Some Simple and Pleasant Algebra of Short Run Monetary Control

Case M. Sprenkle
Department of Economics
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College of Commerce and Business Administration
University of Illinois at Urbana-Champaign
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Department of Economics
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Abstract

An operating target is suggested for a monetary authority in a simple world. In this world currency movements should be weighted by the required reserve ratio on demand deposits. With this weighting of currency, control of the money supply is independent of the required reserve ratio, so that reserve requirements can be kept low without giving up control. This result holds in a Poole type world of an optimal weighting on money changes and interest changes for an intermediate target, and the optimal target weight for the interest rate is also given for such an intermediate target. The simple model is expanded for control of broad money and interest rates.
This is a report from a simple and pleasant world. In this simple and pleasant world the monetary authorities have as a final target nominal income (or some linear function of it). They have read Poole and have as an intermediate target

\[ a\Delta M_t + (1-a)\bar{I} = 0 \]

where \(-\) indicates changes, \(M_t\) is narrow money, \(I\) is some interest rate(s), and \(a\) is a weight chosen optimally to minimize final target deviations as taught by Poole. The monetary authorities have chosen open market operations as their one and only tool with which to meet their target. However, they have found two related problems in conducting their policy. Although they see \(I\) all the time, they only see \(M_t\) every so often, and when they do see it, they often find out it is not what they expected. Thus, they keep missing their target and in addition find that they have to work very hard (do a lot of open market operations) to get back on target. This leads to the other problem. They work very hard every so often but most of the time they have nothing to do since they have not seen a new \(M_t\). This would not be so bad except that other government agencies are complaining of gross inefficiencies, and there have even been calls for privatization.

In order to achieve better control and, of course secondarily, to lessen the complaints about their inefficiency, a new control technique has been proposed---an operating target. The monetary authorities from their own balance sheet know (well, almost know) both the amount of reserves and currency issued on a daily basis. Although they have heard
of other central banks that had tried somewhat similar techniques through something called base control or reserve control in olden days, they believe this violates the spirit of Poole's idea which is to use all the information available.

Their proposed operating target is

$$2 \quad R + \lambda_1 C + \lambda_2 I = 0$$

where \( R \) is reserves, \( C \) is currency issued, and \( \lambda_1 \) and \( \lambda_2 \) are weights to be determined optimally. Since in this simple world, the only reserve requirement is on demand deposits, and since there are no excess reserves and commercial banks are certainly prohibited from borrowing, then \( R = rD \) and the operating target is

$$3 \quad rD + \lambda_1 C + \lambda_2 I = 0.$$

The monetary authorities know that income does not change from one day to the next so that changes in the demand for currency and demand deposits are

$$4 \quad \dot{C} = k_c \ddot{I} + u_c$$
$$\dot{D} = k_d \ddot{I} + u_d$$

where \( u_c \) and \( u_d \) are shock terms.

Putting equations (4) into (3) results in
where \( \phi_1 = rk_d + \lambda_1 k_c + \lambda_2 \). Changes in \( M_1 \) are

\[
\dot{M}_1 = \bar{C} + \bar{D} = \phi_2 \ddot{I} + (u_c + u_d)
\]

where \( \phi_2 \) is \( k_c + k_d \). Given equation (6) the operating target is

\[
\phi_3 \ddot{I} + \alpha (u_c + u_d) = 0
\]

where \( \phi_3 = 1 - \alpha + \alpha \phi_2 \).

Given equation (5), equation (7) is

\[
-\phi_3 \left( \frac{ru_c + \lambda_1 u_c}{\phi_1} \right) + \alpha (u_c + u_d)
\]

and the operating target is

\[
u_c \left( \alpha - \frac{\phi_3 \lambda_1}{\phi_1} \right) + u_d \left( \alpha - \frac{\phi_3 r}{\phi_1} \right).
\]

Now equation (8) seems particularly pleasant because it can be made zero if \( \alpha \phi_1 = \phi_3 \lambda_1 = \phi_3 r \), and this immediately suggests that

\[
\lambda_1 = r
\]

and with \( \lambda_1 = r \), then \( \alpha \phi_1 = \phi_3 r \), or
\( \lambda_2 = \bar{r} \left( \frac{1 - \theta}{\theta} \right) \).

The proposed operating target then seems very nice indeed. The monetary authorities can now control their intermediate target perfectly, and, of course secondarily, keep busy each day as new information comes in from their balance sheet. In addition to all this and the virtue of its simplicity, there are still more virtues. The monetary authorities have noted that \( \lambda_1 = \bar{r} \) is independent of \( \theta \) so that even for some old-timers who believe that \( M_1 \) should be the sole intermediate target, the appropriate level for \( \lambda_1 \) is still \( \bar{r} \). Of course, \( M_1 \) could be controlled perfectly given \( \lambda_1 = \bar{r} \) if only \( \theta \) were set at one, and thus \( \lambda_2 \) at zero. If only the old-timers had known that neither base or reserve control would work, but that instead currency changes must be weighted by the reserve requirement on demand deposits!

But there is even still more virtue in this proposed operating target. The monetary authorities have calculated the variance of both the interest rate and \( M_1 \) if the operating target is used. If \( \sigma_c^2, \sigma_d^2, \) and \( \sigma_{cd} \) are the variances and covariance of the currency and demand deposit shock terms then

\[
\sigma_r^2 = \frac{\sigma_c^2 + \sigma_d^2 + 2 \sigma_{cd}}{\left( \phi + \frac{1 - \theta}{\theta} \right)^2}
\]

and
Of the two variances the monetary authorities have found $\sigma^2_{M_1}$ to be the most interesting. In the past some of the old-timers who wanted to control $M_1$ only and not knowing to set $\lambda_1 = r$, implored the monetary authorities to raise the required reserve ratio to 100 percent. Of course, the commercial banks did not view this proposal with great favor. The resultant setting of $r$ by the monetary authorities had become something of a compromise even in this simple world, pushed higher because of the supposed better control and should lower by the plight of the banks. Now from equation (12) it is clear that control of the money supply is completely independent of the level of $r$. No longer must there be compromise in this simple world since $r$ can be set as low as needed without reducing control of $M_1$.

This result for the setting of $r$ is particularly important for the monetary authorities because they have recently noted some bothersome complexities arising in their simple world. It seems some banks, and even some non-banks!, have started issuing new kinds of liabilities which seem to be treated by the populace as close substitutes for currency and demand deposits. Furthermore the monetary authorities expect this trend to continue and increase in importance. This has even led to some disarray amongst the monetary authorities some of whom have suggested a new broader definition of money, called $M_2$, to use in their Poole control. The development of these new substitutes has raised other problems of fairness for banks and other financial institutions.
Some have even mumbled about something called "level playing fields."
But with this new found result for r, the monetary authorities believe they have a simplifying (naturally) way out.

They have found that with $M_2$, their new intermediate target will be

\[(1') \quad \beta N_2 + (1-\beta) \bar{I} = 0\]

where $\beta$ is not necessarily equal to the $\alpha$ for $M_1$, but is just as simply determined. They would define $M_2$ to include all these new substitute assets, $\Sigma x_i$, each of which has a demand

\[(4') \quad \bar{x}_i = k_i \bar{I} + u_i,\]

and each of which could have a required reserve ratio of $r_i$. Their proposed operating target will now be

\[(3') \quad \beta D + \sum_{i=1}^{n} r_i \bar{x}_i + \lambda_1 \bar{C} + \lambda_2 \bar{I}.\]

Without reviewing all the simple algebra we can just note that with $\phi_1$ now equal to $rk_d + \Sigma r_i k_i + \lambda_1 k_c + \lambda_2$, with $\phi_2$ now equal to $k_c + k_d + \Sigma k_i$, and with $\phi_3$ now equal to $1-\beta + \beta \phi_2$, the new target will be

\[(8') \quad u_c\left(\beta - \frac{\phi_1}{\phi_1}\right) + u_d\left(\beta - \frac{\phi_4}{\phi_1}\right) + \sum_{i=1}^{n} u_i\left(\beta - \frac{\phi_i r_i}{\phi_1}\right).\]
Now equation \((8')\) is still pleasant, although not quite so simple as equation \((8)\), because by sight it can be made zero if

\[(9') \quad \lambda_1 = r = r_1\]

for all \(r_1\) and if

\[(10') \quad \lambda_2 = r \left( \frac{1-\beta}{\beta} \right).\]

Now previous to their finding that \(r\) can be very low and not lessen the control of money, these results might have been impossible to achieve since it might not be possible to have required reserve ratios the same for all these new substitute assets. But since \(r\) can be very low indeed, there seems to be no reason not to have the same \(r\) for all assets in \(M_2\). This, of course, would have the further advantage of being a "level playing field." They have noted and even emphasized that including an asset in \(M_2\) which has no reserve requirement would not be good because it would enter equation \((8')\) as \(u_j \beta\) and there would be no way to perfectly control their target. Thus \(M_2\) should include only assets which have a (low) reserve requirement and should exclude all others. They are aware that the future may bring the introduction of still more new types of assets which may be substitutes for those assets in \(M_2\), but they feel comfortable in being able to adjust simply to this possibility if it does occur.

So life is simple and pleasant in this world. The monetary authorities will have just enough work to do and it will be evenly
distributed over time, if they adopt their new operating target
technique. They will now be able to meet their intermediate target
completely and as a result their final target will be better met. The
banks will be happy playing on a level field and having lower reserve
requirements. There will be just enough in the way of remaining
arguments to make life interesting. Some old-timers will continue to
insist on controlling only money, others on controlling only interest
rates, and there will be continuing arguments as to whether they should
use $M_1$ or $M_2$ or, in the future, some $M_3$. But such arguments seem just
enough to fill the monetary authorities still substantial breaks during
their working days with exciting conversation.

It has been noted that reports from simple and pleasant worlds
often provide useful insights for our complex and sophisticated one.
Might this be the case here, or can nothing useful be gained? We might
do well to consider the possible effects of several complexities before
making any decisions as to the virtues of using the simple world's
monetary authorities' techniques in our own world.

We might note that in our world we do not quite see reserves and
currency in the hands of the public on a day-to-day basis since we
cannot observe vault cash behavior. In addition our banks do have
excess reserves and do borrow from the Federal Reserve. Changes in
these items on a day-to-day basis, however, are quantitatively very
small and thus should not significantly affect the results. There will
simply be some small remaining uncertainty not present in the simple
world.
In our world economic agents are supposed to be very sophisticated indeed. If the Federal Reserve were to use the simple world's control technique, this would undoubtedly alter the behavior of agents in our world. In particular, the demand functions for the various assets in \( M_1 \) or \( M_2 \) might shift. This would mean that the \( k_i \)'s and the \( u_i \)'s would shift from what they are at present. The Federal Reserve would have to be careful in adjusting to its new policy not to use previous estimates for the \( k_i \)'s and \( u_i \)'s. Once the new policy is in effect, however, there is no reason to believe that behavior will change on a day-to-day basis in reaction to day-to-day Federal Reserve behavior since this behavior will be unknown at the time. Thus sophisticated and forward looking agents will not negate this behavior.

Finally we might note the rather cavalier way in which the interest rate is treated by the simple world's monetary authorities. Since they use the symbol \( \bar{I} \) for changes in the interest rate in their Poole target and as determinants of the demands for all the monetary assets, they must be assuming the same \( \bar{I} \) for both situations. In our more complex world, the demand functions for monetary assets must depend on the structure of interest rates or interest rate differentials as well as the level of some interest rate. This suggests a serious difficulty in using the simple results. However, it should be noted that this can be overcome if each interest rate is a linear function of "the" interest rate, by an appropriate change in the \( k \) variable for each asset.

Although this by no means exhausts the potential problems stemming from the use of the simple model, there seems to be at least a
possibility that yet again a simple world may prove instructive for our own. Any comments?