WAVE PROPAGATION IN ONE DIMENSIONAL CONFINED GRANULAR MEDIA

BY

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THESIS

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ABSTRACT

Ordered granular media, such as a one dimensional aligned chain of spherical particles, have unique dynamic properties arising from the contact interaction of their individual grain constituents. A primary result of this contact, the formation of solitary waves in dynamically loaded one dimensional granular arrays, has been studied at great length. This thesis is the first to consider the effects of different types of lateral confinement on the dynamic response of such one dimensional granular chains. Chains of metallic beads are placed under lateral confinement in one of two ways: (a) by having a metal tube with a diameter mismatch shrink fit around the chain and (b) by being embedded in a geopolymer matrix. A geopolymer is a room temperature curable ceramic based on earth minerals. Since the homogeneous geopolymer response is not well characterized, initially in this work a potassium-based geopolymer developed at the Department of Materials Science at the University of Illinois at Urbana-Champaign was examined to determine its dynamic properties. The dynamic response and nonlinear properties of the granular chains either laterally confined or embedded in a geopolymer were then investigated using a modified Split Hopkinson Pressure bar (SHPB). Although a dependence on granular chain length was seen, it was found that confinement does not significantly affect the primary pulse caused by the solitary wave as it passes through the chain, at least for chains exceeding 5-6 grains in length (the length necessary for the formation of a solitary wave in an unconfined chain). However, the “late-time” response, i.e. the force transmitted after the primary pulse, was seen to be strongly affected by the presence of confinement. This is explained by the alteration of the frictional characteristics on the lateral surfaces of the one dimensional bead array. In addition, curing conditions of the matrix were also seen to affect the chain response.
ACKNOWLEDGMENTS

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CHAPTER 1: INTRODUCTION

Granular chains, one dimensional ordered systems of contacting spheres, exhibit some intriguing properties in regards to stress wave mitigation and wave (re)direction when they are subjected to dynamic loading. Extensive research work has been done, by Nesterenko (1983; 1984), Lazaridi and Nesterenko (1985), Shukla et al., (1990; 1992), Shukla (1991), Coste et al. (1997), Daraio et al. (2006) among many others, to examine the effects of stress waves in ordered granular media. The unique properties of ordered granular media arise because of the interaction between individual grains through an inherently nonlinear contact mechanism. The contact points between grains can be approximated, at least in the elastic case, by the Hertz model for non-adhesive contact between two spheres. The Hertz contact model (Hertz, 1882; Johnson, 1985) makes a number of assumptions in its derivation, namely that:

- The strains are within the elastic regime,
- The radius of contact between the spheres is much less than the radius of either sphere,
- The surfaces are continuous and initially non-conforming,
- The surfaces are frictionless.

For the case of elastic contact between two identical spheres, the resulting force-displacement relationship is strongly nonlinear and is given by,

$$d = \left( \frac{9F^2}{16RE^2} \right)^{1/3}$$

(1.1)
where $d$ is the indentation depth at a given force $F$, $R$ is the sphere radius and $E$ the sphere elastic modulus. When the distance between the centers of the spheres becomes greater than the sum of the radii of the spheres, they are no longer in contact and there is no force generated. For granular media, this means that they can withstand compressive loads, but have no tensile strength. Schematically this highly nonlinear interaction is shown in Figure 1.1(a).

![Figure 1.1](image)

**Figure 1.1** (a) Plot illustrating the nonlinear relationship between force and displacement. (b) Illustration of contacting process and definition of $d$ and $\delta$ (from On (2011)).

It should be noted that the Hertzian model assumes purely elastic behavior, while granular media in realistic applications can be loaded with forces extending well into the plastic regime. Therefore, some recent efforts (e.g., On (2011), Wang et al. (2012)), have began studying the plastic contact response of granular media as well.

A unique property of granular media, which is a consequence of Equation (1.1), is the formation of a solitary wave, or soliton, in them when they are loaded dynamically. A solitary
wave, discovered by John Scott Russell in 1834, is a characteristic pulse that maintains its shape and speed as it travels through a medium. Both the shape and the pulse depend on the chain geometry and maximum force rather than the details of the applied loading. Research has been performed to examine solitary waves as they travel through granular media, and attempts have been made to attenuate or control them. In addition to one dimensional systems, experiments and analysis have been performed on branching chains of granular media and even two dimensional arrays, Figure 1.2 presents illustrations from the work of other researchers studying these topics.

Figure 1.2 Example of (a) branched one dimensional granular chains studied by Ngo et al. (2012), (b) two dimensional system studied by Awasthi et al. (2012)

However, in order to obtain practical material designs that exploit the solitary wave properties of granular media, complex bead arrangements will be required. This may necessitate a two dimensional or even three dimensional constraint of the granular beads. One way of accomplishing this is to embed a chain or array of connecting beads in some type of matrix material. In such a case we could produce much more complex granular systems involving
networks of beads, an example of which is shown schematically in Figure 1.3, that may be optimized for particular wave mitigation applications. Another way would be to apply lateral confinement, as opposed to an all-around matrix. In this work we would like to study both these situations since they could affect the development of solitary waves generated in the ordered granular chains.

Figure 1.3 Conceptual example of network granular medium for targeted energy transfer. One option for manufacturing such a structure would be to embed the granular network in a matrix material.

The introduction of new variables – the matrix material and/or the lateral confining pressure – may have drastic effects on the already observed response of granular chains – namely the development of solitary waves. For example, the presence of a matrix surrounding the grains may potentially amplify or suppress the unique properties of granular chains. In part, differences are expected to occur because in addition to the granular bead contact points, load transfer along the chain could occur through the matrix as well. Therefore it is of great interest to study the
influence of a matrix material and/or lateral confinement on the development of solitary waves in one dimensional granular media.

The objectives of the present thesis are, therefore, to:

- Compare wave propagation in unconfined one dimensional granular chains (our benchmark state for this work) to that in chains laterally confined by a shrink-fit metallic tube;
- Investigate the properties of realistic matrix materials that are based on ceramics, such as geopolymers;
- Study the waves developed in one dimensional granular chains fully embedded in a geopolymer matrix, and specifically answer the question of whether solitary waves do develop in such a case, and how they compare to solitary waves in unconstrained systems.
CHAPTER 2: EXPERIMENTAL METHODS

Experiments were performed to evaluate the mechanical properties of 1D chains of beads, geopolymer ceramic, beads embedded in geopolymer, and beads confined by aluminum tubes. The pure geopolymer was evaluated using quasi-static and ultrasonic testing. One dimensional chains of beads embedded in various media, or laterally confined in a surrounding tube, were tested using a Split Hopkinson Pressure Bar (SHPB).

2.1 Quasi-static Testing

Disks with a diameter of 1.57 cm (0.62 inches) composed of potassium-based geopolymer with ten weight percent chopped carbon fiber were tested using an MTS Alliance RT/30 machine. Samples were compressed at a rate of 0.25 mm (0.01 inches) per second. In addition, the compression platens were compressed with no sample in a blank test that was used to correct for the compliance of the loading machine and gripping fixture used. The force-displacement curve during this blank test was recorded and used to measure the compliance of the MTS machine, that is, the amount of deformation that occurs in the machine itself for a given load. The force-displacement data from subsequent samples was then adjusted for the machine compliance by subtracting the compliance data from the raw data.

The samples with a thickness of 1.57 cm (0.62 inches) were compressed continuously to failure. Subsequently, samples of thicknesses of 0.762, 0.635, and 0.381 cm (0.3, 0.25, and 0.15 inches) were loaded and unloaded at increasing levels of compression in order to evaluate the elastic modulus of the material. These samples were compressed to a force of 20 kN, then relaxed to 10 kN, then compressed to 30 kN and relaxed to 20kN, and so on. This process was
repeated until failure. The force-displacement curves adjusted for compliance were used to calculate the stress-strain curves for the geopolymer samples.

2.2 Ultrasonic Testing

Disks of the same geopolymer were also tested using a JSR PR35 Pulser/Receiver Ultrasonic Machine, shown in Figure 2.1. The basic function of the machine is to find the time taken for a wave to travel through a sample of known thickness. The wave speed of the sample material can be calculated by dividing the thickness of the sample by the time taken for the wave to travel through it. Using different sensors, both dilatational and shear wave speeds can be measured.

Figure 2.1 The pulser/receiver used to perform ultrasonic testing
The machine consists of an excitation piezoelectric pad and a receiving piezoelectric pad, and can be used in two different modes – through and echo. In through mode (Figure 2.2), the excitation pad and receiving pad are placed on either side of the sample. The excitation pad oscillates at an ultrasonic frequency, sending waves through the sample into the receiving pad. An oscilloscope displays both the excitation and receiving signals. From the oscilloscope, the time for the wave to travel through the sample can be calculated. The data can be refined by subtracting the time for the wave to travel from the excitation to receiving pad with no sample in between, calculated by placing the pads in contact with each other. Figure 2.3 shows an oscilloscope screen shot recorded during the ultrasound experiment with each pulse labeled and the time difference between them, i.e., the wave travel time, shown on the right hand side of the screen.

Figure 2.2 Test setup for through mode of ultrasonic testing
In echo mode (Figure 2.4), only the excitation pad is placed against the sample. The oscilloscope displays the excitation signal and the echo signal that reflects from the far boundary of the sample back through the sample to the excitation pad. The time derived from the oscilloscope in this mode is halved before dividing by the sample thickness, since the wave travelled through the distance twice. Both echo and through modes can be used to find dilatational and shear wave speeds in the sample.
All four measurement combinations (through and echo, and dilatational and shear) were performed on discs of pure geopolymer with a diameter of 1.57 cm (0.62 inches) and thickness of 0.38 cm (0.15 inches). Once the dilatational ($c_d$) and shear ($c_s$) wave speeds of the sample were calculated, additional dynamic mechanical properties of the sample were found from the following equations (assuming a linear elastic homogeneous and isotropic response):

$$\mu = \delta \cdot c_s^2$$  \hspace{1cm} (2.1)

$$\nu = \frac{1}{2} \left( \frac{c_d}{c_s} \right)^2 - 1$$  \hspace{1cm} (2.2)

$$E = 2\mu(1 + \nu)$$  \hspace{1cm} (2.3)

where $\mu$ is the shear modulus, $\delta$ is the density, $\nu$ is the Poisson ratio, and $E$ is the elastic modulus of the material.
2.3 Split-Hopkinson Pressure Bar

The use of a Split Hopkinson Pressure Bar to evaluate the dynamic response of 1D chains of beads has been treated in great detail in On (2011). A brief overview will be provided here. The dynamic properties of materials can also be found using a Split Hopkinson Pressure Bar, otherwise known as a Kolsky bar (Follansbee, 1995). The specimen to be tested, in this case a 1D array of metal beads held in a holder or in a matrix, is placed between two metal bars of known properties – in this case C350 maraging steel (see Figure 2.5). A high strain rate pulse is created by impacting the far end of one of the bars. The resulting pulse travels along the impacted bar (called the incident bar), through the specimen, and finally into the second (or transmitted) bar. The impact in the incident bar is caused by using a tank of pressurized air to shoot a striker of the same material and diameter as the incident bar at high speed. Strain gauges on the incident and transmitted bars record the elastic deformation of the bars as the pulses pass through. Because the properties of the incident and transmitted bars are known, the forces on either side of the sample can be calculated.

![Figure 2.5 Schematic of a Split Hopkinson Pressure Bar (not to scale). The specimen consists of a one dimensional array of metal beads in a cylinder or matrix.](image)

The process of pulse propagation through the SHPB is illustrated in the x-t diagram of Figure 2.6. At the moment of impact, a compressive wave travels outward from the point of impact in both directions. As the wave reaches the free end of the striker, it reflects as a tensile...
wave, which follows the compressive wave a few microseconds behind. This combination of a compressive and tensile wave a few microseconds apart creates the compressive pulse. When this pulse reaches the interface between the incident bar and the specimen, part reflects back into the incident bar as a tensile pulse, and part travels through the specimen and into the transmitted bar. There is additional reflection of the wave at the interface between the specimen and the transmitted bar. Strain gauges mounted on the incident and transmitted bars record the pulses as they pass. Depending on the specimen tested, the transmitted pulse can have a significantly smaller magnitude than the incident pulse. For some experiments we used what is known as a momentum trapping device (Nemat Nasser et al., 1991) which provides a tensile pulse immediately following the compressive loading pulse. In these experiments the incident bar is then “pulled back” once the compressive loading is over.

Figure 2.6 x-t diagram showing the propagation of pulses through the SHPB

Figure 2.7 shows an example of raw data recorded by the strain gauges of the SHPB without a momentum trap, illustrating the incident compressive pulse, $\varepsilon(t)$, the tensile reflected
pulse, $\varepsilon_R(t)$, and the transmitted compressive pulse, $\varepsilon_T(t)$, of much smaller magnitude. By recording these three signals the input and output force to the sample can be measured through:

$$F_{in} = A_{bar} E_{bar} (\varepsilon_i - \varepsilon_r)$$  \hspace{1cm} (2.4)

$$F_{out} = A_{bar} E_{bar} \varepsilon_t$$  \hspace{1cm} (2.5)

where $F_{in}$ is the force applied to the specimen by the incident bar, $F_{out}$ is the force transmitted through the specimen, $A_{bar}$ is the cross-sectional area of the Hopkinson bar, and $E_{bar}$ is the elastic modulus of the Hopkinson bar. Figure 2.8 shows the same type of raw data for an experiment with a momentum trap. Notice that the incident compressive pulse is now followed immediately by a tensile pulse.

![Figure 2.7 Raw data illustrating pulses recorded by strain gauges when a momentum trap is not used](image)

**Figure 2.7** Raw data illustrating pulses recorded by strain gauges when a momentum trap is not used
In the present work, the SHPB was used to examine the dynamic response of different types of samples consisting of one dimensional arrays of metallic beads which are meant to represent an ordered granular medium. In all cases one dimensional chains of low carbon steel beads from McMaster-Carr (product number 96455K54) were used. However, the main goal of this work is to identify the influence of lateral confinement on such a one dimensional system. Therefore, three types of experiments were conducted: (i) unconfined beads placed in a tube holder which was slightly larger in diameter than the beads, (ii) beads that were laterally confined by shrink fitting them into tube holders of a smaller diameter than the beads, and (iii) one dimensional arrays of beads fully embedded in a geopolymer matrix. The following sections describe how each type of sample was created.

**Figure 2.8 Raw data illustrating pulses recorded from strain gauges when a momentum trap is used**
2.3.1 Unconfined Beads

Unconfined beads were tested using the methodology developed by On (2011), which essentially consists of rolling the beads into a metal holder of the appropriate length. The inner diameter of the holder is large enough so that the beads never compress against the sides of the holder during an experiment. Experiments on unconfined beads were done using the SHPB as described above. The primary difference between the unconfined experiments conducted here and those of On (2011) is that in the study of On (2011) a momentum trapping mechanism was used, whereas here both momentum trapped and non-momentum trapped experiments were conducted and the results of the two were compared.

2.3.2 Beads under Confinement

Low carbon steel metal beads with a diameter of 9.6 mm and aluminum tubes with an interior diameter of 9.4 mm (diameter mismatch of 0.2 mm) were purchased from McMaster Carr. The beads were then shrink fitted in the aluminum tubes. A stand (Figure 2.9) was created to aid in the construction of the samples. Two cylindrical holes were drilled into a metal block. The first hole was larger than the outer diameter of the aluminum tube. The second deeper hole was larger than the diameter of the beads.

![Figure 2.9 Stand used to aid in confinement of beads](image)

3 cm
The aluminum tubes were cut to a length of 3.81 cm (1.5 inches), the length of 4 beads (Figure 2.10). They were then heated at 600°C for 15 minutes and placed upright onto the stand. Five steel beads were then dropped into the tube (Figure 2.11). The setup was left alone for 20 minutes while the tube cooled and constricted around the beads. The final samples consisted of five beads confined by a metal tube the length of four beads, leaving a half bead protruding from both ends. The final assembly was then inserted in the specimen area of the SHPB as shown in Figure 2.12.

Figure 2.10 Illustration of how the beads fit in confinement
Figure 2.11 Beads and tube in the stand

Figure 2.12 Confined beads ready for testing in the SHPB
2.3.3 Beads Embedded in Matrix

Low carbon steel beads were also embedded in a geopolymer matrix as manufactured by Mr. Shinhu Cho of the Material Science and Engineering Department at the University of Illinois at Urbana-Champaign. A potassium-based geopolymer was created and mixed with ten weight percent chopped carbon fiber to provide more stiffness and strength to the matrix. One dimensional chains of low carbon steel beads were then embedded in the matrix, and the sample was left to cure for different amounts of time, namely 12 hrs, 24 hrs, 36 hrs, and 48 hrs. Two different methods were used to embed the beads in the matrix over the course of this research. The first method involved a two-step process: the beads were constrained so they were aligned and in contact, and a small amount of geopolymer was used to fill the interstitial spaces and bind the beads together. This segment was then placed in the center of a larger cylindrical mold, which was subsequently filled with additional geopolymer. As with the beads confined by aluminum tubes, the samples were arranged so that a half bead remained exposed on both ends as seen in Figure 2.13.

![Figure 2.13 Beads embedded in geopolymer using the two stage method](image)
The second manufacturing method employed only one step: the beads were aligned and arranged in contact in a rectangular mold, which was then filled with geopolymer in one step resulting in the type of sample seen in Figure 2.14.

![Beads embedded in geopolymer using the single stage method](image)

**Figure 2.14 Beads embedded in geopolymer using the single stage method**

As will be seen in the subsequent results, the curing time of the geopolymer matrix has a significant effect on the properties of the sample. Curing times of 36 hours and longer were found to produce samples that have consistent properties. Results of the experiments on the confined and unconfined one dimensional granular arrays are presented in the next chapter.
CHAPTER 3: RESULTS AND DISCUSSION

3.1 Unconfined Beads

Extensive work has been done studying nonlinear wave formation in one dimensional plastically deforming chains of metal beads by On (2011). The main focus of On (2011) was to examine the primary pulse traveling through the chain when impacted by the SHPB described above. The results showed the formation of a solitary wave after a length of 5 beads, and a dependence of wave speed on impact force of the primary pulse. In this work, similar experiments to those of On (2011) were performed. However, here configurations both with and without a momentum trap were used. The observed results did not have significant differences in the primary pulse, but did differ in the presence of trailing pulses that followed the primary pulse.

Figure 3.1 below shows the transmitted force (the force recorded in the transmitted bar of the SHPB) from two identical samples of 5 brass beads tested one after the other in the same SHPB setup, with the only difference between them being the use or not of a momentum trap on the incident bar end. In this case, the experiment involving the momentum trap had a slightly higher incident pulse. This in turn creates a slightly higher transmitted pulse for the non-momentum trap case, as seen in Figure 3.1. However, the most significant difference between the two cases is not in the primary pulse, but rather in a series of trailing pulses with decreasing magnitude that occurs in the case without a momentum trap. This “late time” effect after the passage of the primary pulse was seen consistently in many experiments with different materials and loadings. For example, Figure 3.2 shows the same results for a set of steel beads.
Figure 3.1 Comparison of transmitted signals for a 5 bead brass chain loaded with and without a momentum trap.

Figure 3.2 Comparison of transmitted signals for a 5 bead steel chain loaded with and without a momentum trap.
When a momentum trap is not used, the incident bar remains pressed against the first bead throughout the duration of the experiment even after the passage of the primary pulse. When a momentum trap is used, the tensile pulse immediately following the primary compression (see Figure 2.8) pulls the incident bar back (to the left) a significant amount leaving a gap between the first bead on the chain – which is still moving to the right – and the incident bar end. However, as the nonlinear wave reflects in the chain, the beads start moving to the left and the first bead can re-impact the incident bar end and then reflect back into the chain, thus creating additional “impacts”. This is more likely to happen in the non-momentum trapped case where the incident bar has not been pulled back.

To verify this process, as series of high speed imaging experiments designed to capture the later time response in the bead chain were performed. The experiments involved combining a high speed camera (Redlake Motion Pro camera, frame rate: 44,000 fps, resolution: 512 by 64 pixels) with the SHPB loading. The camera was triggered by the incident strain gauge signal. Figure 3.3. below shows selected frames from the high speed video taken during an experiment on 5 beads without a momentum trap. The incident bar is on the left side of the figure and the transmitted bar is on the right. In each frame, the last bead is pressed firmly against the transmitted bar. In the first frame the fourth bead is contacting the fifth, then is separated in the second frame, and in contact again in the last frame. Throughout this process, the incident bar is essentially stationary. Therefore the reflected pulses that cause leftwards motion are again reflected from the incident bar causing additional waves to propagate down the chain. These are recorded as the subsequent peaks in the non-momentum tapped case in Figure 3.1.
Figure 3.3 (a) Selected high speed images from a non-momentum trapped experiment on a 5 brass bead chain. (b) Close up of the region on the transmitted bar side. The first frame occurs as the primary pulse passes through the beads, the second occurs 50 microseconds after the first, and the third occurs 50 microseconds again after the second.

In contrast, in Figure 3.4 when a momentum trap is used, the incident bar jumps back away from the sample after the tensile part of the incident pulse reaches the sample. Because the incident bar moves away from the sample, the beads are not as tightly constrained laterally as in the case without a momentum trap. As the beads jostle back and forth after the primary pulse moves through (very clearly visible in the motion of the video), they shift toward the incident bar, since they have space to move in that direction. Because the beads shift away from the
transmitted bar after the initial pulse, the jostling of the beads is not recorded by the transmitted bar. The transmitted bar, then, only records the initial primary pulse and no subsequent pulses, as seen in the momentum trapped curves in Figures 3.1 and 3.2.

Figure 3.4 (a) Selected high speed images from a momentum trapped experiment on a 5 brass bead chain. Note the pullback of the incident bar in this case. (b) Close up of the region on the transmitted bar side. The first frame occurs 50 microseconds before the primary pulse passes through the beads, the second frame occurs as the pulse passes through, and the third frame occurs 50 microseconds after the pulse.
The difference in response between experiments with and without a momentum trap is obvious when observing transmitted pulses in SHPB analysis. The actual cause of these differences is difficult to prove, but we believe the high speed images of the samples confirm the process described above. However, there is still room for further examination in this issue, although this is not the primary focus of this effort. The high speed images were only able to be captured at a rate of one frame per 20 μs. However, some trailing transmitted pulses have a width of only 15 μs. It is possible that some of the phenomenon was completely missed by the high speed imaging.

3.2 Pure Matrix

Before the effects of embedding beads in geopolymer can be fully understood, the properties of the geopolymer itself must be ascertained. Disks of geopolymer were tested under quasi-static and ultrasonic conditions to determine the properties under both static and dynamic loading conditions.

3.2.1 Quasi-static Testing

As described in the previous chapter, disks of geopolymer were tested under quasi-static compression. A blank test of the compression apparatus found the stiffness of the machine to be 65.8x106 N/m (376,000 lbf/in), meaning that the compression platens of the machine itself deformed one millimeter for every 65,800 N of compressive load. Figure 3.5 shows the force-displacement results recorded both for a blank test (red curve) and for a typical geopolymer experiment (blue curve). The machine compliance was then removed from the geopolymer experimental data to produce a compliance corrected curve. Figure 3.6 shows the results of this
compliance correction procedure by comparing the stress-stain curve for a geopolymer sample with diameter and thickness of 15.7 mm (0.62 inches) before and after correction. This machine compliance was accounted for in all the following experiments in order to find the true deformation of the samples for the recorded load. The (corrected) geopolymer elastic modulus was found to be 6.9 GPa from this monotonic loading experiment.

Figure 3.5 Load-displacement curves from a blank (i.e., no sample) compression experiment and a typical geopolymer compression experiment.
Testing was subsequently done on thinner disks of geopolymer with repeated loading and unloading cycles to get a more accurate measure of elastic modulus. Each disk was loaded to a target threshold, then unloaded to the starting point of the next cycle. This was repeated five times. This test was performed on disks of thickness 7.87, 6.35, and 3.81 mm. In all cases compliance correction was performed. Table 3.1 summarizes the loading profile used in these loading-unloading experiments.

**Figure 3.6 Raw stress-strain data and compliance corrected stress-strain curve for a geopolymer sample.**

**Table 3.1 Loading profile used in quasi-static loading-unloading experiments on neat geopolymer.**

<table>
<thead>
<tr>
<th>Loading Number</th>
<th>Start Force (N)</th>
<th>End Force (N)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>2000</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>5000</td>
</tr>
<tr>
<td>3</td>
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<td>10000</td>
</tr>
<tr>
<td>5</td>
<td>3000</td>
<td>13000</td>
</tr>
</tbody>
</table>
Figure 3.7 Compressive quasi-static loading and unloading profile for a 7.87 mm thick geopolymer disk.

Figure 3.8 Compressive quasi-static loading and unloading profile for a 6.35 mm thick geopolymer disk.
Figures 3.7, 3.8 and 3.9 show the loading-unloading history from the experiments conducted on the 7.87, 6.35 and 3.81 mm thick disks, respectively. The loading portion is shown in blue and the unloading in red. The slope of each loading and unloading was calculated and averaged to find the elastic modulus of each specimen. The values obtained were 7.4 GPa, 7.0 GPa, and 7.2 GPa, respectively. All are close to the modulus of 6.9 GPa found from the monotonically loaded 15.7 mm (0.62 inch) thick disk that had a single load to failure. From this data, we can say with confidence that the static elastic modulus of the geopolymer is close to (an average) 7.2 GPa. Figure 3.10 shows a combination of all the quasi-static curves shown above. From the monotonic loading experiment we can also extract the failure stress of the material, which in this case is about 80 MPa.
Figure 3.10 Collection of compressive stress-strain curves obtained for the geopolymer used in this work. Black curve represents monotonic loading experiment. The remaining results are collections of loading-unloading-reloading experiments.

3.2.2 Ultrasonic Testing

Five geopolymer disks of diameter 15.7 mm were tested using ultrasonic testing, as described in section 2.2. Three disks had a thickness of 3.81 mm (0.15”) and two had a thickness of 2.54 mm (0.1”). Table 3.2 below shows all the data collected and the calculations performed to obtain the sample wave speeds.
Table 3.2 Raw data from ultrasound experiments on pure geopolymer material.

<table>
<thead>
<tr>
<th>Samples</th>
<th>Time (microseconds)</th>
<th>Wave speed (m/s)</th>
<th></th>
</tr>
</thead>
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<tr>
<td></td>
<td>Dilational Through</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Run 1   Run 2  Run 3 Run 4  Run 5  Run 6 Avg c_d</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thick #1</td>
<td>1.68    1.76  1.72  1.68  1.7  1.65 1.3583333 2767</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thick #2</td>
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<td></td>
</tr>
<tr>
<td>Thick #3</td>
<td>1.68    1.59  1.64  1.66  1.59 1.64 1.2933333 2926</td>
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<td></td>
</tr>
<tr>
<td>Thin #1</td>
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<td></td>
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<tr>
<td>Thin #2</td>
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<td></td>
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<tr>
<td></td>
<td>Dilational Echo</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Run 1   Run 2  Run 3 Run 4  Run 5  Run 6 Avg c_d</td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td>Run 1   Run 2  Run 3 Run 4  Run 5  Run 6 Avg c_s</td>
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<td></td>
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The wave speeds were then averaged together and input into the equations given in Chapter 2. Using all of the data, the dynamic properties of the geopolymer were found to be an elastic modulus of 10 GPa, a shear modulus of 4.2 GPa, and a Poisson ratio of 0.19. However, calculating the properties using only the data from through or only echo yielded slightly different
values. Using only through data, the properties were elastic modulus of 11.7 GPa, shear modulus of 4.8 GPa, and Poisson ratio of 0.21. Using only echo data, the properties were elastic modulus of 7.4 GPa, shear modulus of 2.9 GPA, and Poisson ratio of 0.26. The elastic modulus found from only echo data matches the range of the elastic modulus found from quasi-static testing.

3.3 Beads in Confinement

After studying the beads and the geopolymer on their own, it is of interest to study one dimensional granular chains embedded in a matrix. The results of such experiments are presented in Section 3.4. As an alternative of “partially confined” beads we also investigated the case of beads with lateral, although not all-around, confinement induce by a shrink fit tube. Multiple samples of unconfined (section 3.1) low carbon steel beads were tested using an SHPB and the results were compared to low carbon steel beads confined by aluminum tubes tested with an SHPB, discussed below.

3.3.1 Transmitted Force

With an average incident pulse of 30 kN, chains of 5 beads in confinement showed significant and consistent absorption of energy. Figure 3.11 shows a collection of incident bar records for a series of confined and unconfined experiments on steel beads. As can be seen, the incident signals are almost identical in each case which allows for direct comparison of the transmitted signal as a measure of energy absorbed in each case.
Figure 3.11 Incident signals of a series on confined and unconfined SHPB experiments on one dimensional steel chains of 5 beads

Figure 3.12 shows the corresponding transmitted signals recorded in each of these 8 experiments. The average transmitted force of 8 kN shows a roughly 75% reduction in magnitude of the pulse entering and the pulse exiting the samples. An incident pulse of 80 μs resulted in a series of four transmitted pulses of decreasing magnitude. The first pulse had a length of 80 μs and a maximum peak of 10 kN. The second pulse had a length of 55 μs and a peak of 5 kN while the following pulse had the same length and peak, but a different shape. It rose more sharply on the front end and fell more gradually on the tail end.
When impacted with nearly identical incident pulses, the transmitted pulses for the steel beads in confinement were very similar to the transmitted pulses of steel beads with no confinement. They have the same number, shape, and magnitude of trailing pulses. However, after the first two pulses, the confined pulses began to take longer than the unconfined. After 150 μs, the confined pulses lag behind the unconfined pulses while still maintaining the same magnitude and basic shape. This means that laterally confining one dimensional chains of beads will slow the wave speed of the transmitted force, but not change its overall shape. Most likely this is a consequence of the friction force generated at the confined interface. Unfortunately it was not possible to directly measure the laterally confining pressure, or even vary it beyond the
one value possible with the shrink fit used here. From literature data on comparable systems (Chen and Ravichandran, 1997) an estimate of the lateral pressure generated is around a few 10s of MPa. It is expected that as the confining pressure will increase, these friction induced delaying effects will also increase. One possible way to increase the confinement is to use a larger radius mismatch. However the thermal difference we could obtain between beads and tube in order to achieve the shrink fit is already at 527°C, which is close to the maximum capability of the furnace used. However, both these aspects could be the focus of a future effort.

3.3.2 Wave Speed Nonlinearity

It is a known property of granular one dimensional chains that their solitary wave speed has a non-linear relationship with force magnitude. We wanted to investigate to what extent this nonlinear relation is affected by lateral confinement. Therefore, samples of confined beads were tested at various force magnitudes to investigate whether this property holds true for granular chains confined by a solid bulk material. Figures 3.13 and 3.14 show the measured average wave speed in a chain of 5 confined low carbon steel beads (computed as the transit time across the chain divided by its length) plotted versus either the maximum incident or the maximum transmitted force, respectively.
Figure 3.13 Measured average wave speed in a laterally confined chain of 5 low carbon steel beads versus maximum incident force.

Although unconfined steel samples show a nonlinear force relation (On, 2011), for the force loadings tested here there was no discernible nonlinear relationship between force and wave speed. However, limitations of the experimental apparatus prevented testing at lower impact forces. Because we have no data for impact forces between 0 and 25 kN, we cannot determine if there is a truly non-linear relationship between wave speed and force for confined beads. The same inconclusive result was found when comparing wave speed to transmitted force (Figure 3.14). Additional experiments at lower loading rates (with a different device than the SHPB) would be needed to resolve this issue.
3.4 Beads in Matrix

The last focus of this work was to study the response of beads fully embedded in geopolymer matrix of some type. It is expected that in such a case significant energy transfer will occur between beads aided by the presence of the matrix. Figure 3.15 shows a schematic of this energy transfer process. This process is expected to reduce the nonlinear effects of the granular medium. The experiments described here were meant to probe this issue in order to see how the occurrence of nonlinear waves would be affected by the presence of a matrix entirely surrounding the beads.
As the material preparation process being developed by our collaborators in the Materials Science and Engineering department evolved over time, we examined the SHPB results for geopolymer matrices cured using various times and methods. With the two stage method of manufacture, the response of beads embedded in geopolymer cured in air for 24 and 36 hours was compared to that of beads without a matrix. The curing time was seen to have a drastic effect on the transmitted force results. Figure 3.16 shows the transmitted force for a chain of 16 low carbon steel beads for the unconfined case and for matrix cure times of 24 hrs and 36 hrs. The geopolymer cured for 24 hours resulted in a transmitted pulse higher than beads with no matrix, while the geopolymer cured for 36 hours resulted in a transmitted pulse with a lower magnitude than beads with no matrix. One theory to explain this phenomenon looks at the water content of the geopolymer. Geopolymer begins full of water, which evaporates as it cures. After 24 hours of curing, water still remains in the geopolymer, which helps to transfer energy in the dynamic loading situation. As the water continues to evaporate, it leaves behind voids, which absorb energy, resulting in a lower transmitted pulse under dynamic loading as the curing time increased.
The same type of experiment was performed on samples created with the one step method. The bead-laden geopolymer was cured in air, as well as in a desiccator, for various lengths of time. Figure 3.17 shows a collection of transmitted force curves from samples involving the single stage curing process. The results from this test are inconclusive. Curing in a desiccator instead of air, as well as curing for longer periods of time, decreases the water content of the geopolymer. There does not appear to be a consistent trend, however, between water content and the magnitude of the transmitted force. In fact, the samples with the highest water content (cured for 12 hours in air) and the lowest water content (cured for 48 hours in a desiccator) had the most similar results. From this data, we can draw two conclusions. First, embedding beads in a geopolymer matrix does affect the energy transfer and absorption
properties of the chain of beads. The shape of the transmitted pulse itself changes. Second, it is clear that changes in the manufacturing process of the geopolymer have a drastic effect on the properties of the composite. Continuing to develop a reliable manufacturing process that produces materials with consistent quality and properties should be a priority moving forward.

Figure 3.17 Transmitted force curves for 16 low carbon steel bead chains embedded in geopolymer created using the one step process

One aspect of interest is also the dependence of the chain response on chain length. Therefore, a series of experiments were conducted with 5 bead and with 14 to 16 bead chains, with and without a geopolymer matrix. Figure 3.18 shows the transmitted force response for a 5 bead brass chain with and without a geopolymer matrix. A similar response as in the previous results is seen, but only two curves are shown in order to point out the main features. The
primary pulse is similar in shape among the two cases, although the peak load differs in the case of the matrix being present. The trailing peaks in the no matrix case are as explained in Section 3.1, and result from the rebounding of beads. In the signal with the matrix, later peaks are also present although it is unlikely they are related to bead rebounds, since the beads are confined in the matrix.

![Transmitted Pulses](image)

**Figure 3.18** Transmitted signals from 5 bead chains illustrating the different responses with and without geopolymer

There still are forces that could be transmitted after the peak, however, since in this case the geopolymer would fail fairly early on in the deformation (recall that it has about 80 MPa compressive failure strength) and the residual debris in between the SHPB would provide some resistance. Figure 3.19 shows a photograph of the sample in the SHPB (it is supported by a foam enclosure) after impact. The debris between the bars is clearly visible.
After loading, when the chain had failed as shown above, the beads were examined for signs of plastic deformation and contact. Figure 3.20(a) shows a brass bead that has been loaded when unconfined. The plastic zone developed there is seen as a flat region on one end of the bead. Figure 3.20(b) shows the corresponding image from a bead taken from a failed chain that had been embedded in a geopolymer and was loaded at the same loading level. Smaller amounts of plasticity are visible, but there is some permanent deformation visible. This shows firstly that the beads had most likely been in contact to begin with and secondly that the existence of the matrix diminishes the total force on the beads, since some of the load is carried by the matrix itself.
Figure 3.20 (a) Impact area of a brass bead not in geopolymer (b) Impact area of a brass bead embedded in geopolymer

As the chain length increases the differences between the case with matrix and without become less pronounced, as seen in Figure 3.21. Here again the data agree with that shown earlier, but we have isolated only two curves in order to make the comparison more obvious.

Figure 3.21 Transmitted pulses from longer chains of beads both with and without geopolymer
Ordered granular media have unique properties because of the contact interaction between individual grains. Extensive research has been done to study one dimensional granular chains under dynamic loading. This thesis has investigated the effects of introducing different types of confinement to granular chains. Some type of confinement will be necessary when designing and building two and three dimensional ordered granular networks.

For the case of unconfined chains, the “late-time” response is strongly influenced by the incident pulse. For all cases, the transmitted response contains a primary pulse caused by the plastic solitary wave as it travels through the chain. However, experiments performed without a momentum trap also presented trailing pulses that occurred after the primary pulse due to rebounding of the beads. This bead jostling or rebounding occurs in cases both with and without momentum traps, but the nature of the momentum trap – pulling the incident bar away from the beads – prevents this rebounding energy from being recorded by the SHPB.

Under lateral confinement, the primary pulse is again not noticeably affected, but the “late-time” response is delayed. We theorize that the effect of lateral friction, amplified by the confinement, delays the transit of the solitary waves through the chain. Additionally, we cannot determine at this time whether lateral confinement preserves the nonlinear response inherent to granular media or not. More experimentation needs to be done at lower impact force magnitudes (below 20 kN) to ascertain this.

In the fully confined case, where beads are embedded in the geopolymer matrix, the “late-time” response is again affected. Later peaks are still present, but they are not as regular as
the unconfined case. They may still be from the jostling of beads, but attenuated by the confinement of the matrix. It is also possible, however, that the response could come from resistance from residual debris after the geopolymer matrix fails. In addition, matrix manufacturing and curing conditions have been shown to be significantly influential in the response of the embedded chain. Developing a more consistent method of preparing the matrix is key for continuing progress. Lastly, longer chains seem to experience less of an effect from the presence of a matrix, perhaps because significant dissipation occurs even in the unconfined chains of significant length.
REFERENCES


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