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THE EFFECT OF RAINSTORM MOVEMENT ON URBAN DRAINAGE NETWORK RUNOFF HYDROGRAPHS

BY

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DISSEPTION

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Civil Engineering in the Graduate College of the University of Illinois at Urbana-Champaign, 2012

Urbana, Illinois

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ABSTRACT

Prediction and estimation of runoff has been a long-standing topic of hydrology for the purpose of water resources management both in terms of quality and quantity, flood control, ecology, and environmental considerations. It is well recognized that surface runoff from a watershed due to a rainstorm varies with the hydro-meteorological characteristics of the rainfall and the physiographic properties of the watershed. The direct influence of rainfall movement on the shape of a runoff hydrograph has been known for a long time. However, previous studies and research regarding moving rainstorms are mostly based on a specific catchment, which was either synthetic or real. The study of a single specific catchment makes it difficult to extrapolate our understanding of the effects of storm movement to different watersheds. Therefore, instead of focusing on a specific catchment or rainfall condition, this study seeks to establish a generalized relation between rainstorm movement and runoff hydrographs based on network configuration.

This study utilizes a conceptual model based on characteristic timescales to investigate the effects of storm movement on the flood peak flows, and the underlying process controls. A broad theoretical framework is developed that uses characteristic time and space scales associated with stationary rainstorms as well as moving rainstorms. This study explores the relations between network configurations and hydrograph sensitivity to storm kinematics; storm speeds, storm directions, and storm sizes. The configuration of the drainage network is simulated with Gibbs’ model. The peak response is investigated with different rainstorm conditions and network configurations. The results show that the effect of the direction and speed of the rainstorm movement significantly depends on network properties. The relation between storm kinematics and the peak discharge response is dependent on network configuration; accordingly network efficiency.

Mass balance analysis in an urban watershed indicates that rainfall infiltrated to pervious areas might contribute to direct runoff hydrograph, thereby offering an explanation for the long hydrograph tail. Width functions are obtained from urban drainage networks and applied to obtain distinct response functions for Direct Connected
Impervious Areas (DCIA), Isolated Impervious Areas (IIA), and pervious areas combined with excess and infiltrated amount of rainfall. This methodology addresses the mass balance error observed in runoff hydrographs in urban watersheds based on two assumptions regarding the contribution of pervious areas to runoff hydrographs. The results show improvement in the estimation of runoff hydrographs and suggest the need to consider the flow contribution from infiltrated rainfall amount in pervious areas to the flow discharge hydrograph. The results also imply that additional contribution from flow paths such as pipe infiltration needs to be considered. In addition, this study investigates the applicability of stochastic network models to urban drainage network in terms of runoff hydrographs. The actual network is replaced by stochastic networks from the Monte-Carlo simulation and the hydrologic response function is developed using synthetic width functions from Gibbs’ model. The results indicate that the simulated network with the stochastic network model can be a good approximation of an actual network in terms of runoff hydrographs at the outlet of the watershed.

Finally, by introducing the Equivalent Stationary Storm (ESS) compared with moving rainstorms, this study evaluates the effect of rainstorm movement on the peaks discharge response. This study shows that the drainage networks in urban areas have wide range of network configuration and they can be highly inefficient in terms of drainage time compared with natural channel networks. However, the result shows that inefficient networks are less sensitive to rainstorm movement and as a consequence, they potentially contribute to mitigate the effect from rainstorm movement in urban catchments.

This research evaluates the effect of rainstorm movement and also reproduces the discharge hydrograph based on the network configuration. Therefore, the framework of this study strongly suggests a generalized relation between the storm movement and hydrologic response of an urban catchment based on its network configuration. It also implies an optimal balance between network efficiency and safety to storm kinematics that leads to potential improvement in urban drainage networks.
어머니, 아버지께

To Mother and Father
ACKNOWLEDGEMENTS

This research would not have been possible without the guidance and support of my advisor Dr. Arthur R. Schmidt. His invitation in late April 2008 directed my journey so far. It also would not have been possible without support from the Metropolitan Water Reclamation District of Greater Chicago as part of the University’s work on Chicago’s Tunnel and Reservoir Plan project. A special thanks to the members of the Tunnel and Reservoir Project (TARP) group led by Professor Marcelo Garcia and its supporting graduate students and staff, namely; Joshua Cantone, Andrea Zimmer, Nam Jeong Choi, Michelle Hollander, Kia Alexander, Gricelda Ramirez, Hao Luo, Yun Tang, Andrew Erickson, Nick Stepina, Blake Landry, Nils Oberg, and Robin Ray. I was fortunate to have you in my office. Thank you for all your kind helps. Thank you the members of my doctoral committee, Praveen Kumar and Murugesu Sivapalan for their guidance and direction. Their contributions are gratefully acknowledged. Especially, I appreciate the contribution of Professor Murugesu Sivapalan for the conceptual study of my research for rainstorm movement. In Korea, I want to acknowledge my former advisors, Dr. Il Won Seo, and Dr. Young-Oh Kim in Seoul National University for their continuing support and encouragement for me. I would like to thank my classmate Seung-Yup Rieu for inspiring me to study abroad in January 2007, whereafter my journey got started.

I acknowledge my parents, Marae Kim and Giseok Seo, without whom I would not be where I am today. They have kept me in the right direction and provided me with all the support as much as they could. My sisters, Wonkyung, and Wonsuk, thank you for reminding me that family is the most important thing even in the most difficult times. My wife, Jia Yoon, my daughter Eunsung, and my son Eunho, I appreciate you being by my side.
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1. INTRODUCTION

Prediction and estimation of runoff has been being a long-standing topic of hydrology for the purpose of water resources management both in quality and quantity, flood control, ecological and environmental consideration. It is well recognized that the surface runoff from a watershed due to a rainstorm varies with the hydro-meteorological characteristics of the rainfall and the physiographic properties of the watershed (Yen and Chow, 1969). The factors affecting a stream flow hydrograph can be categorized into (a) watershed characteristics; (b) storm precipitation dynamics; (c) infiltration; and (d) antecedent conditions (Singh, 1997). The watershed characteristics can be surficial or subsurficial. Surface features include area, shape, channel network, slope, vegetation, roughness and land use. Subsurface features are soil texture, structure and type; porosity; stratigraphy; hydraulic conductivity; and geological controls. Storm precipitation characteristics include amount, intensity and duration, and velocity and direction storm movement. Among these factors affecting the shape of hydrographs, more realistic and ‘true’ precipitation has been considered one of the most important factors in successful hydrologic modeling (Larson, 1974; Niemczynowicz and Bengtsson, 1996). Niemczynowicz (1988) showed that rainfall data for runoff simulation can be complemented by rainfall movement properties: areal and kinematic properties. For a given amount of rainfall and duration, the temporal and spatial distributions of the rainfall vary with the movement of a rainstorm, which results in significant difference in response at the outlet of a watershed in terms of runoff hydrographs.

The direct influence of rainfall movement on the shape of the runoff hydrograph has been known for a long time (Maksimov, 1964; Yen and Chow, 1968; Wilson et al., 1979; Jensen, 1984; Niemczynowicz, 1991; Singh, 1998). Typically, compared with storms moving upstream, downstream direction results in: (a) late peak; (b) greater peak discharge; (c) steeper rising limb; (d) shorter base time (Roberts and Klingerman, 1970). For this reason, various approaches; analytical (Marcus, 1968; Jensen, 1984; Singh, 1998, 2002a, b), numerical (Surkan, 1974; Foroud et al., 1984; Jensen, 1984; Stephenson, 1984; Ngirane-Katashaya and Wheater, 1985; Niemczynowicz, 1991; Andersen et al., 1991;
Faures et al., 1995; de Lima and Singh, 2002; Morin et al., 2006; Liang, 2010) and experimental (Yen and Chow, 1968, 1969; Marcus, 1968; Roberts and Klingerman, 1970; de Lima et al., 2003; de Lima and Singh, 2003) approach were introduced to evaluate the effect of rainfall movement on the runoff hydrograph of at the outlet of the watershed. Researchers have considered the rainstorm directions (Marcus, 1968; Yen and Chow, 1969; Surkan, 1974; Foroud et al., 1984; Jensen, 1984; Niemczynowicz, 1991; Anderson et al., 1991; Singh, 1998; de Lima and Singh, 2002; de Lima and Singh, 2003; de Lima et al., 2003; Morin, 2006) and speed (Marcus, 1968; Yen and Chow, 1969; Surkan, 1974; Foroud et al., 1984; Jensen, 1984; de Lima and Singh, 2002). Rainfall intensities (Yen and Chow, 1969; Morin, 2006), surface slopes (Yen and Chow, 1969; de Lima et al., 2003) as well as shapes of hyetographs (de Lima and Singh, 2002) were also considered as factors affecting the runoff hydrographs. The majority of the research investigations were based on an synthetic or planar watershed (Marcus, 1968; Yen and Chow, 1969; Foroud et al., 1984; Niemczynowicz, 1991; Anderson et al., 1991; Singh, 1998; de Lima and Singh, 2002; de Lima and Singh, 2003; de Lima et al., 2003) while actual watersheds were also investigated with runoff models (Surkan, 1974; Anderson et al., 1991; Niemczynowicz, 1991; Morin, 2006; Nunes et al., 2006).

In contrast to previous studies on the effect of moving rainstorms, which have been primarily focused on storm dynamics (rainstorm directions, speed, intensity), this study seeks to investigate the relation between the effect of moving rainstorms and the channel network, which is also one of the most important factors affecting a runoff hydrograph (Singh, 1997). Surkan (1969) generated a synthetic hydrograph from a reconstructed network as a special form of directed graph having no loops and forming a tree structure. Kirby (1976) showed that network topology is potentially significant in the prediction of basin hydrograph. In order to capture the relation between the river network and the characteristics of runoff, the geomorphologic theory of the unit hydrograph was introduced by Rodriguez-Iturbe and Valdes (1979) and Gupta et al. (1980). Kirby (1976) and Gupta and Waymire (1983) also suggested an approach looking at the relation between the network geometry and hydrologic response based on the width function. The width function is virtually identical to the shape of the instantaneous unit hydrograph (IUH) and can be transformed to the IUH under appropriate assumption of hydro-
dynamic dispersion relationship (Troutman and Karlinger, 1985). The distance axis of the width function converts to time axis at ease (Naden, 1992). However, attenuation of the flood peak due to channel storage is highly important and therefore, a full routing method is introduced (Van de Nes, 1973), which eventually led to the development of a width function based IUH (WFIUH) (Naden, 1992; Franchini and O’Connell, 1996; Da Ros and Borga, 1997).

Previous studies were mostly based on a specific watershed; hence, properties of the watershed being studies were invariant. In contrast, this study takes account of various conditions of a watershed in terms of channel network configuration. Therefore, in order to categorize and classify channel networks, this study utilizes stochastic network models. Since the random walk model was used to construct a river network by Leopold and Langbein (1962), Scheidegger (1967a) suggested directional self-avoiding walk (DSAW) allowing only downwards directions. Troutman and Karlinger (1992) suggested a more generally applicable stochastic network model based on Gibbs measure (Ising, 1925; Kindermann and Snell, 1980); the models are based on a single parameter as well as two parameters (Karlinger and Troutman, 1994). Gibbs’ measure is based on Boltzmann distribution (Rinaldo et al., 1998) and it has two properties: one is maximum entropy and the other is a Markov random field (Kinderman and Snell, 1980). Gibbs’ model (Troutman and Karlinger, 1992) has a control over the sinuosity of the network and provides a way to categorize networks based on sinuosity.

1.1. Motivation and idea

Most rainstorms are moving (Singh, 1997) and more interestingly, they have directional preferences (Huff, 1977, 1979; Shearman, 1977; Upton, 2001). However, the effect of rainstorm movement is rarely considered in the design process of urban drainage networks. Urban drainage network does not self-evolve, but it is designed and remodeled periodically. To think of the history of modernized drainage system in urban areas, the speed of the modification of artificial drainage network in urban areas can be much faster compared with the evolutionary process in natural river networks. Accordingly, in view of maximizing the effectiveness of urban drainage network and minimizing potential avoidable risks, it is necessary to consider the effect of the moving rainstorms.
Considering the effect of moving rain possibly needs to be utilized to improve design processes for urban drainage networks. A general relation between the storm movement dynamics and the hydrograph is crucial in order to accomplish this. Previous studies and research are mostly based on a specific catchment (either synthetic or real) where watershed properties were given as constant values. Therefore, it was difficult to extend the effect of storm movement to different watersheds with different watershed properties. In this regard, this study seeks a generalized approach that considers the effect of rainstorm movement; the idea is that the effect of rainstorm movement might be different for a different configuration of channel network. In order to investigate the effect of rainstorm movement in different channel networks, it is needed to categorize or classify the network. Therefore, this study utilizes Gibbs’ model (Karlinger and Troutman, 1992), which has control over the channel network sinuosity. This study also utilizes a stochastic network model to reproduce the hydrologic response of a catchment; instead of an actual drainage network, a network generated by Gibbs’ model is tested to reproduce the hydrologic response of a catchment. Stochastic models have been developed to mimic natural river networks. The idea is that the response of an actual network statistically can be represented by the stochastic network model.

1.2. Objectives and scope
This research involves understanding how storm movement affects the hydrologic response of a watershed, specifically in urbanized watersheds. Among various factors that affect the shape of a hydrograph, it is necessary to clarify and narrow down to small number of factors needed to consider for this study. Hence, the focus of the research is reduced to investigate the relation between effect of rainstorm movement and one of the characteristics of a watershed, a channel network. To categorize the network and represent the network characteristics, a stochastic network model, Gibbs’ model, is introduced. Gibbs’ model is a 1-parameter model suggested by Troutman and Karlinger (1992). The network property is represented by a parameter value ($\beta$) of Gibbs’ model. This study investigates network configuration of urban catchments compared with natural river network. Furthermore, two synthetic catchments (square and circular) are introduced in this study to investigate the relation between network configuration and the
effect of moving rainstorms. Realizations of the stochastic network model are tested with synthetic moving storm with different direction, speed, and length scale.

The scope of this study continues to the development of a hydrologic response function based on WFIUH in urban watersheds. The unique response of a channel network can be presented by a width function. WFIUH is introduced and developed for an urban drainage network to directly convert a width function from a drainage network to a runoff hydrograph. Since WFIUH based on the width function that retains spatial information in it, spatial information of a watershed can be regarded. Spatial distribution imperviousness is incorporated to develop WFIUH for an urban catchment considering distinctively different flow dynamics of flows in pervious and impervious areas. This study considers various flow paths possible in urban catchments as well as characteristic excess rainfalls in pervious and impervious areas to develop a hydrologic response function based on WFIUH at the outlet of the catchment.

The third objective of this study is to show that the response function of actual network can be obtained from a stochastic network model. A synthetic width function for an urban catchment is obtained from Gibbs’ model. The hydrologic response from the original pipe network and a stochastic network model with Monte-Carlo simulation are compared with each other. For a specific given network configuration, if a stochastic model successfully reproduces the original network in terms of hydrologic response, it will be also possible to relate the effect of rainstorm movement based on the network configuration.

Finally, this study investigates the effect of rainstorm movement on peak discharge response of urban drainage networks compared with equivalent stationary storm (ESS) depending on network configuration. A synthetic circular watershed is introduced in order to avoid any bias due to geometry. The drainage network is generated with Gibbs’ model.

1.3. Broad outline of thesis

First, this study investigates, in general terms, the effects of storm movement on the resulting flood peaks, and the underlying process controls (Chapter 3). For this purpose, this study utilizes a broad theoretical framework that uses characteristic time and space scales associated with stationary rainstorms as well as moving rainstorms. For a stationary rainstorm the characteristic timescales that govern the peak response include
two intrinsic timescales of a catchment and one extrinsic timescale of a rainstorm. On the other hand, for a moving rainstorm, two additional extrinsic scales are required; the storm travel time and storm size.

In Chapter 4, this study explores the relations between network properties and the effect from moving rainstorms in terms of the peak response and time to centroid of hydrographs. A simple conceptual rectangular catchment is introduced with different configurations of drainage network simulated by Gibbs’ stochastic model. Simple cases of rainstorms moving with upstream and downstream directions and different speeds are considered in order to investigate the effect of rainstorm movement on urban drainage network runoff hydrographs. In addition, the efficiency of the urban pipe networks are examined compared with natural river networks in Chapter 4.

In Chapter 5, this study investigates the relations between network configurations and hydrograph sensitivity to storm kinematics; storm speeds, storm directions as well as storm sizes. A synthetic circular catchment is utilized in order to avoid biases that depend on catchment geometry. The configuration of drainage network is simulated with Gibbs’ stochastic network model. The peak discharge response is investigated with different rainstorm conditions and network configurations.

In Chapter 6, this study utilizes the width function to yield a response function of a drainage network at the outlet. A width function can be obtained from drainage networks directly. The width function can be regarded as a straightforward interpretation of the network response containing the effect of changes in geometric factors specified by shape and connectivity of drainage networks. This chapter addresses the mass balance error observed in runoff hydrographs in urban watersheds by two simple assumptions regarding the contribution of pervious areas to the runoff hydrograph. In this chapter, a framework for rainfall-runoff analysis in urban watersheds based on the width function is introduced with two types of width functions obtained from pervious and impervious areas, respectively. This study utilizes detailed spatial information of imperviousness ratio in an urban catchment obtained from the orthoimages, Light Detection and Ranging (LIDAR) data, and street data of Geographic Information System (GIS). Width functions are obtained from urban drainage networks and applied to obtain distinct response functions for Direct Connected Impervious Areas (DCIA), Isolated Impervious Areas
(IIA), and Pervious Areas combined with excess and infiltrated amount of rainfall. The width functions for pervious and impervious areas combined with proposed assumptions provide with quantification of the contribution from each area to runoff hydrographs. The model framework suggested in this chapter also enables us to evaluate the role of IIA in urban catchments.

In Chapter 7, the possibility for a stochastic network to replace an actual existing urban drainage network in terms of outlet hydrograph is investigated. The actual network is replaced by stochastic networks from Monte-Carlo simulation and the instantaneous unit hydrograph based on the width function (WFIUH) is derived using the synthetic width function averaged from the generated networks with Gibbs’ model. The applicability of stochastic network in urban catchment implies that once the single value of \( \beta \) is estimated for an urban catchment, the flow discharge hydrograph of the catchment can be estimated based on the value of \( \beta \) even if we are lacking detailed layout of the drainage network.

Chapter 8 investigates the effect of rainstorm movement on the peak discharge response of urban drainage networks compared with stationary rainfall depending on network configuration. A synthetic circular watershed is introduced to avoid biases from geometry and the drainage network of the watershed is simulated by Gibbs’ model. This study utilizes two types of the Equivalent Stationary Storm (ESS). The rate of change of the peak discharge response for moving rainstorm is examined with respect to ESS.

1.4. Contributions of the research

This study contributes to achieve a better understanding of rainfall-runoff processed in urbanized areas. In particular, the original contributions of this research are listed as follows:

1. Characteristic timescale and space scales are identified and their interactions for the description of the peak discharge response with moving rainstorms are clarified compared with stationary storms (Chapter 3).

2. The configuration of urban drainage networks in Chicago is investigated compared with natural river networks (Chapter 4).

3. The relation between storm speed and direction and the change in peak discharge hydrographs is evaluated using the synthetic width function (WFIUH) (Chapter 7) and the Equivalent Stationary Storm (ESS) (Chapter 8).
discharge is examined combined with the network configuration and network efficiency (Chapter 4; Chapter 5; Chapter 8).

4. Rainfall-runoff response functions in urban areas are developed for all contributing areas distinctively based on the width function from a drainage network and the applicability of stochastic network models are examined in terms of the hydrograph (Chapter 6; Chapter 7).

5. The effect of moving rainstorms in terms of peak response is evaluated utilizing the Equivalent Stationary Storm (ESS) (Chapter 8).
2. LITERATURE REVIEW

The literature review is composed of four sections: Review on the effect of moving rainstorm, stochastic channel network models, the Geomorphologic Instantaneous Unit Hydrograph (GIUH) and rainfall-runoff modeling in urban catchments.

2.1. Effect of moving rainstorm on runoff hydrograph

Storm pattern, areal extent, and movement are normally determined by the type of storm (McCuen et al., 2002). For example, storms associated with cold fronts (thunderstorms) tend to be more localized, faster moving, and of shorter duration, whereas warm fronts tend to produce slowly moving storms of broad areal extent and longer durations. All three of these factors determine the areal extent of precipitation and how large a portion of the drainage area contributes over time to the surface runoff.

Storm movement is the norm rather than the exception (Singh, 1997). Most rainstorms are moving rather than stationary. Shearman (1977) found that 60% of the storm speeds were greater than 4.2 m/s. Marshall (1980) investigated 219 storms in the UK and found that 26% of the storms moved with a speed of 0-8.3 m/s, 60% with a speed of 8.3-16.7 m/s and 14% with a speed greater than 16.7 m/s. Marshall (1975) suggested a stochastic model considering storm movement considering rainfall producing mechanism as well as the watershed. Most storms comprise rain bands or cells with individual velocities and growth and decay cycles within storm fields. One important point to consider is that most of the previous studies on the direction and speed of the rainstorm showed that a directional preference exists.

Directional preferences of moving rainstorms

The direction of storm movement and cloud movement do not necessarily coincide with each other (Changnon and Vogel, 1981) because storm cells have a circulation different from the main direction of movement (Dixon, 1977). There have been efforts to identify the direction and speed of storm movement from radar (Austin and Houze, 1970, 1972; Hill et al. 1977), from a set of isohyetal maps (Clayton and Deacon, 1971) or directly
from a set of stations (Niemczynowicz and Dahlblom, 1984; Niemczynowicz, 1984a, b, 1987; Diskin, 1987, 1990). A statistical analysis on the kinematics of rainstorm movement was started by Huff (1979) who investigated the storm movement in Illinois. Data were available from 16 gages for all or most of the period from 1949 through 1974. In Illinois, heavy rainstorms are usually produced by one or more squall lines or squall areas traversing a basin or other area of interest. Each system (squall line or squall area) consists of a number of individual convective entities, and these entities have a motion that is strongly related to the wind field in which they are imbedded. These entities are often referred to as raincells. The distribution shows that the most frequent raincell movements are from WSW through W to WNW (240-299°) which counted for 42% of the total number analyzed in the study. Of the total, 84% exhibited motion with a westerly component.

<table>
<thead>
<tr>
<th>Azimuth (degrees)</th>
<th>Percent of storms (%)</th>
<th>Azimuth (degrees)</th>
<th>Percent of storms (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-29</td>
<td>4</td>
<td>180-209</td>
<td>6</td>
</tr>
<tr>
<td>30-59</td>
<td>2</td>
<td>210-239</td>
<td>16</td>
</tr>
<tr>
<td>60-89</td>
<td>2</td>
<td>240-269</td>
<td>22</td>
</tr>
<tr>
<td>90-119</td>
<td>2</td>
<td>270-299</td>
<td>20</td>
</tr>
<tr>
<td>120-149</td>
<td>2</td>
<td>300-329</td>
<td>13</td>
</tr>
<tr>
<td>150-179</td>
<td>4</td>
<td>330-359</td>
<td>7</td>
</tr>
</tbody>
</table>

Shearman (1977) investigated the direction and speed of storm rainfall patterns near London. The idea is that if there exists a preferred direction of storm movement across a drainage basin and this direction coincides with the trunk sewer, a traditional areal reduction factor might not be appropriate. Also, if the most frequent storm speed were similar to the speed of flow of water in the sewer, flooding could occur due to a reinforcement effect, and the storm causing the flood might more commonly than the rarer, more severe, storm originally used to design the sewer. The main purpose of the study was 1) to investigate the relation between the movement of storms and other relevant meteorological parameters, and 2) to investigate the frequency of storms moving in different directions and the distribution of the speed of movement for each direction.
Wind measurement at the 700 mb level, which is closely related to the storm movement, was investigated by Folland and Shaw (1979). Folland and Shearman (1980) later discussed about the difference between atmospheric ‘storm movement’ and the associated ‘storm rainfall pattern movement’ at the ground. Upton (2002) using rain-gauge data investigated all rainstorms between February and December in 2000 and found the preferred directions to be towards the North-East and South-East.

**Figure 2.1** (a) Percent frequency of occurrences of 700 mb winds from direction shown, from Crawley radiosonde, against direction for 1 and 2 h storms (Shearman, 1977) (b) Circular histogram showing the frequency of the estimated directions of 107 storms (Upton, 2001)

*Effect of moving rainstorm on runoff hydrographs*

Many researchers have carried out studies on various aspects of the relation between rainstorm movement and hydrologic response. Yen and Chow (1968, 1969) pioneered this topic of research based on experiments. The experiments were performed on an impervious square watershed with the raindrop production equipment in a laboratory (Chow and Harbaugh, 1965). The geometry of the watershed basically has a drainage network with only downstream directions; this can be categorized as a Scheidegger network (Scheidegger, 1967a). Different conditions are given as two rainfall intensities, four surface slopes, fourteen velocities of rainstorm and four directions in longitudinal as well as in lateral direction. The moving rainstorms were made to have uniform intensities and to move at constant speed. Dimensional analysis showed that the flow can be
expressed as

\[ \frac{Q}{iA} = F\left(\frac{t}{T}, s_x, \frac{W_x}{i}, \frac{W_y}{i}\right) \]  

(2.1)

where \( Q \) is surface runoff, \( t \) is time, \( i \) is intensity, \( A \) is area, \( s_x \) is longitudinal slope, \( W_x \) is speed of rainstorm in longitudinal directions, \( W_y \) is speed in lateral direction, and \( T \) is duration of the rainfall at a point on the watershed.

Figure 2.2 Geometry of watershed tested (Yen and Chow, 1969)

The results of Yen and Chow (1968, 1969) showed that stationary rainstorm gives the largest peak flow compared to moving storm because the amount of rainfall falling in the watershed is greatest for a given duration. Comparing the results with downstream and upstream moving storm, the peak is greater for downstream moving storm than upstream moving one for given slope, speed, and intensity of the rainstorm. The time of occurrence of peak discharge and half recession time (the time between the peak and the half of the peak discharge) are shorter for downstream moving storms compared with upstream moving storms. Later, the dimensional analysis approach was adopted by Townson and Ong (1974) and Niemczynowicz (1984a) for moving rainstorms. Roberts and Klingeman (1970) investigated the influence from spatial variability of rainfall with a laboratory experiment and showed that compared with storms moving upstream, downstream direction results in: (a) later peak; (b) greater peak discharge; (c) steeper rising limb; (d) shorter base time. Yen and Chow (1969) performed laboratory study about the surface runoff with moving rainstorms.
Surkan (1974) performed computer simulation experiments in order to evaluate the effects of moving rainstorm over drainage network. Storms were modeled as polygons surrounding areas of uniform intensity. The system of programs makes it possible to simulate the response of any channel network to storms with boundaries that are numerically specified by line segments. In this experiment, different values of storm directions, speed, as well as size of storms in the direction of movement were considered. The results include the peak values of the simulated flow as a function of direction and speed. The sensitivity of peak flow and average flow rates to downstream direction is maximized when the speed of the storm is comparable with or equal to the average channel flow speed. The sensitivity to changes in direction and speed is greatest when the width of a storm is smaller than the size of a watershed. The condition for resonance type of enhancement of peak flow from a storm of given configuration and intensity is the coincidence of velocity vectors for the storm motion and average flow in the channel.

Figure 2.3 Geometric layout and relative starting positions of the channel network area (square) and moving rainstorm coverage (rectangle) (a) in 45° direction opposite to channel flow and (b) in the 225° direction coinciding with channel flow (Surkan, 1974)

Sargent (1981, 1982) evaluated the effects of moving synthetic storms over hypothetical rectangular watersheds of which areas range from 0.5 km² to 32 km². The storms examined have velocities of 0-17 m/h, five directions (0, 45°, 90°, 135°, 180°) and a fixed
duration of two hours. It was shown that the runoff volume decreased for increasing storm speed but was independent of storm direction and flow velocity. Rainstorms moving faster than flow velocity resulted in reduction of the peak flow.

Foroud et al. (1984) investigated the effect of moving rainstorm with a hypothetical rainstorm of 50 years recurrence interval, 75 mm depth and 4 hours duration on a study watershed, Yamaska S.E. (209.4 km²), located southeast Montreal Island. Compared with equivalent stationary rainstorm (ESRS), the peak flow caused by a rainstorm moving in downstream direction with a speed equal to channel velocity was shown to be 27.5 percent higher and the peak flow caused by a rainstorm moving in upstream direction with the same speed was shown to be 21.7 percent smaller. These differences reduced to be 10.5 percent and 8.5 percent, respectively with the storm speed increasing. The time to peak flow was found to be independent of the storm velocity for all storms moving in downstream direction, providing that the storm velocity exceeded the stream velocity.

Niemczynowicz (1984a, b) studied a conceptual watershed comprising 12 subcatchments with moving storm condition. The conduits connecting subcatchments had same length, diameter and slope. The peak discharge was shown to increase with increasing storm duration and to decrease with increasing storm speed. The directional bias (DB), the value of duration when the bias is equal to zero (TBO), and the width of the storm band (LRO) necessary for TBO condition to occur were defined as

\[
DB = \frac{Q_{ps} - Q_{px}}{Q_{ps}}
\]

(2.2)

\[
TBO = t_c + \frac{L}{v_r}
\]

(2.3)

\[
LRO = L + t_c v_r
\]

(2.4)

where \(Q_{ps}\) is the peak discharge for stationary storms, \(Q_{px}\) is the peak discharge for storms moving in the x-direction, \(t_c\) is the time of concentration, \(L\) is the length of the catchment, and \(v_r\) is the speed of a moving storm. The maximum DB is obtained from \(Q_{pu}\) (upstream direction) and \(Q_{pd}\) (downstream direction).

Jenson (1984) examined the influence of storm movement and its direction on the shape, peak, and time to peak of the runoff hydrograph with a time area model (TAM). TAM is based on a kinematic wave equation. The peak flow was expressed as a function of a
storm speed, duration and travel distance to the outlet. Using synthetic urban catchments, Ngirane-Katashaya and Wheater (1985) investigated the effects of storm velocity on the runoff hydrograph for different catchment areas with a distributed nonlinear rainfall-runoff model. To avoid the interaction between catchment geometry and storm orientations, a circular catchment was introduced.

Figure 2.4 Synthetic urban catchment configuration: (a) catchment shape; (b) catchment drainage network (Ngirane-Katashaya and Wheater, 1985)

Tabios et al. (1988) tested the effect of rainstorm movement with synthetic watersheds; three different watershed areas of 7.77, 1295 and 2590 km² of two different shapes: one elongated, and the other del-shaped with storms moving across, downstream and upstream directions. It was shown that the effect of storm movement on hydrograph peak and peak time may be significantly different for different watershed shape and size and storm size, speed and direction.

Niemczynowicz (1991) investigated rainfall movement and its influence on the runoff generation process in urban runoff simulation models based on observation from Lund, Sweden. He studied the hydrologic response of three cases of storm movement: downstream, upstream, and stationary and also introduced the storm direction frequency to describe the dominant direction of the rainfall in the region. He found that convective rainfall may have distinct kinetic behavior in terms of frequent direction and velocity. Anderson et al. (1991) investigated a narrow artificial catchment (30 m by 2600 m) with
built-in pipe drains. The result shows up to 10% difference in peak flows with respect to the direction of storms.

Faures et al. (1995) performed a hydrologic model analysis on hydrographs for an actual watershed geometry with different wind directions and compared the results with observed data and a ‘no wind’ model runs. The result shows the difference in peak flow rate with direction of the wind up to 60%.

Singh (1998) studied the effect of storm movement on planar flow based on the kinematic wave equation following similar approach with Jenson (1984). He concluded that the direction of storm movement has a significant influence on the peak flow, the time to peak flow, as well as the shape of the overland flow hydrograph.

Watts and Calver (1991) found that the differences in the directional bias reached a maximum at a storm speed and direction similar to the average peak channel flow velocity based on a physically-based rainfall-runoff model tested to an idealized channel network.

Ogden et al. (1995) investigated the effect of storm movement using finite element runoff model (Richardson, 1989; Julien and Moglen, 1990; Ogden and Julien, 1993). The kinematic time to equilibrium (Henderson and Wooding, 1964; Julien and Moglen, 1990; Saghaian and Julien, 1995) was adopted to provide a characteristic runoff response time which incorporates the combined effects of land surface parameters such as runoff plane roughness \( n \), length \( L \), and slope \( S_0 \), and rainfall intensity \( i \). The equation can be derived from continuity equation for overland flow (Chow et al., 1988) and Manning’s equation.

\[
t_e = \left( \frac{nL}{S_0^{1/2} t^{2/3}} \right)^{3/5}
\]

A dimensionless speed, \( u_m \) of rainstorm was defined (Ogden et al., 1995), which produces the greatest effect of storm movement on the peak discharge.

\[
u_m = \frac{u_s}{2} = \frac{L}{2t_e} = \frac{A^{0.2} S_0^{0.3} q^{0.4}}{2n^{0.6}}
\]

where \( A \) is the area and \( u_s \) is a dimensionless speed of storm defined as \( L/t_e \).

Lima and Singh (2002, 2003) performed numerical and experimental approaches to
investigate the influence of storm movement on overland flow with upstream and downstream rainfall movement direction and speed of the order of overland flow velocity. They observed considerable differences in runoff peaks depending on storm patterns and direction and found that the sensitivity of peak response to rainstorm movement decreases as storm speed increases.

A physically-based distributed model (Nunes et al., 2005) was applied to evaluate the consequences of storm movement on runoff and erosion from the Alenquer basin (120 km²) in Portugal. Liang (2010) utilized unpublished experimental data collected from the Watershed Experimentation System (WES) with a V-shaped synthetic catchment developed by the late Professor Ben C. Yen at the University of Illinois at Urbana-Champaign to compare the results from the kinematic wave model; the result showed that the downstream moving storms with \( L_s/L_c < 1 \) increase the peak discharges to a limited extent compared with stationary storms, but the kinematic wave model overestimates the increase in the peak discharge.

Spatial variability of rainfall
The existence and importance of spatial rainfall variability has been long recognized (Osborn and Hickok, 1968; Larson and Peck, 1974; Woldenberg, 1984; Gupta and Mesa, 1988). Efforts to evaluate the spatial variability of rainfall have been made, sometimes compared with compared with stationary and uniform rainfall; Mul et al. (2009) showed that the assumption of uniform spatial distribution of rainfall can easily cause over- or underestimation of the runoff discharge. Arnaud et al. (2002) tested non-uniform rainfall inputs compared with uniform rainfall pattern in terms of peak flows with a distributed hydrologic model. The results showed that the difference ranges from 10-80% depending on the pattern of the rainfall. Babin (1995) examined the sensitivity of annual area mean runoff with actual spatial rainfall variability compared with uniform and stationary rainfall (area-averaged case) and showed that the spatial variability and temporal correlation of rainfall appear to have little impact on the annual area mean runoff. Syed et al. (2003) investigated the geometric measures of thunderstorm rainfall in explaining runoff response from the watershed and showed the importance of spatial variation and volume of the storm core.
The effort to evaluate the effect of rainfall variability on runoff hydrograph includes applying new approaches in rainfall-runoff modeling. Smith et al. (2004) examined the effect of spatial variability of rainfall with a distributed model compared with a lumped model and suggested the use of distributed modeling approach. Cudennec (2007) suggested that the Width Function based Instantaneous Unit Hydrograph (WFIUH) approaches developed by Hung and Wang (2005a, b) would be a good approach analyzing rainfall variability.

2.2. Stochastic channel network model
The basic idea of this study is that introducing a stochastic network model that would help general description of the relation between the rainfall variation and the hydrologic response. To identify and categorize the network according to its characteristics, stochastic network models are adopted in this study. Earlier work done by May (1976) with the logistic equation shows that complexity births from cooperation between and chance and determinism. This argument was previously limited in the physical sciences to the quantum description of phenomena going on at a microscopic scale (Nicolis and Prigogine, 1989). Abrahams (1984) investigated the evolution of channel networks with a variety of methods including conceptual and simulation of models, as well as monitoring of small-scale badland and experimental drainage basins and concluded that the use of stochastic models seems unavoidable in the study of channel networks. Paik (2006) demonstrated that evolutionary dynamics driven by a flow gradient and subject to proximity constraint, that is, the matter and energy can traverse only through a continuum, in the presence of inherent randomness, give rise to a tree topological organization. If the river network is considered as a complex system, there can be a moment of bifurcation when the probability dominates the future (May, 1976; Nicolis and Prigogine, 1989; Strogatz, 2000).

The stochastic channel network model has been an active research area for many years. Randomness built into these models provides various ways of understanding and describing the variability in the network (Barndorff-Nielsen, 1998). There were many efforts to develop random topology models starting in the 1960s. Leopold and Langbein (1962) adopted a random walk model for construction of stream networks. They
concluded that random walk model represents a most probable network in a structurally and lithologically homogeneous region. They also found that the Hack’s exponent of a random walk model is 0.64 which is higher than Hack (1957) found for natural streams. Scheidegger (1966; 1967a, b) proposed a random walk river network model allowing two directions towards downstream, which is very simple but shows a power law distribution (Takayasu, 1990) and reveals the essential features of natural river formation (Nagatani, 1993b). Karlinger and Troutman (1989) also suggested a random walk model with equal probability in all possible directions. A more general model was suggested by Troutman and Karlinger (1992, 1994) based on Gibbs’ distribution.

Considering the amount of geological data collected in the past, it has been shown that the river networks present a self-similar behavior (Rodríguez-Iturbe and Rinaldo, 1996). This is reflected in the power laws in the distribution of quantities like contributing area or stream lengths (Tarboton et al., 1988), including the well-known empirical Hack’s law (1957).

In order to describe river networks in general, it is needed to categorize these networks from each model in terms of sinuosity, similarity dimension and so forth. One of the efforts is multifractality. Multifractality, which originated from theory of measure, is useful in description of distribution of physical quantities over geometric supports and geometric supports can be a fractal. The idea that a fractal measure may be represented in terms of intertwined fractal subsets having different exponent opens a new realm for the applications of fractal geometry to physical system (Feder, 1988). Meneveau and Sreenivasan (1987) showed that observations of fully developed turbulence are very well described by the binomial multiplicative process with $p = 0.7$ which leads to $f(\alpha)$ spectrum that shows the observed multifractal spectrum of the dissipation field.

For river networks, Tarboton et al. (1988) showed that the river network itself has fractal nature. De Bartolo et al. (2000, 2006) showed that river network has a multifractal behavior based on box-counting method (Block et al., 1990). Nagatani (1993b) looked at the flow and width distribution of the Scheidegger network and found that multifractality exists in the flow distribution in the network. In this research, however, the maximum fractal dimension of the $f-\alpha$ spectrum was 1 because of the fixed the width of the network the measuring scale was that of length. Ishida and Nasu (2008) showed that the minimum
value of the Lipschiz-Hölder exponent represents the fractal properties of the most concentrated part of the domain and the maximum value of $\alpha$ represents the fractal properties of the most rarefied part of the domain. The multifractal analysis (Nagatani, 1993b) showed that the Scheidegger network is closest to natural river networks in terms of fractal dimension of network. Rigon et al. (1993) and Ijjasz-Vasquez et al. (1993) showed that the natural river networks tend to organize themselves in terms of minimum total energy expenditure. Rinaldo et al. (1998) argued that both chance and necessity are equally important ingredients for dynamic origin of channel networks. Paik (2006) and Paik and Kumar (2008) argued the orientation of tree topological structures of all dissipative system in nature in terms of inherent randomness.

2.3. Geomorphologic instantaneous unit hydrograph (GIUH)

It is frequently taken for granted that more complex models are more physically realistic. However, it is also true that with greater complexity comes a greater risk of losing insights into the physical processes as a result of getting lost in excess detail (Troutman and Karlinger, 1985). In turn, it is not necessarily true that the simplified model is always sound in terms of insight into the physical phenomena. Nevertheless, the basic concept that explains most of the phenomena cannot be emphasized more. One of the basic ideas is that river water follows the network of the flow paths. It is obvious that the network is one of the most important factors in determining the hydrologic response of a drainage network whether natural or artificial.

One of the first efforts to relate the response of a catchment and the geomorphologic characteristic is the GIUH (Rodriguez-Iturbe and Valdes, 1979; Gupta el al., 1980). The basic idea of the GIUH is that when a unit instantaneous impulse is injected throughout a channel network, the distribution of arrival times at the basin outlet is affected both by the underlying natural order in the morphology of the catchment and the hydraulic characteristics of the flow along the channel themselves (Franchini and O’Connel, 1996). In the original approach of Rodriguez-Iturbe and Valdes (1979), the underlying natural order in the morphology is represented by the Horton ratios which are based on a classification of the channel network of the catchment according to Strahler’s ordering scheme (Strahler, 1957), where the holding time of a drop of water within a stream of a
given order is represented by means of an exponential law. This introduces a conceptualization of the true flow dynamics. As a consequence of this hypothesis, the average holding time of a drop within a stream of a given order is proportional to the average length of all the streams of that order, and the proportionality factor is the velocity of the water, which is considered uniform throughout the drainage basin. Gupta and Waymire (1983) argued that the hydrologic response of a basin should be closely linked to the width function and therefore information about this response might be lost by grouping channel segments according to the Strahler ordering scheme. Troutman and Karlinger (1985) also argued that the averaging process does not consist of explicit evaluation of a conditional expectation. The other formulation of instantaneous unit hydrograph (IUH) was proposed based on the geomorphologic characteristics utilizing the width function. This approach, in which the area is considered to be the mass and network to be the pathways to the outlet, can be regarded as the direct interpretation of the network response without any significant assumption. Mesa and Mifflin (1986) and Naden (1992) coupled the width function with the convective diffusion equation and evaluated the hydrodynamic dispersion represented by two parameters, celerity and longitudinal diffusivity. These parameters are dependent on the local slope, discharge and geometry of the channel, which means at least, the order of magnitude of the parameter can be physically determined (Franchini and O’Connel, 1996). Troutman and Karlinger (1985) and Karlinger and Troutman (1985) proposed the IUH based on a finite number of topologic features rather than using the complete width function. They indicated that the shape of IUH is, properly scaled, identical to that of the width function and that the width function has the shape of a Weibull distribution (Troutman and Karlinger, 1984). Naden (1992) applied the IUH based on the width function to the River Thames considering the spatial variation of the soil types and rainfall. The width function is derived based on the network and spatial variation is presumably built into the function values, which enables the consideration of the spatial variation of the hydrologic characteristics, soil types, rainfall and so on, in terms of the width function. Franchini and O’Cornell (1996) made a comparison between two types of the geomorphologic IUH, GIUH proposed by Rodriguez-Iturbe and Valdes (1979) and IUH which are based on the width function. The comparison was based on a the natural river, the River Tyne, UK and
showed that the GIUH velocity parameter lacks physical interpretation, in contrast to the hydraulic parameters of the IUH that are based on the width function, and which have been seen to be physically consistent. Da Ros and Borga (1997) also compared two types of models in deriving an IUH from digital elevation model (DEM) data and showed that the model based on the width function is able to reduce the variation on the simulated response caused by the different grid sizes. Rinaldo and Rodriguez-Iturbe (1998) also suggested the width-function GIUH based on the work presented by Marani et al. (1991). Agnese et al. (1998) investigated a scale-invariance property of the probability distribution of the topological width function and related peak of the width function with the peak of the hydrologic response of a natural watershed. In urban catchments, the imperviousness should be taken into consideration in modeling the flow at the outlet of the catchment.

Robinson et al. (1995) using a theoretical framework to investigate the two independent processes of hillslope and channel network transport and showed that geomorphologic dispersion varies with catchment sizes. Hall et al. (2001) relaxed the assumption of GIUH introducing a variable velocity instead of a characteristic velocity based on the kinematic wave approximation and developed the Geomorphoclimatic IUH (GCIUH). D’odorico and Rigon (2003) showed analytically how the difference in flow velocity existing between hillslopes and channels affects the probability distribution of travel times based on WFIUH. Gandolfi and Bischetti (1997) showed that the drainage network identification affects the hydrologic response. The drainage network identification is especially important for small basins, where the IUH of the basin do not coincide with the drainage network IUH. Giannoni (2000, 2003) developed a semi-distributed model based on two essential parts of drainage system: hillslopes and channel network with two kinematic scales.

Maidment (1993, 1996) proposed a distributed hydrologic model based on travel time calculated explicitly. Muzik (1996) proposed a distributed unit hydrograph based on kinematic wave routing for both channel and overland flow, utilizing Geographic Information System (GIS). Lee and Yen (1997) suggested a GIUH with varying travel times for overland and channel flows in a stream-ordering subbasin system based on kinematic wave. Lee et al. (2008) also utilized GIUH to develop GIUH for time-varying
rainfall intensity. Rodriguez et al. (2003) applied this approach to an urban catchment to derive IUH which is climatic-dependent. Hung and Wang (2005a, b) developed an algorithm to generate the synthetic width function based on self-similarity in natural river networks in order to produce the hydrologic response with WFIUH. Cleveland et al. (2008) utilized a particle-tracking approach for parameterizing unit hydrographs from topographic information instead of width function, which showed results comparable to observed data. Saghafian et al. (2002) utilized Digital Elevation Model (DEM) to generate a time-area relation based on a kinematic-based travel time, which constitutes total runoff hydrograph. Du et al. (2009) also utilized GIS to calculate the travel time distribution at the outlet of the watershed to develop a distributed rainfall-runoff model. Gironas et al. (2009) transformed the width function to IUH considering the spatial distribution of the imperviousness with different complexity in flow velocity.

2.4. Rainfall-Runoff modeling in urban catchments

An urban watershed is unique in that it has pervious and impervious surfaces which have different hydrodynamic properties. Knowledge of the contributions to urban drainage runoff from both pervious and impervious surfaces is crucial for hydraulic design of stormwater systems as well as for modeling non-point source pollution (Boyd et al., 1993). Heterogeneity in soil structure as well as various types of vegetation makes it difficult to model runoff from pervious areas than that from impervious areas. The runoff from pervious areas also depends on antecedent conditions of soil moisture. Collecting all required information including the network data to build a physically-based model utilizing the whole topology can be time-consuming. Instead, using a geomorphological model was shown to be efficient alternative (Lhomme et al., 2004). Rodriguez et al. (2003), however, argued that an assumption of constant velocity in GIUH can be invalid in urban settings considering engineering design practice instead of compromise between the increase in water depth and the decrease in the slope in natural rivers.

In terms of spatial rainfall distribution, urban area shows influences on rainfall intensity. Huff (1975) showed that the frequency distribution of heavy rainfalls may vary significantly between urban, suburban, and rural areas in large urban–industrial regions and that this may necessitate reevaluation of sewer design storm parameters in use. The
study of urban-induced effects on the frequency of heavy rainstorms has revealed a pronounced increase in the occurrence of storms producing 25 mm or more of rain (Huff, 1977).

It is typically assumed that the rainfall amount infiltrating into pervious areas does not contribute to runoff hydrographs until saturation and excess rainfall occur (Boyd et al., 1993, 1994; Crobeddu et al., 2007; Gironas et al., 2009; Cantone 2010). After saturation, the slope of the runoff depth versus the rainfall depth becomes one, which means that all rainfall falling on the basin, including the pervious areas, starts to contribute to runoff. However, in a combined sewerage system, the infiltrated water takes a more complicated flow path than in the rural system. No sewer system can be 100 percent effective in excluding infiltration from groundwater and surface water, and exclusion efficiency usually declines with age (Santry, 1964). Gregory et al. (2006) investigated that soil compaction during the construction of structural foundations can reduce the moisture loss out of the urban hydrologic system and it indicates that the contribution to the runoff hydrograph increases. Davies et al. (2001) investigated factors influencing structural deterioration and collapse of rigid sewer pipes along with uncertainties and deficiencies in current understanding of these deteriorating processes. Fenner (1990) pointed out that age is not always the reason for sewer failure.

Pipe infiltration can be one of the possible flow paths of infiltrated water to the main drainage network. Butler and Davies (2004) recognized that the infiltrated water in pervious areas also infiltrates back into the combined sewer and contributes to the measured sewer runoff. Weiss et al. (2002) investigated 34 combined sewer systems in Germany and found that sewer inflow due to pipe infiltration is widely underestimated and more than 2/3 of the water passing through the waste water treatment plant can be attributed to infiltration inflow. De Benedittis and Bertrand-Krajewski (2005) calculated the contribution from infiltration inflow in a sewer system to be 30% of dry weather flow. Vaes et al. (2005) also showed the importance of quantifying infiltration rate into sewer pipes. These studies emphasize the pervious areas in urban area should be treated with greater attention in hydrologic modeling.

The presence of impervious land-cover is one of the unique characteristics in urban watersheds. Morgan and Busbey (1993) estimated impervious cover in an urban
watershed using SPOT, 10 meter-panchromatic, satellite digital data compared with air photos. Hernandez et al. (2000) investigated the changes in runoff response of the watershed due to changes in land cover. Chormanski et al. (2008) examined the impact of different methods for estimating impervious surface cover on peak response with a fully distributed rainfall-runoff model and the results showed that detailed information on the spatial distribution of imperious surfaces obtained from remotely sensed data produces substantially different estimates of peak discharges than traditional approaches based on expert judgement of average imperviousness depending on land use. Goldshleger et al. (2009) also utilized remote sensing data including aerial photographs and satellite images to estimate impervious areas. Rodriguez et al. (2000) directly coupled land use data from GIS with rainfall-runoff model. Mejia and Moglen (2010a, b) utilized a geomorphologic unit hydrograph assuming a two-parameter inverse Gaussian travel time distribution for both hillslopes and channels and showed that the spatial pattern of imperviousness can be an influential factor in shaping the hydrologic response of an urbanizing basin. Alley and Veehuis (1983) and Boyd et al. (1994) reinforced the importance of Effective Impervious Area (EIA) or directly connected impervious area (DCIA) in urban watersheds. Lee and Heaney (2003) showed that the runoff hydrograph in urban areas can be over-predicted without considering DCIA that has the same concept with EIA. DCIA is one of the important concepts in land use practice and low impact development (EPA, 2011) and used as runoff coefficient (Garotti and Barbassa, 2010). Pappas et al. (2008, 2011) showed that the existence of impervious surfaces in upstream area affects the sediment regime in pervious areas during runoff processes. Shuster et al. (2008) examined how the factors of impervious extent, connectivity might affect the runoff production at small spatial scale in a laboratory setting.
3. EFFECT OF STORM MOVEMENT ON FLOOD PEAKS: ANALYSIS FRAMEWORK BASED ON CHARACTERISTIC TIMESCALES

The aim of this chapter is to investigate, in general terms, the effects of storm movement on the resulting flood peaks, and the underlying process controls. For this purpose, a broad theoretical framework is utilized that uses characteristic time and space scales associated with stationary rainstorms as well as moving rainstorms. For a stationary rainstorm the characteristic timescales that govern the peak response include two intrinsic timescales of a catchment (hillslope and channel timescale) and one extrinsic timescale of a rainstorm (duration). On the other hand, for a moving rainstorm, two additional extrinsic scales are required; the storm travel time and storm size. The relation between the peak response and the timescales appropriate for a stationary rainstorm can be extended in a straightforward manner to describe the peak response for a moving rainstorm. However, the interdependencies between rainfall duration and storm travel time makes the behavior of the peak response for a moving rainstorm fundamentally different from that of a stationary rainstorm. This chapter shows that the relation between peak response and characteristic timescales also depends on the relative size of the rainstorm with respect to catchment size. For moving rainstorms, we show that the augmentation of peak response arises from both effect of superimposed responses from subcatchments (resonance condition) and effect of increased responses from subcatchments due to increased duration (interdependency), which results in maximum peak response when the moving rainstorm is slower than the channel flow velocity.

3.1. Introduction

In June 2008, eastern Iowa experienced the largest flood ever recorded, including the flooding of Cedar Rapids in the Cedar River basin and Iowa City in the Iowa River basin, respectively. Although the 2008 rains, in themselves, were not the worst in history, the resulting flood was the worst in some areas. Compared with the summer flood in 1993,
which produced much longer flood durations and higher summer runoff volumes, the flood in June 2008 had a relatively short duration (Bradley, 2010). Krajewski and Mantilla (2010) discussed three possible contributing factors for this historical 2008 flood: the severe winter that preceded the floods, the high-intensity rainstorms of late May and early June, and the possibility of a “perfect storm” where the timing and location of rain combined to maximize flood intensity at certain locations. For example, in the Cedar River basin, runoff resulting from rain that fell in the upper watershed on June 8 moved downstream to coincide with rain falling the lower part of the watershed on June 12. These consecutive storms combined to produce an otherwise unexpected, rapid rise of the river and a single well-defined and extremely large flood peak in Cedar Rapids on June 13. The travel time of runoff from the upper watershed to the downstream confluence took four days, which coincided with the time difference of two rainstorms occurring on June 8 and June 12, respectively. This is a clear and compelling example that shows how rainstorm movement in the same direction as runoff can exacerbate the magnitude of flood peaks.

Indeed, it has been known for a long time that storm movement is the norm rather than the exception (Singh, 1997). Most storms tend to be moving storms (Shearman, 1977; Marshall, 1980) with generally preferred directions in different seasons (Huff and Vogel, 1976; Huff, 1979; Shearman, 1977; Upton, 2001). For example, Huff (1979) found that in Illinois 84% of severe heavy rainstorms exhibited motion with a westerly component. The direct influence of rainfall movement on the peak and shape of the runoff hydrograph has been long recognized (Maksimov, 1964; Yen and Chow, 1968; Wilson et al., 1979; Jensen, 1984; Niemczynowicz, 1991; Singh, 1998). In general, a storm moving in the downstream direction shows a late peak, greater peak discharge, steeper rising limb and shorter base time compared with a storm moving upstream.

However, previous studies and research regarding moving rainstorms are mostly based on a specific catchment, which was either synthetic or real. The study of a single specific catchment makes it difficult to generalize our understanding of the effects of storm movement to different watersheds. Therefore, instead of focusing on a specific catchment or condition, this study seeks to establish a conceptual model based on characteristic timescales to investigate the effect of rainstorm movement on the Peak Discharge.
Response (PDR), defined as the ratio of flood peak to average rainfall intensity. Viglione et al. (2010) quantified the contribution of spatio-temporal variability of rainfall and runoff coefficient as well as hillslope and channel velocities, as well as storm movement to the resulting flood peaks. Robinson and Sivapalan (1997) derived the PDR analytically in terms of a ratio of two timescales; the duration of the rainfall and the characteristic timescale of the catchment response used to define a triangular instantaneous unit hydrograph (IUH) of the catchment. However, possible partial activation of a catchment by a moving rainstorm means that additional timescales are required to describe the problem.

This study presents a theoretical framework for a hypothetical catchment with characteristic intrinsic timescales and extrinsic timescales associated with the rainstorm in order to examine the effect of moving rainstorms on the peak response. The intrinsic timescales of a catchment include the catchment response timescale and travel timescale. The extrinsic timescales for a rainstorm include its duration and travel time of the storm. The key aims of this study are (a) to examine how the various timescales and length scales associated with moving rainstorms and catchment responses manifest in a catchment’s peak response, (b) the differences and similarities in the interactions of timescales and length scales between stationary rainstorms and moving rainstorms and (c) the implications of these timescales and length scales for the PDR.

3.2. Stationary rainstorm

Test catchment

Surkan (1974) showed that peak discharge is most sensitive to the direction and speed of the moving rainstorm, especially when the storm sizes in the direction of storm progress are much smaller than the dimensions of the catchment. In this case the catchment is partially activated to produce the response because of smaller length scale of the moving rainstorms.

In order to calculate the PDR at the outlet for moving rainstorms with sizes that are smaller than catchment size, a hypothetical catchment composed of \( N \) consecutive subcatchments \((C_1, C_2, \ldots, C_N)\) is considered in this study, as shown in Figure 3.1.
The number of subcatchments is for computational reasons only and not related to any timescales in this chapter. Here, three length scales of the catchment are given; a longitudinal length scale of the catchment $L_{cl}$, a transverse length scale of the catchment $L_{ct}$, and a length scale of a subcatchment $L_{sc}$. The number of subcatchments, $N = \frac{L_{cl}}{L_{sc}}$, depends on the smallest length scale of the moving rainstorm under consideration compared with the length scale of the catchment in order to satisfy the assumption of IUH that rainfall uniformly covers the entire catchment.

Given a uniform hillslope velocity, $v_h$ and a uniform channel velocity, $v_c$, two intrinsic timescales can be derived for each subcatchment; $t_h = \frac{L_{ct}}{v_h}$ and $t_c = \frac{L_{ct}}{v_c}$. The timescale, $t_h$ represents a response timescale of a hillslope and $t_c$ represents the channel travel time of the flow to the outlet of the catchment. The response from each subcatchment is obtained assuming a triangular IUH as follows.
\[ u(t) = \begin{cases} 
  \frac{u_p t}{t_p}, & 0 \leq t \leq t_p \\
  \frac{u_p}{t_h - t_p}, & t_p \leq t \leq t_h \\
  0, & t \geq t_h 
\end{cases} \tag{3.1} \]

where \( t_p \) is time to peak of a triangular IUH and \( u_p = 2L_{sc}/(L_{cl} t_h) \).

The PDR for stationary storms

A stationary rainstorm refers to a storm that covers the entire catchment with a uniform intensity of rainfall excess. Given a hypothetical catchment with two intrinsic timescales, one additional extrinsic timescale is necessary to describe the PDR; the storm duration, \( t_r \).

Assuming for simplicity that the intensity of rainfall excess, \( i \), is constant over the storm duration, the discharge hydrograph for a subcatchment is given as following equation (Robinson and Sivapalan, 1997b):

\[ \frac{q_e(t)}{i} = \begin{cases} 
  S(t), & 0 \leq t \leq t_r \\
  (S(t) - S(t - t_r)), & t \geq t_r \end{cases} \tag{3.2} \]

where \( q_e \) is the discharge hydrograph for each subcatchment and \( S \)-hydrograph of the triangular IUH for each subcatchment can be derived from Equation 3.1 as

\[ S(t) = \begin{cases} 
  \frac{u_p t^2}{2t_p}, & 0 \leq t \leq t_p \\
  \frac{u_p}{t_h - t_p} + \frac{u_p}{t_h - t_p} \left( t_h(t - t_p) - \frac{1}{2}(t^2 - t_p^2) \right), & t_p \leq t \leq t_h \\
  1, & t \geq t_h \end{cases} \tag{3.3} \]

The discharge hydrograph at the outlet of the entire catchment can then be obtained by summation of the discharge hydrographs from the subcatchments, taking into account the travel times between two consecutive subcatchments, which is given as \( t_c \) divided by the number of subcatchments, \( N \).

\[ \frac{q_o(t)}{i} = \sum_k^{N} \frac{q_e(t-(k-1)t_c/N)}{i} \tag{3.4} \]

Robinson and Sivapalan (1997b) derived the PDR for a triangular IUH as a function of a ratio of the two timescales.

\[ s(t_r) = \begin{cases} 
  \frac{(t_r/t_h)(2 - (t_r/t_h))}{}, & t_r \leq t_c \\
  1, & t_r \geq t_c \end{cases} \tag{3.5} \]

where \( s(t_r) \) is the PDR.
Figure 3.2 The Peak Discharge Response (PDR) as a function of three timescales, $t_r$, $t_h$, and $t_c$ for stationary storms: (a) PDR as a function of $t_r$ when $t_c=0$; (b) PDR as a function of $t_r$ when $t_c=16$

Figure 3.2 shows the PDR for a stationary rainstorm as a function of three timescales, $t_r$, $t_h$, and $t_c$. When $t_c=0$, the discharge hydrograph from each subcatchment exactly overlaps and the relation between the PDR shown in Equation 3.4 is reduced to Equation 3.5 for the whole catchment, as shown in Figure 3.2 (a). Compared with the cases when $t_c > 0$, the PDR has its maxima when $t_c = 0$. When $t_c > 0$, the PDR decreases compared to same duration of rainfall $t_r$ with $t_c = 0$, as shown in Figure 3.2 (b). This is caused by the increased time of concentration for the entire catchment due to the travel time through the catchment to the outlet of the catchment, $t_c$. Figure 3.2 also shows the rainfall duration that is necessary for the PDR to reach the equilibrium condition, when PDR =1. When $t_c = 0$, the PDR reaches equilibrium when $t_r = t_h$. When $t_c > 0$, rainfall duration $t_r$ for the equilibrium condition is

$$t_r = t_h + \frac{L_{cl}-L_{sc}}{L_{cl}} t_c$$

(3.6)

which corresponds to the time of concentration of the entire catchment increased by a channel travel time, $t_c$. If the number of the subcatchments is sufficiently large, the equilibrium condition reduces to

$$\frac{t_r}{t_h} = 1 + \frac{t_c}{t_h}$$

(3.7)

Equation 3.5 indicates that the PDR can be simply expressed as a function of a ratio of
two timescales with the catchment timescale, \( t_h \) as a denominator. This study thus has introduced two ratios of timescales, \( t_c/t_h \) and \( t_r/t_h \) since both \( t_c \) and \( t_r \) are important in the description of the PDR.

\[
s_{st} \sim f \left( \frac{t_c}{t_h}, \frac{t_r}{t_h} \right)
\]  

(3.8)

where \( s_{st} \) is the PDR for a stationary storm. Figure 3.3 shows that for a given rainfall duration, \( t_r \), the PDR has its maximum when \( t_c = 0 \). This relation for the PDR follows the vertical gray dashed lines as \( t_r \) increases. The white dash-dot lines in Figure 3.3 are lines for the PDR to reach equilibrium, which are given by Equation 3.6. For example, the PDR shown in Figure 3.2 (a) corresponds to the gray dashed line in Figure 3.3 when \( t_c = 0 \).

![Figure 3.3 The Peak Discharge Response (PDR) for stationary rainstorms as a function of \( t_r/t_h \) and \( t_c/t_h \)](image)

### 3.3. Moving rainstorms

A moving rainstorm refers to a storm that covers a fraction or entire region of a
catchment with uniform rainfall (excess) and constant intensity and the storm moves with a constant speed in the longitudinal direction of the catchment. Compared with a stationary storm, a moving storm (Figure 3.4) involves two extra extrinsic scales; one length scale, $L_s$ and one timescale, $t_s$ which is equal to $L_{cl}/v_s$. $L_s$ is the length scale of the storm in the direction of the storm movement and $v_s$ is the storm speed. The timescale, $t_s$ depends on both the length scale of the catchment and storm speed. The storm duration, $t_r$, which is regarded as an independent variable for a stationary storm, is now a function of $v_s (L_s/v_s)$ and is related to $t_s$ with the ratio of the two length scales; $L_{cl}$ and $L_s$. In Figure 3.4, it is assumed that the point at the center of each subcatchment represents the subcatchment in terms of intensity and duration of the rainfall. For example, the subcatchment does not take any precipitation until the rainstorm reaches the center point. Once the rainstorm reaches the center, the rainfall is considered to be uniformly falling over the corresponding subcatchment.

Figure 3.4 A conceptual catchment composed of $N$ consecutive subcatchments with a moving rainstorm; the center point of each subcatchment presents the intensity and duration of the rainfall that the subcatchment receives.
With the assumption of the point precipitation being made for each subcatchment, Figure 3.5 illustrates the differences as well as similarities that can be found in the synthesis of hydrographs from the subcatchments for a stationary storm and for a moving storm. Additionally, Figure 3.5 shows how the hydrographs from the subcatchments combine to produce the total response of the entire catchment. For stationary storms, the hydrograph from each subcatchment is delayed by $t_c$ as shown in Figure 3.5 (a). When $t_c = 0$, all the hydrographs from the subcatchments overlap, as shown in Figure 3.5 (b). The case shown in Figure 3.5 (a) for stationary storms corresponds to that of Figure 3.5 (c) for moving storms, except that the first response from subcatchment $C_1$ is delayed by $(N-1)/N \cdot t_s$ and the timescale between the discharge hydrographs from subsequent subcatchments is reduced to $(t_c-t_s)/N$ as shown in Figure 3.5 (c).

Figure 3.5 The synthesis of hydrographs from subcatchments for stationary and moving storms
As $t_s$ increases; rainstorms become slower, $(t_c - t_s)/N$ is reduced and eventually it becomes zero when $t_s = t_c$. Therefore, the case of Figure 3.5 (b) for stationary storm corresponds to that of Figure 3.5 (d) for moving storms, but the hydrographs are delayed by $(N-1)/N \cdot t_c$. Figure 3.5 (d) illustrates how a moving storm exacerbates the flood peak as if $t_c = 0$ for stationary storms. A moving rainstorm drives the catchment into the condition that all the subcatchments of the catchment are superimposed. It implies that relatively shorter duration of moving rainstorm makes the catchment to reach the equilibrium discharge compared with stationary storms. The exact superposition occurs when the storm is moving with the same speed as the channel flow. For stationary storms, the condition that $t_s = t_c$ for moving storms corresponds to $t_c = 0$. This condition is called a resonance condition (Ngirane-Katashaya and Wheater, 1985).

As shown in Figure 3.5 (c), if $t_c > t_s$ (the storm moves faster than the channel flow), the order of discharge hydrographs from subcatchments for a moving storm is the same as for a stationary storm. When the $t_c = t_s$ (the storm moves at the same speed as the channel flow) as shown in Figure 3.5 (d), then the discharge hydrographs from all subcatchments exactly overlap, which corresponds to a stationary storm that has $t_c = 0$. For $t_s > t_c$ (the storm moves slower than the channel flow), the order of hydrographs is reversed, as shown in Figure 3.5 (e). Therefore, the discharge hydrograph at the outlet for a moving storm can be obtained as follows, depending on $t_s$ and $t_c$.

$$q_o(t) = \begin{cases} 
\sum_{k=1}^{N} q_e\left(t - \frac{(N-k)}{N} \cdot t_s - \frac{(k-1)}{N} \cdot t_c\right), & t_s < t_c \\
\sum_{k=1}^{N} q_e\left(t - \frac{(k-1)}{N} \cdot t_s + \frac{(N-k)}{N} \cdot t_c\right), & t_s > t_c \\
\sum_{k=1}^{N} q_e\left(t - \frac{(N-1)}{N} \cdot t_c\right), & t_s = t_c
\end{cases}$$

(3.9)

Due to interdependencies between the rainfall duration, $t_r$ for moving storms and $t_s$ with the ratio of the two length scales; $L_{cl}$ and $L_s$, the PDR can be expressed as a function of two timescale ratios ($t_s/t_h$, $t_c/t_h$), and one length scale ratio ($L_s/L_{cl}$).

$$s_{mv} \sim f\left(\frac{t_c}{t_h}, \frac{t_s}{t_h}, \frac{L_s}{L_{cl}}\right)$$

(3.10)

where $s_{mv}$ is the PDR for moving storms.

Figure 3.6 depicts the PDR for moving storms as a function of $t_s/t_h$, $t_c/t_h$ and $L_s/L_{cl}$ obtained from Equation 3.9; it illustrates that The PDR for moving storms is highly
dependent on the size of the storms. When the relative size of the storm \((L_s/L_{cl})\) is equal to 0.25, the maximum PDR is obtained with \(t_s/t_h\) close to \(t_s/t_h\) which is shown as a narrow band in Figure 3.6. If \(L_s/L_{cl} < 0.75\), the PDR increases and reaches to maximum PDR, then it decreases as \(t_s/t_h\) increases. But, when \(L_s/L_{cl} = 0.75\), the peak reaches the maximum peak, but it no longer decreases as \(t_s\) increases. If the size of the storm is equivalent to the catchment \((L_s/L_{cl} = 1)\), the PDR increases up to the equilibrium discharge eventually, and it does not decrease as \(t_s\) increases once it reaches equilibrium as shown in Figure 3.6.

![Figure 3.6 The PDR (Peak Discharge Response) for moving storms as a function of \(t_s/t_h, t_c/t_h\) and \(L_s/L_{cl}\)](image)

Figure 3.7 depicts the PDR for moving storms for specific storm sizes and the resonance condition; it illustrates the discrepancy between the resonance condition (solid line) and the actual condition for maximum PDR (dashed line). The white solid line in Figure 3.7 represents the resonance condition \((t_s/t_c=1)\) and the PDR has its maximum values in the vicinity of this line. However, the result shows that resonance condition is incongruent with the actual maximum response. The discrepancy becomes larger as \(t_c/t_h\) becomes
smaller before the peak response reaches the equilibrium discharge. Once the PDR reaches the equilibrium discharge, the resonance condition coincides with the actual condition that produces the maximum PDR. Figure 3.7 shows that the condition that a catchment reaches equilibrium changes with the relative storm size \((L_s/L_c)\) as well as \(t_s/t_h\) and \(t_s/t_h\). The white dots in Figure 3.7 represent the 90% of the PDR. As the storm size \((L_s/L_c)\) increases from 0.25 (Figure 3.7 (a)) to 1 (Figure 3.7 (d)), the width between upper and lower boundaries of 90% of the PDR becomes larger. When \(L_s/L_c \geq 1-L_c/L_c\) (Figure 3.7 (c) and (d)), the upper boundary no longer exists because the peak does not decrease once it reaches its maximum.

Figure 3.7 The PDR for moving storms; resonance conditions (solid), actual conditions for the maximum PDR (dash) and 90% PDR boundaries (dot) for storm size of (a) \(L_s/L_c = 0.25\); (b) \(L_s/L_c = 0.5\); (c) \(L_s/L_c = 0.75\); (d) \(L_s/L_c = 1\)
3.4. Results and discussion

Effects of storm sizes

Figure 3.8 The Peak Discharge Response (PDR) for moving storms as a function of $(t_c-t_s)/t_h$, $t_r/t_h$ and $L_s/L_{cl}$; here $L_s/L_{cl}$ is equal to 0.5

The similarities between stationary and moving storms, which can be found during the synthesis of the hydrographs from the subcatchments (Figure 3.5), enable us to extend the relationship of the PDR for stationary storms to moving storms in a simple manner. As shown in Figure 3.5, the timescale, $t_c$ for stationary storms can be replaced by $t_c-t_s$ for moving storms. It should be noted that $t_c-t_s$ can be negative. Figure 3.8 depicts the PDR for moving storms as a function of two ratios of the timescale, $t_r/t_h$ and $(t_c-t_s)/t_h$. The PDR in Figure 3.8 is exactly the same as for stationary storms (Figure 3.3) if $(t_c-t_s)/t_h > 0$. For $(t_c-t_s)/t_h < 0$, the PDR is symmetric with respect to the y-axis ($t_r/t_h$). However, due to interdependencies between $t_s$ and $t_r$, the PDR for moving storms follows the gray dashed lines in Figure 3.8 instead of vertical lines as is the case for stationary storms (Figure 3.3).
The slope of this line is equal to -$L_s/L_{cl}$. The white dotted line in Figure 3.8 is where the PDR reaches the equilibrium discharge and given as the following equation:

$$\frac{t_r}{t_h} = 1 + \frac{L_{cl} - L_{sc}}{L_{cl}} \cdot \frac{|t_c - t_s|}{t_h}$$  \hspace{1cm} (3.11)

Figure 3.9 shows how the PDR changes as the relative size of rainstorm with respect to catchment size increases. Figure 3.9 (a) and Figure 3.9 (b) show the PDR for moving storms when $L_s/L_{cl} = 0.25$; the rainstorm covers at most 25% of the catchment. The gray dotted line in Figure 3.9 (b) ($t_c = 160$ and $t_h = 40$) corresponds to the gray dashed line in Figure 3.9 (a) ($t_c = 4t_h$). For a steeper slope ($L_s/L_{cl} = 1$ in Figure 3.9 (c)), all the PDR eventually reaches equilibrium as $t_c - t_s$ decreases (indicating the storm becomes slower). The effect of the storm size with respect to the catchment size on the PDR is summarized in Table 3.1; the relative storm size, $L_s/L_{cl}$ works as a threshold determining whether or not the catchment ultimately reaches equilibrium. Figure 3.9 also shows when the catchment reaches the equilibrium in terms of $t_c$ and $t_h$. For example, when $L_s/L_{cl} = 1$, the system reaches equilibrium if $t_c \geq t_h$ as shown in Figure 3.9 (c) and also as shown in Figure 3.7 (d).

<table>
<thead>
<tr>
<th>Storm sizes</th>
<th>Description of the PDR for moving storms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_s/L_{cl} &lt; 1$</td>
<td>The PDR is highly sensitive to rainstorm speed: $t_s$ and $t_r$</td>
</tr>
<tr>
<td>$L_s/L_{cl} = 1$</td>
<td>The PDR no longer decreases with increasing $t_s$ and $t_r$</td>
</tr>
<tr>
<td>$L_s/L_{cl} &gt; 1$</td>
<td>The PDR reaches equilibrium with increasing $t_s$ and $t_r$</td>
</tr>
<tr>
<td>$L_s/L_{cl} \gg 1$</td>
<td>The relation tends to be that for stationary rainstorms and the PDR becomes independent of $t_s$</td>
</tr>
</tbody>
</table>

Table 3.1 The effect of the size of rainstorms with respect to the size of a catchment on the PDR

Figure 3.9 (b) illustrates that the variation of the peak response is large. As the storm becomes slower, the peak response increases first and then it decreases when $L_s/L_{cl} < 1$. This is consistent with the results of Surkan (1974) who concluded that the peak discharge is quite sensitive to change with the moving rainstorm especially when the storm size is smaller than the catchment size. In contrast, when $L_s/L_{cl} > 1 - L_{sc}/L_{cl}$, the PDR does not decrease once it reaches equilibrium as $t_c - t_s$ decreases. There is a moment when the slope of the PDR is equal to the line of equilibrium given by the following equation:

$$\frac{t_r}{t_h} = 1 + \frac{L_{cl} - L_{sc}}{L_{cl}} \cdot \frac{|t_c - t_s|}{t_h}$$
\[
\frac{L_s}{L_{cl}} = 1 - \frac{L_{sc}}{L_{cl}} \tag{3.12}
\]

In this case, the PDR no longer decreases as \( t_c - t_s \) decreases even if the maximum PDR does not reach equilibrium.

![Diagram](image)

**Figure 3.9** The impact of the rainstorm sizes on the Peak Discharge Response (PDR) for moving storms when \( t_h = 40 \): (a) when \( L_s/L_{cl} = 0.25 \); (b) the corresponding PDR vs \( t_c - t_s \); (c) when \( L_s/L_{cl} = 1 \); (d) the corresponding PDR vs \( t_c - t_s \)

If the storm size keeps increasing, the PDR for moving storms tends to be equivalent to stationary storms. Figure 3.10 shows the PDR for moving rainstorms when \( L_s/L_{cl} = 40 \). In
Figure 3.10, the gray lines for the PDR approximate vertical lines, which are for stationary storms as shown in Figure 3.3. Hence, the PDR becomes independent of the storm speed and no longer a function of \( t_s \). In this case, \( t_r \) is no longer a dynamic duration by storm movement, instead it is an internal duration of rainfall depending on the life cycle of a rainstorm.

The results indicate that even if the storm size is smaller than the catchment length scale \( (L_s/L_{cl} < 1) \), the PDR can reach the equilibrium states as shown in Figure 3.6, Figure 3.8 and Figure 3.9 (a) depending on \( t_c/t_h \). This result is consistent with that of Lee and Huang (2007) who found that runoff can attain equilibrium discharge for storms moving downstream even though the storm length is smaller than the catchment and the duration is less than the time to equilibrium for stationary storms.

**Figure 3.10** The Peak Discharge Response (PDR) for moving storms as a function of \( (t_c - t_s)/t_h \) and \( t_c/t_h \) when \( L_s/L_{cl} = 40 \) given as the gray dashed lines
Timescales and length scales required to describe the PDR for moving storms

From the previous results, it can be inferred that a stationary storm can be viewed as a special case of moving storms when the storm sizes are much larger than the catchment size. As mentioned earlier, it is necessary to distinguish the internal duration of a rainstorm depending on its own cycle of development from the dynamic duration of rainfall due to rainstorm movement. Table 3.2 lists the timescales and length scales required to describe the behavior of the PDR for both stationary and moving storms. For stationary storms, three timescales are required to describe the PDR including the internal duration of the rainfall. For moving storms, the difference between the two timescales, \( t_c \) and \( t_s \) is decisive for the description of the PDR as is \( t_c \) for stationary storms. The length scale ratio, \( L_s/L_{cl} \) is also crucial for moving storms because it works as a threshold to determine whether or not the catchment reaches equilibrium. It should be noted that all the previous results and discussions of the PDR for moving storms are valid only if \( t_s \) and \( t_c \) are less than \( t_r, \) internal.

Table 3.2 The PDR as a function of the timescales and length scales for stationary storms and moving storms

<table>
<thead>
<tr>
<th>Properties of a catchment</th>
<th>Stationary storms</th>
<th>Moving storms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Timescales</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hillslope</td>
<td>( t_h = L_{cl}/v_h )</td>
<td>( t_h = L_{cl}/v_h )</td>
</tr>
<tr>
<td>Channel flow</td>
<td>( t_c = L_{sc}/v_c )</td>
<td>( t_c = L_{sc}/v_c )</td>
</tr>
<tr>
<td>Storm duration</td>
<td>( t_r, ) internal</td>
<td>( t_r, ) dynamic = ( L_s/v_s )</td>
</tr>
<tr>
<td>Storm travel time</td>
<td>-</td>
<td>( t_s = L_{sc}/v_s )</td>
</tr>
<tr>
<td>Length scales</td>
<td></td>
<td>( L_s/L_{cl} )</td>
</tr>
<tr>
<td>Storm size</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Interdependency</td>
<td>-</td>
<td>( t_s = t_c )</td>
</tr>
<tr>
<td>The PDR* as a function of</td>
<td>( t_d/t_h, t_s/t_h )</td>
<td>( t_d/t_h, t_s/t_h, L_s/L_{cl} )</td>
</tr>
<tr>
<td>Resonance condition</td>
<td>( t_c = 0 )</td>
<td>( t_c &lt; t_s ) if ( t_c &lt; t_h + t_c )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( t_c = t_s ) if ( t_c \geq t_h + t_c )</td>
</tr>
</tbody>
</table>

* The peak discharge response normalized by a rainfall intensity

Resonance effects

As shown in Figure 3.3 and Figure 3.8, the relations of the PDR and the timescales for both stationary and moving storms are exactly the same if \( t_d/t_h \) is replaced by \( (t_c-t_s)/t_h \). For stationary storms the PDR has its maxima when \( t_c = 0 \). Following this logic one could
expect that the maximum PDR for a moving storm can occur when \( t_c = t_s \). This effect has been called a “resonance effect” (Ngirane-Katashaya and Wheater, 1985) and previously regarded as valid and effective (Surkan, 1974; Foroud et al., 1984; Niemczynowicz, 1984a, b; Ngirane-Katashaya and Wheater, 1985; Watts and Calver, 1991; Singh, 1997).

In contrast to stationary storms, \( t_r \) is no longer an independent variable for moving storms.
These two timescales are related by the storm size and the catchment size in the direction of the storm progress; \( t_r = \left( \frac{L_s}{L_{cl}} \right) t_c \). Figure 3.11 (a) shows the discharge hydrograph at the outlet for various \( t_c \) for given \( t_s \). If \( t_s \) is given as a constant, the PDR has its maximum values when \( t_s (\approx 40) \) is equal to \( t_c (\approx 40) \). Figure 3.11 (b) shows the behavior of the PDR as a function of \( t_c \) where the PDR has its maximum when \( t_s = t_c \) (resonance condition). However, even though \( t_s > t_c \), the peak response can be greater than that of resonance condition. Figure 3.11 (c) and Figure 3.11 (d) show the discharge hydrograph and the PDR as a function of \( t_s \), which show that the maximum PDR does not occur when \( t_s = t_c \). The PDR has its maximum values when \( t_s > t_c \) (Figure 3.11 (d)). As shown in Figure 3.5, the hydrograph from each subcatchment depends on both \( t_h \) and \( t_r \). Accordingly, as \( t_s \) increases, \( t_r \) increases, which results in increase of the peak response. The maximum peak response from a moving rainstorm arises from a combination of two effects; resonance condition and varying duration as a function of storm travel timescale. The augmentation of peak response results from both effect of overlaying the responses from subcatchments and effect of increased responses from subcatchments due to increased duration. The effect of overlaying responses form subcatchments is maximized with the resonance condition. However, the effect of increasing response with increasing duration (slowing rainstorm) keeps escalating until the catchment reaches the equilibrium discharge. Therefore, the peak response is maximized when the storm is a little bit slower than the channel flow velocity and the effect of resonance condition is surpassed by increasing duration. The maximum peak response nevertheless occurs in the vicinity of resonance condition but when the moving rainstorm is slower than the channel flow.

Given same amount of rainfall (the storm duration), the peak response for moving storms is higher than that of stationary storms as shown in Figure 3.12. In Figure 3.12 (a), the thick gray line is the PDR for stationary storms when \( t_c = 0 \). As mentioned earlier, when \( t_c = 0 \) in stationary storms, the PDR has its maximum. However, for moving storms, the PDR can reach the maximum even if \( t_c > 0 \) because the resonance condition depends on two timescales; \( t_c \) and \( t_s \) as shown in Figure 3.12 (a). The PDRs for moving storms are tangential to the maxima for stationary storms for the resonance condition. Moreover, Figure 3.12 (a) shows that the maximum peak response for moving storms happens after the resonance condition when \( t_s > t_c \). Figure 3.12 (b) compares the PDRs for both...
stationary and moving storms when the storm size is 1/4 of the catchment given \( t_c = 40 \) and \( t_h = 20 \). If the rainfall duration is less than the time to equilibrium for stationary storms, there is a duration interval for which the same rainfall duration produces much larger peak responses for moving storms. Beyond this interval, as \( t_s \) increases, rainstorm moves much slower than the channel flow. The delay of rainstorm movement results in fragmentation of the contributions from subcatchment and the peak response decreases to be smaller than that of stationary storms and eventually flattens out.

Figure 3.12 The PDR \((t_h= 20, L_s/L_{cl}= 0.25)\): (a) for moving storms as a function of \( t_s/t_h \) where the gray line is the PDR for stationary storms with \( t_c=0 \); (b) Comparison between the PDR for moving and stationary storms given \( t_c \) and \( t_h \)

### 3.5. Conclusions

This chapter investigated the peak discharge response of a hypothetical catchment under moving rainstorm conditions utilizing a broad theoretical framework with characteristic time and space scales. This study is based on assumptions including constant storm speed, constant channel flow velocity of which variability would affect the resonance condition. Moreover, this study does not consider within storm pattern and antecedent condition of rainfall, which would affect the complex behavior of peak response from moving rainstorms. In spite of all simplified assumptions, this study clarifies where the variability of peak response of a moving rainstorm originates from in terms of various time and
space scales. The variability of the peak discharge response for a hypothetical catchment is shown to be a function of the intrinsic timescales of the catchment as well as the extrinsic timescales of stationary and moving storms. We show that three timescales are required to describe the PDR for a stationary storm; two intrinsic timescales ($t_h$, $t_c$) and one extrinsic timescale ($t_r$). The PDR can be expressed as a function of two timescale ratios, $t_c/t_h$ and $t_r/t_h$, for stationary storms. A moving storm requires additional extrinsic time and space scales; a relative storm size with respect to the catchment ($L_o/L_{cl}$) and a travel time of storms ($t_s$). For moving storms, the PDR can be expressed as a function of two timescale ratios, $t_c/t_h$, $t_s/t_h$ and one length scale ratio $L_o/L_{cl}$.

With the framework suggested in this chapter, we propose that the relation between the PDR and timescales for a stationary storm can be extended for a moving storm. However, interdependencies between two extrinsic timescales of a rainstorm (the duration and storm travel time) make the behavior of the PDR for a moving storm fundamentally different from that of a stationary storm. In addition, the interdependency proves to be significant in the examination of the resonance condition. We showed that the augmentation of peak response for moving rainstorms results from both effect of overlaying the responses from subcatchments and effect of increased responses from subcatchments due to increased duration. For this reason, the peak response is maximized when the storm is a little bit slower than the channel flow velocity and the effect of resonance condition is surpassed by increasing duration. We also show that due to the resonance condition a moving rainstorm can produce a higher peak compared with a stationary storm given the same rainfall duration.
4. THE EFFECT OF RAINSTORM MOVEMENT AND ITS RELATION WITH NETWORK PROPERTIES IN URBAN CATCHMENTS*

This chapter explores the relations between network properties and the effect from moving rainstorms in terms of the peak response and time to centroid of hydrographs. A simple conceptual rectangular catchment is introduced with different configurations of drainage network simulated by Gibbs’ stochastic model. The efficiency of the urban pipe networks vary widely compared with natural river networks, hence, Gibbs’ model can be an appropriate approach to represent the network properties in urban drainage system. Simple cases of rainstorms moving with upstream and downstream directions and different speeds are considered in order to investigate the effect of rainstorm movement on urban drainage network runoff hydrographs. The results indicate that the effect of the direction and speed of the rainstorm movement varies significantly depending on the network properties. The relation between storm speed and direction and the change in the peak runoff is dependent on the network configuration and network efficiency. In contrast to previous studies, this study indicates that the speed and direction of the rainfall movement that produces the maximum peak discharge changes depending on the network configuration.

4.1. Introduction

Prediction and estimation of runoff has been being a long-standing topic of hydrology for the purpose of water resources management both in quality and quantity, flood control, ecological and environmental issues. It is well recognized that the surface runoff from a

watershed varies with the hydro-meteorological characteristics of the rainfall and the physiographic properties of the watershed (Yen and Chow, 1969). The factors affecting a flow hydrograph can be categorized into (a) watershed characteristics; (b) storm precipitation dynamics; (c) infiltration; and (d) antecedent conditions (Singh, 1997). For a given amount of rainfall and duration, the temporal and spatial distributions of the rainfall vary with the movement of a rainstorm, which results in significant difference in response at the outlet of a watershed in terms of runoff hydrographs.

Storm movement is the norm rather than the exception (Singh, 1997). Moreover, there exists a directional preference of rainstorm movement. Huff (1976) found that 84% of severe heavy rainstorms exhibited motion with a westerly component. Although the directional preferences of moving rainstorms have been being widely recognized (Huff, 1976; Shearman, 1977; Upton, 2001), the storm movement and subsequent rainfall variation are hardly regarded in the design process of urban drainage networks. In view of maximizing the effectiveness of urban drainage network and minimizing potential avoidable risks, it is necessary to consider the effect of the moving rainstorms. The effect of moving rain possibly needs to be utilized to improve design processes for urban drainage network. A general relation between the storm movement dynamics and the hydrograph is crucial in order to accomplish this.

The direct influence of rainfall movement on the shape of the runoff hydrograph has been recognized for a long time (Maksimov, 1964; Yen and Chow, 1968; Wilson et al., 1979; Jensen, 1984; Niemczynowicz, 1991; Singh, 1998). Researchers have considered the rainstorm directions (Marcus, 1968; Yen and Chow, 1969; Surkan, 1974; Foroud et al., 1984; Jensen, 1984; Niemczynowicz, 1991; Anderson et al., 1991; Singh, 1998; de Lima and Singh, 2002, 2003; de Lima et al., 2003; Morin, 2006) and speed (Marcus, 1968; Yen and Chow, 1969; Surkan, 1974; Foroud et al., 1984; Jensen, 1984; de Lima and Singh, 2002). Rainfall intensities (Yen and Chow, 1969; Morin, 2006), surface slopes (Yen and Chow, 1969; de Lima et al., 2003) as well as shapes of hyetographs (de Lima and Singh, 2002) were also considered as affecting factors to the runoff hydrographs. Typically, compared with a storm moving upstream, a storm moving downstream direction shows later peak, greater peak discharge, steeper rising limb and shorter base time.

However, previous studies and research related to moving storms generally focused on
the response of a specific catchment rather than generalizing to a variety of range of catchments. Therefore, it is difficult to extrapolate the results from these studies describing the effect of storm movement to different watersheds. In contrast, this study explores the idea that the effect of rainstorm movement might be different for different network configurations. Although the relation between topology of a network and the hydrologic response has been recognized and led to development of Geomorphological Instantaneous Unit Hydrograph (GIUH) model (Rodriguez-Iturbe and Valdes, 1979; Gupta et al., 1980), a direct relation between the effect of the moving rainstorm and the configuration of network has not been pursued.

In order to identify and categorize the network according to its configuration, stochastic network models are adopted in this study; the Scheidegger model (Scheidegger, 1967a, b), the Uniform model (Leopold and Langbein, 1962; Karlinger and Troutman, 1989) and Gibbs’ model (Troutman and Karlinger, 1992) based on Gibbs’ measure (Ising, 1925; Kindermann and Snell, 1980). In contrast to the random models (Scheidegger, 1967a, b; Karlinger and Troutman, 1989), which were based on an assumption of the absence of environmental controls, Gibbs’ model introduced a control over the overall sinuosity of a network. This enables us to gain insight into geomorphologically significant factors such as drainage network efficiency, drainage density, and variation of network topology with basin shape and relief (Troutman and Karlinger, 1992).

4.2. Methodology

Stochastic network model

A stochastic network model based on Gibbs’ distribution suggested by Troutman and Karlinger (1992) is utilized in this study to identify and categorize a network. The Scheidegger model and Uniform model are the special cases of Gibbs’ model that correspond to Gibbs’ parameter ($\beta$) equal to infinity and zero, respectively.

In the following description, terminology based on graph theory is used; a graph is acyclic if it has no cycles. A tree is a connected acyclic graph in which any two points (or vertices) are connected by exactly one simple path and a spanning tree is an acyclic tree connecting all the points in the network without loops (or cycles). Two spanning trees are adjacent if one may be obtained from the other by randomly select one point defining one
new direction from that point to produce a new spanning tree. The notion of adjacency corresponds to the idea of “minimal change” in graph structure discussed by Aldous (1987); a change in direction of a link in this study. To generate a stochastic network model, a Markov chain is defined with the spanning trees $S_t$ as the state space. Let $s$ belong to $S_t$ and two trees $s_1$ and $s_2$ be adjacent. Then the transition probability from $s_1$ to $s_2$ can be defined as follows (Troutman and Karlinger, 1992).

$$R_{s_1s_2} = \begin{cases} 
    d_s^{-1} \min \left\{ 1, e^{-\beta[H(s_2) - H(s_1)]} \right\} & s_2 \in N(s_1) \\
    1 - \sum_{s \in N(s_1)} R_{s_1s} & s_2 = s_1 \\
    0 & \text{otherwise}
\end{cases} \quad (3.1)$$

where $N(s_1)$ is the set of trees adjacent to $s_1$, and $\beta$ is a parameter that represents the extent to which the sinuosity of the network is reflected in generation of the new spanning tree, $s_2$. For example, when $\beta$ is equal to zero, the sinuosity has nothing to do with the transition probability and the transition probabilities are same in all possible directions, which is identical to the uniform distribution. The maximum degree of the points in $S_t$, $d_s$, is defined as

$$d_s = \max_{s \in S_t} |N(s)| \quad (3.2)$$

The degree, $d_s$, means the maximum number of direction that can be selected except the existing direction. The Markov chain with this transition probability has a stationary distribution given as Gibbs’ distribution (Troutman and Karlinger 1992), which has the form of Gibbs’ measure where $\beta H(s)$ represents a positive energy function.

$$P_{\beta} \{s\} = \left[C(\beta)\right]^{-1} e^{-\beta H(s)} \quad (3.3)$$

where $s$ belongs to $S_t$. $C(\beta)$ is a normalization factor defined to make the sum of the distribution be one.

$$C(\beta) = \sum_{s \in S_t} e^{-\beta H(s)} \quad (3.4)$$

where $s$ is a tree, a component of $S_t$. $H(s)$ is a measure of the sinuosity of a given spanning tree, $s$.

$$H(s) = \sum_{d \in D(B)} \xi_s(d) - \sum_{d \in D(B)} \xi_b(d) \quad (3.5)$$

where $d$ is a point of a finite and connected graph $B$ and $D(B)$ is the set of the total points.
of $B$. $\zeta_i$ is the distance to an outlet along $s$ from $d$ while $\zeta_B$ is the shortest distance to the outlet from $d$.

The Scheidegger model and uniform distribution model are also utilized in this study to examine their relation with Gibbs’ model and also to explore the network properties of natural river networks. The Scheidegger (1967b) model is a directed self-avoiding random walk (DSAW) model on a lattice with flow from each point being allowed in only two orthogonal and downstream directions, each with equal probability. Four directions used in this study. The Uniform model is defined on a lattice with flow allowed in each of four possible directions with equal probability. The Uniform model is generated by starting from the outlet point and proceeding upstream with uniform distribution until it reaches to the boundary of the network or the network cannot be extended because all adjacent points have been visited before. Then, a point is selected randomly among the points not visited before and the same procedure is repeated as before. It can be started from the outlet point again, but the reason an unvisited point is used here is to reduce the total time needed to generate the network. Again, the steps are repeated until having every single point in the whole domain visited. The Uniform model can be also generated with Gibbs’ model when $\beta$ equals to zero (meaning every direction has equal probability). On the other hand, the Scheidegger model can be represented as the other extreme of Gibbs’ model when $\beta$ tends to infinity.

The procedure used in this study in order to generate a Gibbs’ network given a parameter, $\beta$ is as follows: First, start from a network, $s_1$ generated by the Uniform model and randomly select a point, $d$ in the network and assign a new flow direction from $d$ to generate adjacent network $s_2$. Second, check whether the new network, $s_2$ is acyclic. If not, repeat the first step. Third, draw a random probability $x$ between zero and one and check that $x$ is greater than $e^{-\beta \Delta H}$ where $\Delta H$ is equal to $H(s_2) - H(s_1)$. If this holds, then take $s_2$ as a new network. In the next step use $s_2$ as the starting network and repeat these steps sufficiently number of times that the resulting tree has the distribution close to the stationary Gibbs’ distribution. In this study, the number of iteration to produce Gibbs’ network model was set to two thousand, which is large compared to the total number of points in the network matrix, sixty four points.
Test catchment and rainfall

In this study, the test basin is represented by a square lattice (8 by 8) with the bottom rightmost point as an outlet point. In order to assess the effect of rainstorm movement, two types of rainfall are considered with different directions and speeds. First rainfall input is a stationary rainfall that is uniform throughout the basin. Second rainfall input is moving rainstorm with different speeds. A rainfall strip with unit width (1/8 of the total width of the basin) moving in a given direction is assumed (Figure 4.1). Two different directions are examined; one moving to the downstream and the other moving to the upstream as shown in Figure 4.1 (a) and (b), respectively.

The width function is used to define hydrologic response of the catchment, defined as follows: For a given flow distance $\xi$, the width function $L(\xi)$ is equal to the number of points at distance $\xi$ from the outlet. Under the assumption of a simple routing concerning
convection only through the channel network with a spatially uniform flow velocity, the width function can be scaled to be identical to the shape of unit hydrograph (Troutman and Karlinger, 1985).

\[ L(\xi) = N\{v(\xi)\}, \xi = 1, \ldots, \xi_L \]  

(3.6)

where \( L(\xi) \) is the width function for network which can be obtained by counting the number of the grid points with distance \( \xi \) from the outlet. \( v(\xi) \) is the grid point of which distance from the outlet is equal to \( \xi \). \( \xi_n \) is the longest distance from the outlet. A width function is newly defined as a time-dependent function incorporating a rainfall excess given distance \( \xi \) at time \( t \).

\[ L'(\xi, t) = \sum_{i,j} r_{ij}(\xi, t), \xi = \xi_1, \xi_2, \ldots, \xi_n \]  

(3.7)

where \( r_{ij}(\xi, t) \) is the rainfall intensity at a grid point \((i, j)\) of which distance from the outlet is \( \xi \) at time \( t \). Then the response function at the outlet for each time step can be expressed as follows assuming convection only for channel routing.

\[ Q(t) = \int L'(\xi, \tau) \cdot \delta(t - \tau - \xi / c) d\xi d\tau \]  

(3.8)

where \( c \) is a constant propagation celerity. In a discrete form, the Equation 8 can be written as

\[ Q(t) = \sum_{i=1}^{m} \left( \sum_{j=1}^{n} L'(\xi_j, \tau_i) \cdot \delta(t - \tau_i - \xi_j / c) \Delta \xi \right) \Delta \tau \]  

(3.9)

### 4.3. Width function and Hack’s exponent for stochastic networks

For each stochastic network model, one hundred networks are randomly generated and the corresponding width functions are obtained. Figure 4.2 shows a realization of the Uniform, Scheidegger, and Gibbs’ network model, respectively and their corresponding width functions. For the Scheidegger model, the distance from the outlet is exactly the same with the shortest distance from the outlet at every grid point, which means that there is no sinuosity in the Scheidegger network model. The number next to the each grid point is the distance from the outlet. In Figure 4.2 (b), the distance is equal to the shortest distance from the outlet at every point in the network for the Scheidegger model.
Figure 4.2 Realization of the stochastic network model and width function for uniform instantaneous rainfall: (a) Uniform (b) Scheidegger (c) Gibbs ($\beta = 1.0$)

The Uniform model is highly sinuous and less realistic than natural river network, while in terms of peak and distance to the peak, the Scheidegger model shows the greatest efficiency and shortest distance from the outlet because $H(s)$ is always zero in the Scheidegger model. Gibbs’ model theoretically ranges from the Uniform to the Scheidegger model depending on the value of $\beta$.

Hack’s exponent (Hack, 1957) is one of the measures that reveal geometrical characteristics of a network. Hack’s exponent describes the relation between the length of longest flow path and the area of the watershed. The length of the longest flow path is a key component of the width function. Table 4.1 lists the values of the Hack’s exponent,
characterizing the topology of the networks generated by the Uniform model, Gibbs’ model with $\beta$ and the Scheidegger model, respectively. For each model, one hundred networks are generated from which an average value of the Hack’s exponent is obtained with 95% confidence interval. Hack’s exponent was given as 0.6 for natural river networks (Hack, 1957). Troutman and Karlinger (1992) estimated $\beta$ of the natural watersheds in Montana and found that the estimated values of $\beta$ lie between 0.2 and 2.21 and the averaged value of estimated $\beta$ is equal to 0.92 (order of $10^0$). These results indicate that Gibbs’ model with $\beta$ greater than $10^0$ produces networks geometrically similar to natural rivers while smaller values of $\beta$ indicate less efficient network (longer flow paths).

<table>
<thead>
<tr>
<th>Table 4.1 Hack’s exponents of stochastic network models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>0.70</td>
</tr>
<tr>
<td>±0.07</td>
</tr>
</tbody>
</table>

4.4. Effect of moving rainstorms

*Uniform stationary rainfall*

First, for a given duration, stationary rainfall uniformly distributed over whole basin is considered. This provides the reference condition to which moving storm response will be compared. Figure 4.3 shows the width function averaged over one hundred simulations of network with uniform stationary rainfall and the box plots show the variation of the width function at a given distance from the outlet. The width function of Gibbs’ network model with $\beta$ of $10^{-4}$ is similar to that of the Uniform network model (Figure 4.3 (a) and (c)). Similarly, as $\beta$ increases in Gibbs’ network, the width function of Gibbs’ model becomes close to that of the Scheidegger network model (Figure 4.3 (b) and (d)). Although both Gibbs’ model with small $\beta$ and the Uniform model show high variation of the width function, the variation decreases in Gibbs’ model as $\beta$ increases and there is no uncertainty in the Scheidegger model.
Figure 4.3 Averaged width function of a stochastic network model over 100 simulations on an 8 by 8 lattice: (a) Gibbs ($\beta = 10^{-4}$) (b) Gibbs ($\beta = 10^0$) (c) Uniform (d) Scheidegger

Figure 4.4 (a) and (b) depict the response at the outlet, $Q(t)$ averaged from 100 simulations of Gibbs’ network model for different values of $\beta$ from $10^{-4}$ to $10^0$ and the Scheidegger network model with a unit instantaneous rainfall and a uniform stationary rainfall over a given duration, $t_d$, respectively. $Q(t)$ is obtained from $L(\xi)$ assuming constant channel velocity. In Figure 4.4 to Figure 4.9, $Q(t)$ and $t$ are normalized with the peak value and maximum duration of $Q(t)$ from the Scheidegger model with unit instantaneous rainfall, respectively. In Figure 4.4, the peak decreases while the time to the peak increases as $\beta$ increases.
Figure 4.4 Averaged response of Gibbs’ network over 100 simulations against $\beta$: (a) unit instantaneous rainfall input (b) uniform stationary rainfall input ($t_d = 8$)

Figure 4.5 Averaged response of Gibbs’ network over 100 simulations against $\beta$ with rainstorm: (a) moving downstream and (b) moving upstream

Rainstorm moving downstream and upstream

Second, moving rainstorm with two directions (downstream and upstream) and the same rainfall duration as the uniform case is investigated to evaluate the effect of the direction of the rainstorm to the response of the networks. The total amount of rainfall is equal to the case of a uniform stationary rainfall. Moving rainstorm is represented by a moving vertical band as previously shown in Figure 4.1. The response, $Q(t)$ of the network with rainstorm moving downstream and upstream are shown in Figure 4.5 (a) and (b), respectively. The peak of the response, $Q(t)$ with rainstorm moving downstream is greater than that with the uniform stationary rainfall with as shown in Figure 4.4 (b) and smaller with rainstorm moving upstream. Instead of the time to the peak response, the time to the centroid of the response function is selected in order for robustness. The peak and the
time to the centroid of $Q(t)$ are compared for three cases (Figure 4.6): the uniform stationary rainfall, the rainstorm moving downstream, and the rainstorm moving upstream. The difference in the peak discharges among the three cases is largest for $\beta$ greater than $10^{-1}$ (Figure 4.6 (a)). Typical values for $\beta$ for natural watershed are in the order of $10^0$ indicating that the natural river networks are sensitive to the direction of the rainstorm in terms of peak flows. Figure 4.6 (b) shows that regardless of the direction of the rainstorm, the time to the centroid is the same for all values of $\beta$.

![Image](image.png)

Figure 4.6 Changes in response with different types of network: (a) peak response and (b) time to the centroid of the response against $\beta$

Moving rainstorm with different speed

In order to look at the effect of speed of the moving rainstorm, four different travel times are considered. The duration of the moving rainstorm event is greater than previous cases. The speed of the rainstorm movement and intensity decrease as the travel time increases. The fastest speed considered is when the time to travel one grid size $t_{TR}$ is equal to one and the slowest speed is when $t_{TR}$ is equal to four. Therefore all the speeds of rainstorm considered in this study are made to be equal to or less than the flow velocity. Singh (1997) showed that the effect of storm speed on peak discharge is much less for rapidly moving storms than for storms moving at about the same speed as the channel flow velocity, which is called ‘resonance condition’ (Ngirane-Katashaya and Wheater, 1985; Watts and Calver, 1991; Singh, 1997). The rainfall intensity is uniformly decreased as the travel time increases so that the total amount of the rainfall is same with the previous
cases.

Figure 4.7 Normalized peak response with (a) downstream, (b) upstream moving storm and time to the centroid of $Q(t)$ with (c) downstream, (d) upstream moving storm against different $\beta$ and storm travel times $t_{TR}$

Figure 4.7 illustrates the peak of the width function and the distance to the centroid with different travel times. As shown in Figure 4.7, the peak response rapidly decreases in case of rainstorm moving downstream especially in the interval of $\beta$ greater than $10^{-1}$. In case of rainstorm moving upstream, the peak steadily decreases as the travel time increases. On the contrary, in terms of the time to the centroid of the response, the moving rainstorm has less effect in case of the rainstorm moving downstream than in case of the rainstorm moving upstream. For the rainstorm moving upstream, the speed of the rainstorm movement significantly affects the time to the centroid of the response but the difference
along with $\beta$ is negligible. As is shown in Figure 4.7, the drainage networks with $\beta$ greater than $10^2$ are more sensitive to the speed and direction of the moving rainstorm in terms of the peak response. As discussed earlier, the network properties of natural watershed are close to the network simulated by Gibbs’ model with $\beta$ greater than $10^0$. This means that the natural watershed can be sensitive to the speed and direction of the moving rainstorm.

Results from Figure 4.7 with $\beta$ greater than equal to $10^0$ are consistent with Singh’s (1997) definition of resonance condition where the greatest peak discharge is for the storms moving with the same speed as the channel velocity. However, the network properties (represented by $\beta$) also effect the resonance condition. As seen in Figure 4.8, the relations among peak discharge, time to centroid and rainstorm speed and direction vary with the value of $\beta$. Figure 4.8 shows the response of different network configurations and storm speed and directions normalized to an instantaneous unit rainfall over the entire watershed with the Scheidegger network. In Figure 4.8 (a), the sensitivity of peak highly depends on $\beta$; for the storm travel time ($t_{TR}$) equal to three, the peak can increase as well as decrease depending on the network ($\beta$). For the travel time ($t_{TR}$) equal to two, the peak flow of a network with lower $\beta$ increases up to 38%, while the peak response of the network with higher $\beta$ that is greater than $10^0$ is approximately equal to uniform stationary response. When the storm travel time ($t_{TR}$) is largest, the response from the networks with higher $\beta$ shows the greatest decrease in terms of the peak response. As shown in Figure 4.8 (b) for upstream moving rainstorms, the peak flow monotonically decreases as travel time as well as $\beta$ increases. The time to the centroid increases monotonically as travel time and $\beta$ increase as shown in Figure 4.8 (c) and Figure 4.8 (d).

The result of a slower-moving storm is that the duration for which rain occurs somewhere in the watershed is longer than for a faster-moving rainstorm. The decreasing peak response in Figure 4.8 (a) is partly caused by decreased rainfall intensity due to the increasing storm travel time. In order to isolate the effect of the network has on the peak discharge and travel time, the uniform stationary rainfall with the duration equal to the total time the catchment is receiving rainfall was used to normalize the moving storm results in Figure 4.9.
Figure 4.8 Percent change of peak response with (a) downstream, (b) upstream moving storm and percent change of time to the centroid of $Q(t)$ with (c) downstream, (d) upstream moving storm compared to the unit instantaneous rainfall case

For example, when travel time through a unit distance, $t_{TR}$ is equal to two, the peak response is compared with that from the uniform stationary rainfall with total duration ($t_d$) equal to sixteen. In Figure 4.9 (a), the percent change of the peak discharge varies depending on $\beta$ and the travel time. The percent change of the peak increases as $\beta$ increases when the storm travel time is less than or equal to two. However, when the storm travel time is greater than two, this relation is reversed and the percent change of the peak decreases as $\beta$ increases. This is related to the resonance effect that the storm is moving with the same speed as the flow velocity (Niemczynowicz, 1984a, b; Ngirane-Katashaya and Wheater, 1985; Watts and Calver, 1991; Singh, 1997). In a Schdeidegger network on a square grid, the length of the longest path is double the distance traveled by a storm moving parallel to one of the flow directions. When $t_{TR}$ is equal to two, the travel time of the storm is equal to the travel time of the flow through the longest flow path of
the Scheidegger network and it results in the greatest increase in peak discharge.

![Diagram](image)

Figure 4.9 Percent change of peak response with (a) downstream, (b) upstream moving storm and percent change of time to the centroid of $Q(t)$ with (c) downstream, (d) upstream moving storm compared to the uniform stationary rainfall conditions with corresponding durations.

When the travel time is greater than two, the percent in the peak discharge due to the upstream moving rainstorm become less sensitive to $\beta$ as shown in Figure 4.9 (b). For the downstream moving storm, the time to the centroid is controlled by the duration of the storm and not affected by the geometry ($\beta$) or the storm travel time (Figure 4.9 (c)). However, for an upstream moving storm, the time to the centroid is strongly dependent on the storm travel time with the largest increases for the large values of $\beta$ (Figure 4.9 (d)).

### 4.5. Drainage networks in Chicago

The drainage pipe networks of twelve catchments from the highly urbanized Chicago
area in the United States are examined to estimate the parameter of Gibbs’ model ($\beta$) (Figure 4.10, Table 4.2) that best represent these basins. The pipe network is reconstructed on a lattice as shown in Figure 4.11 based on the distances between streets and blocks. The number next to a point is the distance ($\xi$) from the outlet to that point. The breadth of the line is proportional to the maximum value of width function at that segment. The boundary and the outlet location are used to simulate the network with Gibbs’ model, which is then used to develop the corresponding width function. The width functions from hundred realizations of Gibbs’ model with a given $\beta$ are averaged with respect to the distance from the outlet as shown in Figure 4.12.

The value of $\beta$ that provided the best representation of the network was determined using the Nash-Sutcliffe model efficiency coefficient (Nash and Sutcliffe, 1970), $E$ comparing the averaged width function from simulated networks and the actual width function. A value of $E$ closest to one provides the best match between the real network and the simulated network.

$$E = 1 - \frac{\sum_{i=1}^{n}(L_0(\xi) - L_s(\xi))^2}{\sum_{i=1}^{n}(L_0(\xi) - \bar{L}_o(\xi))^2}$$

(3.10)

where $L_0$ is the actual width function and $L_s$ is the width function from the simulation.

The results are counterintuitive because the estimates of $\beta$ range widely compared to natural river networks. As shown in Figure 4.13 (a) and (b), the best fit value of $\beta$ for CDS-51 is estimated as 0.04. The estimates for twelve catchments range from $10^{-2}$ to $10^2$ as shown in Table 4.2. Four catchments have values of $\beta$ of the order of $10^{-2}$ and five catchments have $\beta$ of the order of $10^{-1}$. As previously mentioned, Troutman and Karlinger (1992) estimated $\beta$ of the natural watersheds in Montana and found that the estimated values of $\beta$ lie between 0.2 and 2.21 and the averaged value of estimated $\beta$ is equal to 0.92 (order of $10^0$). It can be inferred from the results that the urban drainage networks in Chicago have wide range of values for $\beta$; from $10^{-2}$ to $10^2$. The efficiency of the urban pipe networks varies widely compared with natural river networks, hence, Gibbs’ model can be an appropriate approach to represent the network properties in urban drainage system. It has been recognized that urban drainage system is more efficient when it is compared to that of the nature especially in terms of the drainage time. However, in terms
of the network configuration, manmade drainage systems can be less efficient than natural river networks.

![Figure 4.10 Location of 12 catchments in Chicago](image)

**Table 4.2 Estimated $\beta$ for the drainage pipe networks in Chicago**

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Area (km$^2$)</th>
<th>Slope (%)</th>
<th>Grid size (m)</th>
<th>Flow direction</th>
<th>Goodness of fit ($E$)</th>
<th>O($\beta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDS-07</td>
<td>2.25</td>
<td>0.96</td>
<td>183</td>
<td>SE</td>
<td>0.80</td>
<td>10$^{-1}$</td>
</tr>
<tr>
<td>CDS-11</td>
<td>2.28</td>
<td>0.72</td>
<td>116</td>
<td>S</td>
<td>0.92</td>
<td>10$^{-2}$</td>
</tr>
<tr>
<td>CDS-16</td>
<td>0.34</td>
<td>0.66</td>
<td>85</td>
<td>S</td>
<td>0.97</td>
<td>10$^{-2}$</td>
</tr>
<tr>
<td>CDS-17</td>
<td>1.66</td>
<td>0.87</td>
<td>28</td>
<td>N</td>
<td>0.90</td>
<td>10$^{-1}$</td>
</tr>
<tr>
<td>CDS-20</td>
<td>3.35</td>
<td>0.71</td>
<td>91</td>
<td>W</td>
<td>0.94</td>
<td>10$^{2}$</td>
</tr>
<tr>
<td>CDS-25</td>
<td>1.17</td>
<td>0.92</td>
<td>76</td>
<td>S</td>
<td>0.96</td>
<td>10$^{-2}$</td>
</tr>
<tr>
<td>CDS-26</td>
<td>0.61</td>
<td>0.63</td>
<td>101</td>
<td>SW</td>
<td>0.99</td>
<td>10$^{-1}$</td>
</tr>
<tr>
<td>CDS-28</td>
<td>1.62</td>
<td>0.69</td>
<td>101</td>
<td>SW</td>
<td>0.90</td>
<td>10$^{2}$</td>
</tr>
<tr>
<td>CDS-32</td>
<td>1.51</td>
<td>0.67</td>
<td>207</td>
<td>SW</td>
<td>0.96</td>
<td>10$^{-1}$</td>
</tr>
<tr>
<td>CDS-34</td>
<td>14.7</td>
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<td>223</td>
<td>E</td>
<td>0.94</td>
<td>10$^{0}$</td>
</tr>
<tr>
<td>CDS-36</td>
<td>0.82</td>
<td>1.05</td>
<td>116</td>
<td>NE</td>
<td>0.90</td>
<td>10$^{-2}$</td>
</tr>
<tr>
<td>CDS-51</td>
<td>3.42</td>
<td>1.05</td>
<td>61</td>
<td>S</td>
<td>0.91</td>
<td>10$^{-2}$</td>
</tr>
</tbody>
</table>
Figure 4.11 The pipe network of CDS-51 (a) reconstructed on a lattice (16 by 21) and (b) corresponding width function

Figure 4.12 Realization of Gibbs’ network and the corresponding averaged width function over 100 simulations for CDS-51: (a) a realization for $\beta = 10^{-4}$; (b) corresponding width function; (c) for $\beta = 10^0$; (d) corresponding width function
Depending on the network property, $\beta$ and the direction of movement, a moving rainstorm can initiate significant biases in terms of the peak discharge. Huff (1979) found that in Illinois, 84% of the storms investigated exhibited motion with a component from west to east. Huff’s results, coupled with the results from the preceding section, which show an increase in peak discharge for downstream-moving storms, imply the potential for bias in estimation of peak discharge if stationary storms are assumed.

![Figure 4.13 Estimation of $\beta$ based on the Nash-Sutcliffe model efficiency coefficient for CDS-51: (a) Comparison of actual width function of CDS-51 with the width functions averaged over 100 simulations of the stochastic network for each $\beta$; (b) estimation of $\beta$ based on the Nash-Sutcliffe model efficiency](image)

Based on the previous results, an adjustment to the peak response to urban drainage networks is made (Table 4.3). In order to examine this effect, we assumed that typical direction of the rainstorm movement is aligned along West-East and the speed of rainstorm has the same order with the speed of flow. Also, the length scale of the rainstorm is assumed to be 1/8 of the catchment length scale in the direction of the storm movement. This means storms are moving downstream in subcatchment CDS-7, CDS-34 and CDS-36 while rainstorms are moving upstream in CDS-20, CDS-26, CDS-28 and CDS-32. This variety of storm direction relative to the network and the network property ($\beta$) can result in significant differences in the peak response compared to uniform stationary rainfall events. The result shows that the change in peak discharge from a storm moving at the same speed as the flow ranges from (+) 55.7% (CDS-34, storm
moving downstream) to (-) 31.9% (CDS-20 and CDS-28, storms moving upstream).

Table 4.3 Percent change of peak response assuming a typical rainstorm moving from west to east and same order of speed for rainstorm and surface flow

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Flow Direction</th>
<th>Rainstorm Direction</th>
<th>O(β)</th>
<th>Percent Change of Peak Response (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDS-07</td>
<td>SE</td>
<td>Downstream</td>
<td>$10^{-1}$</td>
<td>(+)52.8</td>
</tr>
<tr>
<td>CDS-11</td>
<td>S</td>
<td>-</td>
<td>$10^{-2}$</td>
<td>-</td>
</tr>
<tr>
<td>CDS-16</td>
<td>S</td>
<td>-</td>
<td>$10^{-2}$</td>
<td>-</td>
</tr>
<tr>
<td>CDS-17</td>
<td>N</td>
<td>-</td>
<td>$10^{-1}$</td>
<td>-</td>
</tr>
<tr>
<td>CDS-20</td>
<td>W</td>
<td>Upstream</td>
<td>$10^{2}$</td>
<td>(-)31.9</td>
</tr>
<tr>
<td>CDS-25</td>
<td>S</td>
<td>-</td>
<td>$10^{-2}$</td>
<td>-</td>
</tr>
<tr>
<td>CDS-26</td>
<td>SW</td>
<td>Upstream</td>
<td>$10^{-1}$</td>
<td>(-)29.9</td>
</tr>
<tr>
<td>CDS-28</td>
<td>SW</td>
<td>Upstream</td>
<td>$10^{2}$</td>
<td>(-)31.9</td>
</tr>
<tr>
<td>CDS-32</td>
<td>SW</td>
<td>Upstream</td>
<td>$10^{-1}$</td>
<td>(-)29.9</td>
</tr>
<tr>
<td>CDS-34</td>
<td>E</td>
<td>Downstream</td>
<td>$10^{0}$</td>
<td>(+)55.7</td>
</tr>
<tr>
<td>CDS-36</td>
<td>NE</td>
<td>Downstream</td>
<td>$10^{-2}$</td>
<td>(+)42.1</td>
</tr>
<tr>
<td>CDS-51</td>
<td>S</td>
<td>-</td>
<td>$10^{-2}$</td>
<td>-</td>
</tr>
</tbody>
</table>

4.6. Conclusions

This chapter explores the relation between network properties and the effect from moving rainstorms. Gibbs’ stochastic network model is utilized in this study in order to identify and categorize a network depending on its network configuration. Compared with the natural watershed, it is found that twelve urban drainage networks in Chicago area have a wide range of network property (β). These results are counterintuitive and indicate that the urban drainage network can be less efficient than natural river networks. The results also show that the effect of the direction and speed of the rainstorm movement varies depending on the network properties. The relation between storm speed and direction and the change in the peak runoff is dependent on the network configuration and network efficiency. In contrast to previous studies, this study shows that the speed and direction of the rainfall movement that produces the maximum peak discharge changes depending on the network. Consequently, the wide range of network configuration in urban catchments reveals that the assumption of stationary rainfall can cause biases when it is applied to urban drainage system.
Chapter 4 showed the effect of channel network configurations on the hydrographs with moving rainstorms. This chapter explores further the relations between channel network configurations and hydrograph sensitivity to storm kinematics with different storm speeds, storm directions as well as storm sizes. A synthetic circular catchment is utilized in order to prevent directional biases due to catchment geometry. The drainage network of the test catchment is simulated with Gibbs’ stochastic network model for a given network configuration ($\beta$). The peak response of the catchment is investigated with different configurations of drainage network combined with different conditions of storm kinematics. The results indicate that the relation between storm kinematics and the peak discharge response is dependent on the network configuration; accordingly network efficiency in terms of total drainage time of a network. In contrast to previous studies, this study shows that the storm kinematics that produces the maximum peak discharge depends on the network configuration. The results show that the most inefficient network (a network with lowest $\beta$) produces the most insensitive and lowest peak response to rainstorm movement compared with efficient networks. In contrast, efficient networks produce higher peak responses and are sensitive to storm kinematics. The resonance condition for a two-dimensional drainage network is defined utilizing the Scheidegger network as a function of network configuration. The results show that the resonance condition does not produce the maximum peak discharge and the maximum peak occurs when the storm is slower than the flow.

### 5.1. Introduction

A runoff hydrograph from a watershed depends on the physiographic properties of the watershed and the hydrometeorologic characteristics of rainstorm (Yen and Chow, 1969). For a given amount total amount of rainfall, the temporal and spatial distribution of the
rainfall is determined by the movement of the rainstorm. In June 2008, eastern Iowa experienced the largest flood including the flooding of Cedar Rapids in the Cedar River basin and Iowa City in the Iowa River basin, respectively. The peak discharge at Iowa City was greatest since the completion of the Coralville Dam in 1958 in Iowa River and the peak discharge at Cedar Rapids was greatest ever since the records began and doubled the previous record flooding in 1993 (Linhart and Eash, 2010). The flood in June 2008 had a relatively short duration compared with the summer flood in 1993 (Bradley, 2010), which produced much longer flood durations and higher summer runoff volumes. However, the peak discharge in 2008 almost doubled the peak discharge in 2008. Krajewski and Mantilla (2010) discussed the possibility of a “perfect storm” where the timing and location of rain combined to maximize flood intensity at certain locations. The Iowa flood in 2008 was not caused by a single moving rainstorm, but still it is a clear example that shows how rainstorm movement in the same direction as flow can exacerbate the magnitude of flood peaks.

The direct influence of storm movement on the runoff hydrograph has been recognized for a long time (Maksimov, 1964; Yen and Chow, 1968; Wilson et al., 1979; Jensen, 1984; Niemczynowicz, 1991; Singh, 1998). A non-dimensional relation for the peak response and characteristics of moving rainstorm was introduced by Yen and Chow (1968), which pioneered the fundamental understanding of the relation between runoff hydrographs and moving rainstorms. Niemczynowicz (1984b) utilized this non-dimensional relation for urban catchments and it was also utilized by Ogden et al. (1995) in a simplified form.

In addition, there often exists a directional preference of rainstorm movement. For example, Huff (1979) found that 84% of severe heavy rainstorms in Illinois exhibited motion with a westerly component. Although the directional preferences of moving rainstorms have been widely recognized (Huff, 1979; Shearman, 1977; Upton, 2001), the storm movement and subsequent rainfall variation are hardly regarded in the design process of urban drainage networks. It is necessary to consider the effect of rainstorm movement in order to minimize the avoidable potential risk and to maximize the effectiveness of urban drainage network. Understanding the interaction between the rainstorm movement and the hydrologic response is crucial in order to accomplish this.

However, previous studies and research related to moving storms generally focused on
the response of a specific catchment rather than being generalized for a wide range of watershed properties. Especially in terms of channel network configuration, previous studies of Ngorane-Katashaya and Wheater (1985) and Watts and Calver (1991) were based on only one configuration of the network that belongs to the Scheidegger network. Therefore, it is difficult to extrapolate the results from these studies describing the effect of storm movement to different watersheds. Surkan (1969) investigated the effect of network geometry and developed synthetic hydrographs with directed graphs on a rectangular grid. In this regards, this study explores the idea that the effect of rainstorm movement might be different for different network configurations. In particular, this study seeks to establish a more generalized relation between storm kinematics and the peak response based on network configuration. Instead of the non-dimensional relation developed by Yen and Chow (1968), this study utilizes the results from conceptual studies in Chapter 3, which is based on characteristic time and space scales to describe the effect of rainstorm movements on the peak response depending on the configuration of drainage networks.

In order to generate the drainage network on a rectangular grid for a given network configuration, stochastic network models are adopted in this study; the Scheidegger model (Scheidegger, 1967a, b), the Uniform model (Leopold and Langbein, 1962; Karlinger and Troutman, 1989) and Gibbs’ model (Troutman and Karlinger, 1992) based on Gibbs’ measure (Ising, 1925; Kindermann and Snell, 1980). In contrast to the random models, which were based on an assumption of the absence of environmental controls, Gibbs’ model holds a control over the overall sinuosity of a network depending on the value of a parameter, \( \beta \). We show that urban drainage networks have wide range of network configuration compared with natural river networks in Chapter 4.

The objectives of this study are (a) to investigate the relation between the network configuration and the hydrograph sensitivity to storm kinematics; storm directions, speeds as well as storm sizes; (b) to define the resonance condition on a spanning tree type network in two dimensional spaces; and consequently (c) to achieve better understanding of the effect of moving rainstorms especially in urban areas, which would imply the potential improvement of the urban drainage system adapted to the regional characteristics of the frequent moving rainstorms.
5.2. Methodology

Test catchment and moving rainstorm

In this study, a synthetic circular catchment is introduced with an outlet at the south bottom in order to investigate the effect of rainstorm movement (Figure 5.1). The catchment is composed of 41 by 41 grids. The grid size is 100 meters and the diameter of the test catchment (the catchment length scale, $L_c$) is 4 kilometers. Storm direction is given as an angle with respect to the downstream direction; the direction from the center of the circular catchment to the outlet as shown in Figure 5.1 (a). The flow velocity, $v_c$ is assumed to be constant in this study. The shape of the rainstorm field is assumed to be a semi-infinite band for which the size is given by the width of the rainstorm field, $L_s$ in the direction of storm movement. The rainstorm field starts from the point $P_1$ and ends at $P_2$. The total travel distance $L_t$ is given as $L_c + L_s$ for the simulation of runoff hydrographs. For example, Figure 5.2 depicts the rainstorm progression when the storm speed, $v_s = 5$ m/s and the storm size is 400 meters; the total distance that the rainstorm travels in this case is equal to 4400 meters. The storm speed $v_s$ is also assumed to be constant in this study.

![Figure 5.1 Model configuration in a circular watershed](image)

In this study, the rainfall may partially cover the catchment. In order to obtain the hydrologic response at the outlet of the catchment, the width function is utilized as discussed in Chapter 4. For a given flow distance from the outlet, $\xi$, the width function...
$L(\xi)$ can be defined as follows:

$$L_\beta(\xi) = N\{d(\xi)\}, \quad \xi = \xi_1, \xi_2, \ldots, \xi_n$$

(5.1)

where $L_\beta(\xi)$ is the width function for network generated by Gibbs’ model for a parameter value of $\beta$. The width function can be obtained by counting the number of the grid points with distance $\xi$ from the outlet. $d(\xi)$ is the grid point of which distance from the outlet is equal to $\xi$. $\xi_n$ is the longest distance from the outlet. A width function is newly defined as a time-dependent function incorporating a rainfall excess given distance $\xi$ at time $t$.

$$L_\beta^t(\xi, t) = \sum_{(i, j)} r_{ij}(\xi, t). \quad \xi = \xi_1, \xi_2, \ldots, \xi_n$$

(5.2)

where $r_{ij}(\xi, t)$ is the rainfall intensity at a grid point $(i, j)$ of which distance from the outlet is $\xi$ at time $t$. The rainfall intensity is assumed to be constant in this study. Then the response function at the outlet for each time step can be expressed as follows assuming convection only for channel routing.

$$Q(t) = \int \int (\Delta x)^2 L_\beta^t(\xi, \tau) \cdot \delta(t - \tau - \xi / v_c) \, d\xi \, d\tau$$

(5.3)

where $v_c$ is a flow velocity. In a discrete form, the Equation 5.3 can be written as

$$Q(t) = \sum_{i=1}^{m} \left( \sum_{j=1}^{n} L_\beta^t(\xi_j, \tau_i) \cdot \delta(t - \tau_i - \xi_j / v_c) \Delta \xi \right) \Delta \tau \Delta x^2$$

(5.4)

Figure 5.2 Rainstorm progression ($\theta = \pi / 4$, $v_s = 5$ m/s, $L_s = 400$ m)
Network configuration

Ngirane-Katashaya and Wheater (1985) and investigated the response to moving storms over a circular catchment that has a fixed configuration of network, which can be categorized as a Scheidegger network. Watts and Calver (1991) estimated the runoff hydrograph with moving rainstorms from a single pipe network, which also is a Scheidegger network. However, in Chapter 4, we show that urban drainage networks in Chicago area have wide range of network configuration compared with natural channel networks. Therefore, in contrast to the previous studies, this study introduces various network configurations presented with Gibbs’ model (Figure 5.3).

![Figure 5.3 Generation of networks in a circular watershed with the Scheidegger model, the Uniform model, and Gibbs’ model (41 by 41 grids)](image)

Gibbs’ model (Troutman and Karlinger, 1992) is based on Gibbs’ measure (Ising, 1925; Kindermann and Snell, 1980). Compared with the random models; the Scheidegger model (Scheidegger, 1967b) and Uniform (Leopold and Langbein, 1962; Karlinger and Troutman, 1989), which were based on an assumption of the absence of environmental controls, Gibbs’ model has a control over the overall sinuosity of a network depending on
a parameter, $\beta$. This enables us to gain insight into geomorphologically significant factors such as drainage network efficiency, drainage density, and variation of network topology with basin shape and relief (Troutman and Karlinger, 1992). In this study, the Scheidegger model and the Uniform model are utilized as a reference to Gibbs’ model (Figure 5.4).

Figure 5.4 Realization of networks in grids: (a) the Uniform network; (b) Gibbs ($\beta = 10^{-3}$); (c) Gibbs ($\beta = 10^{-2}$); (d) Gibbs ($\beta = 10^{-1}$); (e) Gibbs ($\beta = 10^0$); and (f) the Scheidegger

The Scheidegger model (Scheidegger, 1967a, b) is a directed self-avoiding random walk model on a lattice with flow from each point being allowed in only two orthogonal and downstream directions, each with equal probability. The Scheidegger network has no sinuosity because it is directed to downstream directions only. The Scheidegger model is equivalent to Gibbs’ model with $\beta$ being infinity. The Uniform model is defined on a lattice with flow allowed in each of four possible directions with equal probability, hence, it is highly sinuous compared with the Scheidegger model; the uniform model is equivalent to Gibbs’ model with $\beta$ being zero. Therefore, theoretically, the Scheidegger
model and the Uniform model belong to Gibbs’ model depending on the parameter values
of $\beta$. The Gibbs’ network is generated from the Uniform network and the procedures for
generation of the Uniform network and Gibbs’ network are described in Chapter 4 in
detail.

Figure 5.4 illustrates typical realizations of the stochastic network models; the
Scheidegger model, Gibbs’ model, and the Uniform model. Figure 5.4 also presents two
index values averaged from 100 realizations for each network. These index values are the
normalized sinuosity ($H'$) and the minimum Lipschitz-Hölder exponent ($a_{\text{min}}$) (Feder,
1993). $a_{\text{min}}$ is the fractal dimension of dominant part (Ishida and Nasu, 2008). Sinuosity
of a network ($H$) can be defined as follows (Troutman and Karlinger, 1992):

$$H(s) = \sum_{d \in D(B)} \xi_s(d) - \sum_{d \in D(B)} \xi_B(d)$$

(5.5)

where $s$ is a spanning tree, $d$ is a point of a finite and connected graph $B$ and $D(B)$ is the
set of the total points of $B$. $d_s$ is the distance to an outlet along $s$ from $d$ while $d_B$ is the
shortest distance to the outlet from $d$. The normalized sinuosity is obtained as follows:

$$H'(r) = \frac{H(r)}{\sum_{d \in D(B)} \xi_B(v)}$$

(5.6)

The minimum Lipshitz-Hölder exponent ($a_{\text{min}}$) is obtained in the process of multifractal
analysis and exactly related to the fractal dimension of a single river in a given network
(See Appendix B). As $\beta$ increases, Gibbs’ network becomes less sinuous; the averaged
normalized sinuosity decreases and tends to be zero. The averaged minimum Lipshitz-
Hölder exponent also decreases as $\beta$ increases as shown in Figure 5.4.

One of the well-known empirical laws is the Hack’s law (Hack, 1957), in which the
following relation holds between the length, $L$ of the mainstream and the area, $A$ of the
drainage basin area.

$$L \propto 1.89A^{0.6}$$

(5.7)

In general, for any non-fractal objet, the following relation holds between its length $L$,
area $A$, and volume $V$.

$$L \propto A^{1/2} \propto V^{1/3}$$

(5.8)

If there is any quantity, $X$ that increases by $2^D$ when we change its size by the factor of 2,
$X$ is $D$-dimensional. This quantity, again, satisfies the relation (Takayasu, 1991)

$$L \propto X^{1/D}$$

(5.9)
Equation 5.7 can be written in the form of 5.9 as
\[ A^{1/2} \propto L^{1.2} \quad (5.10) \]
Equation 5.10 shows that the fractal dimension of the natural river is 1.2 based on the Hack’s law. Takayasu (1991) showed that the fractal dimension of the natural rivers falls in the range 1.1 to 1.3, with a mean value of 1.2. From the minimum Lipshiz-Hölder exponent shown in Figure 5.4, the result indicates that the Scheidegger network is closest to the natural river in terms of the fractal dimension of rivers.

5.3. Results and discussion

Resonance condition in two-dimensional space

The peak response is given as a function of relative storm speeds, \( v_s/v_c \), relative storm sizes, \( L_s/L_c \), and storm direction, \( \theta \) for a given network configuration, \( \beta \). As previously discussed in Chapter 3, two timescales are involved in resonance condition; the rainstorm travel timescale, \( t_s \) and flow travel timescale, \( t_c \). When \( t_s/t_c = 1 \), it is called resonance condition. If the resonance condition produces the maximum peak response, it is called resonance effect (Surkan, 1974; Foroud et al., 1984; Niemczynowicz, 1984a, b; N girane-Katashaya and Wheater, 1985; Watts and Calver, 1991; Singh, 1997). The results in Chapter 3 indicate that the resonance condition does not produce the maximum peak due to interdependencies between the storm travel time (storm speed) and rainfall duration; slow storm produces longer rainfall duration. As previously mentioned in Chapter 3, the resonance condition is obtained when the travel timescale of rainstorm is equal to travel timescale of flow:
\[ t_s = t_c \quad (5.11) \]
where \( t_s \) is travel timescale of rainstorms and \( t_c \) is travel timescale of flow as discussed in Chapter 3.

For a storm moving from the north, the equidistant line for the Scheidegger network in in Figure 5.5 (a) shows that the flow travel distance in the direction of the moving storm is exactly equal to the rainstorm travel distance, which is the catchment size, \( L_c \). Therefore, in terms of relative storm speed, the resonance condition for the Scheidegger network (Figure 5.5 (a)) is obtained when \( v_s/v_c = 1 \) when the storm is moving in the downstream direction (\( \theta = 0 \)). As \( \beta \) decreases in Gibbs’ network, flow travel distance increases, hence,
resonance condition is obtained when $v_s/v_c < 1$. Presumably, the Uniform network will produce the longer average flow distance from the outlet compared with other networks referred in this study.

Figure 5.5 Equidistant line from the outlet from a realization of (a) the Scheidegger; (b) Gibbs ($\beta=10^6$); (c) Gibbs ($\beta=10^4$); (d) Gibbs ($\beta=10^2$); (e) Gibbs ($\beta=10^4$); (f) the Uniform
Figure 5.6 The resonance condition in two-dimensional spaces in terms of relative storm speed ($v_s/v_c$); (a) the average flow distance in the flow direction normalized by the Scheidegger network; (b) resonance condition depending on network configuration.

In order to evaluate the resonance condition in two-dimensional spaces, the ‘average flow distance’ is utilized in this study. In addition, utilizing the Scheidegger network that has a resonance condition when $v_s/v_c = 1$, the average flow distance normalized by the Scheidegger network is obtained as

$$\bar{L}_f = \frac{\left(\sum_i d_{ij}\right)}{\left(\sum_i d_{ij}\right)_{sch}}$$  \hspace{1cm} (5.12)

where $d_{ij}$ is the distance from a point $(i, j)$ on a network to the outlet and the subscript ‘sch’ denotes the Scheidegger network. From Equation 5.12, the resonance condition in a circular watershed can be obtained by average flow distance normalized by average flow distance of the Scheidegger network as follows:

$$\left(\frac{v_s}{v_c}\right)_{res} = \frac{1}{\bar{L}_f}$$  \hspace{1cm} (5.13)

Figure 5.6 (a) illustrates how the average flow distance changes and consequently, how the resonance condition varies (Figure 5.6 (b)) depending on the configuration of the network. For example, for a Gibbs’ network with $\beta$ being equal to $10^{-1}$, the resonance condition in terms of relative storm speed is given as: $v_s/v_c = 0.95$ as shown in Figure 5.6 (b). When $\beta = 10^{-4}$, the resonance condition is given as $v_s/v_c = 0.50$; the resonance condition is obtained with a rainstorm two times slower than the flow velocity. The resonance condition for the Scheidegger network is given as $v_s/v_c = 1$. 

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The results for the PDR ($Q_p/iA$) as a function of storm speed, direction and network configuration ($\beta$) for different storm sizes of $L_s/L_c = 1/8, 1/4, 1/2$ and 1, are plotted in Figure 5.7, Figure 5.8, Figure 5.9, and Figure 5.10, respectively. The results consistently show that the maximum peak response is obtained when the storm speeds are slower than the resonance condition (Figure 5.6), which is consistent with the results in Chapter 3, due to the interdependencies between storm speed and duration.

When the storm sizes are smaller than the catchment size, moving rainstorms do not necessarily produce the equilibrium discharge regardless of the storm speed. However, storms with $L_s/L_c < 1$ still can produce the equilibrium discharge due to the resonance condition as shown in Figure 5.9. The results show that when the storm sizes are equivalent to the catchment size, slow storms produce the equilibrium discharge and the peak response decreases as the storm speed increases (Figure 5.10). However, storm sizes smaller than $L_s/L_c = 1/2$ do not produce the equilibrium discharge (Figure 5.7, Figure 5.8) because the network expands in two dimensions instead of one dimension; in a one-dimensional catchment, regardless of storm size, the moving storm can produce the equilibrium discharge due to resonance condition as shown in Chapter 3.

![Figure 5.7 Peak response as a function of storm orientation ($L_s/L_c = 0.125$): (a) $\beta = 10^0$; (b) $\beta = 10^{-1}$; (b) $\beta = 10^{-2}$; (b) $\beta = 10^{-3}$](image-url)
Figure 5.8 Peak response as a function of storm orientation ($L_s/L_c = 0.25$): (a) $\beta = 10^0$; (b) $\beta = 10^{-1}$; (b) $\beta = 10^{-2}$; (b) $\beta = 10^{-3}$

Figure 5.9 Peak response as a function of storm orientation ($L_s/L_c = 0.5$ ): (a) $\beta = 10^0$; (b) $\beta = 10^{-1}$; (b) $\beta = 10^{-2}$; (b) $\beta = 10^{-3}$
Storm size not only determines the magnitude of the peak response but also interacts with other factors. Surkan (1974) showed that peak discharge is most sensitive to the direction and speed of the moving rainstorm, especially when the storm sizes in the direction of storm progress are much smaller than the dimensions of the catchment. In this case, the catchment is partially activated to produce the response because of smaller length scale of the moving rainstorms. The results shown in Figure 5.7, Figure 5.8, Figure 5.9, and Figure 5.10 are consistent with the results of Surkan (1974) in that as storm sizes are smaller ($L_s/L_c < 1$) and the storm is moving in the downstream direction; $\theta = 0$, and $\pi/4$, the peak response does not behave monotonically with the speed of rainstorm, $v_s/v_c$. Under these conditions, the resonance condition becomes an important factor because the maximum response is obtained in the vicinity of the resonance condition. In contrast, for the storms moving in the transversal direction ($\theta = \pi/2$) and upstream direction ($\theta = 3/4 \pi$, and $\theta = \pi$), the peak response monotonically decreases as relative storm speed, $v_s/v_c$ increases and the resonance condition is not related with the maximum peak response. When the storm sizes are greater than the catchment size ($L_s/L_c \geq 1$), the peak reaches the
equilibrium discharge as $v_c/v_c$ tends to be 0, which is consistent with the results of Chapter 3 in that the peak response does not decrease as $t_e$ increases once it reaches the maximum peak response.

When the storm sizes are relatively small ($L_s/L_c = 1/8$), the maximum peak response is reached when the storm is not moving in exact downstream direction as shown in Figure 5.7 (a), where the peak is larger when $\theta = \pi/4$ compared with the peak response when $\theta = 0$. Figure 5.11 (a) shows the movement of a storm ($L_s/L_c = 1/8$) in the direction of $\theta = \pi/4$. As previously shown in Figure 5.5, when the storm direction is $\theta = \pi/4$, the equidistant line is parallel to the moving storm band especially when $\beta > 10^{-1}$, which results in higher peak response with the moving rainstorm in the same direction.

Figure 5.11 The effect of arrangement of equidistant line (Scheidegger network): (a) a storm moving ($L_s/L_c = 0.125$) in direction of $\theta = \pi/4$; (b) the hydrographs at the outlet

In area A, storm is located over an iso-distance line, meaning that all the pipes where the storm is occurring are equi-distant from the outlet. In contrast, in area B, the storm crosses several iso-distance lines, meaning that the pipes where the storm is occurring span multiple distances from the outlet. The former case (location A) results in a sharp peak in the resulting hydrograph. Furthermore, as the storm moves across the left half of the catchment, this process is repeated. In contrast, the latter case (location B) results in a
broader but lower peak as shown in Figure 5.11 (b). Depending on the storm size and direction, and network configuration ($\beta$), the number of iso-lines that the storm crosses is changing, thereby affecting the shape of the hydrograph at any instant and therefore the overall runoff response. Although the above result is counterintuitive because the direction of $\theta = 0$ does not produce the maximum peak response, it implies that the peak response of a drainage network spanning in two-dimensional spaces can be significantly affected by the size and direction of rainstorm movement and network configurations. As the storm size becomes larger (for example when $L_s/L_c = 1/4$), the effect from the storm direction dominates other factors and the maximum peak response occurs when $\theta = 0$ as shown in Figure 5.8.

![Figure 5.12 Peak response as a function of storm sizes ($\theta=0$): (a) $\beta = 10^0$; (b) $\beta = 10^{-1}$](image)

The results show that the maximum peak response results from interactions between storm sizes, direction and speed, and the network configuration. In general, peak response decreases as $\beta$ decreases because flow travel distance increases as $\beta$ decreases, which results from the temporal dispersion of responses from the catchment as shown in Chapter 4. However, when the storm is moving downstream, a network with smaller $\beta$ (Figure 5.12 (b)) produces higher peak compared with one with larger $\beta$ (Figure 5.12 (a)) depending on the relative storm sizes ($L_s/L_c$). Figure 5.13, Figure 5.14, Figure 5.15, and Figure 5.16 show the peak response as a function of network configuration ($\beta$) for different storm sizes; $L_s/L_c = 1/8$, $L_s/L_c = 1/4$, $L_s/L_c = 1/2$, and $L_s/L_c = 1$, respectively. The results show that the peak response is highly dependent on $\beta$ when the storm is moving downstream ($\theta = 0, \pi/4$). When the storm is moving in transversal direction ($\theta = \pi/2$) or
in upstream direction ($\theta = 3/4 \pi, \pi$), the peak response monotonically increases with increasing $\beta$. Figure 5.13 (a) shows that when $\theta = 0$ (moving downstream), the peak response monotonically decreases with increasing $\beta$. The results show that $\beta$ that produces the maximum peak response is determined by the relative storm speed ($v_s/v_c$); as the storm speed increases, the value of $\beta$ that produces the maximum peak response increases. The result shows that the network configuration ($\beta$) is combined with the resonant storm speed to produce the critical peak response of the drainage network. For a small $\beta$, assuming a constant flow velocity, the network has longer flow travel timescale, which results in resonance condition with a longer storm travel timescale; slower storm speed. As $\beta$ increases, the flow travel timescale is decreased because the flow distance decreases as shown in Figure 5.6 (a), which results in resonance condition with a faster storm. As the storm size increases, the effect from the network configuration diminishes (Figure 5.15, Figure 5.16). When the storm size is equivalent to the catchment size ($L_s/L_c = 1$), the peak response monotonically increases to reach the equilibrium discharge as $\beta$ increases regardless of storm direction and storm speed (Figure 5.16).

![Figure 5.13](image)

**Figure 5.13** Peak response as a function of network configuration ($L_s/L_c = 0.125$): (a) $v_s/v_c = 0.2$; (b) $v_s/v_c = 0.4$; (c) $v_s/v_c = 0.8$; (d) $v_s/v_c = 1$
Figure 5.14 Peak response as a function of network configuration \((L_s/L_c =0.25): (a) v_s/v_c = 0.2; (b) v_s/v_c = 0.4; (c) v_s/v_c = 0.8; (d) v_s/v_c = 1\)

Figure 5.15 Peak response as a function of network configuration \((L_s/L_c =0.5): (a) v_s/v_c = 0.2; (b) v_s/v_c = 0.4; (c) v_s/v_c = 0.8; (d) v_s/v_c = 1\)
Figure 5.16 Peak response as a function of network configuration \((L_s/L_c = 1)\): (a) \(v_s/v_c = 0.2\); (b) \(v_s/v_c = 0.4\); (c) \(v_s/v_c = 0.8\); (d) \(v_s/v_c = 1\)

As mentioned earlier, the spatial distribution of flow distance is highly affected by network configuration and closely related to the effect of moving rainstorm. Figure 5.13 (c) shows that the maximum peak response is obtained when \(\theta = \pi/4\) instead of \(\theta = 0\) when the storm size is relatively small \((L_s/L_c = 1/8)\); the same results is shown in Figure 5.7 (a). This result implies that the peak response can be significantly affected by the direction of rainstorm movement depending on spatial arrangement of equidistant line. Figure 5.5 shows that the equidistant line is parallel to the moving storm band especially when \(\beta > 10^{-1}\), which results in higher peak response for the storms moving in direction of \(\theta = \pi/4\).

**Transition of the peak response depending on \(\beta\)**

As mentioned earlier, the result shows that the network configuration \((\beta)\) is combined with the resonant storm speed to produce the critical peak response of the drainage network. Therefore, the resonance condition for each drainage network depends on its
network configuration. This study assumes a constant flow velocity. However, in reality, the network configuration combined with other factors (e.g., slope, roughness, cross-section geometry) that affect the flow velocity results in the unique resonance condition for each drainage network or catchment. In Chapter 4, we showed that there exists a transition in the peak response depending on the network configuration ($\beta$). Figure 5.13, Figure 5.14, and Figure 5.15 clearly show this. For example, when the relative storm speed is small ($v_s/v_c = 0.2$), and the storm is moving downstream ($\theta = 0$), the maximum peak occurs with smallest $\beta$ ($10^{-4}$) as shown in Figure 5.14 (a). However, when the relative storm speed becomes larger ($v_s/v_c = 0.4$), the maximum peak is obtained with $\beta$ being equal to $10^{-3}$ as shown in Figure 5.14 (b). Eventually, when the relative storm speed is equivalent to the flow velocity ($v_s/v_c = 1$), the maximum peak response is achieved with $\beta$ equal to $10^{-1}$. These results are consistent with previous results in that the resonance condition depends on the network configuration ($\beta$). For a small $\beta$, the network has longer flow travel timescale assuming a constant flow velocity, which results in a resonance condition with longer storm travel timescale (slower storm speed). As $\beta$ increases, the flow travel timescale decreases because the flow distance decreases as shown in Figure 5.6 (a), which results in a resonance condition occurring with faster storm. For example, Figure 5.14 clearly shows that the resonance condition is changing with the storm speed and $\beta$. For a slower storm ($v_s/v_c = 0.2$, Figure 5.14 (a)), the resonance condition is obtained with lower $\beta$. While, for a faster storm ($v_s/v_c = 0.4$, Figure 5.14 (a)), the resonance condition is obtained with $\beta$ being close to $10^{-3}$.

**Storm direction and network configuration**

The results show that the storm direction, combined with the relative storm speed ($v_s/v_c$) significantly affects the peak response. Figure 5.17 shows the results of the peak response as a function of storm speed ($v_s/v_c$), direction ($\theta$) and network configuration ($\beta$) for different storm sizes ($L_s/L_c$) of 1/8. When the storm size and speeds are small ($L_s/L_c = 1/8$; $v_s/v_c = 0.2$), difference in the peak response is not distinctive (Figure 5.17 (a)). In terms of the effect of the network configuration, moving storms of which speed ($v_s/v_c$) is less than 0.5 do not result in much different peak response compared with the peak response caused by faster storms ($v_s/v_c = 0.8$ or $v_s/v_c = 1$).

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The result also indicates that the storm direction significantly affects the peak response combined with network configuration as well as other storm characteristics. In general, the storm moving downstream direction ($\theta = 0$) produces the maximum peak response. However, when the storm sizes are relatively small ($L_s/L_c$), the storm direction of $\theta = \pi/4$ produces the maximum response. Ngirane-Katashaya and Wheater (1985) investigated the effect from moving rainstorm based on a synthetic circular watershed and only one configuration of the drainage network. The drainage network investigated by Ngirane-Katashaya and Wheater (1985) is only composed of downstream directions and categorized as a Scheidegger network. Ngirane-Katashaya and Wheater (1985) examined the storm direction ranging from 0 to $\pi$ (with an increment of $\pi/6$) and found that the maximum peak response is obtained at $\theta = \pi/6$ instead of $\theta = 0$ with a specific storm speed (5 m/s). As mentioned earlier, this result shows that the peak response can be significantly affected by the spatial layout of equidistant line especially when the storm
sizes are small compared to the catchment size and the storm band is moving perpendicular to the equidistant lines. Figure 5.5 shows that when the storm direction is $\theta = \pi/4$, the equidistant lines are parallel to the moving storm band especially when $\beta > 10^{-1}$, which results in higher peak response with the moving rainstorm in the same direction.

As discussed earlier, the Scheidegger network can be regarded as a reference in terms of network configuration as well as resonance condition.

**Combined effect of storm kinematics and network configuration**

The previous results show that the maximum peak response results from interactions between storm sizes, direction and speed, and the network configuration. Figure 5.18 illustrates the isosurface of the PDR ($Q_p/iA = 0.5$) for different storm sizes ($L_s/L_c$) of 1/8, 1/4, 1/2, and 1. Figure 5.18 (a) shows that when the storm size is relatively small ($L_s/L_c = 1/8$), the isosurface for $Q_p/iA = 0.5$ is confined within a narrow area where the direction is $\pi/2$, the storm speed ($v_s/v_c$) is close to 0.7, and network configuration ($\beta$) is greater than $10^{-1}$. When the size of a moving rainstorm is relatively small, the storm direction and speed as well as the network configuration are significant in defining the peak response. The isosurface expands as the storm size increases. Figure 5.18 (b) shows that the resonance condition varies depending on the network configuration ($\beta$): as $\beta$ decreases the area that produces $Q_p/iA$ greater than 0.5 breaks through with relatively low storm speeds smaller than 1. Figure 5.18 (c) illustrates that when the storm size is half of the catchment, storm direction can be the most important factor because, in this case, most of the storms moving downstream ($\theta < \pi/2$) with the same speed or smaller speed than the flow produce the PDR greater than 0.5. In contrast, when the storm is moving in the transverse direction or upstream ($\theta \geq \pi/2$), the peak response depends on the storm speed as well as network configuration. When the storm size is equivalent to the catchment size (Figure 5.18 (d)), most of the moving rainstorms with the same speed or slower speed compared with the flow produce the PDR ($Q_p/iA$) greater than 0.5, but, still the network configuration affects the peak response. Especially when the storm moves upstream, the peak response can be decreased depending on the network configuration as shown in Figure 5.18 (d).
Figure 5.18 Isosurface of the PDR ($Q/A = 0.5$) with different storm sizes of (a) $L_s/L_c = 0.125$; (b) $L_s/L_c = 0.25$; (c) $L_s/L_c = 0.5$; (d) $L_s/L_c = 1$

5.4. Conclusions
This study explores the relations between network configurations and hydrograph sensitivity to storm kinematics; storm speeds, storm directions as well as storm sizes. A synthetic circular catchment is utilized in order to avoid biases due to catchment geometry. The configuration of drainage network is simulated with Gibbs’ stochastic network model. The peak discharge response is investigated with different rainstorm conditions and network configurations.

The results show that the effect of the direction and speed of the rainstorm movement significantly depends on the network properties. This study shows that the network configuration affects the flow traveling timescale, hence, suggests that the resonance condition in terms of the relative storm speed ($v_s/v_c$) can change as a function of network configuration ($\beta$). Therefore, the size and speed of the moving storm that produces the
maximum peak response are dependent on network configuration. The results also show that the maximum peak occurs when the storm is than the flow before it reaches equilibrium, but, still in the vicinity of the corresponding resonance condition. This study also shows that the network configuration is not just affecting the resonance condition but also determining the spatial layout of equidistant line of flow distance from the catchment outlet. Combined with the direction of moving storms, the spatial layout of equidistant line contributes to yield the shape of hydrographs and the peak response. The relation between storm kinematics and the peak response is highly dependent on the network configuration, or network efficiency. This study also shows that the most influential factor can change depending on the size of moving rainstorms.

The results of this study indicate that a network with lowest $\beta$ (most inefficient network) is the least sensitive to storm kinematics and produces the lowest peak response. In contrast efficient networks are sensitive to storm kinematics and the peak response is highly affected. The fact that urban drainage networks have wide range of network configuration compared with natural river networks implies significant differences to storm kinematics in urban catchments depending on their network configuration. In addition, the study implies potential improvement of urban drainage networks in terms of network efficiency as well as sensitivity and safety to rainstorm movement.
6. CONTRIBUTION OF PERVERIOUS AREAS AND ISOLATED IMPERVIOUS AREAS TO HYDROLOGIC RESPONSE IN URBAN CATCHMENTS

A width function can be obtained from drainage networks directly. The width function can be regarded as a straightforward interpretation of the network response. The width function incorporates geometric and topologic characteristics of a watershed including shape and connectivity of drainage networks. This chapter utilizes the width function to yield a response function of a drainage network at the outlet. This chapter addresses the mass balance error observed in runoff hydrographs in urban watersheds by two simple assumptions regarding the contribution of pervious areas to the runoff hydrograph. Rainfall infiltrating into pervious areas has been assumed not to contribute to the runoff hydrograph; Hortonian runoff (Horton, 1933; Chow et al., 1988). However, mass balance analysis in an urban watershed indicates that rainfall infiltrated to pervious areas might contribute to discharge hydrographs of the drainage sewer networks, thereby offering an explanation for the long hydrograph tail commonly observed in runoff from urban storm sewers. In this Chapter, a framework for rainfall-runoff analysis in urban watersheds based on the width function is introduced with two types of width functions obtained from pervious and impervious areas, respectively. This study utilizes detailed spatial information of imperviousness ratio in an urban catchment obtained from digital orthoimages, Light Detection and Ranging (LIDAR) data, and street data analyzed using a Geographic Information System (GIS). Width functions are obtained from urban drainage networks and applied with excess and infiltrated rainfall amounts to obtain distinct response functions for Directly Connected Impervious Areas (DCIA), Isolated Impervious Areas (IIA), and Pervious Areas. The width functions for pervious and impervious areas, combined with proposed assumptions, provide quantification of the contribution from each area to runoff hydrographs. The model framework suggested in this chapter also enables us to evaluate the role of IIA in urban catchments. The results
show improvement in the estimation of runoff hydrographs and suggest the need to consider the flow contribution from infiltrated rainfall in pervious areas to the runoff hydrograph. The results also imply that additional contribution from flow paths such as pipe infiltration needs to be considered.

6.1. Introduction
An urban watershed is unique in that it has a mixture of natural and artificial flow paths, which have different hydrodynamic properties. Knowledge of the contributions to urban drainage runoff from both pervious and impervious surfaces is crucial for hydraulic design of stormwater systems as well as for modeling non-point source pollution (Boyd et al., 1993). Heterogeneity in soil structure as well as various types of vegetation makes it more difficult to model runoff from pervious areas than runoff from impervious areas. The runoff from pervious areas also depends on antecedent conditions of soil moisture. It is typically assumed that the rainfall infiltrating into pervious areas does not contribute to runoff hydrographs and that pervious areas do not contribute until saturation occurs (Boyd et al., 1993, 1994; Crobeddu et al., 2007; Gironas et al., 2009; Cantone 2010). After saturation, the slope of the runoff depth versus the excess rainfall depth becomes one, which means that all excess rainfall on the basin, including the pervious areas, starts to contribute to runoff. However, in a sewered system, the infiltrated water takes a more complicated flow path than in the rural system. Butler and Davies (2004) recognized that the infiltrated water in pervious areas also infiltrates back into the sewers and contributes to the measured sewer runoff. Gregory et al. (2006) investigated that soil compaction during the construction of structural foundations can reduce the moisture loss out of the urban hydrologic system and they indicate that this increases the contribution to the runoff hydrograph. Pipe infiltration can be one of the possible flow paths of infiltrated water to the main drainage network. Weiss et al. (2002) investigated 34 combined sewer systems in Germany and found that sewer inflow due to pipe infiltration is widely underestimated and more than 2/3 of the water passing through the waste water treatment plant can be attributed to infiltration inflow. De Benedittis and Bertrand-Krajewski (2005) calculated the contribution from infiltration inflow in a sewer system in Lyon, France to be 30% of dry weather flow. Vaes et al. (2005) also showed the importance of quantifying
infiltration rate into sewer pipes. These studies emphasize the importance of pervious areas in urban catchments in that they should be treated with greater attention than they are commonly treated with in current practice for hydrologic modeling.

Many researchers have carried out studies on hydrologic response based on the geomorphologic structure of river networks. One of the first efforts to relate the response of a catchment to its geomorphologic characteristics was the Geomorphologic Instantaneous Unit Hydrograph (GIUH) (Rodriguez-Iturbe and Valdes, 1979; Gupta et al., 1980). The GIUH demonstrated that when a unit instantaneous impulse is injected into a channel network, the distribution of arrival times at the basin outlet is affected both by the geomorphology of the catchment, such as stream drainage patterns, and the hydraulic characteristics of the channel flow, such as stream roughness (Franchini and O’Connel, 1996).

The GIUH approach takes account of geomorphologic dispersion separately by ordering of channel networks according to the Strahler ordering scheme (Strahler, 1957), which is a method of classifying stream segment based on the number of tributaries upstream. In contrast, the width function approach incorporates the width function directly from the network, which captures the unique response of the catchment by representing the topology and the metrics of the channel network in a concise form (Moussa, 2008a, b). The width function approach taken in the IUH considerably simplifies the GIUH approach previously discussed by emphasizing the metric representation of the basin instead of the topologic one (Di Lazzaro, 2009). Mesa and Mifflin (1986) and Naden (1992) coupled the width function with the convective diffusion equation to evaluate the hydrodynamic dispersion represented by two parameters, celerity and longitudinal diffusivity. These parameters are dependent on the local slope, discharge and geometry of the channel, which implies that the parameter values can be physically determined (Franchini and O’Connel, 1996). The hydrologic response of a basin should be closely linked to the width function (Gupta and Waymire, 1983) and information about this response might be lost by grouping channel segments (Troutman and Karlinger, 1985).

Although width functions have been applied to rural areas, this work extends their use to urban catchments and further explores the quantification of contribution from pervious and impervious areas composing urban catchments. This chapter develops a framework
using the instantaneous unit hydrograph based on the width function (WFIUF) in order to
examine the contribution from pervious areas in urban catchments. Utilizing the spatial
distribution of imperviousness, this study introduces two types of width functions from
pervious and impervious areas, respectively. Alley and Veehuis (1983) and Boyd et al.
(1994) reinforced the importance of Effective Impervious Area (EIA) or directly
connected impervious area (DCIA) in urban catchments. Lee and Heaney (2003) showed
that the runoff hydrograph in urban areas can be over-predicted without considering
DCIA. This is conceptually analogous to EIA.

DCIA is one of the important concepts in land use practice and low impact development
(EPA, 2011). Therefore, this study incorporates the concept of DCIA and isolated
impervious area (IIA) to capture the flow characteristics in urban catchment. The key
questions of this chapter are: (a) to examine applicability of the WFIUH in urban
drainage networks, incorporating unique characteristics of urban areas; (b) to investigate
the hydrologic response when contribution of the precipitation infiltrated from pervious
areas is considered; and (c) to quantify the contributions from DCIA, IIA and pervious
areas to the flow discharge hydrograph in urban catchments.

6.2. WFIUH for an urban drainage network

Assuming constant flow velocity, the width function can be easily discretized into the
unit hydrograph for a generic watershed by converting the distance to time. However, in
an actual channel, storage capacity of the channel and the variation in travel time along
the flow paths cannot be ignored. The drainage network can be regarded as a series of
individual channel links with length of $\Delta x$, each receiving a lateral inflow, and being
connected to the basin outlet at a distance $x$. The derivation of WFIUH is started from the
equations for conservation of momentum and mass, as shown below.

Conservation of momentum:

$$S_f = S_0 - \frac{\partial y}{\partial x} - \frac{v}{g} \frac{\partial v}{\partial x} - \frac{1}{g} \frac{\partial y}{\partial t} - D_L$$  \hspace{1cm} (6.1)

Conservation of mass:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = i(x, t)$$  \hspace{1cm} (6.2)

In Equations 6.1 and 6.2, $i(x, t)$ is the lateral flow per unit distance and $D_L$ is the energy
dissipation due to the lateral mixing.

The derivation of the advection-diffusion equation for an open channel depends on the assumptions selected (Singh, 1996). One of the most common assumptions in the derivation is to neglect the local and convective acceleration terms ($\partial y/\partial t$, $v\partial v/\partial x$) in the momentum equation. This is the so-called ‘Non-inertial wave’ approximation (Yen, 2001). Without any simplifying assumptions, van de Nes (1973) proposed an approach to obtain analytical solution of the governing equations for a trapezoidal cross section. Following the approach, the advection-diffusion equation can be derived assuming that the main drainage network has a circular cross section. From Equation 6.1 and 6.2,

$$S_f = S_0 - (1 - F^2) \frac{\partial y}{\partial x} - (2v - c) \frac{Bc}{a} \frac{\partial y}{\partial x}$$

where $F$ is Froude number, $B$ ($= \partial A/\partial y$) is water surface width and $c$ ($= \partial Q/\partial A$) is celerity.

Equation 6.3 can be rewritten as

$$S_f = S_0 - \left[1 - F^2 \left(1 - \frac{c}{v}\right)^2\right] \frac{\partial y}{\partial x}$$

$Q$ can be expressed as the sum of initial uniform flow and perturbed flow.

$$Q = Q_I + Q_P$$

Assuming $Q$ is a function of flow depth $y$ and energy slope $S$ ($= \partial y/\partial x$), and that the flow perturbations $Q_P$ can be equated to changes in the uniform flow $Q_I$, a Taylor series expansion of Equation 6.5 can be obtained. Neglecting the higher order terms in the Taylor series expansion of Equation 6.5 yields

$$Q_P = \frac{\partial Q_I}{\partial y} y_P + \frac{\partial Q_I}{\partial S} S_P$$

where $y_P$ and $S_P$ are depth and energy slope associated with flow perturbations. For a circular cross section, the derivatives of the uniform flow with respect to depth and slope can be derived

$$\frac{\partial Q_I}{\partial y} = v_I \left[\frac{3}{4} \frac{d_0^2}{R_I} (1 - \cos \theta_I) - \frac{R_I d_0}{B_I}\right]$$

$$\frac{\partial Q_I}{\partial S} = -\left(1 - \frac{F_I^2 d_0^2}{16 B_I^2} \left(1 - \cos \theta_I - \frac{4R_I}{d_0}\right)^2\right) \frac{Q_I}{2S_0}$$

where $d_0$ is the diameter of a circular cross section and subscript $I$ represents the initial uniform flow condition. Substituting Equation 6.7 and 6.8 into Equation 6.6 yields

$$\frac{Q_P}{B_I} = c y_P - D \frac{\partial y_P}{\partial x}$$
where \( x \) is distance from upstream end of the reach (m) and the celerity, \( c \) (m/s) and diffusion coefficient, \( D \) (m\(^2\)/s) for a circular cross section are obtained in Equations 11 and 12 (See Appendix A).

\[
c = \left[ d_0 (1 - \cos \theta_I) - \frac{4}{3} R_I \right] \frac{\sqrt{2} \varphi_d d_0}{4 B_I^2}
\]

\[
D = \left( 1 - \frac{F_I}{16} \left( \frac{d_0^2}{B_I^2} \left( 1 - \cos \theta_I - \frac{4 R_I}{d_0} \right) \right)^2 \right) \frac{Q_I}{2 S_0 B_I}
\]

The continuity equation for the perturbation can be written as

\[
\frac{\partial Q_P}{\partial y} + \frac{\partial A}{\partial y} \frac{\partial y}{\partial S} = 0
\]

Substituting Equation 6.9 into 6.12 gives the advection-diffusion equation for the perturbation:

\[
\frac{\partial Q_P}{\partial t} = D \frac{\partial^2 Q_P}{\partial x^2} - c \frac{\partial Q_P}{\partial x}
\]

In the case of a semi-infinite uniform channel fed by inflow at the upstream end (\( x = 0 \)), the routing function can be derived from Equation 6.13. When the coefficients \( D \) and \( c \) are considered constant, the solution of Equation 6.13 with the boundary condition, \( Q_p(0, t) = \delta(t) \), \( Q_p(x, 0) = 0 \) and \( Q_p(\infty, t) = 0 \), is given as follows (Van de Nes, 1973; Naden, 1992; Franchini and O’Connel, 1996; Da Ros and Borga, 1997):

\[
u(x, t) = \frac{x}{\sqrt{4 \pi D t^3}} \exp \left[ -\frac{(x-ct)^2}{4Dt} \right]
\]

where \( u(x, t) \) is the impulse response of the advection-diffusion equation, i.e. the time evolution of the discharge at a distance \( x \) from the upstream end when an instantaneous upstream impulse \( \delta(t) \) is introduced (Naden, 1992; Franchini and O’Connel, 1996; Da Ros and Borga, 1997). From the unit impulse response, \( u(x, t) \) in Equation 6.14, an IUH of a catchment can be defined as

\[
h(t) = \int_0^\infty W(x) u(x, t) \, dx
\]

where \( W(x) \) is the width function.

For a discrete distance interval, Equation 6.15 can be written as

\[
\hat{h}(t) = \sum_{i=1}^{n} \frac{\Delta x}{\sqrt{4 \pi D t^3}} W(i \Delta x) \exp \left[ -\frac{(i \Delta x - ct)^2}{4Dt} \right] \Delta x
\]

The diameter and the slope selected to calculate the celerity and the diffusion coefficient of the model are the catchment-representative values to capture the characteristics of the hydrodynamic dispersion. In this chapter, the flow in the main drainage network is
considered to be open channel flow with a circular cross section. A maximum flow value for the circular cross section is defined as 0.8 times the pipe full flow as shown following equation:

\[ Q_o = \frac{d_o^{8/3}S_o^{1/2}}{4n_o} \]  \hspace{1cm} (6.17)

where \( Q_o, d_o, S_o \) and \( n_o \) are the peak discharge, diameter, bottom slope and the roughness at the outlet. The flowrate at each pipe outlet is tested, and if it is greater than the \( Q_o \), the difference between the actual and the maximum flow is delayed to the next time step when the flow becomes smaller than \( Q_o \).

6.3. Application

Study area

The test catchment: CDS-51 in this chapter is a part of the Calumet portion of the Tunnel and reservoir Plan (TARP) system in the Chicago area. TARP is a system of deep tunnels and reservoirs that relieves pollutant load and water volume load to area waterways. CDS-51 is a highly-urbanized catchment in which most of the drainage load is conveyed through the pipe network as shown in Figure 6.1. Accurate estimation of the flow is crucial in operation of the entire system. The watershed captures storm and sanitary flows for a service area of 3.16 km\(^2\). The combined sewerage system of CDS-51 collects inflow from in excess of 800 inlets and conveys it to the outlet of the watershed via a network of 722 pipes ranging in size from 0.15 m to 2.13 m. Dry weather flows are intercepted by two interceptor sewers which convey flow to the Calumet Water Reclamation Plant. When the interceptor sewers and/or treatment plant reach capacity, excess flow is directed towards the combined sewer overflow (CSO) location and conveyed through the dropshaft to the deep tunnel. Table 6.1 summarizes the number of pipes, diameters, lengths and slopes of the pipe network of CDS-51 according to Strahler ordering scheme. From 2007 to 2011, the United States Geological Survey (USGS) used three acoustic flow meters to monitor the inflow from the catchment, the volume of flow partitioned to the CSO, and the amount of inflow entering the drop shaft connected to the deep tunnel at CDS-51.
Figure 6.1 The drainage pipe network; CDS-51 in Chicago

Table 6.1 Conduits of CDS-51 according to the Strahler ordering (Miller et al., 2009)

<table>
<thead>
<tr>
<th>Order</th>
<th>No.</th>
<th>Diameter (m)</th>
<th>Length (m)</th>
<th>Bottom slope (×10⁻³)</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Standard dev.</td>
<td>Mean</td>
</tr>
<tr>
<td>1</td>
<td>449</td>
<td>0.33</td>
<td>0.11</td>
<td>61.62</td>
</tr>
<tr>
<td>2</td>
<td>157</td>
<td>0.46</td>
<td>0.16</td>
<td>59.69</td>
</tr>
<tr>
<td>3</td>
<td>57</td>
<td>0.72</td>
<td>0.23</td>
<td>75.00</td>
</tr>
<tr>
<td>4</td>
<td>51</td>
<td>1.18</td>
<td>0.34</td>
<td>64.30</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>2.06</td>
<td>0.08</td>
<td>98.68</td>
</tr>
</tbody>
</table>

The IUH based on the width function has two parameters, the kinematic wave celerity, \( c \), and the diffusion coefficient, \( D \), which are representative values for a catchment. As shown in the previous section, the parameter values are physically determined from Equation 6.10 and 6.11 based on geometry and slope of a drainage network. The average celerity and diffusion coefficient for CDS-51 were determined as a function of slope, diameter and initial depth of the conduit pipe using Eq. 6.10 and 6.11. The information on the diameter, length and the bottom slope listed in Figure 6.1 is used to calculate the
average diameter in CDS-51. Figure 6.2 and Figure 6.3 illustrate $c$ and $D$ as a function of the diameter, slope and initial depth assumption. The transition coefficient (kinematic wave celerity), $c$ and diffusion coefficient, $D$ are determined with average diameter and slope assuming 20% of the pipe is initially full; $c = 0.43$ m/s and $D = 5.6$ m$^2$/s.

**Figure 6.2** Kinematic wave celerity, $c$ (m/s) as a function of slope, diameter and initial depth: (a) initial depth as 10% of the pipe diameter; (b) 20% of the pipe diameter

**Figure 6.3** Diffusion coefficient, $D$ (m$^2$/s) as a function of slope, diameter and initial depth: (a) initial depth as 10% of the pipe diameter; (b) 20% of the pipe diameter

The diffusion coefficient of CDS-51 is on the order of $10^0$ (5.58 m$^2$/s); the diffusion coefficient is small compared with the diffusion coefficient of natural river networks. The diffusion coefficient in a wide rectangular channel can be obtained as follows (Henderson,
1966; Harley, 1967):

\[ D_r = \frac{c^2 y^2}{2v} = \frac{q}{2S_0} \]  \hspace{1cm} (6.18)

where \( D_r \) is the diffusion coefficient of a wide rectangular channel. Compared with the diffusion coefficient of circular cross section (Equation 6.11), wide rectangular channel has greater coefficient values by the ratio of

\[ \frac{D}{D_r} = 1 - \frac{F_I^2}{16} \left( \frac{d_0^2}{R_I} \left( 1 - \cos \theta_I - \frac{4R_I}{d_o} \right) \right)^2 \]  \hspace{1cm} (6.19)

Franchini and O’Connel (1996) showed that natural rivers have diffusion coefficients of the order of \( 10^3 \).

The width function is a straightforward interpretation of the system response. As the diffusion coefficient increases, the original and unique shape of the width function (Figure 6.4) of the catchment begins to diminish. The shapes of WFIUH with different magnitude of celerity and diffusion coefficient are compared Figure 6.5. The result shows that given a relatively small diffusion coefficient, the original shape of the width function persists in IUH. CDS-51 has a width function that has one small peak and two high peaks as shown in Figure 6.4.

![Figure 6.4 Width function of CDS-51 with grid size of 60.3 meters](image)
Figure 6.5 WFIUH of CDS-51 depending on celerity and diffusion coefficient: (a) for \( D = 20 \text{ m}^2/\text{s} \), \( c = 0.3-0.45 \text{ m/s} \); (b) for \( D = 20 \text{ m}^2/\text{s} \), \( c = 0.7-0.85 \text{ m/s} \); (c) for \( D = 2,000 \text{ m}^2/\text{s} \), \( c = 0.3-0.45 \text{ m/s} \); (d) for \( D = 2,000 \text{ m}^2/\text{s} \), \( c = 0.7-0.85 \text{ m/s} \)

Figure 6.6 shows the runoff hydrographs estimated for CDS-51 with combinations of different values of celerity and diffusion coefficient. The result shows that both celerity and diffusion coefficient are closely related to the resulting shape of hydrographs. Larger values of diffusion coefficient (\( D = 2,000 \text{ m}^2/\text{s} \)) tend to smooth out the shape of hydrographs and reduce the peak response compared with the hydrograph produced with a small diffusion coefficient (\( D = 20 \text{ m}^2/\text{s} \)).

Figure 6.7 (a) shows that the peak of WFIUH is highly dependent on celerity when the diffusion coefficient is relatively small (\( D = 20 \text{ m}^2/\text{s} \)). While when the diffusion coefficient is relatively large (\( D = 2,000 \text{ m}^2/\text{s} \)), the peak of WFIUH steadily increases.
Figure 6.7 (b) shows the peak discharge response for a rainfall event on 7 Jan 2008. In contrast to the peak of WFIUH (Figure 6.7 (a)), the peak of the runoff hydrograph does not greatly fluctuate compared with the peak of WFIUH, but it still depends on the diffusion coefficient (Figure 6.7 (b)). In terms of the time to the peak, both WFIUH and the runoff hydrograph show monotonically decreasing timing to the peak with increasing celerity (Figure 6.8).

Figure 6.6 Sensitivity of WFIUH: (a) estimated runoff hydrograph for a rainfall event (7 Jan 2008) with $D = 20 \text{ m}^2/\text{s}$; (b) with $D = 2,000 \text{ m}^2/\text{s}$

Figure 6.7 Sensitivity of WFIUH: (a) peak response of IUH; (b) simulated peak discharge (7 Jan 2008 storm)
Figure 6.8 Sensitivity of WFIUH: (c) time to the peak of IUH; (d) time to the simulated peak discharge (7 Jan 2008 storm)

Mass balance error and flow paths in urban drainage system

Typically, infiltrated amount of rainfall is assumed not to contribute to discharge hydrographs in direct runoff modeling in a relatively short timescales. Especially in urban catchments, it is assumed to be lost out of the system. Based on that assumption that the infiltrated rainfall do not contribute, the result of a rainfall-runoff analysis is shown in Table 6.2 and Figure 6.9; the mass balance in terms of volume of water for a rainfall event on 7 Jan 2008 in CDS-51. The hydrologic analysis is performed utilizing WFIUH described previously. The resulting hydrograph is compared with IUHM (Cantone, 2010) as well as monitored runoff discharge, which shows disagreement with observed data in excess rainfall runoff estimation (Figure 6.9). The difference with observation is up to 19% (Table 6.2) in terms of volume of water. Specifically, the model simulates the peak discharge, but, the model fails to reproduce the long tail observed as shown in Figure 6.9. The comparison of the results in Table 6.2 implies that a part of the infiltrated amount of rainfall can eventually contribute to the runoff hydrograph. The result strongly suggests that it is necessary to reevaluate the typical assumption for the infiltrated rainfall. This study examines the possible flow paths in urban drainage system in order to review this.
Figure 6.9 Flow hydrograph estimated with WFIUH and Illinois Urban Hydrologic Model (IUHM, Cantone (2010)) for a rainfall event on 7 Jan 2008 in CDS-51

Table 6.2 Mass balance analysis of a rainfall event on 7 Jan 2008 in CDS-51

<table>
<thead>
<tr>
<th>Rainfall/Runoff</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rainfall Total</td>
<td>100</td>
</tr>
<tr>
<td>Loss</td>
<td></td>
</tr>
<tr>
<td>Infiltration</td>
<td>46</td>
</tr>
<tr>
<td>Depression Storage</td>
<td>3</td>
</tr>
<tr>
<td>Excess Rainfall Runoff</td>
<td></td>
</tr>
<tr>
<td>Simulated</td>
<td>51</td>
</tr>
<tr>
<td>Observed</td>
<td>70</td>
</tr>
</tbody>
</table>

Butler and Davis (2004) illustrated the flow paths of an urban water system; combined sewerage as shown in Figure 6.10. One of the flow paths for the infiltrated rainfall to the combined sewerage system (i.e. main drainage network) is pipe infiltration. The pipe infiltration can be one of the possible flow paths of infiltrated water in pervious areas to the main drainage network in urban catchments (Weiss et al., 2002; De Benedittis and Bertrand-Krajewski, 2005; Vaes et al., 2005). Therefore, this study introduces two
assumptions for pervious areas (Figure 6.11): a portion of infiltrated rainfall contributes to the discharge of the main drainage network (Assumption 1) and the second is that the rest of the infiltrated water percolates into aquifer section being lost out of the system (Assumption 2). Among the infiltrated rainfall, the ratio of the contributing amount of rainfall to main drainage network, $r_b$ is an unknown variable and it needs to be calibrated.

Figure 6.10 Urban water system: combined sewerage (Butler and Davies, 2004)
Figure 6.11 Two assumptions for infiltrated water paths in pervious areas

While, the rainfall amount falling on impervious areas takes two types of flow paths depending on whether the impervious area is directly connected to the main drainage network or not. The impervious area directly connected to the main drainage network is called Directly Connected Impervious Area (DCIA) (or effective impervious area) and the other impervious areas are called Isolated Impervious area (IIA). The rainfall falling on DCIA directly drains to the drainage pipe network. However, the rainfall on IIA flows through pervious area to reach the drainage network and infiltrates into subsurface until the soil saturates and excess rainfall occurs. EPA (2011) suggested a set of formula to calculate the area of DCIA depending on catchment-average imperviousness ratio and land use (watershed section criteria). This study takes account of IIA utilizing a highly detailed imperviousness map developed by Crosa-Rivarola (2008) based on orthoimages, Light Detection and Ranging (LIDAR) measurement, and street data from geographic information system (GIS). A detailed impervious map is crucial because it enables us to distinguish IIA from DCIA in impervious areas and also to develop width function for pervious and impervious separately.
Crosa-Rivarola (2008) investigated the spatial variability that can be found in urban catchments and developed a highly-detailed imperviousness map of CDS-51 based on three filters: the orthoimages through image processing filter, Light Detection and Ranging (LIDAR) data through LIDAR filter, and street data through street filter with Geographic Information System (GIS) as shown in Figure 6.12. The imperviousness map shows that the IIA can be separated from the DCIA. However, IIA is not clearly distinguishable from DCIA when buildings and houses are connected with narrow pathways. In this case a threshold distance can be used to make buildings and houses connected with these narrow pathways be categorized as IIA instead of DCIA. If the widths of the connecting paths are smaller than a given threshold distance, these connections are ignored and the area is categorized as IIA. Figure 6.13 shows the procedures for this purpose and Figure 6.14 shows the actual application on impervious map of CDS-51 when the threshold distances are 0, 0.9 m, 1.8 m, and 2.7 m.
Table 6.3 lists the resulting $r_c$, which is defined as the ratio of IIA with respect to total impervious area using impervious map (Crosa-Rivarola, 2008). The result shows that at least 23% of the total impervious area is categorized as IIA and it also shows that the ratio of IIA is highly dependent on the threshold values. As the threshold distance increases, the ratio of IIA ($r_c$) increases; IIA ranges from 25% (no threshold distance) to 80% (threshold distance of 2.7 meters) depending on threshold distance.

![Figure 6.13 Estimation of IIA with a threshold distance: (a) original impervious map; (b) shrinking boundaries by given threshold distance; (c) selecting newly defined DCIA; (d) finalized DCIA](image)

Table 6.3 Estimation of Isolated Impervious Areas (IIA) and $r_c$ depending on a threshold distance for CDS-51

<table>
<thead>
<tr>
<th>Cell size</th>
<th>Threshold distance (m)</th>
<th>DCIA (km$^2$)</th>
<th>IIA (km$^2$)</th>
<th>$r_c = \text{IIA} / \text{IA}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1.34</td>
<td>0.40</td>
<td>23</td>
</tr>
<tr>
<td>1</td>
<td>0.9</td>
<td>0.77</td>
<td>0.96</td>
<td>56</td>
</tr>
<tr>
<td>2</td>
<td>1.8</td>
<td>0.75</td>
<td>0.98</td>
<td>58</td>
</tr>
<tr>
<td>3</td>
<td>2.7</td>
<td>0.35</td>
<td>1.38</td>
<td>80</td>
</tr>
</tbody>
</table>
EPA (2011) adopted the ‘Sutherland Equations’ from Sutherland (2000) to calculate area of DCIA from the ratio of Impervious Area (IA) for Massachusetts communities depending on watershed criteria as shown in Table 6.4. The Sutherland Equations are only valid where IA (%) is greater than one; Therefore, EPA assumed DCIA (%) to be zero where the IA (%) within a given land use is less than one. Table 6.5 lists the percentage of DCIA calculated with Sutherland Equation (EPA, 2011). From the DCIA percentage values obtained as shown in Table 6.5, the ratio of IIA, $r_c$ (IIA/IA) values are obtained (Table 6.6). The results indicate that for the imperviousness ratio (IA) of 54% in CDS-51, the $r_c$ can be estimated to be 27% assuming watershed selection criteria as ‘Average: medium density residential area’. Therefore, in this study, IIA of 23% is used (without threshold distance) for CDS-51 (Table 6.3). The comparison between the results from Sutherland Equation and the results from impervious map shows that all connected impervious areas in impervious map should be regarded as DCIA.
### Table 6.4 Sutherland equations to determine DCIA from IA (EPA, 2011)

<table>
<thead>
<tr>
<th>Watershed selection criteria</th>
<th>Assumed land use</th>
<th>DCIA (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average: Mostly storm sewered with curb &amp; gutter, no dry wells or infiltration, residential rooftops not directly connected</td>
<td>Commercial, Industrial, Institutional, Open land, and Med. density residential</td>
<td>DCIA = 0.1 (IA)^1.5</td>
</tr>
<tr>
<td>Highly connected: Same as above, but residential rooftops are connected</td>
<td>High density residential</td>
<td>DCIA = 0.4 (IA)^1.2</td>
</tr>
<tr>
<td>Totally connected: 100% storm sewered with all Impervious Area (IA) connected</td>
<td>Low density residential</td>
<td>DCIA = 0.04 (IA)^1.7</td>
</tr>
<tr>
<td>Somewhat connected: 50% not storm sewered, but open section roads, grassy swales, residential rooftops not connected, some infiltration</td>
<td>Agricultural: Forested</td>
<td>DCIA = 0.01 (IA)^2</td>
</tr>
</tbody>
</table>

### Table 6.5 Directly Connected Impervious Areas (DCIA, %) calculated with the EPA equation

<table>
<thead>
<tr>
<th>IA(%)</th>
<th>DCIA(%)</th>
<th>Highly connected</th>
<th>Totally connected</th>
<th>Somewhat connected</th>
<th>Mostly disconnected</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>9</td>
<td>15</td>
<td>20</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>30</td>
<td>16</td>
<td>24</td>
<td>30</td>
<td>13</td>
<td>9</td>
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<tr>
<td>50</td>
<td>35</td>
<td>44</td>
<td>50</td>
<td>31</td>
<td>25</td>
</tr>
<tr>
<td>60</td>
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<td>54</td>
<td>60</td>
<td>42</td>
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<tr>
<td>90</td>
<td>85</td>
<td>89</td>
<td>90</td>
<td>84</td>
<td>81</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>
### Table 6.6 Area ratio of IIA ($r_c$) calculated with the EPA equation

<table>
<thead>
<tr>
<th>IA(%)</th>
<th>$r_c$(%) obtained by the EPA equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average*</td>
</tr>
<tr>
<td>10</td>
<td>68</td>
</tr>
<tr>
<td>20</td>
<td>55</td>
</tr>
<tr>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td>40</td>
<td>37</td>
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<tr>
<td>50</td>
<td>29</td>
</tr>
<tr>
<td>60</td>
<td>23</td>
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<tr>
<td>70</td>
<td>16</td>
</tr>
<tr>
<td>80</td>
<td>11</td>
</tr>
<tr>
<td>90</td>
<td>5</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
</tr>
</tbody>
</table>

Width functions for pervious and impervious area

One of the advantages of utilizing the width function for hydrologic analysis is that it incorporates the spatial variability of the watershed properties and precipitation (e.g. imperviousness) as shown in Chapter 4 and Chapter 5. Viglione et al. (2010) quantified the contribution of spatio-temporal variability of rainfall and runoff coefficient as well as hillslope and channel velocities, as well as storm movement to the resulting flood peaks. The spatial distribution of watershed properties and precipitation significantly affects the model’s capability of flow estimation. For hydrologic modeling in urban watersheds, the imperviousness ratio is an important factor that distinguishes urban catchments from natural catchment. However, the imperviousness ratio is often represented as an average value for one catchment. Imperviousness ratios for sub-areas within a catchment are typically obtained by assigning impervious values corresponding to the type of land use in the sub-area.

Utilizing the imperviousness map developed by Crosa-Rivarola (2008), this study develops two width functions, one for pervious and one for impervious area, respectively. With the imperviousness map explicitly obtained from the orthoimagery, the imperviousness ratio is averaged to each grid cell as shown in Figure 6.15 (b). Combined with the drainage network built on grid, width functions are developed for pervious areas.
and impervious areas. Figure 6.16 shows two resulting width functions for pervious and impervious areas obtained from spatial distribution of imperviousness ratio in Figure 6.15 (b). Figure 6.16 also shows that utilizing the catchment-average imperviousness ratio can cause biases in estimation of exact response in terms of the width function. In this study, two width functions from both pervious and impervious areas are utilized to obtain the response function at the catchment outlet. The advantage of this approach is that response functions can be distinctively derived for both areas depending on the hydrodynamic properties (celerity and diffusion coefficient) of corresponding areas.

Figure 6.15 Imperviousness map of CDS-51 (0 for pervious and 1 for impervious area): (a) from orthoimagery (Crosa-Rivarola, 2008); (b) imperviousness ratio averaged to grid cells
Development of hydrologic response function in urban catchments based on width functions for pervious and impervious areas

In this section, a hydrologic response functions is developed utilizing the width function for pervious and impervious areas distinctively in urban catchments. Based on flow paths in pervious areas (Figure 6.11) and impervious areas (Figure 6.14), we develop a framework incorporating various flow paths in urban drainage networks. Aronica and Cannarozzo (2000) suggested a semi-distributed model based on separate definition of the hydrological response of subcatchment and the drainage network. The response of the catchment is obtained as a convolution of two response functions for the main drainage network and a grid cell, respectively. Figure 6.17 illustrates the framework of this study where $n$ represents each grid cell and the main drainage network is represented by thick arrows. The excess rainfall falling on impervious areas is assumed to be drained into the main drainage network immediately. Hence, the flow paths for impervious areas are identical to the main drainage network. Paths for pervious areas are divided into two; one for infiltrated and the other for excess rainfall. The first path is the subsurface flow path

Figure 6.16 Two normalized width functions for pervious and impervious areas obtained from the imperviousness map in CDS-51

\[
\begin{align*}
W_{\text{normalized}} & = 0.04 \\
\text{distance (km)} & = 0 \\
0 & \leq W_{\text{normalized}} \leq 0.04
\end{align*}
\]
taken by the infiltrated rainfall. A portion of the infiltrated rainfall eventually contributes to the main drainage network (Assumption 1) and the remainder of infiltrated rainfall is lost out of system (Assumption 2). The second path taken by excess rainfall from pervious areas is the same as the flow paths for the impervious areas: the flow path through the main drainage network. In Figure 6.17, DCIA describes impervious areas (e.g. roadways and roofs with attached roof drains) where the runoff flows directly into the drainage system (Alley and Veenhuis, 1987; Lee and Heaney, 2003; EPA, 2011). While IIA defines areas that are indirectly connected to the drainage system and cause flows to be routed through pervious areas, DCIA accounts for no additional flow translation between the impervious areas and the network.

\[
\Delta x
\]

\[
\Delta x'/2
\]

**Figure 6.17 Response functions from excess rainfall and infiltrated rainfall contributing to runoff hydrographs**

In order to account for the different flow paths from pervious and impervious areas, the WFIUH defined by Equation 6.16 can be written as follows:

\[
h_i(t) = \sum_{j=1}^{n_{w}} (W_i(j \Delta x) \cdot f(j \Delta x, t) \cdot g_i(t)) \Delta x \tag{6.20}
\]

where \(i=1\) for contribution from excess rainfall in DCIA, 2 for excess rainfall in IIA, 3 for excess rainfall in pervious areas (ExPerv), and 4 for infiltrated rainfall in pervious areas.
(InPerv). \( W_1 \) and \( W_2 \) are the same width functions obtained from impervious area and \( W_3 \) and \( W_4 \) are the same from pervious area, respectively (Figure 6.16). \( n_w \) is the maximum distance of the width function. \( f \) is a response function of the main drainage network and \( g \) is a response function defined in a cell (Figure 6.17). From Equation 6.16, the response from the main drainage network is given as

\[
f(i \Delta x, t) = \frac{i \Delta x}{\sqrt{4 \pi D_1 t^3}} \exp \left[ -\frac{(i \Delta x - ct)^2}{4D_1 t} \right]
\]

where \( c_1 \) and \( D_1 \) are celerity and diffusion coefficient for the main drainage network. The response function in a cell, \( g_i \) from excess rainfall in DCIA, IIA as well as in pervious areas (ExPerv) is given as

\[
g_i(t = 0) = 1, \text{ otherwise } 0; \text{ for } i = 1, 2, 3
\]

The response function \( g_4 \) is from the infiltrated rainfalls in pervious areas (InPerv) by Assumption 1. Mejia and Moglen (2010a) assumed a two-parameter inverse Gaussian travel time distribution for both hillslopes and channels to derive a geomorphologic unit hydrograph for a natural watershed. In this study, \( g_4 \) is assumed to have the same form with Equation 6.21 which is a solution of an advection-diffusion equation.

\[
g_i(t) = \frac{\Delta x}{\sqrt{16 \pi D_2 t^3}} \exp \left[ -\frac{(\Delta x - 2c_2 t)^2}{16 D_2 t} \right]; \text{ for } i = 4
\]

where \( c_2 \) and \( D_2 \) are celerity and diffusion coefficient of the flow path, through which the infiltrated rainfall in pervious areas contributes to the main drainage network. Given the length of \( f \) and \( g \) as \( M_f \) and \( M_k \), respectively, the convolution for discrete time steps can be obtained as

\[
(f * g)[k] \equiv \sum_{m=0}^{\max(M_f,M_k)-1} f[m] g[k - m], 0 < k < M_f + M_k - 2
\]

The response at the outlet can be obtained as the sum of the convolution of the response function from each area and the corresponding precipitation.

\[
Q(t) = \sum_{i=1}^{n_c} h_i * I_i
\]

where \( i=1 \) for DCIA, 2 for IIA, 3 for ExPerv, and 4 for InPerv, respectively. Excess rainfall and infiltrated rainfall for corresponding pervious and impervious areas are defined in Table 6.7 where \( I_{imperv} \) denotes the excess rainfall amount considering depression storage only in impervious areas, \( I_{ExPerv} \) denotes the excess rainfall considering depression storage as well as infiltration. \( I_{InPerv} \) is infiltrated amount of
rainfall. In Table 6.7, \( r_i \) is impervious ratio of the watershed and \( r_c \) is the area of IIA divided by total impervious area. \( r_b \) is contributing ratio of infiltrated water to runoff by Assumption 2.

**Table 6.7 Precipitation separately assigned for four contributions in urban catchments**

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Soil condition</th>
<th>Unsaturated</th>
<th>Saturated</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCIA</td>
<td>( I_1 = (1 - r_c)I_{imperv} )</td>
<td>( I_1 = (1 - r_c)I_{imperv} )</td>
<td></td>
</tr>
<tr>
<td>IIA</td>
<td>( I_2 = 0 )</td>
<td>( I_2 = r_cI_{imperv} )</td>
<td></td>
</tr>
<tr>
<td>ExPerv</td>
<td>( I_3 = 0 )</td>
<td>( I_3 = I_{ExPerv} )</td>
<td></td>
</tr>
<tr>
<td>InPerv</td>
<td>( I_4 = \left( 1 + \frac{r_i r_c}{1 - r_l} \right) r_b I_{InPerv} )</td>
<td>( I_4 = r_b I_{InPerv} )</td>
<td></td>
</tr>
</tbody>
</table>

In this chapter, the area of DCIA is estimated from the impervious map developed from orthoimagery (Crosa-Rivarola, 2008). The average diameter and slope of the drainage network are adopted to calculate the hydrodynamic properties of the main drainage network. The celerity, \( c_1 \) and diffusion coefficient, \( D_1 \) used for calculation of the response functions of the main drainage network are calculated by Equation 6.10 and 6.11, respectively assuming 20% of the pipe is initially full.

However, the flow path of the infiltrated water to the main drainage network is not explicitly identified for calculation of the delayed response function of pervious area, \( g_2 \) as shown in Figure 6.17. For the infiltrated rainfall in pervious area, the flow path is assumed to consist of subsurface paths until it reaches a main drainage network. Therefore, the celerity of pervious contribution flow is assumed to be the same order of hydraulic conductivity of the soil; \( 10^{-3} \) m/s based on the ranges of values for hydraulic conductivity (Table 6.8). The two unknown parameters; the diffusion coefficient for delayed response from infiltrated amount of rainfall, \( D_2 \) in Equation 21 and the contributing ratio of infiltrated rainfall, \( r_b \) in Table 6.7 need to be calibrated using observation.

Four sets of observed runoff hydrographs and precipitation data from CDS-51 are used in this study as shown in Table 6.9. The flow meters and precipitation gages were operated
by the USGS from 2007 to 2011 in order to monitor the flow discharge amount into the TARP dropshafts in Chicago. For Event 2 during August 2007, four rainfall gages operated by the Illinois State Water Survey (ISWS) are used because the precipitation records from the USGS gage are not available.

**Table 6.8** Hydraulic conductivity in nature (Bear, 1988)

<table>
<thead>
<tr>
<th>Relative Permeability</th>
<th>Aquifer</th>
<th>Unconsolidated Sand &amp; Gravel</th>
<th>Unconsolidated Clay &amp; Organic</th>
<th>Consolidated Rocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pervious</td>
<td>Good</td>
<td>Well Sorted</td>
<td>Peat</td>
<td>Highly Fractured Rocks</td>
</tr>
<tr>
<td>Semi-Pervious</td>
<td>Poor</td>
<td>Well Sorted Sand or Sand &amp; Gravel</td>
<td>Layered Clay</td>
<td>Oil Reservoir Rocks</td>
</tr>
<tr>
<td>Impervious</td>
<td>None</td>
<td>None</td>
<td>Fat / Unweathered Clay</td>
<td>Fresh Sandstone</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Limestone, Loess</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Fresh Granite</td>
</tr>
</tbody>
</table>

**Table 6.9** Four sets of observed runoff hydrograph and precipitation data from USGS and ISWS in CDS-51

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
<th>Duration (hr)</th>
<th>Flow meter</th>
<th>Precipitation</th>
<th>Parameter Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2007-04-25</td>
<td>24</td>
<td>USGS</td>
<td>USGS</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2007-08-22</td>
<td>28</td>
<td>USGS</td>
<td>ISWS</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2008-01-07</td>
<td>15</td>
<td>USGS</td>
<td>USGS</td>
<td>x</td>
</tr>
<tr>
<td>4</td>
<td>2009-04-27</td>
<td>33</td>
<td>USGS</td>
<td>USGS</td>
<td></td>
</tr>
</tbody>
</table>

**6.4. Results and discussion**

_Parameter estimation and resulting hydrographs_

Two coefficients; the contributing ratio, $r_b$ and diffusion coefficient of infiltrated water, $D_2$ are calibrated by the goodness of fit criteria with observed data (Event 3 in Table 6.9). Figure 6.18 shows the location of the estimated values of these unknown parameter values that maximize the goodness of fit. The model efficiency indicates how accurately the model reproduces the observed results. The efficiency, $E$ ranges from $-\infty$ to 1. If $E$ is
close to 1, the model better simulates the observation. The contributing ratio of infiltrated rainfall in pervious areas, \( r_b \) is estimated as 0.55. It implies that 55% of infiltrated water eventually contributes to the runoff hydrograph. Table 6.10 list the values of parameter set for CDS-51. The celerity and diffusion coefficient of main drainage network are calculated based on geometry of the network (Table 6.1). The average imperviousness ratio, \( r_i \) and the area ratio of IIA, \( r_c \) are obtained from impervious map (Table 6.3; Figure 6.15; Figure 6.14). Figure 6.18 shows the goodness of fit index (Nash and Sutcliffe, 1970) depending on coefficients. Two unknown coefficients: the diffusion coefficient, \( D_2 \) and the contributing ratio, \( r_b \) of infiltrated water are calibrated using observed data (7 Jan 2008 event) (Figure 6.18).

![Figure 6.18 Estimation of diffusion coefficient, \( D_2 \) and contributing ratio of pervious area, \( r_b \) using Jan 2008 storm](image)

Table 6.10 Parameter values estimated for CDS-51

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Area (km²)</th>
<th>( \Delta x ) (m)</th>
<th>( c_1 ) (m/s)</th>
<th>( D_1 ) (m²/s)</th>
<th>( r_i )</th>
<th>( r_c )</th>
<th>( D_2 ) (m²/s)</th>
<th>( r_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDS-51</td>
<td>3.42</td>
<td>156</td>
<td>0.43</td>
<td>5.58</td>
<td>0.54</td>
<td>0.23</td>
<td>5.6 X 10⁻¹</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Figure 6.19 compares the estimated runoff hydrographs by WFIUH with the observed
data when the contribution from infiltrated rainfall in pervious areas is ignored. It also assumes that 100% of impervious area of the watershed contributes to runoff without consideration of DCIA and IIA. Figure 6.20 shows the runoff hydrographs estimated by WFIUH when the contribution from pervious areas is accounted for. Figure 6.20 shows improvements in the estimation of flow hydrograph especially for the long tail observed when the contribution from infiltrated rainfall amount is considered. The goodness of fit criteria of Nash-Sutcliffe is increased when contribution from pervious areas before saturation is taken into account as shown in Table 6.11. Especially the model is able to reproduce the long tail in the observation. Although the goodness of fit for Event 1 is decreased when contribution from pervious areas are considered, the model starts to simulate the long tail which can be observed in the measurement as shown in Figure 6.20.

Figure 6.19 Comparison with the observed hydrographs when the contribution from infiltrated rainfall is ignored for the storms in: (a) April 2007; (b) August 2007; (c) January 2008; (d) April 2009
Figure 6.20 Comparison with the observed hydrographs when the contribution from infiltrated rainfall is considered for the storms on: (a) April 2007; (b) August 2007; (c) January 2008; (d) April 2009

Table 6.11 Comparison between a runoff hydrograph considering contribution from impervious areas only and one that considers contribution from both pervious and impervious areas

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
<th>Contribution from impervious areas only</th>
<th>Contribution from both pervious and impervious areas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$E^*$</td>
<td>Peak ratio**</td>
</tr>
<tr>
<td>1</td>
<td>2007-04-25</td>
<td>0.89</td>
<td>0.86</td>
</tr>
<tr>
<td>2</td>
<td>2007-08-22</td>
<td>0.21</td>
<td>1.06</td>
</tr>
<tr>
<td>3</td>
<td>2008-01-07</td>
<td>0.70</td>
<td>0.96</td>
</tr>
<tr>
<td>4</td>
<td>2009-04-27</td>
<td>0.72</td>
<td>0.90</td>
</tr>
</tbody>
</table>

* Nash-Sutcliffe model efficiency; ** $Q_{max, observed}/Q_{max, simulated}$
Figure 6.21 Contribution from pervious and impervious areas for the storm on 7 Jan 2008 in CDS-51: (a) flow discharge; (b) contributing ratio to total flow

Figure 6.21 (a) shows the contribution of pervious and impervious areas to total flow with time for the storm event on January 2008 (Event 3). The contribution from each area changes with time. For a short duration after the storm event begins, the contribution from impervious areas dominates. But, after the rainfall event, the contribution from pervious areas starts to dominate as shown in Figure 6.21 (a). The result shows that the long tail in the discharge hydrograph is originated from the contribution of pervious areas. Figure 6.21 (b) illustrates the variation of the contributing ratio with time.

Quantifying the contributions from pervious and impervious areas and the role of IIA

The modeling framework in urban catchments suggested in this study is able to quantify the contributions from pervious and impervious areas. In order to quantify the contribution of the pervious and impervious areas to the runoff hydrographs, additional model runs are performed with test rainfall events of a synthetic triangular hyetograph in CDS-51. The two test events have the same duration of 10 hours and the maximum intensities of the rainfall are 10 mm/hr (no excess rainfall) and 12 mm/hr (with excess rainfall), respectively. The contribution of DCIA, IIA, ExPerv and InPerv can be separately quantified by Equation 6.23. Figure 6.22 depicts the resulting flow discharges
per unit area of DCIA, IIA, InPerv, and ExPer, respectively. The results illustrate how the contribution ratio of each area changes with rainfall intensity and time. The results show that the contribution from DCIA dominates initially while the contribution from InPerv slowly increases. The contribution from DCIA shortly diminishes after the rainfall stops. While, the contribution from InPerv shows a slower response and consequently a longer tail. The slow response from InPerv mainly contributes to the long tail of the total runoff discharge hydrograph. Before excess rainfall occurs, ExPerv and IIA do not contribute to flow discharge and the runoff hydrograph is composed of contributions from DCIA and InPerv only (Figure 6.22 (a)). Once excess rainfall occurs, InPerv and IIA start to contribute the total runoff hydrograph (Figure 6.22 (b)). The contribution of DCIA, IIA, and ExPerv grows at rates that are proportional to corresponding areas with increasing rainfall intensity.

Figure 6.22 Flow discharge per unit area in CDS-51 with a triangular hyetograph and maximum intensity of (a) \( I = 10 \) mm/hr; (b) \( I = 12 \) mm/hr and the contributing ratio of each area with (c) \( I = 10 \) mm/hr; (d) \( I = 12 \) m/hr
This study utilizes Green and Ampt method (Green and Ampt, 1911) to estimate the infiltrated amount of rainfall as well as excess rainfall in pervious areas. IIA affects the runoff hydrograph especially before saturation occurs. Figure 6.23 compares the flow discharge per unit area with a synthetic triangular hyetograph when IIA is considered and
when IIA is totally ignored and, therefore, DCIA is regarded to be exactly equal to Impervious Area (IA) before saturation; maximum intensity = 10 mm/hr. When IIA is ignored \( (r_c = 0) \) (Figure 6.23 (a)), all impervious areas are regarded as DCIA immediately response. However, when IIA is accounted for \( (r_c = 0.5) \), IIA does not contribute to the runoff hydrograph because the rainfall falling on IIA infiltrates before saturation occurs. As a result Figure 6.23 (b) shows a reduced peak discharge and a thick and long tail compared with the result shown in Figure 6.23 (b) when IIA is ignored. If the soil saturates and excess rainfall occurs, IIA as well as rainfall amount dropping on saturated pervious areas (ExPerv) starts to contribute the runoff hydrograph (Figure 6.24 (b)). Once saturation occurs and ExPerv and IIA start to contribute to runoff hydrograph; all the areas contribute to produce runoff, the peak of hydrograph does not show much difference as shown in Figure 6.24 (a) and Figure 6.24 (b). However, IIA affects the shape of the hydrograph depending on rainfall intensity and also produce a thick and long tail compared with the case when it is ignored. The contributing ratio as shown in Figure 6.21, Figure 6.22, Figure 6.23, and Figure 6.24 is the ratio of contribution to the hydrographs at given time and the contributing ratio of the loss out of system is obtained from contribution of InPerv with the coefficient \( r_b \) calibrated from the observed data.

**Contributing ratio of infiltrated rainfall**

The results from the rainfall runoff model proposed in this study strongly suggest that the contribution from infiltrated rainfall should not be ignored in urban catchments. The contributing ratio of infiltrated rainfall is given as \( r_b \) (Table 6.7). The values of \( r_b \) for three catchments (CDS-17 and CDS-36; Figure 6.25) in Chicago are estimated in this study. The parameter values for \( D_2 \) and \( r_b \) for CDS-17 and CDS-37 are shown in Figure 6.26. Table 6.12 lists the parameter values estimated for three catchments in Chicago; CDS-17, and CDS-36, and CDS-51. Parameter values of \( c_1 \) and \( D_1 \) are obtained from geometry of the pipe cross section and slope and \( r_i \) and \( r_c \) are calculated based on the impervious map. The imperviousness ratio, \( r_i \) for CDS-17 and CDS-36 are calculated based on land use because the impervious map for CDS-17 and CDS-36 are not developed. Because of the same reason, the ratio of IIA, \( r_i \) obtained for CDS-51 is used for CDS-17 and CDS-36. The result shows that the contribution from infiltrated rainfall
$(r_b)$ varies among catchments. Compared with CDS-51, CDS-17 and CDS-36 have relatively small contribution from infiltrated rainfall. The estimation of flow discharge hydrographs for CDS-17 and CDS-37 is shown in Figure 6.27 and Figure 6.28.

Figure 6.25 Urban catchments in Chicago: (a) CDS-17; (b) CDS-36

Figure 6.26 Calibration of parameters $(D_2$ and $r_b)$ for (a) CDS-17 with an event on 27 Jul 2007; and (b) CDS-36 with an event on 4 Sep 2008
Table 6.12 Parameter values estimated for CDS-17, CDS-36, and CDS-51

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Area (km²)</th>
<th>Δx (m)</th>
<th>$c_1$ (m/s)</th>
<th>$D_1$ (m²/s)</th>
<th>$r_i$</th>
<th>$r_c$</th>
<th>$D_2$ (m²/s)</th>
<th>$r_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDS-17</td>
<td>1.26</td>
<td>89</td>
<td>0.76</td>
<td>1.86</td>
<td>0.52</td>
<td>0.23</td>
<td>1.0 X 10⁻²</td>
<td>0.17</td>
</tr>
<tr>
<td>CDS-36</td>
<td>0.65</td>
<td>118</td>
<td>0.34</td>
<td>4.65</td>
<td>0.51</td>
<td>0.23</td>
<td>5.0 X 10⁻²</td>
<td>0.22</td>
</tr>
<tr>
<td>CDS-51</td>
<td>3.42</td>
<td>156</td>
<td>0.43</td>
<td>5.58</td>
<td>0.54</td>
<td>0.23</td>
<td>5.6 X 10⁻¹</td>
<td>0.55</td>
</tr>
</tbody>
</table>

* $r_c$ estimated for CDS-51 is used for CDS-17 and CDS-36

The contributing ratio of infiltrated rainfall ($r_b$) can be a representative index of a catchment indicating a current condition and status of an urban drainage network. The contributing ratio of infiltrated rainfall to the drainage network is closely related to the age of the drainage network, inappropriate installation of pipe connection. Various sources can affect the contributing ratio of combined sewer systems, including footing/foundation drains, roof drains or leaders, downspouts, drains from window wells, outdoor basement stairwells, drains from driveways, groundwater/basement sump pumps, and even streams. These sources are typically improperly or illegally connected to combined sewer networks, via either direct connections or discharge into sinks or tubs that are directly connected to the sewer system.

![Figure 6.27 Estimation of flow discharge hydrographs in CDS-17 for storm events on (a) 8 Jul 2008; (b) 19 Jul 2008; (c) 4 Sep 2008; (d) 12 Sep 2008](image)
Figure 6.28 Estimation of flow discharge hydrographs in CDS-36 for storm events on (a) 8 Jul 2008; (b) 19 Jul 2008; (c) 4 Sep 2008; (d) 12 Sep 2008

6.5. Conclusions
In this chapter, WFIUH is adapted to account for the contribution of distinct pervious and impervious areas utilizing the spatial distribution of the imperviousness ratio in an urban catchment. Accounting for pervious and impervious areas separately enables us to take account of the unique hydrodynamic properties for each contribution. This paper introduces two assumptions regarding pervious area contribution to the hydrograph. The first assumption employs a contribution from pervious areas to the runoff hydrograph before saturation along subsurface flow path occurs. The second assumption acknowledges that some of the infiltrated rainfall is eventually lost out of the system.
Explanation of the observed hydrograph can be improved significantly with this framework. Specifically, the suggested approach is able to reproduce the long tails observed in the urban runoff hydrograph which could not be explained by the conventional approach that does not account for the infiltrated rainfall. The results show that a large portion (55%) of the infiltrated water eventually contributes to the runoff hydrograph in an urban catchment. The ratio of infiltrated water indirectly implies the state of the combined sewage system depending on various conditions; for example, the aging, connectivity, drain material, and fractures of the conduits.

Based on the two simple assumptions for pervious areas, this paper also distinguishes the contribution from DCIA and IIA. By introducing DCIA and IIA, and dividing the runoff contribution from pervious areas into two components: infiltrated rainfall (InPerv) and excess rainfall (ExPerv), we are able to quantify the contribution of each area. As a result, this approach shows the important role of IIA in that IIA reduces the direct runoff contribution from impervious areas to the total runoff hydrographs. The framework of this study strongly suggests the flow contribution from pervious areas to the total runoff hydrograph, even before saturation occurs in urban areas. Consequently, this chapter shows that runoff prediction must account for the various and complicated flow paths from pervious areas to the main drainage network in urban catchments.
7. APPLICATION OF STOCHASTIC MODELS TO URBAN DRAINAGE NETWORKS

In this chapter, the possibility for a stochastic network to replace an actual existing urban drainage network in terms of hydrographs is investigated. Instead of the actual network, stochastic networks from Monte-Carlo simulation are utilized and the discharge hydrograph is estimated with the synthetic width function from the generated networks. The result shows that the simulated network with the stochastic network model can be a good approximation of an actual network in terms of the width function and, consequently, the runoff hydrograph at the outlet of the watershed. In this study, the characteristic property of a network or configuration of a network is given as a value of parameter, $\beta$ of Gibbs’ model. The applicability of stochastic network in urban catchment implies that once the single value of $\beta$ is estimated for an urban catchment, the flow discharge hydrograph of the catchment can be estimated based on the value of $\beta$ even if we are lacking detailed layout of the drainage network. As discussed in Chapter 4, urban catchments have wide range of network configuration compared with natural river networks, which implies variation of the flow discharge hydrographs in urban catchments depending on $\beta$. Moreover, combined with the results shown in Chapter 3, Chapter 4, and Chapter 5, the network property ($\beta$) is not just being utilized to estimate the discharge hydrograph of a catchment, but also can be a key link to evaluate the effect from moving rainstorms in urban catchments.

7.1. Introduction

Stochastic modeling of river networks has been an active area to describe and present the natural river networks. A river network can be considered as a complex system evolving in time. It is possible to distinguish any form of a complex system in nature when its pattern distinguishes itself from others. Nicolis and Prigogine (1989) described the characteristic behavior of a complex system: symmetry breaking, multiple choices, and correlation of macroscopic ranges. The emergence of the concept of space in a system where it was originally not possible to be perceived is called symmetry breaking.
Multiple choices imply a bifurcation where chance is the only one factor that determines the future in a deterministic system (May, 1976; Nicolis and Prigogine, 1989; Strogatz, 2000). The system ends up with macroscopic correlation, which is a pattern that distinguishes itself from others. A river network has pattern that distinguishes itself in terms of fractal dimension (Hack, 1957; Tarboton et al., 1988). Evolutionary dynamics driven by a flow gradient and subject to a proximity constraint, that is, that matter and energy can traverse only through a continuum, give rise to a tree topological organization in the presence of inherent randomness (Paik, 2006). Every river network is unique, which implies that the bifurcation of deterministic evolution of the natural river networks is dominated by the probability due to inherent randomness in nature.

Exploring a way to describe complex river networks pioneered the development of the stochastic network model. Leopold and Langbein (1962) suggested a random walk model with equal probability for downwards directions. Karlinger and Troutman (1989) proposed a random walk model with two postulates that describes the topologic configuration and uniform probability. A stochastic network model based on Gibbs’ distribution (Troutman and Karlinger, 1992) was suggested based on Gibbs’ measure (Gibbs’ measure (Ising, 1925; Kindermann and Snell, 1980); the models are based on a single parameter as well as two parameters (Karlinger and Troutman, 1994). The Gibbs’ measure has two properties: one is maximum entropy and the other is a Markov random field (Kinderman and Snell, 1980).

The interest in river networks and the original implication that the energy of the system corresponds to flow discharge resulted in the development of IUH methods based on the topology of river networks. The initial attempt was made by Rodriguez-Iturbe and Valdes (1979) and Gupta et al. (1980) as a form of Geomorphologic Instantaneous Unit Hydrograph (GIUH) with an assumption for the travel time distribution, which was then improved by introducing the solution of the advection-diffusion equation in an open channel (Rinaldo et al., 1991; Marani et al., 1991; Naden, 1992). Van de Nes (1973) developed a routing approach for a distributed model which started the formulation of the width function-based IUH (WFIUH) without any significant assumption that simplifies the Saint-Venant equation. Mesa and Mifflin (1986) and Naden (1992) coupled the width function with the convective diffusion equation and evaluated the hydrodynamic
dispersion represented by two parameters, celerity and longitudinal diffusivity, then applied it to a natural river network to estimate the runoff hydrograph. Troutman and Karlinger (1985) and Karlinger and Troutman (1985) proposed an IUH based on a finite number of topologic features rather than using the complete width function. They indicated that the shape of the IUH is, when properly scaled, identical to that of the width function and that the width function has the shape of a Weibull distribution (Troutman and Karlinger, 1984). Naden (1992) applied the WFIUH to a natural watershed, the River Thames, and considered the spatial variation of the soil types and rainfall. The width function is derived from a spatially-branching network, and spatial variation is presumably built into the function values. This enables consideration of the spatial variation in soil types, rainfall, and other hydrologic variables, in terms of the width function (Naden, 1992). Franchini and O’Connell (1996) made a comparison between the GIUH and the WFIUH which was based on a natural river in the United Kingdom, the River Tyne. The results showed that the GIUH velocity parameter lacks physical interpretation, in contrast to the hydraulic parameters of the WFIUH and have been seen to be physically consistent. Hung and Wang (2005a, 2005b) developed an algorithm to generate the synthetic width function based on self-similarity in natural river networks in order to produce the hydrologic response with WFIUH.

The aims of this chapter are (a) to evaluate the possibility of replacing an actual network with the networks generated by a stochastic network model in terms of hydrologic response at the outlet and (b) to examine the implication of the similarity found between the stochastically generated networks and actual drainage networks in urban areas. In this chapter, Gibbs’ model (Troutman and Karlinger, 1992) is utilized to generate the stochastic network for urban catchments in Chicago. Then, the synthetic width function is obtained using the stochastic network to calculate the hydrologic response function based on the WFIUHs developed in Chapter 6.

### 7.2. Methodology

*Gibbs’ model*

The uniform distribution model (Leopold and Langbein, 1962) is a stochastic network model assuming uniform probability for all directions in the generation of a network.
Gibbs’ network model sets probability based on the Gibbs measure, which originated from the Ising model (Ising, 1925) in which a probability measure by an energy function $U$ for a ferromagnetic material from observation was given as

$$P(\omega) = \frac{1}{Z} e^{-\frac{1}{kT}U(\omega)}$$  \hspace{1cm} (7.1)$$

where $\omega$ is a state of a system, $k$ is a constant and $Z$ is a normalizing constant. Gibbs’ measure has two properties: maximum entropy and a Markov random field, and any positive measure with the property of a Markov random field can be regarded as a Gibbs’ measure with an appropriate energy function (Kindermann and Snell, 1980).

In the stochastic network model based on Gibbs measure that was suggested by Troutman and Karlinger (1992), a Markov chain is defined with the spanning trees $S$ as the state space. A tree is a graph in which any two vertices are connected by exactly one simple path and a spanning tree is an acyclic tree connecting all points in the network without loops or cycles. Let $s$ belong to $Ss$ and two trees $s_1$ and $s_2$ be adjacent if one may be obtained from the other. To do this, a point must be randomly selected in $s_1$ and a new direction defined from that point will be a new spanning tree, $s_2$. Then the transition probability from $s_1$ to $s_2$, $R_{S_1S_2}$, can be defined as follows (Troutman and Karlinger, 1992):

$$R_{S_1S_2} = \begin{cases} r^{-1} \min \left\{ 1, e^{-H(s_2) - H(s_1)} \right\} & s_2 \in N(s_1) \\ 1 - \sum_{s \in N(s_1)} R_{s}s & s_2 = s_1 \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (7.2)$$

where $N(s_1)$ is the set of trees adjacent to $s_1$, $H$ is sinuosity, and $\beta$ is a parameter that represents the extent to which the sinuosity of the network is reflected in generation of the new spanning tree, $s_2$. Therefore, the transition probability is given as Gibbs’ measure and the energy function in the Gibbs measure is defined as the change of the sinuosity in Gibbs’ model.

The procedure used in this chapter to generate a Gibbs network given a value of $\beta$ follows that of Barndorff-Nielsen (1998). First, start from a uniform network generated by the uniform distribution model, $s_1$, and randomly select a point, $v$, in the network and assign a new flow direction from $v$ to generate neighboring network, $s_2$. Second, check whether the new network, $s_2$, has any loop inside the network. If it does, go back to the first step. If it does not have any loop, draw a random value $x$ between 0 and 1 and check that $x$ is
greater than \( e^{\beta \Delta H} \) where \( \Delta H \) is equal to \( H(s_2) - H(s_1) \). If this holds, then take the \( s_2 \) as a new network. Third, let \( s_2 \) be equal to \( s_1 \) and repeat the above steps a sufficiently large number of times until the resulting tree has a distribution close to the stationary Gibbs’ distribution.

**Hydrologic response in an urban watershed**

Van de Nes (1973) developed a distributed model and suggested a fundamental approach for determining the WFIUH, and derived the celerity and the dispersion coefficient for trapezoidal channel geometry. Naden (1992) suggested an approach based on the width function associated with the solution of the advection-diffusion equation in a natural river basin assuming wide rectangular channel geometry. In the case of a semi-infinite uniform channel fed by inflow at the upstream (\( x=0 \)), the routing function is derived from the linear advection-diffusion equation given as

\[
\frac{\partial Q_p}{\partial t} = D \frac{\partial^2 Q_p}{\partial x^2} - c \frac{\partial Q_p}{\partial x} \quad (7.3)
\]

where \( Q_p \) is flow perturbation (\( m^3/s \)), \( D \) is the diffusion coefficient (\( m^2/s \)), \( c \) is the celerity of the flood wave (\( m/s \)), \( t \) is time (s) and \( x \) is distance from upstream end (m). Assuming that the drainage network consists of pipe with circular cross sections, the celerity and the diffusion coefficient can be derived as

\[
c = d_0 \left( 1 - \cos \theta_I \right) - \frac{4}{3} R_I \frac{v_I d_0}{4 B_I} \quad (7.4)
\]

\[
D = C_1 \frac{Q_I}{2 S_0 B_I} \quad (7.5)
\]

where

\[
C_1 = 1 - \frac{v_I^2 d_0^2}{16 B_I^2} \left( 1 - \cos \theta_I - \frac{4 R_I}{d_0} \right)^2 \quad (7.6)
\]

where \( d_0 \) is the diameter of the circular cross section, \( v_I \) is the initial flow velocity, \( B_I \) (\( B = \partial A/\partial y \)) is the initial water surface width, \( \theta_I \) is the initial angle of the water surface, \( R_I \) is the initial hydraulic radius, \( S_0 \) is a channel slope and \( F \) is Froude number. When the coefficients \( D \) and \( A_t \) are considered to be constant, the solution to equation (3) with the boundary condition \( Q(0, t) = \delta(t) \), \( Q(x, 0) = 0 \) and \( Q(\infty, t) = 0 \), is given (Naden, 1992; Franchini and O’Connel, 1996; Da Ros and Borga, 1997) as
\( u(x, t) = \frac{x}{\sqrt{4\pi Dt^3}} \exp \left[ -\frac{(x-ct)^2}{4Dt} \right] \) (7.7)

where \( u(x, t) \) is the impulse response of the advection-diffusion equation, i.e. the time evolution of the discharge at a distance \( x \) from the upstream end when an instantaneous upstream impulse \( \delta(t) \) is introduced. With the unit impulse response, \( u(x, t) \) given as in equation (6), the IUH of a catchment can be defined as

\[
h(t) = \int_0^\infty W'(x)u(x, t)\,dx
\]

(7.8)

where \( W(x) \) is the width function. In this chapter, the width function from the actual drainage network is replaced by the synthetic width function simulated by Gibbs’ model. Then, the response from the network from Equation 7.8 for discrete time interval can be written as

\[
h(t) = \sum_{i=1}^n \frac{i\Delta x}{\sqrt{4\pi Dt^3}} W_\beta(i\Delta x) \exp \left[ -\frac{(i\Delta x-ct)^2}{4Dt} \right] \Delta x
\]

(7.9)

where the \( W_\beta \) is the synthetic width function obtained from Gibbs’ model with a parameter value of \( \beta \). The diameter and the slope selected to calculate the celerity and the diffusion coefficient of the model are the catchment-representative values to capture the characteristics of the hydrodynamic dispersion. In this study, the flow in the main drainage network is considered to be open channel flow with a circular cross section. A maximum flow value for the circular cross section is defined as 0.8 times the pipe full flow as shown in the following equation:

\[
Q_o = \frac{d_o^{8/3} s_o^{1/2}}{4 n_o}
\]

(7.10)

where \( Q_o, d_o, s_o \) and \( n_o \) are the peak discharge, diameter, bottom slope and the roughness at the outlet. The flow discharge at each pipe outlet is tested, and if it is greater than the \( Q_o \), the difference between the actual and the maximum flow is delayed to the next time step when the flow becomes smaller than \( Q_o \).

As discussed in Chapter 6, this study introduces a hydrologic response function at the outlet considering contribution from infiltrated amount of rainfall and also considering isolated impervious areas in urban watersheds.

\[
h_i(t) = \sum_{j=1}^{n_w} \left( W_{\beta,i}(j\Delta x) \cdot f(j\Delta x, t) \cdot g_i(t) \right) \Delta x
\]

(7.11)

where \( i=1 \) for contribution from excess rainfall in Directly Connected Impervious Areas (DCIA), \( 2 \) for excess rainfall in Isolated Impervious Areas (IIA), \( 3 \) for excess rainfall in
pervious areas (ExPerv), and 4 for infiltrated rainfall in pervious areas (InPerv). $W_{\beta,1}$ and $W_{\beta,2}$ are the same width functions obtained from impervious area and $W_{\beta,3}$ and $W_{\beta,4}$ are the same from pervious area, respectively. In this chapter, catchment-average imperviousness ratio is used to calculate the width functions for pervious and impervious areas instead of impervious map developed by Crosa-Rivarola (2008). $n_w$ is the maximum distance of the width function. $f$ is a response function of the main drainage network and $g$ is a response function defined in a cell (Figure 6.17). From Equation 7.9, the response from the main drainage network is given as

$$f(i\Delta x, t) = \frac{(i\Delta x)}{\sqrt{4\pi D_1 t^3}} \exp\left[-\frac{(i\Delta x - c_1 t)^2}{4D_1 t}\right]$$  \tag{7.12}

where $c_1$, $D_1$ are celerity and diffusion coefficient of the main drainage network, respectively. The response function in a cell, $g_i$, is from excess rainfall in DCIA, IIA as well as in pervious areas (ExPerv).

$$g_i(t = 0) = 1, \text{ otherwise } 0; \text{ for } i = 1, 2, 3$$  \tag{7.13}

The response function $g_4$ is from the infiltrated rainfall amount in pervious areas (InPerv). $g_4$ is assumed to have the same form with Equation 7.12 which is a solution of an advection-diffusion equation for the drainage network.

$$g_i(t) = \frac{\Delta x}{\sqrt{16\pi D_2 t^3}} \exp\left[-\frac{(\Delta x - 2c_2 t)^2}{16D_2 t}\right]; \text{ for } i = 4$$  \tag{7.14}

where $c_2$, $D_2$ are celerity and diffusion coefficient of flow paths for the infiltrated rainfall. Given the length of $f$ and $g$ as $M_f$ and $M_k$, respectively, the convolution for discrete time steps can be obtained as

$$(f \ast g)[k] \overset{\text{def}}{=} \sum_{m=0}^{\text{max}(M_f, M_k)-1} f[m] g[k - m], 0 < k < M_f + M_k - 2$$  \tag{7.15}

The response at the outlet can be obtained as the sum of the convolution of the response function from each area and the corresponding precipitation.

$$Q(t) = \sum_{i=1}^{n_c} h_i \ast I_i$$  \tag{7.16}

where $i=1$ for DCIA, 2 for IIA, 3 for ExPerv, and 4 for InPerv, respectively.

### 7.3. Application

**Study area**

As a case study, a highly-urbanized catchment, Calumet Drop Shaft (CDS)-51, in which
most of the drainage load is conveyed through a pipe network, is investigated as shown in Figure 7.1. CDS indicates the catchment as well as the drop shaft that are connected to the deep tunnel of the Tunnel and Reservoir Plan (TARP) System in Cook County, IL to relieve the load both in environmental and flood control aspects. The TARP system is monitored, maintained and operated by Metropolitan Water Reclamation District of Greater Chicago (MWRDGC).

The watershed, CDS-51, captures storm and sanitary flows for a service area of 3.16 km². The combined sewerage system of CDS-51 collects inflow from more than 800 inlets and conveys it to the outlet of the watershed via a network of 722 pipes ranging in size from 15 cm to 2.13 m at the outlet. Dry weather flows are intercepted by two MWRDGC interceptor sewers which convey the flow to the Calumet Water Reclamation Plant. When the treatment plant reaches capacity, flow in the 2.13 m diameter pipe is driven towards the combined sewer overflow (CSO) outfall and conveyed to the TARP. From 2007 to 2011, the United States Geological Survey (USGS) used three acoustic flow meters to monitor the inflow from the catchment, the volume of flow partitioned to the CSO, and
the amount of inflow entering the drop shaft connected to the deep tunnel at CDS-51.

Table 7.1 Conduits of CDS-51 with respect to the Strahler ordering (Miller et al., 2009)

<table>
<thead>
<tr>
<th>Order</th>
<th>No.</th>
<th>Diameter (m)</th>
<th>Length (m)</th>
<th>Bottom slope (x10^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Standard dev.</td>
<td>Mean</td>
</tr>
<tr>
<td>1</td>
<td>449</td>
<td>0.33</td>
<td>0.11</td>
<td>61.62</td>
</tr>
<tr>
<td>2</td>
<td>157</td>
<td>0.46</td>
<td>0.16</td>
<td>59.69</td>
</tr>
<tr>
<td>3</td>
<td>57</td>
<td>0.72</td>
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<td>75</td>
</tr>
<tr>
<td>4</td>
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<td>1.18</td>
<td>0.34</td>
<td>64.3</td>
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<tr>
<td>5</td>
<td>8</td>
<td>2.06</td>
<td>0.08</td>
<td>98.68</td>
</tr>
</tbody>
</table>

Table 7.2 Parameter values estimated for CDS-51

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Area (km^2)</th>
<th>Δx (m)</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>c_1 (m/s)</td>
</tr>
<tr>
<td>CDS-51</td>
<td>3.42</td>
<td>156</td>
<td>0.43</td>
</tr>
</tbody>
</table>

The celerity, c_1 and the diffusion coefficient, D_1 are functions of the pipe diameter, slope and initial depth assumption respectively. Table 7.1 lists the conduit information in CDS-51. Considering the average diameter, slope, and coefficient, the celerity and diffusion coefficient of the main drainage network are determined to be 0.43 m/s and 5.58 m^2/s, respectively, provided that 20% of the pipe is initially full. Table 7.2 lists the parameter values estimated for CDS-51. The celerity and diffusion coefficient of main drainage network are calculated based on geometry of the network (Table 7.1). The average imperviousness ratio, r_i and the area ratio of IIA, r_c are obtained from impervious map (Crosa-Rivarola, 2008). Two unknown coefficients: the diffusion coefficient, D_2 and the contributing ratio, r_b of infiltrated water are calibrated using observed data (7 Jan 2008 event). Details for estimation and calibration of parameter values are described in Chapter 6.

Estimation of β
In order to incorporate the stochastic network, which will be generated on a lattice, the
drainage network of CDS-51 was reconstructed on a lattice as shown in Figure 7.2 (a). The number next to each node represents distance to the outlet in terms of the number of links. The width of the network is proportional to the maximum value of the width function at the link normalized by the maxima at the outlet. The size of the grid is 156 meters. The normalized width function from the reconstructed pipe network is shown in Figure 7.2 (b).

Figure 7.2 Drainage network of CDS-51: (a) reconstructed on a lattice; (b) corresponding width function

Figure 7.3 shows a realization of the stochastic network obtained using Gibbs’ method for a given $\beta$ and corresponding width function. The information needed to generate the stochastic network includes the boundary of a network, location of the outlet, and $\beta$. The uniform model generates networks which are highly sinuous (Troutman and Karlinger, 1992). As $\beta$ tends to zero, Gibbs’ model becomes equivalent to the uniform model. Figure 7.3 (a) shows the network generated with $\beta$ equal to $10^{-4}$, which is the most sinuous, and the corresponding width function which has the lowest peak and longest distance. As $\beta$ increases, the network becomes less sinuous, while the corresponding width function has a higher peak and a shorter total distance. Figure 7.3 (c) shows the network generated with $\beta$ equal to $10^{0}$; here the corresponding width function has greater peak than that of the actual width function. In this case, $\beta$ can be estimated using the synthetic width function from the simulated network obtained from Gibbs’ model.
Figure 7.3 A realization of Gibbs’ model with a given $\beta$ and its corresponding width function for CDS-51 compared with the actual width function in dotted line: (a) realization with $\beta = 10^{-2}$; (b) corresponding width function; (c) realization with $\beta = 10^{-1}$; (d) corresponding width function; (e) realization with $\beta = 10^{0}$; (f) corresponding width function

The procedure to estimate $\beta$ in this chapter is as follows: first, for a given $\beta$, 100 stochastic networks are generated with Gibbs’ model. Then, the averaged width function for each distance from the generated stochastic networks is obtained. Finally, the value of $\beta$ that generates the closest width function to the actual width function and maximizes the goodness of fit index is selected as a representative value for a given catchment.
7.4. Results and discussion

Figure 7.4 Comparison of actual width function of CDS-51 with the width functions averaged over 100 simulations of the stochastic network for each $\beta$

Figure 7.4 depicts the actual width function and a synthetic width function averaged over 100 simulations of stochastic networks for a given $\beta$ generated with Gibbs’ model. The value of $\beta$ is estimated so that it maximizes the goodness of fit with the actual width function of CDS-51. Figure 7.5 shows the variation of a synthetic width function from the stochastic network generated for a given $\beta$. As $\beta$ increases, the variation in both the range of distance as well as the magnitude of the variation decreases at each ordinate. Therefore, this variation must be considered in the process of estimating the value of $\beta$. Nash-Sutcliffe model efficiency is considered to be most appropriate goodness of fit measures available (Legates and McCabe, 1999). In this study, the Nash-Sutcliffe model efficiency as a goodness of fit index was modified by the number of the outliers. The number of outliers is obtained as follows: for a given $\beta$, minimum and maximum values for each abscissa are obtained after 100 simulations. The ordinates of the actual width function that are greater than the maximum value or less than the minimum value are counted as the outliers. Figure 7.6 shows the comparison between the actual width function and the minimum and maximum range of values for each $\beta$, while Figure 7.7 shows the number of outliers changing with $\beta$. 
Figure 7.5 Variance of the width function at each ordinate from stochastic networks generated for a given $\beta$

Then, the goodness of fit index is modified as follows:

$$E_{\text{Modified}} = f_o \left( 1 - \frac{\sum_{i=1}^{T} (Q'_o - Q'_s)^2}{\sum_{i=1}^{T} (Q'_o - \overline{Q_o})^2} \right)$$

(7.17)

where, $f_o$ is a modification factor depending on the number of outliers.

$$f_o = \begin{cases} 
\frac{\xi_{\text{max, actual}} - N_{\text{outlier}}}{\xi_{\text{max, actual}}} & \text{Modified 1} \\
\left( \frac{\xi_{\text{max, actual}} - N_{\text{outlier}}}{\xi_{\text{max, actual}}} \right)^2 & \text{Modified 2}
\end{cases}$$

(7.18)

where $Q_o$ is actual width function, $Q_s$ is the synthetic width function, $f_o$ is marginal fraction of the number of outliers as a modification factor; absolute value for Modified 1 and squared for Modified 2, $N_{\text{outlier}}$ and $\xi_{\text{max, actual}}$ is the maximum distance of the actual width function. ‘Modified 1’ is marginal fraction of the number of outliers (M1), and ‘Modified 2’ is squared marginal fraction of the number of outliers (M2).
Figure 7.6 Comparison between actual width function and the range from minimum to maximum width function for a given distance obtained from 100 simulation for a given value of $\beta$: (a) for $\beta = 10^0$; for $\beta = 10^{-4}$

Figure 7.7 Number of outliers for each $\beta$

In order to explore the applicability of stochastic in terms of the runoff hydrograph at the outlet of the watershed, a storm event on 7 Jan 2008 is tested. Figure 7.8 shows the runoff estimate with the actual width function previously obtained from the reconstructed network of CDS-51 as shown in Figure 7.2. Figure 7.9 shows the synthetic width function for a given $\beta$ and its corresponding WFIUH. Based on the previously determined value of $\beta$ for CDS-51, the synthetic width function was obtained and used to estimate the runoff hydrograph for the storm event on Jan 2008. The goodness of fit of the hydrograph estimated with the synthetic width function compared with the actual network of CDS-51 is greater than 0.95. Figure 7.10 shows the variation of the goodness of the fit for the runoff estimates based on the synthetic width function for each $\beta$ to the original estimate from the actual width function as well as the ratio of the estimated peak to the observed peak. The goodness of the fit has its maximum values when $\beta$ is equal to $10^{-1}$ which is the
same as the value of $\beta$ previously estimated for CDS-51. However, the goodness of fit does not decrease significantly when $\beta$ is greater than $10^{1}$. The peak ratio of the estimates from synthetic width function also increases as $\beta$ increases, although it remains stable for $\beta$ greater than $10^{0}$.

Figure 7.8 Estimation of the runoff hydrograph on 8 Jan 2008 with actual width function of CDS-51

Figure 7.9 Estimation of the runoff hydrograph on 8 Jan 2008 with a synthetic width function from Gibbs’ model ($\beta=4\times10^{-2}$)
Figure 7.10 Variation of goodness of fit and peak ratio with $\beta$

Figure 7.11 Location of 12 catchments in Chicago

Estimation of $\beta$ for 12 catchments in Chicago areas

In this chapter, the values of $\beta$ for catchments in the Chicago metropolitan area are estimated in order to investigate the applicability of Gibbs’ model to urban drainage networks in addition to CDS-51. These width functions are not used to estimate the flow discharge hydrographs because detailed spatial information is lacking in these catchments.
12 catchments are reconstructed on grids as shown in Figure 7.12. Figure 7.13 shows the difference between the original model efficiency and the modified, where the modified index reveals the optimal value better than the original. Especially for $\beta$ greater than 1, the original index does not show much difference compared with modified indexes. For example, the optimal value of $\beta$ can be easily found in CDS-07, CDS-32, CDS-25, and CDS-26 when the modification is applied to Nash-Sucliffe efficiency index. Using this modified index, the values of $\beta$ for 12 urban watersheds are determined.

Figure 7.12 Drainage networks in Chicago reconstructed on grids
Figure 7.13 Estimation of $\beta$; original Nash-Sutcliffe efficiency coefficient compared with modified efficiency multiplied by marginal fraction of the number of outliers (M1), and modified efficiency multiplied by squared marginal fraction of the number of outliers (M2)

Table 7.3 lists the goodness of fit index of the synthetic width function to the actual one in 12 catchments in Chicago including CDS-51. Except CDS-07, all other catchments have the goodness of fit greater than 0.9 and average goodness of fit value of 0.924 (Table 7.3) for the optimal values of $\beta$. CDS-26 shows the maximum value of 0.987 while CDS-11 shows the minimum of 0.804. In most cases, Gibbs’ model reproduces the width
function of each catchment well and the resulting synthetic width function basically represents the response function of the drainage network. Therefore, it can be inferred from the results that that the stochastic network model can replace the actual urban drainage network in terms of the flow discharge hydrograph at the outlet.

Table 7.3 Goodness of fit of the synthetic width function to actual one for 12 catchments in Chicago (Nash-Sutcliffe)

<table>
<thead>
<tr>
<th>Catchment</th>
<th>$10^{-4}$</th>
<th>$10^{-3}$</th>
<th>$10^{-2}$</th>
<th>$10^{-1}$</th>
<th>$10^{0}$</th>
<th>$10^{1}$</th>
<th>$10^{2}$</th>
<th>$10^{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDS-07</td>
<td>0.54</td>
<td>0.54</td>
<td>0.64</td>
<td>0.78</td>
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<tr>
<td>CDS-11</td>
<td>0.82</td>
<td>0.84</td>
<td>0.92</td>
<td>0.92</td>
<td>0.90</td>
<td>0.89</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>CDS-16</td>
<td>0.93</td>
<td>0.93</td>
<td>0.97</td>
<td>0.95</td>
<td>0.89</td>
<td>0.89</td>
<td>0.89</td>
<td>0.89</td>
</tr>
<tr>
<td>CDS-17</td>
<td>0.53</td>
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<td>0.90</td>
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<tr>
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<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
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</tr>
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<tr>
<td>CDS-51</td>
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<td>0.87</td>
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Table 7.4 Goodness of fit of the synthetic width function to actual one for 12 catchments in Chicago (Modified 1)

<table>
<thead>
<tr>
<th>Catchment</th>
<th>$10^{-4}$</th>
<th>$10^{-3}$</th>
<th>$10^{-2}$</th>
<th>$10^{-1}$</th>
<th>$10^{0}$</th>
<th>$10^{1}$</th>
<th>$10^{2}$</th>
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<td>0.92</td>
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<td>0.64</td>
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<td>0.61</td>
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<tr>
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<td>0.90</td>
<td>0.62</td>
<td>0.62</td>
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<td>0.59</td>
</tr>
<tr>
<td>CDS-20</td>
<td>0.17</td>
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<td>0.70</td>
<td>0.75</td>
</tr>
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<td>CDS-51</td>
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<td>0.85</td>
<td>0.82</td>
<td>0.79</td>
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Table 7.5 Goodness of fit of the synthetic width function to actual one for 12 catchments in Chicago (Modified 2)

<table>
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<th>$10^{2}$</th>
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<td>0.46</td>
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<td>0.92</td>
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<td>0.34</td>
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<td>0.97</td>
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7.5. Conclusions

This chapter investigates the possibility of replacing an actual urban drainage network with stochastic networks in terms of the flow discharge hydrograph at the outlet. This study utilized the hydrologic response function based on width functions discussed in Chapter 6 to predict the runoff hydrograph because the width function can be directly obtained from the stochastic networks generated by Gibbs’ model. The width function can be regarded as a direct interpretation of the network response containing the effect of changes in geometric factors specified by shape and connectivity of drainage networks. The width function is a way to retain spatial information of the watershed properties. The result of this chapter shows that the stochastic network can replace the actual urban drainage network to estimate the flow discharge hydrograph of an urban catchment. The applicability of stochastic network in urban catchment implies that once the single value of $\beta$ is estimated for an urban catchment, the flow discharge hydrograph of the catchment can be estimated based on the value of $\beta$ even if we are lacking detailed layout of the drainage network.

This chapter shows that the hydrologic response of a drainage network can be estimated based on a single value of $\beta$. The network property ($\beta$) is a key link that combines the results from previous chapters about the effect of moving rainstorms. As previously
shown in Chapter 4 and Chapter 5, the effect of rainstorm movement is highly dependent on the network configuration. Especially, the urban drainage networks in Chicago area have a wide range of network configuration ($\beta$) compared with natural river networks as discussed in Chapter 4, which implies that the effect of rainstorm movement varies widely depending on $\beta$. Therefore, combined with previous results, this study proposes a framework, which is able not just to reproduce the flow discharge hydrographs, but also to evaluate the effect of rainstorm movement based on network configuration.
8. EVALUATION OF THE EFFECT OF RAINSTORM MOVEMENT BASED ON THE EQUIVALENT STATIONARY STORM

This chapter investigates the effect of rainstorm movement on the peak discharge response of urban drainage networks compared with corresponding stationary rainfall. A synthetic circular watershed is introduced to avoid biases from interaction between catchment geometry and storm orientation. The drainage network of the watershed is simulated by Gibbs’ model. This study utilizes two types of the Equivalent Stationary Storm (ESS): the average rainfall intensity over the entire catchment (ESSAV) and the point stationary rainfall intensity (ESSQ) to evaluate the effect of moving rainstorms in terms of the peak discharge response. The rate of change of the peak discharge response for moving rainstorm is examined with respect to ESS. The results are consistent with the results of Chapter 3; there exists an interval in which the same rainfall duration produces much larger peak responses for moving storms compared with ESSQ. The augmentation of the peak response by moving rainstorm is dependent on the relative rainstorm speed, size, and direction as well as drainage network configuration of the catchment. The results indicate that the effect of moving rainstorm increases as storm speed approaches resonance condition and the storm size is smaller compared with the catchment size. The results also show that the effect of moving rainstorm heavily depends on storm orientation as well as network configuration. Especially, the result indicates that an efficient network is more sensitive to moving rainstorms in terms of the peak ratio with respect to ESSAV and ESSQ. In contrast, a less efficient network tends to mitigate the effect of rainstorm movement on peak response. The results from Chapter 4 showed the existence of wide range network configuration in urban catchments. In the context of the results in this chapter, a less efficient network existing in urban catchments prove to contribute unexpectedly to reduce the impact from moving rainstorms. In this regard, the results in this study imply a potential improvement in urban drainage networks in terms of efficiency as well as sensitivity to moving rainstorms.
8.1. Introduction

The importance of spatial distribution and interconnectivity of river network on hydrographs has been well recognized. Surkan (1969) investigates the effects of network geometry on hydrographs to develop a distributed network model. Afterwards, this model was used to evaluate the effect of moving rainstorm (Surkan, 1974). Rodriguez-Iturbe and Valdes (1979) established a link between the topological features of river basin geomorphology and hydrologic response to develop Geomorphologic Instantaneous Unit Hydrograph (GIUH). Saco and Kumar (2002a, 2002b) introduces kinematic dispersion with varying celerity in addition to geomorphologic dispersion and hydrodynamic dispersion.

However, the direct relation between the network geometry and moving rainstorms has not been pursued. In previous studies, the effect of rainstorm movement has been investigated with a one-dimensional watershed (Ogden et al., 1995; Singh, 1997, 1998; de Lima and Singh, 2002, 2003), single configuration of network (Surkan, 1974; Niemczynowicz, 1984a, 1991; Ngirane-Katahaya and Wheater, 1985; Watts and Calver, 1991; Nunes et al., 2006), and a V-shaped catchment (Yen and Chow, 1969; Lee and Huang, 2007; Liang, 2010) only. Majority of the synthetic network previously studied (Ngirane-Katahaya, 1985; Watts and Calver, 1991) belongs to the Scheidegger network (Scheidegger, 1967a, b).

This study introduces various configurations of network based on stochastic models. Leopold and Langbein (1962) suggested a random walk model with equal probability for all directions. Scheidegger model (Scheidegger, 1967b) is a random network model only with the directions downstream and equal probability. Scheidegger model reveals the essential features of river formation (Nagatani, 1993b) and Hack’s exponent (Hack, 1957) for the Scheidegger network is close to that of natural river networks as shown in Chapter 4. Troutman and Karlinger (1989) proposed a random walk model with two postulates that describes the topologic configuration and uniform probability. A stochastic network model based on Gibbs’ distribution (Troutman and Karlinger, 1992) was suggested based on Gibbs’ measure (Ising, 1925; Kindermann and Snell, 1980); the models are based on a single parameter as well as two parameters (Karlinger and Troutman, 1994). The results
from Chapter 4, Chapter 5, Chapter 6, and Chapter 7 imply that the network property represented by a parameter ($\beta$) of Gibbs’ model can be a key link that relates the effect of rainstorm movement to the urban drainage network runoff hydrographs. The results of Chapter 4 and Chapter 5 showed that network configuration is a crucial factor in order to evaluate the effect of moving rainstorms. The results of Chapter 6 and Chapter 7 showed that the hydrologic response of an urban catchment can be reproduced given a network configuration ($\beta$).

Foroud et al. (1984) defined the Equivalent Stationary Rainstorm (ESRS) that has same duration and total amount of rainfall. The peak responses of a watershed, Yamaska S.E., located southeast of Montreal Island is investigated with ESRS and compared to rainstorms moving in upstream and downstream directions with different storm speeds. Arnaud et al. (2002) tested non-uniform rainfall inputs compared with uniform rainfall pattern in terms of peak flows with a distributed hydrologic model. The results showed that the difference ranges from 10-80% depending on the pattern of the rainfall.

In this chapter, the effect of moving rainstorm is evaluated in terms of the rate of change of peak response compared with the corresponding ESS. Two types of the ESS (Ngirane-Katashaya and Wheater, 1985) are examined in this study. One is the catchment-average stationary storm (ESSAV) that preserves the total rainfall amount and duration. The other one is point stationary storm (ESSQ) that preserves the total rainfall amount and intensity. Therefore, for a moving rainstorm, ESSAV has smaller rainfall intensity and ESSQ has shorter rainfall duration compared to each other.

**8.2. Methodology**

In order to investigate the effect of rainstorm movement, this study introduces a synthetic circular catchment and the configuration of moving rainstorms as shown in Chapter 5.

*Equivalent stationary storm (ESS)*

In order to investigate the effect of moving rainstorms in terms of the peak response compared with stationary rainstorms, two types of ESS (Ngirane-Katashaya and Wheater, 1985) is utilized in this study (Table 8.1): the average rainfall intensity over the entire catchment (ESSAV) and the point stationary rainfall intensity (ESSQ). ESSAV is a
catchment-averaged rainfall event and the rainfall duration of ESSAV equivalent to the total travel time of the moving rainstorm through the entire catchment. The rainfall duration of ESSQ is based on a point observation that preserves the same rainfall intensity as the moving storm condition, but rainfall duration is shorter compared with ESSAV.

Table 8.1 An example of Equivalent Stationary Storms (ESSAV and ESSQ)
(Ngirane-Katashaya and Wheater, 1985)

<table>
<thead>
<tr>
<th>Time increment</th>
<th>Rainfall intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
</tr>
</tbody>
</table>

* For all nodes

The total volume of rainfall is dependent on rainfall speed and the size of rainstorms. However, the rainfall intensity of ESSAV is a function of rainstorm size only because rainfall speed is inversely proportional to rainfall duration. The duration of ESSAV (Figure 8.2 (a)) is determined by obtains as

\[ t_{r,ESSAV} = \frac{L_c + L_s}{v_s} \]  

(8.1)

where \( t_r \) is duration of ESSAV, \( L_c \) is the catchment size, \( L_s \) is the rainstorm size, and \( v_s \) is the speed of a moving rainstorm. While the duration of ESSQ (Figure 8.2 (b)) is given as follows:

\[ t_{r,ESSQ} = \frac{L_s}{v_s} \]  

(8.2)

The rainfall intensity of ESSQ is the same as the intensity of moving rainstorm. Because the total amount of rainfall for ESSAV should be same with ESSQ, rainfall intensity of ESSAV is obtained as

\[ I_{ESSAV} = \frac{L_s/L_c}{1 + L_s/L_c} I_{ESSQ} \]  

(8.3)

Figure 8.1 illustrates the total volume of rainfall and rainfall intensity of the equivalent
stationary rainfall (ESSAV) depending on the speed \((v_s/v_c)\) and size \((L_s/L_c)\) of moving rainstorms assuming that the moving rainstorms have the same rainfall intensity of 60 mm/hr. When the relative size of a moving storm, \(L_s/L_c\) is 1/8 of the catchment size, the rainfall intensity of ESSAV is 6.7 mm/hr. When the size of a moving rainstorm is equivalent to the catchment size \((L_s/L_c = 1)\), the intensity of ESSAV is 30 mm/hr, which is half of the intensity of the moving rainstorm. Figure 8.2 shows the rainfall duration of two equivalent stationary rainfalls; ESSAV and ESSQ that depends on the speed and size of rainstorms.

![Figure 8.1](image)

**Figure 8.1 Total rainfall volume of moving rainstorms, and rainfall intensity of the equivalent stationary rainfall (ESSAV) depending on the speed \((v_s/v_c)\) and size \((L_s/L_c)\) of rainstorm**

ESSQ preserves the total rainfall volume and intensity of a moving rainstorm, hence, comparison between the moving rainstorm and ESSQ shows the effect of spatial variation of rainfall only. ESSQ with the size and speed as shown in Table 8.1 is equivalent to design storm for common practice of engineering design (Chow et al., 1988). ESSQ also account for different storm sizes and speeds. Therefore, the engineering design storm can be regarded as a special case of ESSQ. In contrast to ESSQ, ESSAV is defined as a rainstorm whose duration and total volume are identical to that of a moving rainstorm, but which occurs simultaneously over entire catchment. Therefore, comparison between the moving rainstorm and ESSAV shows the effect of both temporal and spatial variation.
of rainfall in a catchment within given rainfall duration. In this chapter, the peak discharge response of moving rainstorms is compared to the peak response of ESSAV and ESSQ for different storm speeds \( \frac{v_s}{v_c} \), storm sizes \( \frac{L_s}{L_c} \), storm directions (\( \theta \)) as well as various network configurations (\( \beta \)).

**Figure 8.2** Rainfall duration of the equivalent stationary rainfall (a) ESSAV and (b) ESSQ depending on the speed \( \frac{v_s}{v_c} \) and size \( \frac{L_s}{L_c} \) of rainstorm

### 8.3. Results and discussions

*Peak discharge response with ESSAV and ESSQ*

In Chapter 3, we defined a peak response normalized by intensity and area as the PDR. However, it is not possible to directly compare the PDR from ESSAV and ESSQ because the intensities for ESSAV and ESSQ can be different from each other. The peak responses obtained with ESSAV and ESSQ are shown in Figure 8.3 and Figure 8.4, respectively. Figure 8.5 and Figure 8.6 show the PDR with respect to the catchment area and rainfall intensity obtained with ESSAV and ESSQ, respectively. The result shows that ESSAV produces lower peak response compared with ESSQ due to smaller rainfall intensities, but the catchment reaches the equilibrium discharge if \( \frac{v_s}{v_c} \geq 1 \) regardless of the storm sizes (Figure 8.3; Figure 8.5) due to the increased rainfall duration. The duration of ESSAV reaches the time of concentration of the catchment if \( \frac{v_s}{v_c} = 1 \) and the catchment is close to the Scheidegger network (with \( \beta \) greater than 1), which is consistent with the results of Chapter 3. In Chapter 3, the peak response is given as a function of three
timescales; rainfall duration, $t_r$, hillslope timescale, $t_h$ and channel flow timescale, $t_c$ for stationary storms. If $t_h$ is negligible compared with $t_c$ and $t_r$, the catchment reaches equilibrium when the rainfall duration is equal to the time of concentration of the catchment ($t_r = t_c$). For the Scheidegger network, which is the most efficient network with the shortest flow paths, the longest length of the flow path is equal to the catchment size ($L_c$); the length scale of the flow path for a linear catchment was $L_c$ in Chapter 3. Therefore, for the Scheidegger network, when $v_s/v_c = 1$, $t_r$ (rainfall duration; $L_c/v_s$) is equal to $t_c$ (channel timescale; $L_c/v_c$) and the catchment reaches equilibrium. Figure 8.5 shows that if the network configuration ($\beta = 10^0$) is close the Scheidegger network, the catchment reaches the equilibrium discharge when $v_s/v_c = 1$ regardless of the storm size. The peak response of ESSAV increases as the storm size increases as shown in Figure 8.3 because the rainfall intensity of ESSAV is directly related to rainstorm sizes (Equation 8.7), but, still dependent on network configuration ($\beta$).

Figure 8.3 Peak response of ESSAV depending on storm speed ($v_s/v_c$) and network configuration ($\beta$) for storm size of (a) $L_s/L_c = 1/8$ (b) $L_s/L_c = 1/4$ (c) $L_s/L_c = 1/2$ (d) $L_s/L_c = 1$
Figure 8.4 Peak response of ESSQ depending on storm speed \( \left( \frac{v_s}{v_c} \right) \) and network configuration \((\beta)\) for storm size of (a) \( L_s/L_c = 1/8 \) (b) \( L_s/L_c = 1/4 \) (c) \( L_s/L_c = 1/2 \) (d) \( L_s/L_c = 1 \) 

The result indicates that the peak response of ESSQ is greater than ESSAV for a given storm speed, size and network configuration (Figure 8.4) because the rainfall intensity for ESSQ (60 mm/hr) is always higher compared with ESSAV of which maximum value of rainfall intensity is 30 mm/hr if \( L_s/L_c \leq 1 \) (Figure 8.1 (b)). As mentioned earlier, ESSQ preserves the rainfall intensity of moving rainstorms. However, the duration of ESSQ is smaller than that of ESSAV because duration depends on rainstorm size and speed only; \( t_r = L_s/v_s \). Therefore, in contrast to ESSAV, the size of moving rainstorms determines whether the catchment under ESSQ reaches the equilibrium discharge or not. Figure 8.4 shows that if the rainstorm size is smaller than the catchment size, the peak response of ESSQ does not reach equilibrium due to the smaller size of rainstorm and, accordingly, relatively shorter duration compared with ESSAV. The PDR of ESSQ with respect to intensity and a catchment area is very close to the equilibrium discharge (0.95), but, still does not reach equilibrium even if the rainstorm size is equivalent to the catchment size \((L_s/L_c = 1)\) as shown in Figure 8.6.
Figure 8.5 The PDR of ESSAV depending on storm speed ($v_s/v_c$) and network configuration ($\beta$) for storm size of (a) $L_s/L_c = 1/8$ (b) $L_s/L_c = 1/4$ (c) $L_s/L_c = 1/2$ (d) $L_s/L_c = 1$

Figure 8.6 The PDR of ESSQ depending on storm speed ($v_s/v_c$) and network configuration ($\beta$) for storm size of (a) $L_s/L_c = 1/8$ (b) $L_s/L_c = 1/4$ (c) $L_s/L_c = 1/2$ (d) $L_s/L_c = 1$
The peak response obtained with stationary storms (ESSAV and ESSQ) is independent of the storm direction. As shown in Figure 8.3 and Figure 8.4, the peak responses of ESSAV are smaller compared with ESSQ, but, the differences are negligible when the storm is moving relatively fast (For example, when $v_s/v_c \geq 4$). Compared with ESSAV, the peak response of ESSQ shows large variability depending on storm speed ($v_s/v_c$) as well as network configuration. When moving storms are slow ($v_s/v_c < 0.5$), the peak response of ESSAV is constant regardless of network configuration; the catchment reached equilibrium because ESSAV has much longer duration compared with ESSQ. When the storm size is equivalent to the catchment size and the storm speed is slow ($v_s/v_c = 0.2$), both ESSAV and ESSQ drive the catchment to the equilibrium states regardless of network configuration as shown in Figure 8.3 (d) and Figure 8.4 (d).

![Graph](image)

Figure 8.7 Peak response of moving rainstorms ($L_s/L_c = 0.25$) compared with ESSAV and ESSQ depending on the relative storm speed ($v_s/v_c$) and storm direction ($\theta$) with a specific network configuration ($\beta = 10^0$)

Figure 8.7 compares the peak response of a moving rainstorm ($L_s/L_c = 0.25$) with those of ESSAV and ESSQ depending on storm directions ($\theta$) for a specific network configuration ($\beta = 10^0$). The result shows that for rainstorm moving in downstream direction ($\theta = 0$), the peak response can be increased to be two times larger depending on the relative storm speed ($v_s/v_c$) compared with the stationary storm (ESSAV). Especially, the result shows that the effect of moving rainstorms is maximized in the vicinity of resonance condition.
More specifically, the maximum peak response for moving rainstorms occurs where $v_s/v_c < 1$; when the storm is slower than the channel flow as shown in Figure 8.7. This result is consistent with the result of Chapter 3. In Chapter 3, it was shown that the moving rainstorm produces higher peak response compared with stationary storms by the effect of superposition and the slower storms produce the maximum peak response due to interdependencies between the duration and travel timescale of moving rainstorms. The x-axis ($v_s/v_c$) of Figure 8.7 can be easily replaced with duration, $t_r$ for ESSQ. Then, Figure 8.7 is basically the same with Figure 3.12 (b) in Chapter 3.

**Figure 8.8 Peak response moving rainstorm ($L_s/L_c = 0.25$) compared with ESSAV and ESSQ depending on the relative storm speed ($v_s/v_c$) and storm direction ($\theta$) with network configuration of (a) $\beta = 10^{-1}$; (b) $\beta = 10^{-2}$; (c) $\beta = 10^{-3}$; (d) $\beta = 10^{-4}$**

**Peak ratio and network configuration of catchments**

This study introduces a peak ratio (Ngirane-Katashaya and Wheater, 1985) to evaluate the effect of moving rainstorm in terms of the peak discharge response. The peak ratio ($Q_p/Q_{p, ESS}$) shows how the effect of moving rainstorm on peak response would be when it is compared to the ESS. Therefore, the peak ratio is a function of the kinematic characteristics (storm speed and direction) of a moving rainstorm as well as the size of
storms and configuration of a drainage network (See Appendix C). Table 8.2 and Table 8.3 list the results of the peak ratio with respect to ESSA V and ESSQ, respectively for a relatively efficient network configuration ($\beta = 10^0$). In contrast, Table 8.4 and Table 8.5 list the peak ratio with ESSA V and ESSQ, respectively for a relatively inefficient network configuration ($\beta = 10^{-4}$).

Table 8.2 The peak ratio ($Q_p/Q_{p,\ ESSA}$) with a relatively efficient network configuration ($\beta = 10^0$) depending on the relative storm speed ($v_s/v_c$) and size ($L_s/L_c$)

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<th>Storm speed ($v_s/v_c$)</th>
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</tr>
</tbody>
</table>

Table 8.3 The peak ratio ($Q_p/Q_{p,\ ESSQ}$) with a relatively efficient network configuration ($\beta = 10^0$) depending on the relative storm speed ($v_s/v_c$) and size ($L_s/L_c$)

<table>
<thead>
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<th>Storm speed ($v_s/v_c$)</th>
<th>Storm size ($L_s/L_c$)</th>
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For an efficient network, the peak response with rainstorm movement is maximized near $v_s/v_c = 1$, especially when the storm size is relatively smaller than the catchment size.
compared with both ESSAV and ESSQ. In contrast, for an inefficient network, the peak response with rainstorm movement is maximized with relatively slower storm speeds. The results also show that as storm speed increases, the peak ratio becomes close to one; the effect of moving rainstorm on peak response is negligible with relatively faster storms compared with the ESS. For example, if \( v_s/v_c \geq 4 \), both the peak ratio of ESSAV and ESSQ tend to be one regardless of direction of moving rainstorm and network configuration of drainage network. The result is consistent with conclusion of Singh (1997) in that the effect of storm speed on peak discharge is much less for rapidly moving storms than for storms moving at the same speed as the flow velocity.

In general, the results indicate that the effect of moving rainstorm increases as storm size decreases. The peak ratio compared with ESSAV shows that the peak can be 4.6 times greater with moving rainstorms (Table 8.2) and the peak ratio compared with ESSQ can be 2.7 times greater (Table 8.3) when the storm size is 1/8 of the catchment size. As the storm size increase, the effect of moving rainstorm decreases because the catchment reaches equilibrium states. If the storm size is equivalent to the catchment size (\( L_s/L_c = 1 \)), the effect of moving rainstorm decreases. If \( L_s/L_c = 1 \) and the storm speed is slower than the flow, the peak response easily reaches the equilibrium discharge for all three types of rainstorms; moving rainstorm, ESSAV, ESSQ. Once the catchment reaches equilibrium, the peak response is directly proportional to catchment area and intensity and no longer affected by rainstorm movement. As discussed earlier, the rainfall intensity of ESSQ is constant and the same with moving rainstorms (60 mm/hr). In constrast, the rainfall intensity of ESSAV depends on the size of moving rainstorm and if \( L_s/L_c = 1 \), the rainfall intensity is exactly half of ESSQ or moving rainstorms as shown in Equation 8.7 and Figure 8.1 (b). Therefore, the peak ratio with respect to ESSAV tends to be two in the equilibrium states. For ESSQ, the peak ratio tends to be one because it has same intensity with moving rainstorms. As storms get slower, the catchment reaches equilibrium states and the peak ratio with ESSAV and ESSQ tend to be two and one, respectively. The effect of moving rainstorm on peak response is negligible with relatively faster storms and this result is valid regardless of storm sizes as discussed earlier. Therefore, the peak ratio with both ESSAV and ESSQ tends to be one.
Table 8.4 The peak ratio ($Q_p/Q_{p, ESSA}$) with a relatively inefficient network configuration ($\beta = 10^{-4}$) depending on the relative storm speed ($v_s/v_c$) and size ($L_s/L_c$)

<table>
<thead>
<tr>
<th>Storm speed ($v_s/v_c$)</th>
<th>0.125</th>
<th>0.25</th>
<th>0.375</th>
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<th>0.625</th>
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Table 8.5 The peak ratio ($Q_p/Q_{p, ESSQ}$) with a relatively inefficient network configuration ($\beta = 10^{-4}$) depending on the relative storm speed ($v_s/v_c$) and size ($L_s/L_c$)

<table>
<thead>
<tr>
<th>Storm speed ($v_s/v_c$)</th>
<th>0.125</th>
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The results indicate that effect of moving rainstorm heavily depends on storm orientation and network configuration. The effect of moving rainstorm is magnified especially when the storm moves in downstream direction and the storm speed is close to the resonance condition. For a relatively smaller storms ($L_s/L_c = 1/8$), the maximum peak ratio with respect to ESSAV and ESSQ is obtained when $\theta = \pi/4$ instead of $\theta = 0$. As discussed in Chapter 5, this is due to the spatial arrangement of equidistant line especially with efficient networks. The results also show that the peak ratio compared with ESS is closely related to network configuration (See Appendix C) because the resonance condition is a
function of network configuration as shown in Chapter 5. As the storm speeds are close to resonance condition and the size of moving rainstorm is smaller than the catchment size, the peak ratio becomes sensitive to network configuration. Especially, when \( L_s/L_c = 0.125 \), the peak ratio with respect to ESSAV shows great difference depending on network configuration. For example, if \( v_s/v_c = 0.8 \), and \( \theta = \pi/4 \), the difference among different configuration of network is up to 2.8; the least efficient network (\( \beta = 10^{-4} \)) has the peak ratio of 1.8 and an efficient networks (\( \beta = 10^0 \)) shows the peak ratio of 4.6. As the storm size increases, the difference in peak ratio with respect to ESSAV depending on network configuration decreases. The peak ratio with respect to ESSQ shows that the maximum difference in the peak ratios among different configuration of network is up to 1.2; the least efficient network (\( \beta = 10^{-4} \)) has peak ratio of 1.5, while, one of the efficient networks (\( \beta = 10^0 \)) shows the peak ratio of 2.7 when \( v_s/v_c = 0.8 \). As the storm size increases, the difference among different network configuration diminishes and the peak ratio with respect to ESSQ tends to 1.

Figure 8.9 Peak ratio with respect to ESS for an efficient network (\( \beta = 10^0 \)) and a less efficient network (\( \beta = 10^{-4} \)) with a moving rainstorm (\( L_s/L_c = 0.25 \)): (a) ESSAV; (b) ESSQ

In general, the results show that an efficient network is more sensitive to moving
The result shows that an efficient network is more sensitive to rainstorm movement compared with an inefficient network. As discussed in Chapter 4, the one of the characteristics of urban drainage network is that urban catchments have wide range of network configuration compared with natural river networks. Therefore, the effect of rainstorm movement widely varies depending on network configuration in urban catchments. In urban catchments, there can be highly efficient network in terms of drainage time and flow distance as much as natural river network. Furthermore, there can be a less efficient drainage network that has longer drainage time and flow distance, which are close to a random network (Karlinger and Troutman, 1989). However, in the context of the results in this chapter, a less efficient network in urban catchments unexpectedly contributes to reduce the impact from moving rainstorms in terms of peak discharge response.

8.4. Conclusions

This chapter investigates the effect of moving rainstorm on the peak discharge response compared with stationary storms. To avoid any bias due to catchment geometry, a synthetic circular watershed is introduced. Two types of the ESS (Ngirane-Katashaya and Wheater, 1985) are introduced; one is a catchment-average stationary storm (ESSA V) and the other is a point-based stationary storm (ESSQ). Both ESSA V and ESSQ preserve the total amount of rainfall with moving rainstorms. ESSA V preserves the total duration of rainfall event with respect to the entire catchment and ESSQ preserves the intensity of moving rainstorms. A peak ratio with respect to these equivalent stationary storms is obtained to evaluate the effect of moving rainstorm.

The rate of change of peak response due to moving rainstorm compared with equivalent stationary storms is calculated in this study. The results indicate that the effect of moving rainstorm increases as storm speed is close to resonance condition and the storm size is smaller compared with the catchment size. The results also show that the effect of
moving rainstorm heavily depends on storm orientation as well as network configuration. Especially, an efficient network is more sensitive to moving rainstorms in terms of the peak ratio with respect to ESSAV and ESSQ. In contrast to this, a less efficient network tends to mitigate the effect of rainstorm movement on the peak response. The results in this chapter are consistent with the results from previous chapters: resonance condition in Chapter 3 and 5, the effect of spatial distribution of equidistant lines in Chapter 5, and the importance of network configuration as shown in Chapter 4 and Chapter 5. Especially, the results from Chapter 4 showed existence of wide range network configuration in urban catchments, which enables us to evaluate the effect of moving rainstorm in terms of peak ratio obtained in this study. In the context of the results in this study, a less efficient network existing in urban catchments prove to contribute unexpectedly to reduce the impact from moving rainstorms. In this regard, the results in this study imply a potential improvement in urban drainage networks and it also implies an optimal network configuration in urban catchments in terms of network efficiency as well as sensitivity to rainstorm movement.
9. CONCLUSIONS

This research involves understanding how storm movement affects the hydrologic response of a watershed, especially in urbanized areas. This research mainly focuses on the relation between the effect from rainstorm movement and network configuration.

9.1. The effect of storm movement and network configuration

In this study, Gibbs’ model (Troutman and Karlinger, 1992) is adopted in order to categorize the network and represent the network characteristics. A network configuration is represented by a parameter value ($\beta$) of Gibbs’ model. This study investigates network configuration of urban catchments compared with natural river network. Furthermore, two synthetic catchments (rectangular and circular ones) are introduced in this study to investigate the relation between network configuration and the effect of moving rainstorms. Realizations of the stochastic network model are tested with moving storms with different direction, speed, and length scale.

In Chapter 3, a broad theoretical framework is utilized that uses characteristic time and space scales associated with stationary rainstorms as well as moving rainstorms. For a stationary rainstorm the characteristic timescales that govern the peak response include two intrinsic timescales of a catchment and one extrinsic timescale of a rainstorm. On the other hand, for a moving rainstorm, two additional extrinsic scales are required; the storm travel time and storm size. The relation between the peak response and the timescales appropriate for a stationary rainstorm can be extended in a straightforward manner to describe the peak response for a moving rainstorm. However, the interdependencies between rainfall duration and storm travel time makes the behavior of the peak response for a moving rainstorm fundamentally different from that of a stationary rainstorm. This chapter shows that the relationship between peak response and characteristic timescales also depends on the relative size of the rainstorm with respect to catchment size. For moving rainstorms, we show that the augmentation of peak response arises from both effect of overlaying the responses from subcatchments (resonance condition) and effect of increased responses from subcatchments due to increased duration (interdependencies),
which results in maximum peak response when the moving rainstorm is slower than the channel flow velocity.

In Chapter 4, this study explores the relations between network properties and the effect from moving rainstorms in terms of the peak response and time to centroid of hydrographs. A synthetic rectangular catchment is introduced with different configurations of drainage network simulated by Gibbs’ stochastic model. The efficiency of the urban pipe networks vary widely compared with natural river networks; hence, Gibbs’ model can be an appropriate way to represent the network properties in urban drainage system. In contrast, natural river networks are very efficient and close to the Scheidegger network. Simple cases of rainstorms moving with upstream and downstream directions and different speeds are considered in order to investigate the effect of rainstorm movement on urban drainage network runoff hydrographs. The results indicate that the effect of the direction and speed of the rainstorm movement varies significantly depending on the network properties. In contrast to previous studies, this study indicates that the speed and direction of the rainfall movement that produces the maximum peak discharge changes depending on the network configuration.

In Chapter 5, this study investigates the relations between network configurations and hydrograph sensitivity to storm kinematics; storm speeds, storm directions as well as storm sizes. A synthetic circular catchment is utilized in order to avoid any bias that depends on catchment geometry. The configuration of drainage network is simulated with Gibbs’ stochastic network model. The results show that the effect of the direction and speed of the rainstorm movement highly depends on the network properties. The relation between storm kinematics and the peak discharge response is dependent on the network configuration; accordingly network efficiency. The results indicate that the resonance condition does not produce the maximum peak discharge and the maximum peak occurs when the storm is slower than the flow. The results indicate that a network with lowest $\beta$ (most inefficient network) produces the most insensitive and lowest peak response to storm kinematics.

9.2. Hydrologic response function based on WFIUH for urban catchments

In Chapter 6, we developed a framework for rainfall-runoff analysis in urban watersheds
based on the width function. Width functions are obtained from urban drainage networks and applied to obtain distinct response functions for DCIA, IIA, and pervious areas combined with excess and infiltrated amount of rainfall. The modeling framework suggested in this study is able to reproduce the long tails observed in the urban runoff hydrograph which could not be explained by contribution of the impervious area alone before excess rainfall. It also enables us to quantify of the contribution from each area to runoff hydrographs; especially evaluation of the role of IIA in urban areas. The results show significant improvement in the estimation of runoff hydrographs and suggest the need to consider the flow contribution from infiltrated rainfall in pervious areas to the runoff hydrograph before saturation. The results also imply that additional contribution from flow paths such as pipe infiltration needs to be considered in urban areas.

9.3. Application of stochastic network model to urban catchments

This study shows that the response function of actual network can be obtained from a stochastic network model. A synthetic width function for an urban catchment is obtained from Gibbs’ model. The hydrologic response from the original pipe network and a stochastic network model with Monte-Carlo simulation are compared with each other. For a specific given network configuration, if a stochastic model successfully reproduces the original network in terms of hydrologic response, it will be also possible to relate the effect of rainstorm movement based on the network configuration.

In Chapter 7, the possibility for a stochastic network to replace an actual existing urban drainage network in terms of outlet hydrograph is investigated. The actual network is replaced by stochastic networks from Monte-Carlo simulation and the WFIUH is derived using the synthetic width function averaged from the generated networks with Gibbs’ model. The result shows that the goodness of fit of the resulting hydrographs from the stochastic network and the actual urban drainage network is greater than 0.95, hence, the stochastic network can be used to replace the actual urban drainage network to estimating the flow discharge hydrograph of an urban catchment. The applicability of stochastic network in urban catchment implies that once the single value of $\beta$ is estimated for an urban catchment, the flow discharge hydrograph of the catchment can be estimated based on the value of $\beta$ even if we are lacking detailed layout of the drainage network. In this
chapter, the characteristic property of a network is given as a value of parameter, \( \beta \) of Gibbs’ model. As discussed in Chapter 4, \( \beta \) has wide range of values in urban catchment compared to natural river networks. The wide range of \( \beta \) implies that the flow discharge hydrographs in urban catchments show various characteristics.

### 9.4. Peak ratio with respect to the ESS

Finally, this study investigates the effect of rainstorm movement on peak discharge response of urban drainage networks compared with the ESS depending on network configuration. A synthetic circular watershed is introduced in order to avoid any biases due to catchment geometry. The drainage network is generated with Gibbs’ model. Chapter 8 investigates the effect of rainstorm movement on the peak discharge response of urban drainage networks compared with stationary rainfall depending on network configuration. A synthetic circular watershed is introduced and the drainage network of the watershed is simulated by Gibbs’ model. This study utilizes two types of the ESS. The rate of change of the peak discharge response for moving rainstorm is examined with respect to ESS. The results are consistent with the results of Chapter 3; there exists an interval for which the same rainfall duration produces much larger peak responses for moving storms compared with ESSQ. The augmentation of the peak response by moving rainstorm is dependent on the relative rainstorm speed, size, and direction as well as drainage network configuration of the watershed. The results indicate that the effect of moving rainstorm increases as storm speed is close to resonance condition and the storm size is smaller compared with the catchment size. The results also show that the effect of moving rainstorm heavily depends on storm orientation as well as network configuration. Especially, the result indicates that an efficient network is more sensitive to moving rainstorms in terms of the peak ratio with respect to ESSAV and ESSQ. In contrast to this, a less efficient network tends to mitigate the effect of rainstorm movement on peak response.

The results from Chapter 4 showed existence of wide range network configuration in urban catchments. In Chapter 8, we show that an efficient network is more sensitive to moving rainstorms in terms of the peak response and a less efficient network tends to mitigate the effect of moving rainstorm on peak responses. Therefore, it can be inferred
that a less efficient network existing in urban catchments contributes to reduce the impact from moving rainstorms. In this regard, the results in this study imply a potential improvement in urban drainage networks in terms of efficiency and sensitivity to moving rainstorms. Especially combined with the results shown in Chapter 4, Chapter 5, and Chapter 7, the network property ($\beta$) can be a key link that will be used to evaluate the effect from moving rainstorms as well as to produce the flow discharge hydrographs.

### 9.5. Contributions of the research

Overall, this study achieves a better understanding of rainfall-runoff processes in urbanized areas. The original contributions of this research are listed as follows:

1. Intrinsic and extrinsic timescale and space scales are identified for moving rainstorms and their interactions for the description of the peak discharge response with moving rainstorms are clarified compared with stationary storms (Chapter 3).

2. Urban drainage networks can have wide range of network configuration compared with natural river networks; the network configuration can be inefficient in urban areas in terms of total drainage time (Chapter 4).

3. The relation between storm speed and direction and the change in peak discharge is dependent on the network configuration and network efficiency (Chapter 4; Chapter 5; Chapter 8).

4. Rainfall-runoff response functions in urban areas are developed for all contributing areas distinctively based on the width function from a drainage network and it is able to reproduce the hydrologic response of an urban catchment combined with stochastic network models (Chapter 6; Chapter 7).

5. Equivalent Stationary Storm (ESS) is utilized to evaluate the effect of moving rainstorms in terms of peak response and efficient networks prove to be sensitive to rainstorm movement compared with inefficient networks (Chapter 8).

6. The network property ($\beta$) can be a key link that enables us to evaluate the effect from moving rainstorms as well as to produce the flow discharge hydrographs (Chapter 4; Chapter 5; Chapter 7).
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APPENDIX A: TRANSITION AND DISPERSION COEFFICIENT FOR A CIRCULAR CROSS SECTION

The transition and dispersion coefficient can be obtained during the derivation of advection-diffusion equation for different geometry. For trapezoidal channel cross-section geometry, van de Nes (1973) derived the transition and dispersion coefficient as follows:

\[ A_i = \frac{5}{3} \nu \left( 1 - \frac{4 R_i}{5 B_i} \sqrt{1 + m^2} \right) \]  
(A.1)

\[ D = \frac{Q_i}{2S_i B_i} \left( 1 - \frac{4}{9} F_i \left( 1 - \frac{2 R_i}{B_i} \sqrt{1 + m^2} \right)^2 \right) \]  
(A.2)

Following the same steps, the transition and dispersion coefficient of the advection-diffusion equation for circular cross-section can be derived as follows.

With or without lateral inflows, it is valid that

\[ S_f = S_0 - \left( 1 - F^2 \right) \frac{\partial y}{\partial x} - \frac{2Q}{gA^2} \frac{\partial Q}{\partial x} - \frac{1}{gA} \frac{\partial Q}{\partial t} \]  
(A.3)

Equation A.3 can be rewritten as

\[ S_f = S_0 - \left( 1 - F^2 \right) \frac{\partial y}{\partial x} - \frac{2Q}{gA^2} \frac{cB}{gA} \frac{\partial y}{\partial x} - \frac{1}{gA} \frac{\partial y}{\partial x} \left( -c^2 \frac{\partial y}{\partial x} \right) \]

\[ = S_0 - \left( 1 - F^2 \right) \frac{\partial y}{\partial x} - (c - 2v) \frac{cB}{gA} \frac{\partial y}{\partial x} \]

\[ = S_0 - \left( 1 - F^2 - (c + 2v) \frac{cB}{gA} \right) \frac{\partial y}{\partial x} \]

\[ = S_0 - \left( 1 - F^2 - (c + 2v) \frac{Q^2 B c^2}{gA^3 Q^2} \right) \frac{\partial y}{\partial x} \]  
(A.4)

Substituting \( Q^2 B / gA = F \) and \( cA^2 / Q^2 = c/v^2 \) into Equation A.4 gives

\[ S_f = S_0 - \left( 1 - F^2 \left( 1 - \frac{c}{v} \right)^2 \right) \frac{\partial y}{\partial x} \]  
(A.5)

where

\[ c = \frac{\partial Q}{\partial A} = \frac{\partial Q}{\partial y} \frac{\partial y}{\partial A} = \frac{\partial y}{\partial A} \left( A \frac{\partial y}{\partial y} + v \frac{\partial A}{\partial y} \right) = A \frac{\partial y}{\partial A} \frac{\partial v}{\partial y} + v \]  
(A.6)
Then, Equation A.5 can be rewritten as

\[ S_f = S_0 - \left( 1 - F^2 \left( - \frac{A}{v} \frac{\partial v}{\partial y} \right)^2 \right) \frac{\partial y}{\partial x} \]  

(A.7)

Introducing Chezy equation, Equation A.7 can be written as

\[ S_f = S_0 - \left( 1 - F^2 \left( - \frac{1}{2} \frac{A'}{P'} \frac{AP - AP'}{P^2} \right)^2 \right) \frac{\partial y}{\partial x} \]  

(A.8)

where \( A \) is a cross sectional area of a channel flow and \( P \) is a perimeter and \( A' = \partial A/\partial y \) and \( P' = \partial P/\partial y \).

![Figure A.1 Circular cross-section](image)

Assuming a circular cross section (Figure A.1),

\[ A = \frac{D^2}{8} \left( \theta - \sin \theta \right) \]  

(A.9)

\[ P = \frac{D}{2} \theta \]  

(A.10)

\[ R = \frac{D}{4} \left( 1 - \frac{\sin \theta}{\theta} \right) \]  

(A.11)

\[ y = \frac{D}{2} \left( 1 - \cos \frac{\theta}{2} \right) \]  

(A.12)

\[ B = \frac{D}{2} \sin \frac{\theta}{2} \]  

(A.13)

Therefore, Equation A.8 can be written as
\[
S_f = S_0 - \left(1 - \frac{F^2}{16} \frac{D^2}{B^2} \left(1 - \cos \theta - \frac{4R}{D} \right)^2 \right) \frac{\partial y}{\partial x}
\]

where
\[
C_i = 1 - \frac{F^2}{16} \frac{D^2}{B^2} \left(1 - \cos \theta - \frac{4R}{D} \right)^2
\]

From the Chezy equation,
\[
Q = CR^{1/2} A \left(S_0 - C_i S\right)^{1/2}
\]

Let \(Q\) as a function of \(y\) and \(S\). Then, the Taylor series of \(Q\) is given as
\[
Q = Q_I + y_p \frac{\partial Q_I}{\partial y} + S_p \frac{\partial Q_I}{\partial S} + \ldots
\]

where \(Q_I\) is initial flow rate and \(Q_p\) is perturbed flow rate.

Neglecting higher order terms,
\[
Q_p = y_p \frac{\partial Q_I}{\partial y} + S_p \frac{\partial Q_I}{\partial S}
\]

From Equation A.16,
\[
Q_I = C \left(S_0 - C_i S\right)^{1/2} A^{3/2} P^{-1/2}
\]

Then,
\[
\frac{\partial Q_I}{\partial y} = C \left(S_0 - C_i S\right)^{1/2} \frac{\partial}{\partial y} \left(A^{3/2} P^{-1/2}\right)
\]

From Equation A.9, A.10, A.11, A.12 and A.13,
\[
\frac{\partial Q_I}{\partial y} = C \left(S_0 - C_i S\right)^{1/2} R^{1/2} \left(3 \frac{D^2}{4B} \left(1 - \cos \theta \right) - \frac{RD}{B}\right)
\]

\[
= V_I \left(3 \frac{D^2}{4B} \left(1 - \cos \theta \right) - \frac{RD}{B}\right)
\]

\[
\frac{\partial Q_I}{\partial S} = -C_i \frac{Q_I}{2S_0}
\]

Substituting Equation A.21 and A.22 into A.18 gives the advection-diffusion equation
\[
Q_p = \frac{\partial Q_I}{\partial y} y_p + \frac{\partial Q_I}{\partial S} S_p
\]

or

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\[ \frac{Q_i}{B_i} = c y_i - D \frac{\partial y_i}{\partial x} \]  \hspace{1cm} (A.24)

where the transition coefficient (kinematic wave celerity) and the diffusion coefficient (from Chezy equation) are given as follows:

\[ c = V_i \left( \frac{3}{4} D_i^2 \left( 1 - \cos \theta_i \right) - \frac{R_i D_i}{B_i^2} \right) \]  \hspace{1cm} (A.25)

\[ D = \frac{Q_i}{2S_0 B_i} \left[ 1 - \frac{F^2}{16} \left( \frac{D_i}{B_i} \left( 1 - \cos \theta_i - \frac{4R_i}{D_i} \right) \right)^2 \right] \]  \hspace{1cm} (A.26)

Instead of Chezy equation, Manning’s equation can be used to derive the transition coefficient and the diffusion coefficient in the same way. Then, Equation A.8 can be rewritten as

\[ S_j = S_0 - \left( 1 - F^2 \left( - \frac{2}{3} B R \frac{A^1 P - AP}{P^2} \right) \right) \frac{\partial y}{\partial x} \]  \hspace{1cm} (A.27)

From Equation A.9, A.10, A.11, A.12 and A.13 for a circular cross-section, Equation A.27 can be written as follows:

\[ S_j = S_0 - \left( 1 - \frac{F^2}{9} \left( \frac{D_i}{B_i} \left( 1 - \cos \theta_i - \frac{4R_i}{D_i} \right) \right)^2 \right) \frac{\partial y}{\partial x} \]  \hspace{1cm} (A.28)

\[ = S_0 - C_i \frac{\partial y}{\partial s} \]

where

\[ C_i = 1 - \frac{F^2}{9} \left( \frac{D_i}{B_i} \left( 1 - \cos \theta_i - \frac{4R_i}{D_i} \right) \right)^2 \]  \hspace{1cm} (A.29)

From the Manning’s equation,

\[ Q = K A R^{2/3} (S_0 - C_s S)^{1/2} \]  \hspace{1cm} (A.30)

where \( K = k/n \), \( k \) is a constant (1 for SI units, \( m^{1/3}/s \); 1.49 for US customary units, \( ft^{1/3}/s \)), \( n \) (\( s/m^{1/3} \)) is the Manning’s roughness coefficient. From Equation A.30,

\[ \frac{\partial Q_i}{\partial y} = K (S_0 - C_s S)^{1/2} \frac{\partial}{\partial y} \left( A^{5/3} P^{-2/3} \right) \]  \hspace{1cm} (A.31)

From Equation A.9, A.10, A.11, A.12 and A.13,
\[
\frac{\partial Q_i}{\partial y} = K (S_o - C_i S)^{1/2} R_i^{2/3} \left( \frac{5 D^2}{6 B^3} (1 - \cos \theta_i) - \frac{4 RD}{3 B} \right)_i
\]

Substituting Equation A.33 and A.34 into A.18 gives the advection-diffusion equation

\[
Q_r = \frac{\partial Q_i}{\partial y} y_r + \frac{\partial Q_i}{\partial S} S_r
\]

where the transition coefficient (kinematic wave celerity) and the diffusion coefficient are given as follows:

\[
ce = V_i \left( \frac{5 D^2}{6 B_i^3} (1 - \cos \theta_i) - \frac{4 R_i D}{3 B_i^2} \right)
\]

\[
D = \frac{Q_i}{2 S_o B_i} \left( 1 - \frac{F^2}{9} \left( \frac{D^2}{B^2} \left( 1 - \cos \theta_i - \frac{4R}{D} \right) \right)^2 \right)
\]

### Table A.1 Transition and diffusion coefficient depending on the empirical relations for open channel flow

<table>
<thead>
<tr>
<th>Empirical relation for open channel flow</th>
<th>Transition coefficient (c)</th>
<th>Diffusion coefficient (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chezy</td>
<td>$V_i \left( \frac{3 D^2}{4 B_i^3} (1 - \cos \theta_i) - \frac{R_i D}{B_i^2} \right)$</td>
<td>$Q_i \left( 1 - \frac{F^2}{16} \left( \frac{D^2}{B_i^2} \left( 1 - \cos \theta_i - \frac{4R}{D} \right) \right)^2 \right)$</td>
</tr>
<tr>
<td>Manning</td>
<td>$V_i \left( \frac{5 D^2}{6 B_i^3} (1 - \cos \theta_i) - \frac{4 R_i D}{3 B_i^2} \right)$</td>
<td>$Q_i \left( 1 - \frac{F^2}{9} \left( \frac{D^2}{B_i^2} \left( 1 - \cos \theta_i - \frac{4R}{D} \right) \right)^2 \right)$</td>
</tr>
</tbody>
</table>
APPENDIX B: MULTIFRACTAL OF THE FLOW DISTRIBUTION IN STOCHASTIC NETWORKS

Multifractal measures are related to the study of a distribution of physical or other quantities on a geometric support. The support may be an ordinary plane, the surface of a sphere or a volume, or it could itself be a fractal. Multifractal is useful in describing experimental observations as shown by Frisch and Parisi (1985) and Meneveau and Sreenivasan (1987). In this study, we investigate the distribution of flow (river width) under the condition of continuous unit injection (rainfall) on a drainage network simulated by the Scheidegger model, the Uniform model, and Gibbs’ model. Nagatani (1993) investigated the flow distribution of flow distribution with the Scheidegger model with periodic lateral boundary condition and allowed multiple outlets of the basin. In this study, a circular catchment (41 by 41 grids) with one fixed outlet at the bottom is introduced. Nagatani (1993) used a vertical length scale $L$ with fixed horizontal width as shown in Figure B.1, which resulted in a maximum fractal dimension in $f$-$\alpha$ spectrum of 1.0 that is equal to the fractal dimension of the ‘support’ of the measure (Feder, 1988). In contrast, this study introduces only one designated outlet in the catchment and a boundary like watershed divide in natural streams (Figure 5.3). Therefore, this study utilizes the box-counting method (Block et al., 1990) with a size of $\delta$.

Figure B.1 Scheidegger network with multiple outlets and periodic boundary (Nagatani, 1993)
The partition function (Halsey et al., 1986), \( Z(q) \) is defined as the moments of the flow rate at each point for a given resolution \( \delta \).

\[
Z(q) = \sum_i I_i^q
\]  

(B.1)

For a sufficiently small \( \delta \), the partition function scales as (Figure B.2)

\[
Z(q) \sim \delta^{\zeta(q)}
\]  

(B.2)

![Figure B.2](image)

**Figure B.2 (a) \( Z(q) \) against \( \delta \) and (b) the exponent \( \zeta(q) \) of a Uniform network**

The normalized partition function is defined with the probability of mass distribution in \( i^{th} \) cell compared to the total summation of the flow distribution. Let us define the measure (or the probability or mass) of the content in the \( i^{th} \) cell as

\[
\mu_i = \frac{N_i(\delta)}{N}
\]  

(B.3)

where \( N_i(\delta) \) is the number of points falling in the \( i^{th} \) cell at the resolution \( \delta \). In the same way, we can define the ratio of amount of flow in \( i^{th} \) cell as the measure.

\[
Z'(q) = \frac{\sum_i \left( \frac{I_i}{\sum_i I_i} \right)^q}{\sum_i \left( \frac{I_i}{Z(1)} \right)^q} = \frac{Z(q)}{Z(1)^q} = \sum_i \mu_i^q
\]  

(B.4)

For a sufficiently small \( \delta \), the normalized partition function can be scaled as

\[
Z'(q) \sim \delta^{-\tau(q)}
\]  

(B.5)

The measure with a mass exponent \( d = \tau(q) \) for which the measure neither vanishes nor diverges as \( \delta \to \infty \) is

\[
M_x = \sum_i \mu_i^d \delta^d = N(q, \delta) \delta^d \to \begin{cases} 0, & d > \tau(q) \\ \infty, & d < \tau(q) \end{cases}
\]  

(B.6)
The weighted number of boxes \( N(q, \delta) \) has the form,

\[
N(q, \delta) = \sum_i \mu_i^q \sim \delta^{-\tau(q)} \quad \text{(B.7)}
\]

This is equal to the form of the partition function in Equation B.5 and the mass exponent is given by

\[
\tau(q) = -\lim_{\delta \to 0} \frac{\ln N(q, \delta)}{\ln \delta} \quad \text{(B.8)}
\]

Figure B.3 shows the exponent can be obtained with the slope of the normalized partition function and the how the value of the exponent varies with \( q \) in a uniform network model.

![Figure B.3 Z'(q) against \( \delta \) and the exponent \( \tau(q) \) in a uniform river network](image)

With the Legendre transformation of \( \tau(q) \), we can get \( f-\alpha \) spectrum

\[
f(q) = q\alpha(q) + \tau(q) \quad \text{(B.9)}
\]

where the variable conjugate to \( q \),

\[
\alpha(q) = -\frac{\partial \tau(q)}{\partial q} \quad \text{(B.10)}
\]

The Legendre transformation is from the independent variables \( \tau \) and \( q \) to the independent variables \( f \) and \( \alpha \).

For \( q = 1 \), we find that \( d\tau/dq \) has an interesting value (Feder, 1988),

\[
\left. \frac{\partial \tau(q)}{\partial q} \right|_{q=1} = -\lim_{\delta \to 0} \frac{\sum_i \mu_i \ln \mu_i}{\ln \delta} = \lim_{\delta \to 0} \frac{S_i(\delta)}{\ln \delta} \quad \text{(B.11)}
\]

where \( S_i(\delta) \) is the information entropy of the partition of the measure and \( \alpha_1 \) is the fractal dimension of the set, onto which the measures concentrates and describes the scaling with the box size \( \delta \) of the (partition) entropy of the measure.
\[ \alpha_i = -\partial \tau(q)/\partial q \rvert_{q=1} = f_s \]  

\[ \alpha(q) \text{ against } q \]

**Figure B.4 \( \alpha(q) \) against \( q \) in the Uniform network**

From Equation B.43 and B.5,

\[ \frac{Z(q)}{Z(1)} = \delta^{-\tau(q)} \]  

\[ \tau = -\frac{1}{\ln \delta} \left[ \ln Z - q \ln Z(1) \right] \]  

Then, \( \partial \tau/\partial q \) can be written as

\[ \frac{\partial \tau}{\partial q} = \frac{\partial}{\partial q} \left[ \ln Z \right] + \frac{\ln Z(1)}{\ln \delta} \]  

The maximum value of \( \alpha \) gives the minimum fraction of flow rate and the minimum value of \( \alpha \) gives the maximum fraction of flow rate. The minimum value of \( \alpha(\infty) \) is exactly related to the fractal dimension of a single river.

\[ \alpha_{\text{min}} = \left. \frac{\partial \tau(q)}{\partial q} \right|_{q=\infty} = \left[ \frac{\partial}{\partial q} \left( \ln Z(q) \right) \right]_{q=\infty} - \frac{\ln Z(1)}{\ln \delta} = \left[ \frac{\partial \zeta(q)}{\partial q} \right]_{q=\infty} \]  

In \( Z(1)/\ln \delta \) is equal to \(-\tau(1)\) which is zero in this case because \( Z(1) \) is equal to 1. The first term shows that the rate of increase of the slope of partition function \( Z(q), \zeta(q) \) when \( q \) is sufficiently large is equal to the minimum value of \( \alpha \). Therefore, the minimum value of \( \alpha \) is exactly equal to the fractal dimension of a single channel in the network.
The maximum value of $f(\alpha)$ is equal to 2 which is equal to the dimension of the geometry on which the measure is defined. In this case we have a measure defined on a box of which dimension is 2.

The properties of the stochastic model are investigated in terms of multifractal of the flow distribution in a network. This study utilizes a direct box counting method in order to investigate the multifractality of flow distribution instead of the method used in the previous study based on a one dimensional support. The multifractality clearly shows the difference between Gibbs’ network with different values of $\beta$ as well as the similarity of Gibbs’ network with the Scheidegger network and the Uniform network.
APPENDIX C: EQUIVALENT STATIONARY STORMS COMPARED WITH MOVING RAINSTORMS
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