Squire's theorem for the Rayleigh–Taylor problem with a phase transformation

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We consider the Rayleigh–Taylor problem with a phase transformation. For simplicity, we restrict our attention to base states in which the interface convects with the phases, so that mass exchange between the phases occurs only in response to a disturbance. We find that every unstable three-dimensional disturbance (involving a pair of modes transverse to the interface) is associated with a more unstable two-dimensional disturbance (involving a single mode transverse to the interface) at lower values of the Weber, Froude, Reynolds, Voronkov, and Gurtin numbers. This constitutes an appropriate version of Squire's theorem.

Keywords: Squire's theorem; liquid-liquid phase transformations; dispersion relation.

1. Introduction

Squire (1933) studied the stability of the flow of a viscous fluid between parallel walls subject to three-dimensional disturbances. Based on an analysis of the fourth-order differential equation for the normal perturbation velocity, he concluded that every unstable three-dimensional disturbance is associated with a more unstable two-dimensional modal disturbance at a lower value of the Reynolds number. This result is commonly known as 'Squire's theorem.' The connection between the two- and three-dimensional modes and the associated Reynolds numbers is known as 'Squire's transformation.'

Subsequently, Squire's theorem has been extended to more complicated flow situations. Yih (1955) discussed a more general case in which the density and viscosity are allowed to vary. Pearlstein (1985) considered the variation of temperature and solute concentration. Hesla *et al.* (1986) examined the case of two-stratified, homogeneous, immiscible fluids, with constant surface tension, bounded by two (possibly moving) parallel plates, and generalized their results for flows in unbounded domains and with more than two strata. Yiantsios & Higgins (1988) investigated the linear stability of the plane Poiseuille flow of two superposed fluids with different viscosity, with surface tension taken into account. They concluded that Squire's theorem holds when $n > \sqrt{m}$ but is invalid otherwise, where n is the thickness ratio of the two fluids and m is the viscosity ratio. Moreover, they obtained an extension of Squire's transformation revealing that three-dimensional disturbances have smaller Weber, Froude, and Reynolds numbers than do two-dimensional disturbances. Subbiah & Padmini (1997) then proved the equivalent of Squire's theorem for plane and parallel flows when the compressibility of the fluids are taken into account. In a study of stability of liquid film flowing down a vertical plane, Lin *et al.* (1996) showed that unstable two-dimensional waves can be suppressed by appropriate amplitudes and frequencies of the plate oscillation and that the windows of stability depend on the flow parameters. However, Lin & Chen (1997) later used a numerical approach to demonstrate that three-dimensional disturbances do not always close the stability window opened by two-dimensional disturbances. Lin & Jiang (2002) investigated the reasons underlying the violation of Squire's theorem and concluded that, due to three dimensionality, the increase in the dissipation rate is much larger than that of the Reynolds stress. Tlapa & Bernstein (1970) developed the viscoelastic analogue of Squire's theorem for the steady plane shear flow of an Oldroyd-B fluid, while Ramanan & Graham (2000) generalized that result to a time-dependent inertialess plane shear flow.

For problems involving superposed fluids, Squire's theorem has thus far been considered only for material interfaces. The goal of this work is to establish a form of Squire's theorem for the Rayleigh–Taylor problem that allows for a phase transformation. We approach this goal using the isothermal specialization of a theory, developed by Anderson *et al.* (2006), in which phase interfaces are treated as sharp surfaces of discontinuity in various bulk material properties, are endowed with thermodynamic structure, and are assumed to be capable of sustaining stress. The final governing equations of that theory consist of evolution equations for the bulk phases and the interface. For the bulk phases, the equations are standard and consist of the continuity equation

$$\operatorname{div}\mathbf{u} = 0 \tag{1.1}$$

and the linear momentum balance

$$\varrho\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\operatorname{grad}p_e + \operatorname{div}\mathbf{S}, \tag{1.2}$$

where $\varrho = \varrho^\pm$ is the constant density in the $(\pm)$-phase, p_e is the effective pressure accounting for the potential of the gravitational body force density $-\varrho\mathbf{g}$, and the extra stress in the $(\pm)$-phase is simply

$$\mathbf{S} = 2\mu^\pm\mathbf{D}, \tag{1.3}$$

where $\mu = \mu^\pm$ is the (constant) shear viscosity in the $(\pm)$-phase and $\mathbf{D} = \frac{1}{2}\big(\operatorname{grad}\mathbf{u} + (\operatorname{grad}\mathbf{u})^\top\big)$ is the rate-of-stretch in the bulk phases. By treating the phases as incompressible, we, in effect, model them as large reservoirs with spatially uniform and time-independent densities that are unaffected by mass exchange across the interface. Importantly, the densities of the phases differ. On the interface, the equations consist of the mass balance, the linear momentum balance, and the normal configurational momentum balance. The interfacial mass balance has the form

$$[\![\varrho(V - \mathbf{u}\cdot\mathbf{n})]\!] = 0 \qquad \text{or} \qquad J = \varrho^+(V - \mathbf{u}^+\cdot\mathbf{n}) = \varrho^-(V - \mathbf{u}^-\cdot\mathbf{n}), \tag{1.4}$$

where $[\![\varphi]\!] = \varphi^+ - \varphi^-$ denotes the difference between the interfacial limits of a bulk field in the $(+)$-phase and the $(-)$-phase, $\mathbf{n}$ is the interfacial unit normal directed from the region occupied by $(-)$-phase into the region occupied by the $(+)$-phase, V is the scalar normal velocity of the interface in the direction of $\mathbf{n}$, and J is the mass flux normal to the interface. The interfacial linear momentum balance has the form

$$[\![\mathbf{S}]\!]\mathbf{n} - [\![p_e]\!]\mathbf{n} - J^2[\![1/\varrho]\!]\mathbf{n} = -\gamma K\mathbf{n} - \mathrm{div}_{\mathcal{S}}\mathbb{S}, \tag{1.5}$$

where $\gamma > 0$ is the (constant) interfacial tension, K is the total curvature (i.e., *twice* the mean curvature) of the interface, and the surface extra stress $\mathbb{S}$ is of the classical form (Boussinesq 1913)

$$\mathbb{S} = \lambda(\mathrm{tr}\,\mathbb{D})\mathbb{P} + 2\alpha\mathbb{D}, \tag{1.6}$$

where $\mathbb{P} = \mathbf{1} - \mathbf{n}\otimes\mathbf{n}$ is the interfacial projector, $\mathbb{D} = \mathbb{P}\langle\!\langle\mathbf{D}\rangle\!\rangle\mathbb{P}$ is the interfacial rate-of-stretch (with $\langle\!\langle\varphi\rangle\!\rangle = \frac{1}{2}(\varphi^+ + \varphi^-)$ being the average of the interfacial limits of a bulk field φ in the two phases), and $\lambda + \alpha > 0$ and $\alpha > 0$ are the (constant) dilatational and shear viscosities of the interface. The normal configurational momentum balance has the form

$$\Psi - \mathbf{n}\cdot[\![\mathbf{S}/\varrho]\!]\mathbf{n} + [\![p_e/\varrho]\!] + \tfrac{1}{2}J^2[\![1/\varrho^2]\!] = \langle\!\langle 1/\varrho\rangle\!\rangle\{\kappa V^{\mathrm{mig}} - \beta\Delta_{\mathcal{S}}V^{\mathrm{mig}} + \mathbb{S}:\mathbb{K}\}, \tag{1.7}$$

where Ψ is the (constant) specific free-energy of the (+)-phase relative to that of the (−)-phase, $V^{\mathrm{mig}} = V - \langle\!\langle\mathbf{u}\rangle\!\rangle\cdot\mathbf{n}$ is the migrational velocity of the interface, $\Delta_{\mathcal{S}}$—which is defined for any surface field φ by $\Delta_{\mathcal{S}}\varphi = \mathrm{div}_{\mathcal{S}}(\mathrm{grad}_{\mathcal{S}}\varphi)$—is the *Laplace–Beltrami* operator on the interface, $\mathbb{K}$ is the curvature tensor of the interface, and $\kappa > 0$ and $\beta > 0$ are the (constant) migrational viscosities of the interface. Whereas κ is associated with viscous drag that impedes the local motion of the interface normal to itself, β is associated with viscous drag that impedes the local reorientation of the interface. For further discussion of these viscosities, see Anderson *et al.* (2006)

For a viscous fluid ($\mu^+ \neq 0$ and $\mu^- \neq 0$), the interface conditions (1.4)–(1.7) are supplemented by a kinematical condition,

$$\mathbb{P}[\![\mathbf{u}]\!] = \mathbf{0}, \tag{1.8}$$

enforcing the requirement that the phases not slip with respect to one another.

Chen & Fried (2006) used the theory of Anderson *et al.* (2006) to study the stability of a base state involving a planar interface that convects with the phases at constant velocity subject to a disturbance involving only a single transverse sinusoidal component with wave number k (Figure 1). Assuming that the phases have identical kinematic viscosity

$$\nu = \frac{\mu^+}{\varrho^+} = \frac{\mu^-}{\varrho^-} \tag{1.9}$$

and supposing that there is no mass transport across the interface in the base state,

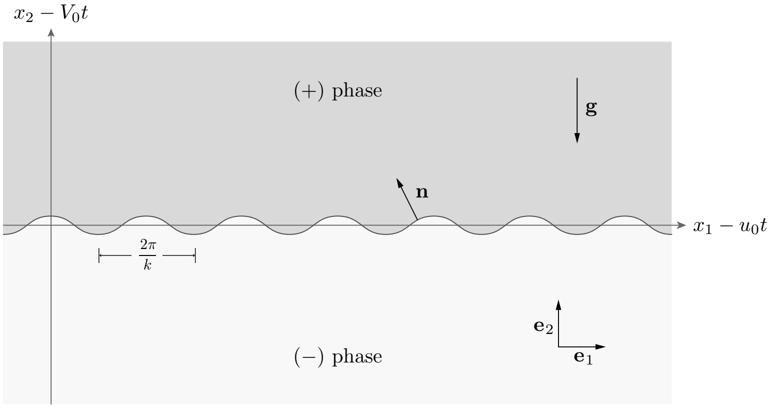

Figure 1. Schematic of the phases and the disturbed interface for the case of a perturbation involving only a single transverse component with wave number k.

this problem leads to a dispersion relation of the form

$$\begin{aligned}
&\frac{2\omega^2 q}{|\mathrm{At}|}\left\{\omega^2 + \frac{4|\mathrm{At}|k^2\omega}{\mathrm{Re}} - \frac{4\mathrm{At}^2k^3(q-k)}{\mathrm{Re}^2} + \left(\frac{k^2}{2\mathrm{We}} - \mathrm{At}\right)\frac{k}{\mathrm{Fr}}\right\} \\
&+ \frac{2\mathrm{Bo}k^2q\omega}{\mathrm{Re}}\left\{q\omega^2 + \left(\frac{k^2}{2\mathrm{We}} - \mathrm{At}\right)\frac{k(q-k)}{\mathrm{Fr}}\right\} - \left\{\frac{1}{\mathrm{Vo}} + \frac{k^2}{\mathrm{Gu}}\right\}\left\{k - \frac{q}{\mathrm{At}^2}\right\}k\omega^3 \\
&+ \left\{\frac{1}{\mathrm{Vo}} + \frac{k^2}{\mathrm{Gu}}\right\}\left\{\frac{4|\mathrm{At}|k}{\mathrm{Re}}\left(\omega - \frac{|\mathrm{At}|k(q-k)}{\mathrm{Re}}\right) + \frac{k^2}{2\mathrm{We}\mathrm{Fr}} - \frac{\mathrm{At}}{\mathrm{Fr}}\right\}k^2\omega(q-k) \\
&+ \frac{\mathrm{Bo}k^3(q-k)}{\mathrm{Re}}\left\{\frac{1}{\mathrm{Vo}} + \frac{k^2}{\mathrm{Gu}}\right\}\left\{\frac{2q\omega^2}{|\mathrm{At}|} + \left(\frac{k^2}{2\mathrm{We}} - \mathrm{At}\right)\frac{|\mathrm{At}|k(q-k)}{\mathrm{Fr}}\right\} = 0, \qquad (1.10)
\end{aligned}$$

where ω is the growth rate of the disturbance, q is defined by

$$q = \sqrt{k^2 + \frac{\mathrm{Re}\,\omega}{|\mathrm{At}|}}, \qquad (1.11)$$

and where At, We, Fr, Re, Bo, Vo, and Gu denote the Atwood, Weber, Froude, Reynolds, Boussinesq, Voronkov, and Gurtin numbers, which, given a characteristic length L and a characteristic time T, are defined by

$$\left.\begin{aligned}
&\mathrm{At} = \frac{[\![\varrho]\!]}{2\langle\!\langle\varrho\rangle\!\rangle}, \qquad \mathrm{We} = \frac{\langle\!\langle\varrho\rangle\!\rangle g L^2}{\gamma}, \qquad \mathrm{Fr} = \frac{L}{gT^2}, \qquad \mathrm{Re} = \frac{L^2}{\nu T}, \\
&\mathrm{Bo} = \frac{\lambda + 2\alpha}{2\langle\!\langle\varrho\rangle\!\rangle\nu L}, \qquad \mathrm{Vo} = \frac{L}{\langle\!\langle 1/\varrho\rangle\!\rangle\kappa T}, \qquad \text{and} \qquad \mathrm{Gu} = \frac{L^3}{\langle\!\langle 1/\varrho\rangle\!\rangle\beta T}.
\end{aligned}\right\} \qquad (1.12)$$

Analysis of (1.10) shows that, as with the conventional Rayleigh–Taylor problem, instability is possible only when the phase with the higher density is above that with the lower density. Of the dimensionless numbers introduced in (1.12), only the Atwood number and and the Weber number influence the value of the cut-off wave number $k_c = \sqrt{2|\mathrm{At}|\mathrm{We}}$ above which perturbations are stable. Further, Chen & Fried (2006) found that increases of the Froude and Boussinesq numbers are stabilizing, while increases of the Atwood, Weber, and Reynolds numbers are destabilizing. The roles of migrational viscosities of the interface, or the associated dimensionless parameters, the Voronkov number and the Gurtin number, are very similar to that of the conventional viscosities as embodied by the Boussinesq number.

Here, we extend the two-dimensional analysis of Chen & Fried (2006) to consider disturbances of the base state involving two transverse components with dimensionless wave numbers k_1 and k_3 and associated length scales L_1 and L_3. This leads to the consideration of nine amplitude equations. However, we find that those equations can be reduced to a system of five equations that, save for the appearance of an effective wave number $\tilde{k} = \tilde{L}\sqrt{k_1^2/L_1^2 + k_3^2/L_3^2}$ and an effective length scale $\tilde{L} = \sqrt{L_1^2 + L_3^2}$, is identical to the amplitude equations obtained by Chen & Fried (2006) for a disturbance involving only a single transverse component. It follows that two-dimensional disturbances of the base state are always more unstable than three-dimensional disturbances. In concert with observations concerning the roles of the Weber, Froude, Reynolds, Boussinesq, Voronkov, and Gurtin numbers, this implies an appropriate version of Squire's theorem.

2. Base state

Consider a stationary base state in which the time-dependent regions $\{\mathbf{x} : x_2 < V_0 t\}$ and $\{\mathbf{x} : x_2 > V_0 t\}$ are occupied by $(-)$-phase and the $(+)$-phase, respectively. The interface then corresponds to the time-dependent surface $\{\mathbf{x} : x_2 = V_0 t\}$. Suppose that the velocity and pressure in the base state are constant and given by $\mathbf{u}_0^{\pm} = u_0\mathbf{e}_1 + v_0^{\pm}\mathbf{e}_2 + w_0\mathbf{e}_3$ and $p_0^{\pm}$, with

$$v_0^- = v_0^+ = V_0. \tag{2.1}$$

As a consequence of (2.1), the base state does not involve a transfer of mass between the phases. However, any perturbation of the base state generally results in an exchange of mass.

For such a base state, the bulk equations (1.1) and (1.2) hold trivially, and the interfacial equations (1.4), (1.5), and (1.7) require that the parameter $p_0^{\pm}$ describing the base state be consistent with the equations

$$[\![p_e]\!] = 0 \qquad \text{and} \qquad \Psi + [\![p_e/\varrho]\!] = 0. \tag{2.2}$$

3. Perturbated state

Following Chen & Fried (2006), we assume that the phases have identical kinematic viscosity ν; cf. (1.9).

(a) *Bulk disturbance*

For an infinitesimal sinusoidal increment, indicated by the subscript 1, to the base state, the velocity and pressure in the $(\pm)$-phase become $\mathbf{u}^{\pm} = \mathbf{u}_0^{\pm} + \epsilon\mathbf{u}_1^{\pm}$ and $p^{\pm} = p_0^{\pm} + \epsilon p_1^{\pm}$, with $\epsilon \ll 1$ and

$$(u_1^{\pm}, v_1^{\pm}, w_1^{\pm}, p_1^{\pm})(x_1, x_2, x_3, t) = (u_1^{\pm}, v_1^{\pm}, p_1^{\pm})(y)\exp\left\{ik_1 x + ik_3 z + \frac{\omega t}{|\mathrm{At}|T}\right\}, \tag{3.1}$$

where

$$x = \frac{x_1 - u_0 t}{L_1}, \qquad z = \frac{x_3 - w_0 t}{L_3}, \qquad y = \frac{x_2 - V_0 t}{\tilde{L}}, \tag{3.2}$$

$k_1 > 0$ and $k_3 > 0$ are the dimensionless wave-numbers of the perturbation in x_1 and x_3 directions respectively, ω is the dimensionless growth-rate of the perturbation,

$$\tilde{L} = \sqrt{L_1^2 + L_3^2}, \tag{3.3}$$

L_1 and L_3 are characteristic length scales in the x_1 and x_3 directions, and T is a characteristic time scale.

Inserting (3.1) into the continuity equation (1.1) and the momentum balance (1.2), neglecting terms of $O(\epsilon^2)$, and cancelling a common factor of $\exp(ik_1 x + ik_3 z + \omega t/|\mathrm{At}|T)$ from each term, we arrive at a system of four ordinary differential equations within each phase. On writing $D = \mathrm{d}/\mathrm{d}y$, that system takes the form

$$\left.\begin{aligned} \frac{ik_1}{L_1}u_1^\pm + \frac{D}{\tilde{L}}v_1^\pm + \frac{ik_3}{L_3}w_1^\pm &= 0, \\ \varrho^\pm\Big\{\frac{\nu}{\tilde{L}^2}(D^2 - \tilde{k}^2) - \frac{\omega}{|\mathrm{At}|T}\Big\}u_1^\pm &= \frac{ik_1}{L_1}p_1^\pm, \\ \varrho^\pm\Big\{\frac{\nu}{\tilde{L}^2}(D^2 - \tilde{k}^2) - \frac{\omega}{|\mathrm{At}|T}\Big\}v_1^\pm &= \frac{D}{\tilde{L}}p_1^\pm, \\ \varrho^\pm\Big\{\frac{\nu}{\tilde{L}^2}(D^2 - \tilde{k}^2) - \frac{\omega}{|\mathrm{At}|T}\Big\}w_1^\pm &= \frac{ik_3}{L_3}p_1^\pm, \end{aligned}\right\} \tag{3.4}$$

with

$$\tilde{k} = \tilde{L}\sqrt{\frac{k_1^2}{L_1^2} + \frac{k_3^2}{L_3^2}}. \tag{3.5}$$

Eliminating $p_1^\pm, u_1^\pm$ and $w_1^\pm$ from (3.4) yields an ordinary differential equation,

$$(D^2 - \tilde{k}^2)(D^2 - \tilde{q}^2)v_1^\pm = 0, \tag{3.6}$$

for $v_1^\pm$, with

$$\tilde{q} = \sqrt{\tilde{k}^2 + \frac{\tilde{L}^2\omega}{|\mathrm{At}|T\nu}}. \tag{3.7}$$

To rule out infinite fluid velocities in the far field, we choose v_1 to be of the form

$$v_1(y) = \begin{cases} A^+\exp(-\tilde{k}y) + B^+\exp(-\tilde{q}y), & y > 0, \\ \\ A^-\exp(\tilde{k}y) + B^-\exp(\tilde{q}y), & y < 0, \end{cases} \tag{3.8}$$

where $A^{\pm}$ and $B^{\pm}$ are unknown amplitudes. In view of (3.8) and $(3.4)_{2-4}$ yield expressions

$$
\left.
\begin{aligned}
u_1(y) &= \begin{cases} -i\dfrac{\tilde{L}k_1}{L_1\tilde{k}}A^+\exp(-\tilde{k}y) - iC^+\exp(-\tilde{q}y), & y>0,\\[2ex] i\dfrac{\tilde{L}k_1}{L_1\tilde{k}}A^-\exp(\tilde{k}y) + iC^-\exp(\tilde{q}y), & y<0,\end{cases}\\[2ex]
w_1(y) &= \begin{cases} -i\dfrac{\tilde{L}k_3}{L_3\tilde{k}}A^+\exp(-\tilde{k}y) - iE^+\exp(-\tilde{q}y), & y>0,\\[2ex] i\dfrac{\tilde{L}k_3}{L_3\tilde{k}}A^-\exp(\tilde{k}y) + iE^-\exp(\tilde{q}y), & y<0,\end{cases}
\end{aligned}
\right\} \tag{3.9}
$$

and

$$
p_1(y) = \begin{cases} \dfrac{A^+\varrho^+\tilde{L}\omega}{|\mathrm{At}|T\tilde{k}}\exp(-\tilde{k}y), & y>0,\\[2ex] -\dfrac{A^-\varrho^-\tilde{L}\omega}{|\mathrm{At}|T\tilde{k}}\exp(\tilde{k}y), & y<0,\end{cases} \tag{3.10}
$$

for u_1, w_1, and p_1 involving additional unknown amplitudes $C^{\pm}$ and $E^{\pm}$. Furthermore, the equation $(3.4)_1$ of continuity gives

$$
\frac{k_1}{L_1}C^+ + \frac{k_3}{L_3}E^+ - \frac{\tilde{q}}{\tilde{L}}B^+ = 0, \tag{3.11}
$$

and

$$
\frac{k_1}{L_1}C^- + \frac{k_3}{L_3}E^- - \frac{\tilde{q}}{\tilde{L}}B^- = 0. \tag{3.12}
$$

In contrast to the two-dimensional analysis conducted by Chen & Fried (2006), in which the bulk portion of the problem involves only four perturbation amplitudes, eight bulk amplitudes enter the three-dimensional problem performed here.

(*b*) *Interfacial disturbance*

We assume that the bulk disturbance is accompanied by an infinitesimal interfacial disturbance of the form $x_2 = V_0 t + \epsilon F(x_1, x_3, t)$. Direct calculations then show that

$$
\left.
\begin{aligned}
\mathbf{n} &\sim -\epsilon\frac{\partial F}{\partial x_1}\mathbf{e}_1 + \mathbf{e}_2 - \epsilon\frac{\partial F}{\partial x_3}\mathbf{e}_3,\\
\mathbb{K} &\sim \epsilon\,\frac{\partial^2 F}{\partial x_1^2}\mathbf{e}_1\otimes\mathbf{e}_1 + \epsilon\,\frac{\partial^2 F}{\partial x_1 x_3}(\mathbf{e}_1\otimes\mathbf{e}_3 + \mathbf{e}_3\otimes\mathbf{e}_1) + \epsilon\,\frac{\partial^2 F}{\partial x_3^2}\mathbf{e}_3\otimes\mathbf{e}_3,\\
K &\sim \epsilon\,\frac{\partial^2 F}{\partial x_1^2} + \epsilon\,\frac{\partial^2 F}{\partial x_3^2}, \qquad V \sim V_0 + \epsilon\,\frac{\partial F}{\partial t},\\
J &\sim J_0 + \epsilon\varrho^{\pm}\left\{\frac{\partial F}{\partial t} + u_0\,\frac{\partial F}{\partial x_1} + w_0\,\frac{\partial F}{\partial x_3} - v_1^{\pm}\right\},\\
V^{\mathrm{mig}} &\sim V_0^{\mathrm{mig}} + \epsilon\left\{\frac{\partial F}{\partial t} + u_0\,\frac{\partial F}{\partial x_1} + w_0\,\frac{\partial F}{\partial x_3} - \langle\!\langle v_1\rangle\!\rangle\right\}.
\end{aligned}
\right\} \tag{3.13}
$$

Consistent with the form (3.1) of the bulk disturbance, we take

$$F(x_1,x_3,t)=H\exp\left\{ik_1x+ik_3z+\frac{\omega t}{|\mathrm{At}|T}\right\},\tag{3.14}$$

where x and z are defined in $(3.2)_{1,2}$ and H is an additional unknown amplitude.

(*c*) *Amplitude equations*

The bulk and interfacial disturbances involve nine unknown amplitudes $A^{\pm}$, $B^{\pm}$, $C^{\pm}$, $E^{\pm}$, and H. However, we have yet to make use of the interface conditions (1.4)–(1.8) expressing mass balance, linear momentum balance, normal configurational momentum balance, and the no-slip condition on the interface. Inserting the assumed forms for the velocity and the interface profile in the mass balance (1.4), using (3.8) and (3.9), and invoking the condition (2.1) in the base state, dropping terms of $O(\epsilon^2)$, and cancelling a common factor of $\exp(ik_1x+ik_3z+\omega t/|\mathrm{At}|T)$ from each term, we obtain

$$\varrho^+(A^++B^+)-\varrho^-(A^-+B^-)=\frac{[\![\varrho]\!]\omega}{|\mathrm{At}|T}H.\tag{3.15}$$

In contrast to the two-dimensional case considered by Chen & Fried (2006), the linear momentum balance (1.5) now gives rise to amplitude equations in the $\mathbf{e}_1,\mathbf{e}_2$, and $\mathbf{e}_3$ directions. In the $\mathbf{e}_1$-direction, (1.5) yields

$$\begin{aligned}\left\{2\varrho^+\frac{k_1}{L_1}+\frac{(\alpha+\frac{1}{2}\lambda)k_1\tilde{k}}{\nu L_1\tilde{L}}\right\}A^+ +\varrho^+\frac{k_1}{L_1}B^+ +\left\{\varrho^+\frac{\tilde{q}}{\tilde{L}}+\frac{(\alpha+\frac{1}{2}\lambda)k_1^2}{\nu L_1^2}+\frac{\alpha k_3^2}{2\nu L_3^2}\right\}C^+\\ +\frac{(\alpha+\lambda)k_1k_3}{2\nu L_1L_3}E^+ -\left\{2\varrho^-\frac{k_1}{L_1}+\frac{(\alpha+\frac{1}{2}\lambda)k_1\tilde{k}}{\nu L_1\tilde{L}}\right\}A^- -\varrho^-\frac{k_1}{L_1}B^-\\ -\left\{\varrho^-\frac{\tilde{q}}{\tilde{L}}+\frac{(\alpha+\frac{1}{2}\lambda)k_1^2}{\nu L_1^2}+\frac{\alpha k_3^2}{2\nu L_3^2}\right\}C^- -\frac{(\alpha+\lambda)k_1k_3}{2\nu L_1L_3}E^-=0.\end{aligned}\tag{3.16}$$

In the $\mathbf{e}_2$-direction, (1.5) yields

$$\left\{\frac{2\nu\tilde{k}^2}{\tilde{L}^2}+\frac{\omega}{|\mathrm{At}|T}\right\}(\varrho^+A^++\varrho^-A^-)+\frac{2\nu\tilde{k}\tilde{q}}{\tilde{L}^2}(\varrho^+B^++\varrho^-B^-)=\frac{\tilde{k}}{\tilde{L}}\left\{[\![\varrho]\!]g-\frac{2\gamma\tilde{k}^2}{\tilde{L}^2}\right\}H.\tag{3.17}$$

Further, in the $\mathbf{e}_3$-direction, (1.5) yields

$$\begin{aligned}\left\{2\varrho^+\frac{k_3}{L_3}+\frac{(\alpha+\frac{1}{2}\lambda)\tilde{k}k_3}{\nu\tilde{L}L_3}\right\}A^+ +\varrho^+\frac{k_3}{L_3}B^+ +\frac{(\alpha+\lambda)k_1k_3}{2\nu L_1L_3}C^+\\ +\left\{\varrho^+\frac{\tilde{q}}{\tilde{L}}+\frac{(\alpha+\frac{1}{2}\lambda)k_3^2}{\nu L_3^2}+\frac{\alpha k_1^2}{2\nu L_1^2}\right\}E^+ -\left\{2\varrho^-\frac{k_3}{L_3}+\frac{(\alpha+\frac{1}{2}\lambda)\tilde{k}k_3}{\nu\tilde{L}L_3}\right\}A^- -\varrho^-\frac{k_3}{L_3}B^-\\ -\frac{(\alpha+\lambda)k_1k_3}{2\nu L_1L_3}C^- -\left\{\varrho^-\frac{\tilde{q}}{\tilde{L}}+\frac{(\alpha+\frac{1}{2}\lambda)k_3^2}{\nu L_3^2}+\frac{\alpha k_1^2}{2\nu L_1^2}\right\}E^-=0.\end{aligned}\tag{3.18}$$

Next, the normal configurational momentum balance (1.7) yields

$$\left\{\frac{2\nu\tilde{k}^2}{\tilde{L}^2}+\left(\kappa+\frac{\beta\tilde{k}^2}{\tilde{L}^2}\right)\frac{\langle\!\langle 1/\varrho\rangle\!\rangle\tilde{k}}{2\tilde{L}}+\frac{\omega}{|\mathrm{At}|T}\right\}(A^+ + A^-)$$
$$+\left\{\frac{2\nu\tilde{k}\tilde{q}}{\tilde{L}^2}+\left(\kappa+\frac{\beta\tilde{k}^2}{\tilde{L}^2}\right)\frac{\langle\!\langle 1/\varrho\rangle\!\rangle\tilde{k}}{2\tilde{L}}\right\}(B^+ + B^-)=\left\{\kappa+\frac{\beta\tilde{k}^2}{\tilde{L}^2}\right\}\frac{\langle\!\langle 1/\varrho\rangle\!\rangle\tilde{k}\omega}{|\mathrm{At}|\tilde{L}T}H. \quad (3.19)$$

Finally, the no-slip condition (1.8) yields

$$\frac{k_1}{L_1}A^+ + \frac{\tilde{k}}{\tilde{L}}C^+ + \frac{k_1}{L_1}A^- + \frac{\tilde{k}}{\tilde{L}}C^- = 0, \quad (3.20)$$

and

$$\frac{k_3}{L_3}A^+ + \frac{\tilde{k}}{\tilde{L}}E^+ + \frac{k_3}{L_3}A^- + \frac{\tilde{k}}{\tilde{L}}E^- = 0. \quad (3.21)$$

4. Comparison to the results of the two-dimensional analysis

In this section, we manipulate the nine amplitude equations (3.11), (3.12) and (3.15)–(3.21) to obtain the amplitude equations obtained in the two-dimensional analysis of Chen & Fried (2006).

We start with the amplitude equations obtained by linear momentum balance in the $\mathbf{e}_1$ and $\mathbf{e}_3$ directions. Noticing the symmetry in the $\mathbf{e}_1$ and $\mathbf{e}_3$-directions, and bearing in mind from (3.5) that $k_1^2/L_1^2 + k_3^2/L_3^2 = \tilde{k}^2/\tilde{L}^2$, direct calculation shows that the sum of (3.16) multiplied by k_1/L_1 and (3.18) multiplied by k_3/L_3 can be reduced to

$$\left\{2\varrho^+\tilde{k}^2+\frac{(\alpha+\frac{1}{2}\lambda)\tilde{k}^3}{\nu\tilde{L}}\right\}A^+ + \varrho^+\tilde{k}^2B^+ - \left\{2\varrho^-\tilde{k}^2+\frac{(\alpha+\frac{1}{2}\lambda)\tilde{k}^3}{\nu\tilde{L}}\right\}A^- - \varrho^-\tilde{k}^2B^-$$
$$+\tilde{q}\tilde{L}\Big\{\frac{k_1}{L_1}(\varrho^+C^+ - \varrho^-C^-)+\frac{k_3}{L_3}(\varrho^+E^+ - \varrho^-E^-)\Big\}$$
$$+\frac{(\alpha+\frac{1}{2}\lambda)\tilde{k}^2}{\nu}\Big\{\frac{k_1}{L_1}(C^+ - C^-)+\frac{k_3}{L_3}(E^+ - E^-)\Big\}=0. \quad (4.1)$$

(Inspection of (4.1) shows that, contrary to an expectation voiced by Chen & Fried (2006), the interfacial dilatational and shear viscosities enter the three-dimensional problem in the same combination, namely via $\alpha + \frac{1}{2}\lambda$, that they enter the two-dimensional problem.) Similarly, adding (3.20) multiplied by k_1/L_1 to (3.21) multiplied by k_3/L_3 and cancelling a common factor of $\tilde{k}/\tilde{L}$ yields

$$\frac{\tilde{k}}{\tilde{L}}(A^+ + A^-)+\frac{k_1}{L_1}(C^+ + C^-)+\frac{k_3}{L_3}(E^+ + E^-)=0. \quad (4.2)$$

Further, suitable combinations of (3.11) and (3.12) yield

$$\left.\begin{aligned} \frac{k_1}{L_1}(\varrho^+C^+ - \varrho^-C^-) + \frac{k_3}{L_3}(\varrho^+E^+ - \varrho^-E^-) &= \frac{\tilde{q}}{\tilde{L}}(\varrho^+B^+ - \varrho^-B^-), \\ \frac{k_1}{L_1}(C^+ - C^-) + \frac{k_3}{L_3}(E^+ - E^-) &= \frac{\tilde{q}}{\tilde{L}}(B^+ - B^-), \\ \frac{k_1}{L_1}(C^+ + C^-) + \frac{k_3}{L_3}(E^+ + E^-) &= \frac{\tilde{q}}{\tilde{L}}(B^+ + B^-). \end{aligned}\right\} \tag{4.3}$$

Substituting $(4.3)_1$ and $(4.3)_2$ into (4.1), we now arrive at

$$\begin{aligned} &\left\{2\varrho^+\tilde{k}^2 + \frac{(\alpha+\frac{1}{2}\lambda)\tilde{k}^3}{\nu\tilde{L}}\right\}A^+ + \left\{\varrho^+(\tilde{k}^2+\tilde{q}^2) + \frac{(\alpha+\frac{1}{2}\lambda)\tilde{k}^2\tilde{q}}{\nu\tilde{L}}\right\}B^+ \\ &\quad - \left\{2\varrho^-\tilde{k}^2 + \frac{(\alpha+\frac{1}{2}\lambda)\tilde{k}^3}{\nu\tilde{L}}\right\}A^- - \left\{\varrho^-(\tilde{k}^2+\tilde{q}^2) + \frac{(\alpha+\frac{1}{2}\lambda)\tilde{k}^2\tilde{q}}{\nu\tilde{L}}\right\}B^- = 0. \end{aligned} \tag{4.4}$$

Then using $(4.3)_3$ for (4.2), we obtain

$$\tilde{k}A^+ + \tilde{q}B^+ + \tilde{k}A^- + \tilde{q}B^- = 0. \tag{4.5}$$

The five amplitude equations (3.15), (3.17), (3.19), (4.4), and (4.5) contain only five amplitudes $A^\pm, B^\pm$ and H. Necessary and sufficient for the system of amplitude equations to possess a nontrivial solution is the requirement that the determinant of the relevant coefficient matrix vanish. This requirement generates a dispersion relation. On replacing $\tilde{L}$ with L and $\tilde{k}$ with k in (3.15), (3.17), (3.19), (4.4), and (4.5) (so that $\tilde{q}$ reduces to q as defined in (1.11)), we find that (3.15), (3.17), (3.19), (4.4), and (4.5) are identical to the five amplitude equations obtained in the two-dimensional analysis by Chen & Fried (2006). Conversely, replacing L, k, and q in (1.10) with $\tilde{L}$, $\tilde{k}$, and $\tilde{q}$ yields the dispersion relation for the three-dimensional perturbation problem considered here. Hence, neither of the individual transverse wave numbers k_1 and k_3 associated with the perturbation plays an individual role in the growth rate of the disturbance. Moreover, for three-dimensional disturbances with $\tilde{L}^2 = L_1^2 + L_3^2$, the parameters defined in $(1.12)_{2\text{–}7}$ are replaced by

$$\left.\begin{aligned} \widetilde{\mathrm{We}} &= \frac{\langle\!\langle\varrho\rangle\!\rangle g\tilde{L}^2}{\gamma}, &\quad \widetilde{\mathrm{Fr}} &= \frac{\tilde{L}}{gT^2}, &\quad \widetilde{\mathrm{Re}} &= \frac{\tilde{L}^2}{\nu T}, \\ \widetilde{\mathrm{Bo}} &= \frac{\lambda+2\alpha}{2\langle\!\langle\varrho\rangle\!\rangle\nu\tilde{L}}, &\quad \widetilde{\mathrm{Vo}} &= \frac{\tilde{L}}{\langle\!\langle 1/\varrho\rangle\!\rangle\kappa T}, &\quad \widetilde{\mathrm{Gu}} &= \frac{\tilde{L}^3}{\langle\!\langle 1/\varrho\rangle\!\rangle\beta T}. \end{aligned}\right\} \tag{4.6}$$

We therefore conclude that, for a base state of the sort considered here, every unstable three-dimensional disturbance (involving transverse dimensionless wave numbers k_1 and k_3 and length scales L_1 and L_3) is associated with a more unstable two-dimensional disturbance (involving a transverse dimensionless wave number $\tilde{k} = \tilde{L}\sqrt{k_1^2/L_1^2 + k_3^2/L_3^2}$ and a length scale $\tilde{L} = \sqrt{L_1^2 + L_3^2}$) at lower values of the Weber, Froude, Reynolds, Voronkov, and Gurtin numbers. Most importantly, in view of the definition $(4.6)_4$ of the Boussinesq number, it follows that two-dimensional disturbances are always more unstable than three-dimensional disturbances. Appropriate versions of Squire's theorem and Squire's transformation therefore hold for the problem considered here.

For completeness, we now summarize the main conclusions of the two-dimensional analysis. As with the conventional Rayleigh–Taylor problem, instability is possible only when the phase with the higher density is above that with the lower density. Of the dimensionless numbers introduced in (1.12), only At and We influence the value of the cut-off wave number $k_c = \sqrt{2|\mathrm{At}|\mathrm{We}}$ above which perturbations are stable. Additionally, whereas increases of Fr and Bo are stabilizing, increases of At, We, and Re are destabilizing. It follows from (1.10) that increases of the density difference between the phases or the gravitational force lead to a more unstable system and, furthermore, that the growth rates of unstable modes are diminished by the dissipative character of various viscosities entering the theory—that is, the shear viscosities of the phases, the dilatational and shear viscosities of the interface, and the migrational viscosities of the interface.

In closing, we emphasize that we have shown only that a version of Squire's theorem holds when mass transport between the phases is absent in the base state. We leave the question of how such transport may influence the stability properties of the system for a future investigation.

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