feedback. The benefits of model-based control also apply to shake table testing, achieving robust and repeatable tracking performance.

9.2 Future Studies

This research has addressed many challenges in dynamic testing frameworks including actuator control for RTHS and shake table testing. A number of exciting research avenues still exist, which will be detailed below.

- Actuator control in the presence of strong coupling. The proposed model-based multi-actuator control strategy accommodated actuator coupling through an identified MIMO model. The multi-actuator system considered in this dissertation had actuators with a large capacity relative to the restoring force of the specimen. Therefore, limited coupling exists and the benefit of modeling the coupling was not conclusively demonstrated in experiment.

- Multi-dimensional RTHS. To date, RTHS involving multiple actuators have focused on structures with all actuators in one plane. For RTHS in multiple dimensions, a geometric transformation is required to convert Cartesian commands to actuator space and vice versa. This geometric transformation presents an additional actuator control challenge.

- Multiple degrees of freedom at one node. For some experimental substructures, it may be necessary to impose horizontal, vertical, and rotational motion at the interface between numerical and experimental substructures. Two actuators could be used in one direction and one actuator in the orthogonal direction to impose these three in-plane motions. With actuators operating in close proximity, it is expected that the degree of actuator coupling would be higher and actuator control algorithms would have to account for the strong coupling.
• Multi-dimensional shake table testing. The study on shake table testing was restricted to a single axis shake table. However, the proposed model-based actuator controller is flexible to accommodate multi-actuator systems including multi-dimensional shake tables.

• Nonlinear actuator control. The proposed model-based controller was developed around a linearized model of the servo-hydraulic system. A model-based feedback control, also linear, was designed to accommodate model inaccuracies. Instead, a nonlinear control strategy could be adapted that directly accounts for model nonlinearities. Such a control scheme would also be able to capture the velocity dependency in the performance of some actuators.

• MR damper idle time. The study on MR damper idle time was limited due to frequent use of the specimen. With longer gaps in use, a more detailed study on idle time could be conducted coupled with a theoretical study of MR fluid, settling time, and mixing.

• Nonlinear numerical structures. The RTHS of this dissertation has concentrated all nonlinearities on the experimental substructure. Nonlinearities could also be modeled numerically; however this would come at the cost of increased computational time. As long as the numerical calculations can be completed within the prescribed time step, nonlinear numerical models are not expected to pose any additional constraints. That is to say, the model-based actuator control was developed independently of the numerical structure.

• Sensor placement. In the RTHS study, the accelerometer and LVDT used for model-based feedback control were located at opposite ends of the actuator. Elastic deformation of the frame under high levels of restoring force led to slight discrepancies in the
measurements. Sensor placement that leads to consistent measurements even under
elastic deformation should be considered for multi-metric control schemes.
REFERENCES


