THREE ESSAYS ON INEQUALITY

BY

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DISSERTATION

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ABSTRACT

This dissertation analyzes inequality focusing on labor market frictions that determine occupational choice and on different educational policies that determine the level of education and human capital accumulation.

The first essay presents an overlapping generations model with three different occupational choices: full-time manager, self-employed or worker, and frictions in the labor market due to labor protection policies. The model shows that labor market frictions can explain differences in employment structures and firm sizes across countries. Developing countries have a high entrepreneur to workforce ratio but low firm size. The main idea is that by excluding less productive workers from the labor market, policies such as minimum wages create "bad" entrepreneurs, in the sense that these individuals would be better off employed as workers. The decreased supply of labor and demand for capital have competitive equilibrium effects in a Lucas "span of control model" (1978), which prevents capital accumulation with a lower number of entrepreneurs but much larger and more productive firms.

The second essay analyses the quantitative effects of labor rigidities in a model with financial frictions. The model is calibrated using U.S. data and a set of experiments is performed to investigate the effects of labor policies on productivity, occupational choice and inequality. The model fits well the U.S. income inequality statistics. The analysis includes a counterfactual exercise using Brazilian labor friction parameters. The results show that both of the labor protection policies considered have a significant impact on inequality.

Finally, the last essay studies the quantitative implications of different education systems and the effects of affirmative action policies on long run inequality and productivity. A large number of countries in Latin America and Asia have adopted education policies based on income and race characteristics to offset lack of opportunities and increase intergenerational mobility. In an overlapping generations model, parents choose the type of child education, public or private school, and young agents choose to become unskilled workers or attend a university and become skilled workers. If attending university, young agents choose between public or private university. Results for the Brazilian economy show that an increase in the quality of public

schools has a higher impact on inequality and output per capita than policies such as the introduction of quotas for black or native students.

To my mother, Maria Szydoski, and my husband, Marcelo Bego, for all their love and support.

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CHAPTER 1

THE EFFECTS OF FRICTIONS IN THE LABOR MARKET ON ENTREPRENEURSHIP AND ECONOMIC DEVELOPMENT

1.1 Introduction

Firm size is an important determinant of growth. Larger firms have higher levels of physical capital and higher investment in R&D (Freeman, 1982; Mowery and Rosenberg, 1989; Chandler, 1990). Therefore, there is a positive and robust relationship between average firm size and growth because larger firms can take advantage of the increasing returns on investment in human capital. Evidence also suggests that the direction of causality is from firm size to growth (Pagano and Shivard (2003)). In this sense, it is important to remember the seminar paper of Lucas (1978). Through a general equilibrium effect, Lucas could explain the differences of firm size across countries. The main idea is that capital accumulation leads to an increase in the marginal productivity of labor, which increases wages. This mechanism generates an increase in the labor force, fewer entrepreneurs and larger firms. In the long run, small firms and self-employment should decrease, concentrating production.

Gollin (2008) found that in most developing countries, small firms and self-employment are the dominant firm forms. On the other hand, in the developed countries fewer entrepreneurs run much larger firms, on average. For instance, in the United States the entrepreneur-workforce ratio is 7.02%, while in Brazil the ratio is about 31%. Gollin also notes that there are some outliners such Italy, Spain and Greece with the entrepreneur-workforce ratio of 23.32%, 17.65% and 35.08%, respectively. These countries are known for their rigid and highly centralized wage-setting system. Therefore, two important questions arise: why do countries have such different firm size distributions and why is the structure of production so rigid?

Hurst and Lusardi (2004) found that financial wealth is important for the choice to become an entrepreneur only for the richest households. Their result changed the common view that initial wealth is an important feature for explaining occupational choice. Quadrini (2008) emphasizes that even though financial constraints are not important for the decision to become an entrepreneur, they are an important determinant of firm size and other entrepreneurial decisions. Hence, frictions in the financial market help to explain differences in the firm size and, consequently, in economic development (Antunes et al, 2008).

The model in this chapter compliments capital market explanations of differences in firm size across countries by focusing on labor force frictions. The idea is that labor market frictions create unemployment, causing unemployed agents to become "bad" entrepreneurs in the sense that they would be better off as workers. Since they cannot be workers, they become self-employed entrepreneurs who operate small firms. An indirect impact is that frictions in the labor force also decrease the share of "good" entrepreneurs, i.e., full time managers with a higher demand for capital, which decreases the overall demand for capital and employment, impeding the competitive equilibrium adjustment mechanism described in Lucas (1978). Therefore, labor market rigidity can help explain why small businesses and self-employment are so persistent in some countries.

This work is related to two main literatures. The first literature focuses on occupational choice between work and being an entrepreneur and on firm size (Lucas, 1978; Quadrini, 2000; Antunes et al, 2008, Hurst and Lusardi, 2004, Cagetti and DeNardi, 2006). We add a third occupational choice: self-employment (see Gollin, 2008). The key difference is that we focus on labor market frictions instead of financial market frictions. The second literature is related to how labor market rigidities affect employment, firm size and growth (Fajgelbaum, 2011; Chor, 2006; Helpman and Itskhoki, 2007, Jovanovich, 1979 and McCall 1970). These papers differ from this work because they focus on trade to explain the effect of labor market frictions on firm size, using search and matching models where the frictions come from the cost of firing/hiring workers and bargaining power. As explained below, the frictions we consider capture differences in government policies such as minimum wages and social security benefits, as well as costs of hiring and firing workers.

The model is based on Antunes, Cavalcanti and Villamil (2008). Agents choose between being a worker or an entrepreneur. The difference is there are two types of entrepreneurs: full-time

managers and self-employed as in Gollin (2008). Self-employed agents divide time between managing a firm and working. The remuneration of both types of entrepreneurs will be strictly increasing and convex with respect to ability, but full time managers' profit is more convex. That is, it is strictly better for higher ability agents to be full time managers instead of self-employed. The other important assumption of the model is that we use a more general definition of entrepreneurial ability than the one considered in Lucas (1978). Ability increases profit as an entrepreneur but also increases labor productivity:

> It is tempting to argue that the most talented people become entrepreneurs because they have the skills required to engage in creative activity. Perhaps so, but this flies in the face of some facts. The man who opens up a small dry-cleaning shop with two employees might be termed an entrepreneur, whereas the half-million-dollar-per-year executive whose suit he cleans is someone else's employee. It is unlikely that the shop owner is more able than the typical executive.
>
> The reverse might be true. As necessity is the mother of invention, perhaps entrepreneurs are created when a worker has no alternatives. Rather than coming from the top of the ability distribution, they are what is left over. This argument also flies in the face of some facts. Any ability measure that classifies John D. Rockefeller, Andrew Carnegie, or, more recently, Bill Gates near the bottom of the distribution needs to be questioned. (Poschke, 2008, apud Lazear, 2005).

Evidence corroborates the assumption that more productive workers are also better entrepreneurs if educational attainment is used as a proxy for entrepreneur ability. Moreover, an important share of agents decides to become entrepreneurs "out of necessity", i.e., because there is no opportunity to work. Poschke (2008) shows that the probability of being an entrepreneur is higher at extreme levels of the productivity distribution.

Finally, is important to note that many studies have highlighted the role of small business mainly in developing countries, but they also attest to the importance of changes in productive structures as central to development and growth (see, for instance, Liedholm and Meade, 1999; Tybout, 2000; Fafchamps, 1994; Hirschman, 1958; Rostow, 1960; Lewis, 1965). The main view is that small family and self-employment businesses would decrease in importance if market rigidities were removed.

The chapter proceeds as follows. Section 1.2 provides some empirical evidence to support our theory. Section 1.3 presents the model. Section 1.4 derives the results and analyzes different government labor market policies. Section 1.5 provides concluding remarks.

1.2 Empirical Evidence

Countries have very different policies with respect to labor. For instance, most countries have a federal minimum wage but for some countries in Europe, such as Italy and Iceland, minimum wages are negotiated in various collective bargaining agreements that are industry or sector based. In this section, an employment rigidity index built by Botero et al. 2004 is used to measure the difference in labor regulations across countries. The employment rigidity index is an average of three different indexes: the difficulty of hiring index, rigidity of hours index and the difficulty of redundancy index. They used data for 85 countries. "Our measures of labor regulation deal with three broad areas: (i) employment laws, (ii) collective relations laws, and (iii) social security laws. In addition, we assembled some data on civil rights laws in different countries" (Botero at al., 2004).

The index runs from 0 (less rigid) to 100 (more rigid) and, as expected, varies greatly across countries. The United States has the lowest index, 0, while Spain, Greece, Brazil, Italy, Argentina and New Zealand have 49, 47, 46, 38, 21 and 7, respectively. The rigidity index considers all types of costs firms incur to hire and fire, such as contracts and notification requirements. The index also considers restrictions on minimum and maximum workweek and the existence of a minimum wage by law or by a collective bargaining agreement, the size of the minimum wage in different sectors, public or private, and in different industries. Our model will use two different policies to capture labor market rigidity. The first is a minimum wage policy and the second is a deadweight labor cost for firms. This second policy captures all obligatory benefits not paid directly for workers and costs due to contracts.

Figure 1a below shows the positive relationship between the employment rigidity index and the entrepreneur-workforce ratio. A simple correlation analysis shows that labor market rigidity alone is responsible for 65% of the variability in the number of entrepreneurs across countries. Figure 1b shows the positive relationship between the employment rigidity index and the percentage of entrepreneurs who are involved in entrepreneurship because they had no other

option for work. Labor market rigidity is responsible for more than 20% of the variability of entrepreneurial activity driven by necessity.
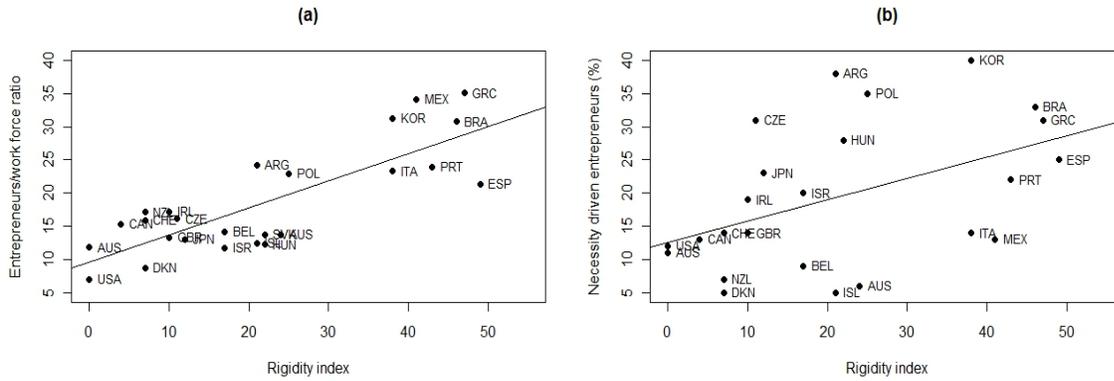

Fig 1.1: Entrepreneur-workforce ratio and necessity driven entrepreneurs (%) versus employment rigidity index. Source: Rigidity employment index: World Bank (2008). Data on labor force structure: LABORSTA, International Labor Organization. Source: Necessity driven entrepreneurial activity: GEM, Global Entrepreneurship Monitor. Countries were selected by data availability, total of 25 countries in both graphs. The straight line represents the regression, both coefficients statistically significant at 5%.

Figure 2a below shows the well-documented inverse relationship between the number of entrepreneurs and average firm size. Firm size varies greatly across countries and is measured by the total number of employees. For instance, in Brazil, 79.9% of entrepreneurial firms in 2008 had no employee. The average size of an entrepreneurial firm in Brazil is 0.9 while in US, for the same period, is 2.7. Firm size also varies greatly across European countries, for Italy, an entrepreneurial firm had 1.3 employees on average in 2008 while the United Kingdom had 2.2. Figure 2b compares firm size and the employment rigidity index. There is a negative relationship between frictions in the labor market and firm size. Rigidities account for more than 68% of variability in the average firm size across countries, indicating that labor market frictions affect not just the decision to become an entrepreneur but also the firm size.
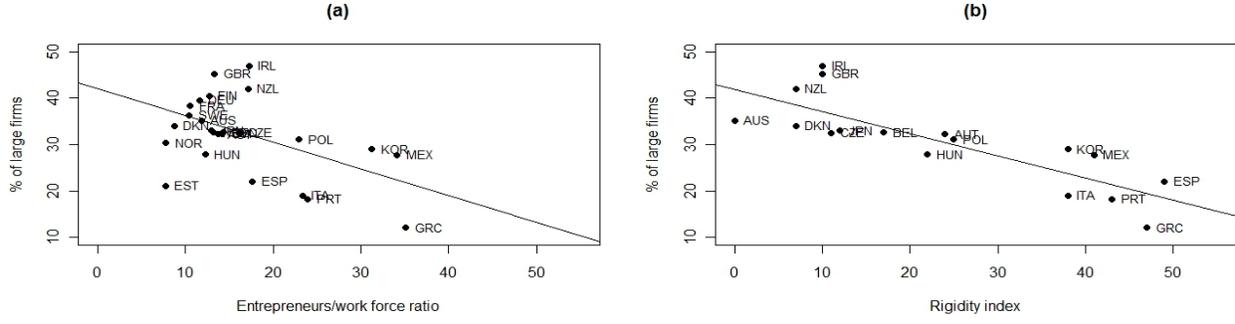
Fig 1.2: Entrepreneur/workforce ratio versus Percentage of large firms. Source: Percentage of large firms (+250 employees): OECD Structural and Demographic Business Statistics SDBS Database. Data on labor force structure: LABORSTA, International Labor Organization. Countries were selected by data availability, total of 26 and 17 countries. The straight line represents the regression; coefficients are statistically significant at 5%, with R^2=0.21and 0.68.

1.3 Model

There is a continuum of measure one individuals each period. There is no population growth. An agent chooses consumption, bequest for her child and occupational choice. The difference in this paper is that agents can choose among three occupational choices: worker, full time manager or self-employed. Agents differ in their ability, $x \in (1,0)$, which is drawn from a continuous cumulative probability distribution function, $\Gamma(x)$, and initial wealth/bequest, b. We consider a more general definition of ability x, where an agent with higher ability is a more lucrative manager if she is an entrepreneur but also more productive if she chooses to be a worker.

1.3.1 Preferences

An agent with ability x and initial wealth/bequest b_t maximizes:

$$u = c_t^\gamma b_{t+1}^{1-\gamma} \tag{1}$$

An agent with ability x^i will make an occupational choice by comparing the profits if she is a full time manager,(fm), self-employed, (se), or the wage as a worker, (l). Her lifetime wealth is:

$$Y_t^i = Y^i(b_t; w_t, r_t) = max_{fm,se,l}[\pi_{fm}^i(b_t; w_t, r_t), \pi_{se}^i(b_t; w_t, r_t), x^i w_t] + (1 + r_t)b_t \tag{2}$$

if $x^i w \geq w_{min}$.

$$Y_t^i = Y^i(b_t; w_t, r_t) = max_{fm,se}[\pi_{fm}^i(b_t; w_t, r_t), \pi_{se}^i(b_t; w_t, r_t)] + (1 + r_t)b_t$$

if $x^i w < w_{min}$.

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Let w_t denote the wage rate per unit of efficiency per hour, r_t be the interest rate and w_{min} be the minimum wage. Hence, the wage rate per hour, $x^i w$, cannot be less than w_{min}. The first impact of the minimum wage policy is that it rules out less productive workers in the formal labor market. Section 1.3 shows that this work force will be self-employed entrepreneurs that work and manage their own firms. They demand just the minimum capital required to start the business. We call them "bad" entrepreneurs in the model because they would be better off as workers, but their low productivity precludes this option.

1.3.2 Technologies

We abstract from financial market frictions to focus on the effects of labor market frictions on occupational choice.

1.3.2.1 Full Time Managers

A full time manager with ability x maximizes:

$$\Pi_{fm}{}^i = x^i k^\alpha n^\beta - wn(1 + \tau) - (1 + r)k \qquad (3)$$

where n is the total efficiency units of labor and τ is the additional cost of labor that firms pay due to obligatory benefits and contract costs that are not paid directly[1] to workers. Notice that since the wage per hour cannot be less than $w_{min,}$ then full time managers will not employ any worker j with $x^j < \frac{w_{min}}{w}$.[2]

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[1] For simplicity, τ is treated in the model as a deadweight loss. Social security benefits and old agents could be added to the model but this would increase complexity but not change the main results.

[2] Minimum wage policy distorts the relative price between units of efficiency of labor as long there exists $x^i > 0$ such that $x^i w < w_{min}$, i.e., the minimum wage policy is effective in the economy. However, the production function for full-time managers (equation 3) treats units of efficiency of labor as perfect substitutes. Therefore, in equilibrium, full time managers maximize profit by choosing zero units of efficiency of labor for all $x^i w < w_{min}$. Let $x^I \in (0,1)$ such that $x^I w = w_{min}$. Mathematically, for given w, r and w_{min}:

$$\max_{k,\{L_i\}_{i\in(0,1)}} xk^\alpha \left(\int_0^1 x_i L_i dx \right)^\beta - w(1 + \tau) \int_I^1 x_i L_i dx - w_{min}(1 + \tau) \int_0^I L_i dx - (1 + r)k$$

From the first order conditions, it is straightforward to show that $L_i > 0$ if and only if $x_i \in [I, 1)$. This means a competitive equilibrium with no unemployment exists if and only if the wage rate increases until $x_1 = \frac{w_{min}}{w}$ where x_1 is the lowest bound for x. Since $x \in (0,1)$, there is no $w \in (0, \infty)$ and no such competitive equilibrium exists.

1.3.2.2 Self Employed Managers

A self-employed manager will divide her time as a manager and worker in her firm. Following Gollin[3] (2008), the time constraint faced by a self-employed manager is given. Let θ be the time the manager works in her firm as a worker; therefore $(1 - \theta)$ is the time that she works as a manager. This entrepreneur with ability x maximizes:

$$\Pi_{se}{}^i = (1 - \theta)x^i k^\alpha (\theta x)^\beta - (1 + r)k \tag{4}$$

1.4 Competitive Equilibrium

1.4.1 Household's Problem

Given the utility function defined in equation (1), the optimal choice for households is to consume proportion γ of their lifetime wealth given by equation (2) and leave bequest (1-γ) of Y_t^i. Therefore, given prices (w_t, r_t), household occupational choice solves (2) and $c_t^{i*} = \gamma Y_t^{i*}$ and $b_t^{i*} = (1 - \gamma)Y_t^{i*}$ (Antunes et. al, 2008)[4].

Before we consider the market clearing conditions, define $\Omega = (0, \infty) \times [x_{min}, x_{max}]$. We assume that bequests might be near zero but not equal to zero. This guarantees positive lifetime wealth and therefore, positive bequests for all agents even in the presence of a minimum wage policy. Finally, let the measure of households in each occupational choice be given by:

$$E_{fm}(w, r) = \{(b, x) \in \Omega : fm = argmax\{Y^i(b_t; w_t, r_t)\},$$

$$E_{se}(w, r) = \{(b, x) \in \Omega : se = argmax\{Y^i(b_t; w_t, r_t)\},$$

$$E_w(w, r) = \{(b, x) \in \Omega : l = argmax\{Y^i(b_t; w_t, r_t)\}.$$

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[3] Gollin (2008) allows self-employed managers to choose additional labor in the market. For simplicity, we assume that self-employed agents choose only capital. This assumption will not change the analytical results, but it might decrease profit and capital demand for some levels of ability. On the other hand, with the imposition of labor market rigidities, this assumption decreases in importance since if labor market frictions are high enough the optimal labor demand for self-employed managers will be zero.

[4] It is possible to define the competitive equilibrium with labor frictions (w_{min}, τ) for a general production function, where the elasticity of substitution among different labor productivity is finite. In this case, equation (2) for $x^i w < w_{min}$ is: $Y_t^i = Y^i(b_t; w_t, r_t) = max_{fm, se}[\pi_{fm}^i(b_t; w_t, r_t), \pi_{se}^i(b_t; w_t, r_t), w_{min}(1 - u(x^i; w_t, r_t))] + (1 + r_t)b_t$, where $u(x^i; w_t, r_t)$ is the unemployment function rate for each $x_i \in (0, I)$. In the definition of the competitive equilibrium, $u(x^i; w_t, r_t)$ is such that, for given w_{min}, total demand of labor for type $x_i \in (0, I)$ equals supply: $w_{min}(1 - u(x^i; w_t, r_t))L_i$.

1.4.2 Market Clearing Conditions

Following Antunes et. al (2008), let Υ_0 be the initial distribution of wealth. Then the market clearing conditions for the labor and capital markets are:

$$\iint_{\in E_{fm}(w_t,r_t)} n(x; w_t, r_t)\Upsilon_t\,(db)\Gamma(dx) = \iint_{(b,x)\,\in\,E_w(w_t,r_t)} x\Upsilon_t(db)\Gamma(dx)$$

$$\iint_{(b,x)\,\in\,E_{fm}(w_t,r_t)\,\cup\,E_{se}(w_t,r_t)} k(b, x; w_t, r_t)\Upsilon_t\,(db)\Gamma(dx) = \iint b\Upsilon_t(db)\Gamma(dx)$$

The law of motion for wealth distribution is:

$\Upsilon_{t+1} = \int P_t(b, A)\,\Upsilon_t(db)$ where $P_t(b_t, A) = \Pr(b_{t+1} \in A/b_t)$.

1.5 Qualitative Results

1.5.1 Model without Labor Market Frictions

First, let's consider the efficient case where w_{min} equals zero. In the appendix, we show that the income for both types of entrepreneurs is strictly increasing and strictly convex in the ability level. Moreover, profits for full-time managers are "more convex" than the total remuneration of self-employed agents.

Assumption 1. $\theta^\beta(1 - \theta)(1 - \alpha)^{1-\alpha} \leq \beta^\beta(1 - \alpha - \beta)^{1-\alpha-\beta}$. (5)

Proposition 1. Under assumption 1, there is no self-employment. Furthermore, there is a unique x* such that for all $x \leq x^*$, the agent prefers to work and for all $x > x^*$ the agent prefers to be a full-time manager.

Assumption 1 guarantees that there are no self-employed managers in this economy, i.e., the parameterization is such that the economy is in the case shown by figure 1.3[5]. This happens because in the equilibrium, if it is optimal for an agent to be a self-employed manager instead of a worker, then it will be optimal to be a full-time manager instead a self-employed manager (see

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[5] If agent is indifferent, assume that she chooses to work.

the Appendix for a complete proof). Note that assumption 1 does not depend on prices: interest rate and wage. This result is consistent with theory: in a long-run equilibrium without frictions, the productive structure will be concentrated, with small family business disappearing and self-employed workers being wage workers (Lucas, 1978, Kuznets, 1966, Schumpeter, 1934). Therefore, without any frictions, in the labor market the inclusion of a self-employment sector is redundant as figure 1.3 below shows:
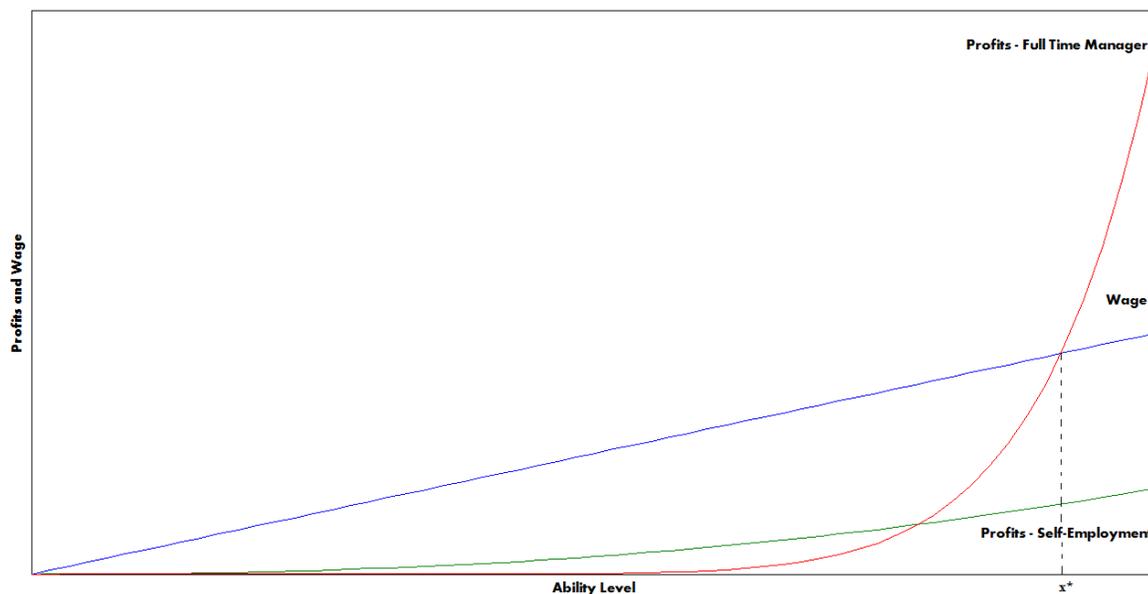
Figure 1.3: Profits for each type of entrepreneur and wage as a function of the ability level.

1.5.2 Model with a Minimum Wage Policy

Despite the simplicity of the labor market frictions incorporated in the model, the occupational choices for different levels of ability differ significantly with respect to the model without frictions in the labor market. Recall that we do not consider any financial market frictions.

Proposition 2: for any $w_{min} > 0, \exists\, x^{**} > 0\ and\ x^* > 0, such\ that$

    i) $\forall\, x < x^{**}$, agents will be self-employed.

    ii) $\forall\, x \in [x^{**}, x^*]$, agents will work.

    iii) $\forall\, x > x^*$, agents will be full-time managers.

Figure 1.4 below shows that, under assumption 1, there are self-employed managers, the "bad" entrepreneurs. With a minimum wage policy, there is a level of ability, x^{**}, such that for all

$x < x^{**}$, agents will be ruled out from the formal labor market. Since their ability is low enough, they would be better off working but since they are unemployed, they will become "bad" entrepreneurs in order to assure a positive income. Even without any borrowing constraints or spread between interest rates, it is not optimal for them be full time managers because their projects will not be sufficiently profitable.

Another interesting implication of introducing a minimum wage policy in the labor market is the general equilibrium effect. Notice that the number of "good" entrepreneurs decreases in figure 1.4. This happens because the less productive workers are not in the labor market, and hence the supply of labor decreases, increasing the equilibrium wage and interest rate. Therefore, the threshold ability x^* is greater in this model. The higher the minimum wage, the higher is this general equilibrium effect.
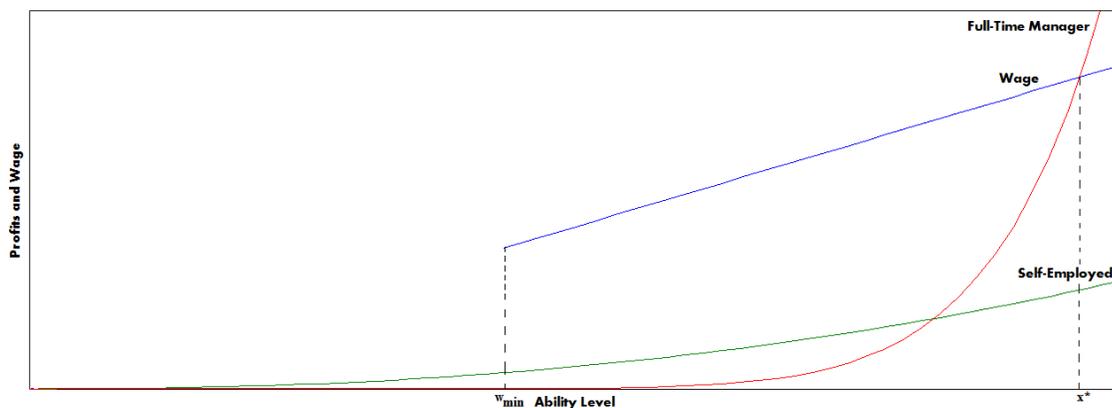

Figure 1.4: Occupational Choice with Frictions in the Labor Market
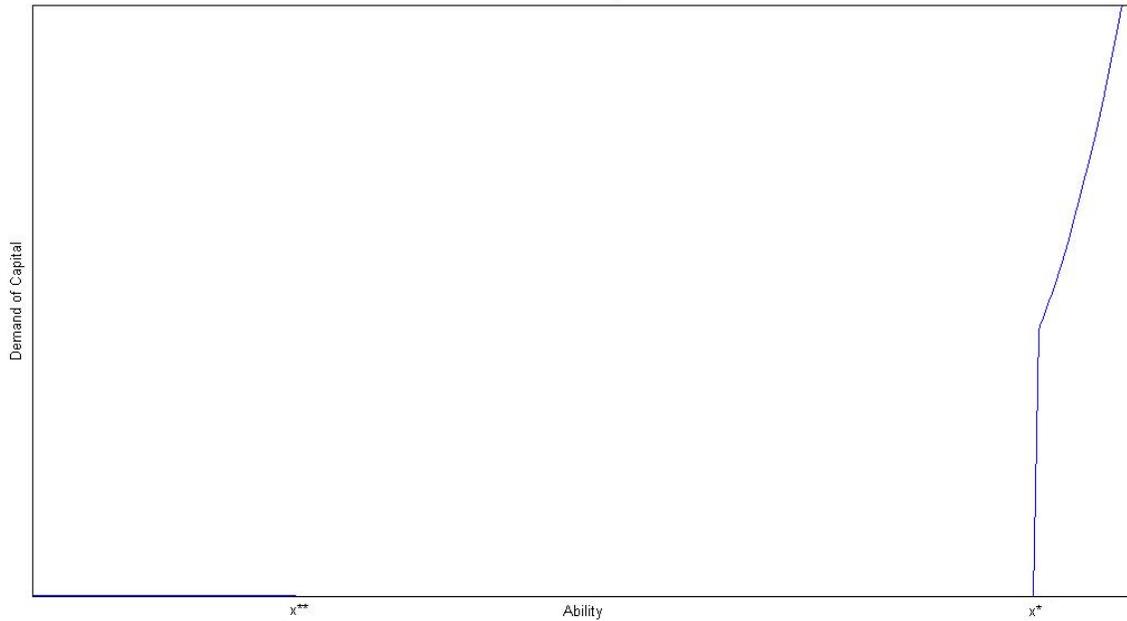
Figure 1.5: Demand of Capital for Different Levels of Ability with Frictions in the Labor Market

Figure 1.5 above shows the capital demand by self-employed and full-time managers.[6] The levels of capital demand vary significantly for these two types of entrepreneurs. In figure 1.5, firm size for self-employed entrepreneurs is slightly greater than zero; they invest just the amount necessary to keep the business open. For $x \in [x^*, x^{**}]$, optimal occupational decision is to be a worker and, therefore, their demand of capital is zero. For $x > x^*$, agents will be full-time managers and the demand for capital jumps. These are the "good" entrepreneurs, in the sense that they are responsible for capital accumulation, higher wages and higher production.

1.5.3 Model with a Deadweight Cost

Now consider another friction in the labor market. Suppose there is no minimum wage policy but there is a distortion between the cost of labor for full time managers and the wage received by workers. This distortion reflects different policies that increase the cost of labor for firms such as hiring and firing costs and penalties, advance notice requirements, contract length and cost, legal

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[6] The demand for capital by both types of entrepreneurs is strictly increasing and convex in the ability level.

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workweek constraints, etc. Let, as before, wx be the total wage received if the agent is a worker but full time managers pay the wage rate: $(1 + \tau)w$.

Assumption 2. $(1 + \tau) > \frac{\beta}{\theta} \left(\frac{(1-\alpha-\beta)^{(1-\alpha-\beta)}}{(1-\theta)(1-\alpha)^{(1-\alpha)}} \right)^{\frac{1}{\beta}}$

Proposition 3. Under assumption 2, there exists a unique $x^* > 0$ and $x^{**} > 0$ such that:

    i.   For all $x < x^{**}$, the agent will be worker.

    ii.   For all $x \in [x^{**}, x^*]$, the agent will be self-employed.

    iii.   For all $x > x^*$, the agent will be a full-time manager.

With the introduction of protective labor laws, threshold ability x* increases, i.e., the number of full time managers decrease due to the decrease in profits. If the increase in labor costs perceived by full-time managers is high enough, proposition 3 shows that, at an optimum, agents with intermediate levels of ability will be self-employed instead of full-time managers, therefore, decreasing the total demand for capital in the economy, as figure 1.7 below shows.
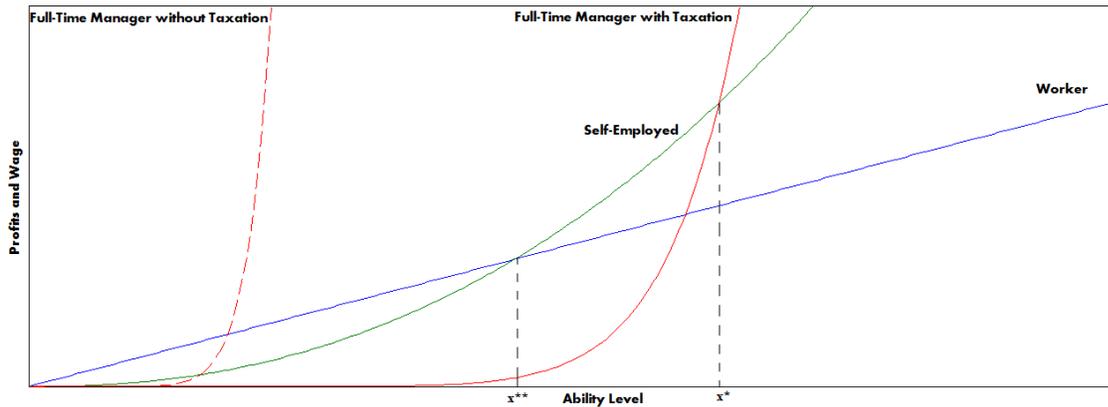

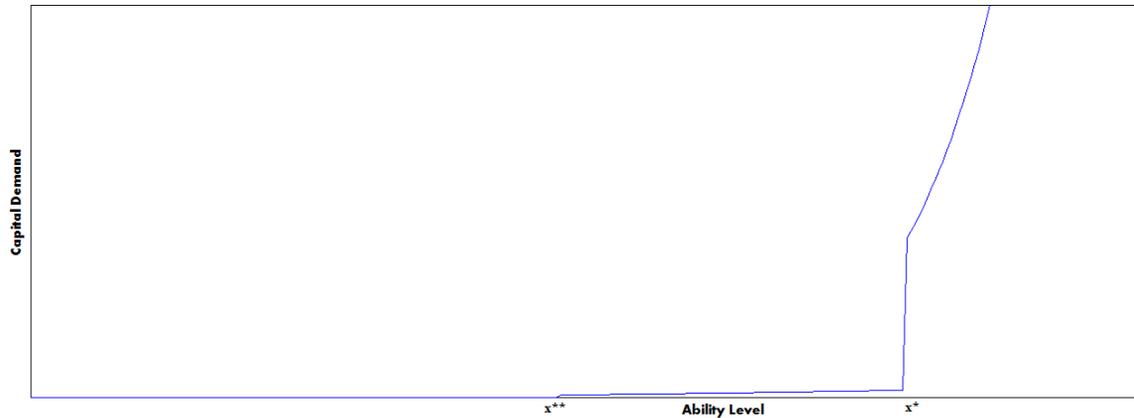Figure 1.6: Occupational Choice with Frictions in the Labor Market.

Figure 1.7: Demand for Capital by Different Levels of Ability with Frictions in the Labor Market

1.6 Concluding Remarks

Despite the simplicity of the model and the labor policies considered, we show that labor market frictions have important effects on occupational choice and firm size. This result is consistent with the empirical facts presented and offers an additional explanation for the differences and rigidity of productive structures across countries. The other main finding of this model is the trade-off between working and being a full-time manager for high ability agents. This trade-off comes directly from the assumption that more able managers will also be more productive workers. In the presence of labor market rigidities, the trade-off is reinforced by a general equilibrium effect that decreases the share of "good" entrepreneurs in the economy, decreasing the total demand for capital and total output. This prevents the economy from shifting to another productive structure with higher a concentration of capital and larger firms.

Finally, it is important to highlight that we focused solely on the cost of labor market rigidities for different agents and how this cost affects their occupational choice and firm size. One could also consider the benefits that labor policies such as social security have on the decision to become worker. Modeling and measuring the cost versus the benefit of different protective labor laws is an important extension of this work, however, that is beyond the goal of this chapter. Policies that increase the benefit of working (net of their cost), would increase the supply of labor and, therefore, in a competitive equilibrium, the wage rate should decrease, offsetting the impact of these benefits in the occupational choice model studied here.

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CHAPTER 2

QUANTITATIVE ANALYSIS OF LABOR MARKET RIGIDITIES ON ENTREPRENEURSHIP AND ECONOMIC DEVELOPMENT

2.1 Introduction

Many studies have confirmed the importance of financial frictions such as intermediation costs and contract enforcement on an agent's decision to become an entrepreneur and on the size of the firm the agent chooses to operate. This chapter introduces financial constraints together with labor market rigidities. Financial frictions decrease the total demand for capital and increase the share of self-employed managers. This quantitative experiment uses the Antunes et. al (2008b) general equilibrium model, which provides a framework for investigating agents with different level of initial wealth, ability and a variety of economic frictions. Initial wealth and financial constraints can explain why managers with high ability but low initial wealth do not become entrepreneurs. Labor frictions can explain "good" and "bad" entrepreneurs, i.e., the decision be self-employed or a full time manager. This chapter investigates how labor market frictions, mainly minimum wage policy and higher labor costs due to rigidity, affect occupational choice, firm size and macroeconomic variables such as output per capita and inequality.

Empirical work has found controversial results about the effects of minimum wage policies. Bell (1997) found significant negative effects on employment in the case of Colombia, and insignificant effects on the labor market in Mexico. Maloney *et al.* (2001) showed that an increase in the minimum wage has a statistically significant impact on the probability that an individual will become unemployed in Colombia. More importantly, the probability is reduced for higher positions in the wage distribution; i.e., minimum wages affect more severely those whose income is close to the minimum. Foguel (1998) found that an increase of about 10 percent in the minimum wage raises the open unemployment rate by 0.5 percentage points in Brazil. On the

other hand, Lemos (2000) found the elasticities of employment with respect to changes in the minimum wage in Brazil were not statistically different from zero.

The baseline model is calibrated to match U.S. key statistics. Then, the baseline model is used to investigate the effect of labor market rigidities as minimum wage policy and social security benefits on occupational choice, firm size, prices, output per capita and inequality. Finally, a counterfactual experiment is done using the Brazilian case. Brazil has one of the highest hiring costs for firms in terms of contracts, bureaucracy, social benefits and taxation. It is also one of the most unequal economies, with an impressive self-employment - workforce ratio of 32%. Brazil also has a federal legal minimum wage that has been increasing recently, although it is still lower in real terms if compared with the US federal minimum wage.

2.2 The Model

As in the previous chapter, the model in an overlapping generations model where agent maximizes utility by choosing optimal consumption and a bequest:

$$u^i(b_t^i, x_i, z_i) = (c_t^i)^\gamma (b_{t+1}^i)^{1-\gamma} \tag{1}$$

The individual state variables for each agent are the wealth or bequest received, b_t^i, entrepreneurial ability, x_i, and labor productivity, z_i. Labor productivity is drawn from a cumulative distribution function $Л(z)$. Entrepreneurial ability is drawn from a cumulative distribution function conditional on labor productivity, $\Phi(x/z)$. Therefore, high skilled workers also have more entrepreneurial ability.

An agent again chooses among three different occupational choices: worker, self-employed or full-time manager. If the agent is a worker, he receives the wage:

$$z^i w_t \text{ if } z^i w_t \geq w_{min}$$

If agent is self-employed, he receives the profit:

$$\Pi_{SE}^i = \max_{\{a,l,k\}} (1-\theta) x^i k^\alpha (\theta x^i)^\beta - (1+r)a - (1+r+\tau)l \tag{2}$$

Subject to:

$$a + l = k \tag{3}$$

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$$\phi\pi^i_{SE} \geq (1 + r + \tau)l \tag{4}$$

$$b^i \geq a \tag{5}$$

Where l is the quantity of loans. Following Antunes et. al (2008), we introduce two financial frictions: an intermediation cost, τ and a level of contract enforcement, ϕ. The intermediation cost ensures that, in the optimum, the entrepreneur will use his own wealth to buy capital, i.e., $l > 0$ if $k > b^i$. Enforcement guarantees that loans will be repaid. Therefore, higher is ϕ, the higher is the quantity of loans available for entrepreneurs.

Finally, if the agent is a full-time manager, he receives the profit:

$$\pi^i_{FM} = \max_{\{a,l,k,\eta\}} x^i K^\alpha_t n^\beta_t - wn_t(1 + \tau_l) - (1 + r)a - (1 + r + \tau)l \tag{6}$$

Subject to:

$$a + l = K \tag{7}$$

$$\phi\pi^i_{FM} \geq (1 + r + \tau)l \tag{8}$$

$$b^i \geq a \tag{9}$$

Full-time managers face both financial constraints and also a deadweight cost of labor due to employment protection laws, τ_l.

2.3 Partial Equilibrium Analysis

In the absence of labor market frictions, there is no self-employment. As pointed out by Antunes et. al (2008), financial frictions distort occupational choice for agents with low enough wealth. When entrepreneurial ability and labor productivity are linked, this distortion is higher since the opportunity cost of being an entrepreneur, which is the wage workers receive, increases with ability.
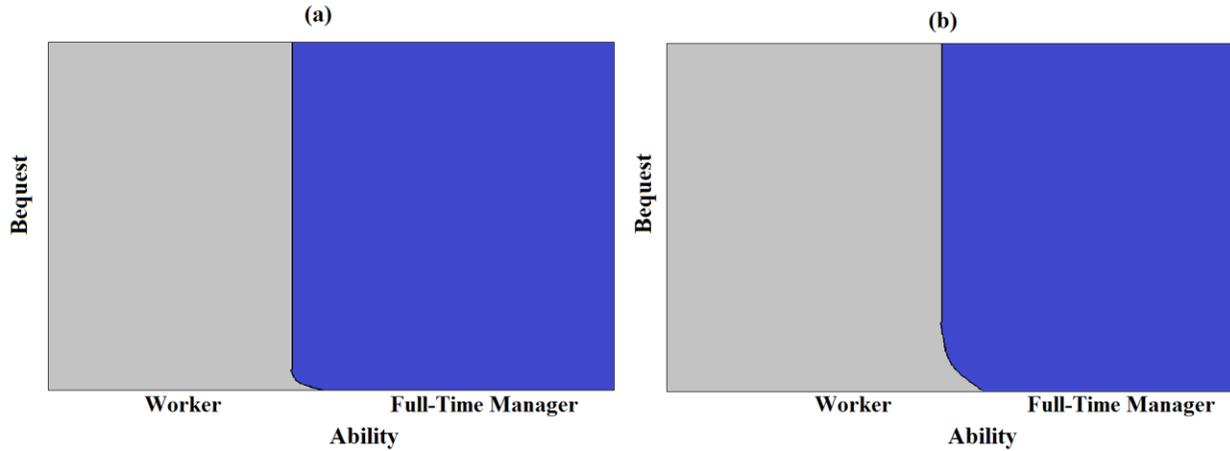
Figure 2.1: Occupational choice with financial frictions when (a) ability and labor productivity are independent and (b) ability and labor productivity are dependent.

By introducing the deadweight cost of labor regulation laws, some agents choose to be self-employed, since the profits of full-time managers decrease. By introducing a minimum wage policy, low skill workers are ruled out from labor market and therefore, must choose self-employed since their ability is not high enough to be full-time manager.
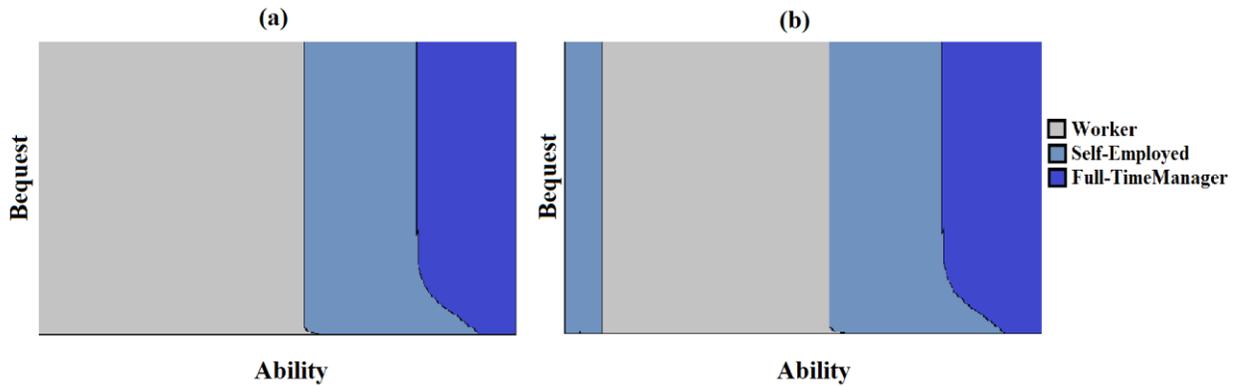
Figure 2.2: Occupational choice with financial frictions and (a) employment protection laws on labor, (b) minimum wage policy and employment protection laws on labor.

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2.4 Competitive Equilibrium

The existence and uniqueness of a stationary competitive equilibrium is proved in Antunes et al (2008). Let the measure of households in each occupational choice be $E_{fm}(w,r)$, $E_{se}(w,r)$ and $E_w(w,r)$ as defined in chapter 1. Then the market clearing conditions and the law of motion that characterizes the stationary competitive equilibrium are:

$$\iint_{(b,z,x)\,\in\,E_{fm}(w,r)} n(x;w,r)\Upsilon(db)Л(dz)\Phi(x/z)(dx) = \iint_{(b,z,x)\,\in\,E_w(w,r)} z\Upsilon(db)Л(dz)\Phi(x/z)(dx)$$

$$\iint_{(b,z,x)\,\in\,E_{fm}(w,r)\,\cup\,E_{se}(w,r)} k(b,x;w,r)\Upsilon(db)Л(dz)\Phi(x/z)(dx) = \iint b\Upsilon(db)Л(dz)\Phi(x/z)(dx)$$

$$\Upsilon = \int P(b,A)\,\Upsilon(db)$$

2.5 The Baseline Model

In order to map the model to US data, it is necessary to calibrate 16 parameters. The financial friction parameters, τ and ϕ, and the parameters for the production function, α and β, were taken from Antunes et al (2008). Gollin (2002) estimated that the percentage of the time a self-employed individual works in his own business equals 57.5%. Finally, τ_L, the cost of labor regulation laws, equals the hiring cost indicator provided by the OECD database. The hiring cost indicator measures all social security payments and payroll taxes associated with hiring an employee as a percentage of salary.

Parameters determined independently	Value	Observations
α	0.35	Capital share based on Antunes et. al, 2008 and Gollin (2002)
β	0.55	Labor share based on Antunes et. al, 2008 and Gollin (2002)
φ	0.26	Investor protection based on Antunes et. al, 2008
τ	0.1926	Intermediation cost based on Antunes et. al, 2008
τ_L	0.08	Hiring cost indicator for US (OECD – Doing Business Dataset)
$1 - \theta$	0.575	% of time self-employers manage their firms (Gollin, 2002)

Table 2.1: Parameters determined independently.

The remaining 10 parameters were jointly determined to match key statistics of US economy: annual real interest rate, percentage of entrepreneurs, percentage of entrepreneurs out of necessity, the quintiles for income distribution, entrepreneur's income Gini index, income Gini index and the percentage of full time managers. Following Aiyagari (1994), let the logarithm of labor income follow a first-order autoregressive process. Therefore, using Tauchen's method, the conditional labor productivities can be approximated by a N-state Markov Chain. This is an useful method since the N state variables for productivity and the N by N transition matrix can be found by calibrating just two parameters: the persistence of a productivity shock (or intergeneration persistence of productivity level) and the variance, ρ_Z and σ_Z^2. The model does not consider intergenerational links between productivity, so instead of the transition matrix, we

use the invariant distribution (unconditional probabilities) generated by ρ_z and σ_z^2. We consider 9 different levels of labor productivity[7].

Entrepreneurial ability is drawn from a cumulative distribution: $\int_{x_{min}^z}^{y} \Phi(x/z) = P(x \leq y/z) = \left(\frac{y - x_{min}^z}{x_{max}^z - x_{min}^z}\right)^{\varepsilon}$ (Chatterjee et al., 2002). For simplicity, only three different cumulative distributions for entrepreneurial ability are considered: $\Phi(x/z_1) = \Phi(x/z_2) = \Phi(x/z_3)$, $\Phi(x/z_4) = \Phi(x/z_5) = \Phi(x/z_6)$, and $\Phi(x/z_7) = \Phi(x/z_8) = \Phi(x/z_9)$. Hence, there are 6 parameters to calibrate for ability: $\left(\varepsilon, \frac{x_{max}^1}{x_{min}^1}, \frac{x_{min}^2}{x_{min}^1}, \frac{x_{max}^2}{x_{min}^1}, \frac{x_{min}^3}{x_{min}^1}, \frac{x_{max}^3}{x_{min}^1}\right)$.

Parameters determined jointly	Value
ρ_z	0.65
σ_z^2	0.25
ε	0.55
x_{max}^1/x_{min}^1	0.5
x_{min}^2/x_{min}^1	1
x_{max}^2/x_{min}^1	2
x_{min}^3/x_{min}^1	10
x_{max}^3/x_{min}^1	15
w_{min}	$0.3w_{mean}$
γ	0.92

Table 2.2: Parameters calibrated jointly by minimizing the weighted sum of square errors.

Table 2.2 above shows the values for the parameters calibrated together in order to match the target values of the US economy, shown in table 2.3 below. The parameters for the labor productivity distribution, ρ_z and σ_z^2, are consistent with the estimates found by Zimmerman

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[7] The choice of the number of states for labor productivity, N, is important. If N is too low it might influence the competitive equilibrium results.

$(1992)^8$. The parameter for the entrepreneurial ability distribution, ε, is 0.55 which is near the estimate of 0.5 found by Chatterjee et al. (2002). The minimum wage was calibrated to match the percentage of self-employed out of necessity in U.S. It is slightly higher than the minimum wage in U.S., which is 20-25% of the average wage. Parameter γ is 0.92. Therefore, the percentage of wealth left as bequest is 8%, which is higher than the 6% found by Antunes et al. (2008b). The lower γ found in this calibration is expected since this model has a lower supply of capital because of the existence of self-employment, therefore, a higher bequest rate is needed in order to match the real annual U.S. interest rate.

Targets	Baseline Model	US Economy
Self-Employed out of necessity (wage<w_{min}) as % of total number of entrepreneurs	10.7%	11%
Full Time Managers as % of total number of entrepreneurs	26.6%	26.25%
% Entrepreneurs	9.8%	9%
Entrepreneur's Income Gini	0.457	0.45
Income Gini Index	0.40	0.4
Income Shares		
First Quintile	4.8%	3.6%
Second Quintile	11%	8.9%
Third Quintile	12.7%	14.8%
Fourth Quintile	22.5%	23%
Fifth Quintile	49%	49.7%
Interest rate	2.7%	2%

Table 2.3: Targets chosen to calibrate parameters in table 2.2. Sources: U.S. Bureau of the Census, Global Entrepreneurship Monitor and Quadrini (1999).

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[8] The vector for the possible labor productivities is Z=[0.5908   0.6738   0.7686   0.8767   1.0000   1.1406   1.3011   1.4840   1.6928] and the respective unconditional probabilities are P(Z)=[0.0377   0.0657   0.1219   0.1756   0.1982   0.1756   0.1219   0.0657   0.0377].

In order to match the targets described in table 2.3 for the Gini income indexes, we gave more weight for the shares of full time manager and self-employed out of necessity than for the quintiles and interest rate.[9] The model matches very well the income distribution in U.S. Interestingly, it also matches the right tail of the income distribution, for instance, in data, the richest 5% have 22.2% of total income. In the model, this share is 23.5%. The mean-median ratio for income in U.S., is 1.61 (Búdria et al, 2002), while in the model, the same ratio is 1.7.

2.6 Quantitative Results

2.6.1 Labor Market Rigidities

In order to investigate the effects of minimum wage policy and the effect of the deadweight cost of labor on inequality, prices and output per capita, we conduct a set of experiments in which we change these policies and keep all the other parameters constant. First, we analyze the effect of an increase in the minimum wage alone. Second, we analyze an increase in the cost of labor. Subsection 2.6.1.3 shows the change in both parameters, using the Brazilian case as an example.

2.6.1.1 Minimum Wage Policy: w_{min}

In this section, the minimum wage is increased to 1.5 and 2 times the baseline model. The main results are showed in table 2.4 below.

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[9] The weights are [0.2 0.2 0.2 0.2 0.1 0.02 0.02 0.02 0.02 0.02] for percentage of self-employment out of necessity, percentage of full-time managers, Entrepreneur's Gini index, Income Gini Index, percentage of entrepreneurs, income quintiles and interest rate, respectively.

	Output per Capita	Total Employment	Wage Rate	Gini Index	% Full Time Managers	Interest Rate
Baseline	100	100	100	0.40	26.6%	2.7%
$1.5w_{min}^{baseline}$	93.2	85.5	107.6	0.46	12.7%	2.7%
$2w_{min}^{baseline}$	72.4	57.2	118.9	0.58	4.7%	3.1%

Table 2.4: Minimum wage policy experiments.

An increase of the minimum wage increases firms' demand for highly skilled labor. The first effect is to increase the wage rate in the economy. On the other hand, the increase in the wage rate decreases the number of full time managers, decreasing the demand for labor and capital. Output per capita decreases and, therefore, the supply of capital decreases. These effects together have very little net effect on the price level in the competitive equilibrium. However, the percentage of full time managers decreases substantially and the percentage of self-employment increases, increasing inequality in the economy.

2.6.1.2 Cost of labor: τ_l

Table 2.4 below shows the results when the labor cost due to labor protection laws is increased 2 and 6 times the level of the baseline economy.

	Output per Capita	Total Employment	Wage Rate	Gini Index	% Full Time Managers	Interest Rate
Baseline	100	100	100	0.40	26.6%	2.7%
$2\tau_l$	96.1	98.6	79.0	0.44	25.5%	3.2%
$5\tau_l$	79.8	84.8	60.0	0.54	12.3%	3.9%

Table 2.5: Minimum wage policy experiments.

When τ_l is doubled, price adjustment in the competitive equilibrium offsets the negative effects on output per capita and total employment. However, the lower wage rate increases inequality in the economy. When $\boldsymbol{\tau_l}$ is increased to five times the baseline model, the effects on output per capita, employment and occupational choice are significant. This occurs because the decrease in the wage rate is high enough to rule out more workers from the labor market, keeping the minimum wage constant. This is an important result. The existence of self-employment due to necessity is caused by the minimum wage policy. However, the size of this type of self-employment is determined by labor protection regulations jointly. The Brazilian case below will illustrate how important it is to consider labor market frictions jointly.

2.6.1.3 The Brazilian Case

Brazil was chosen because it has very high inequality and a significant self-employment rate. It also has one of the highest index scores for entrepreneurs out of necessity: 48%. However, Brazil's minimum wage is 20% lower than U.S. minimum wage. This experiment attempts to quantify how much of Brazilian labor rigidities can account for the differences in occupational choice and inequality between Brazil and U.S.

Table 2.4 below shows the result for the baseline model when the labor rigidity parameters are set for Brazilian economy. The model accounts for all out of necessity self-employment. Note that Brazil's real minimum wage is lower than the U.S. minimum wage, however, due to the higher cost of labor in Brazil, the wage rate is much lower. In the competitive equilibrium, the percentage of less productive workers ruled out in the formal market is higher in Brazil than in US. We find that differences in labor market regulations account for 26% of the difference in output per capita and almost 60% of the difference in average real wage rates. This suggests that financial frictions play an important rule for the choice of full time managers. Labor market rigidities explain 85% of the total self-employment, with higher level of financial frictions, the profit for full-time managers would decrease further, increasing self-employment and decreasing even more output per capita. The results in this experiment also show that labor market rigidities

are closely linked to inequality. An increase of 4 times the size of self-employment is responsible for an increase of more than 10 points of the Gini Index.

	Output per Capita	Ratio Self-Employed/ Workforce	Wage Rate	Gini Index	%Self Employed out of necessity
Baseline	100	7.19%	100	0.40	10.7%
Brazil (data)	22	32%	31.20	0.55	48%
$3.75\tau_l$, $0.7w_{min}$	84	27.28%	65.68	0.51	49%

Table 2.6: Minimum wage policy experiments.

2.7. Conclusion

This quantitative experiment attests to the importance of considering labor market frictions can explain occupational choice and inequality. The model matches very well key statistics on inequality in the U.S. Two policies were analyzed: a minimum wage and the cost of hiring due to labor protection laws. The minimum wage policy alone has little effect on prices but it increases the share of self-employment due to necessity, shifts the demand curve for firms for higher productivity workers and decreases the percentage of full time managers and, therefore, decreases output per capita and increases inequality. A higher cost of labor for firms due to social security benefits, taxation and contracts decreases the wage rate in the competitive equilibrium. This general equilibrium effect on prices might offset the initial increase on the cost of labor and therefore, it has little effect on output per capita. However, if the decrease in the wage rate is high enough, it can rule out more workers from the labor market. The example is the Brazilian economy, where the minimum wage is lower but the share of entrepreneurs due to necessity is much higher because of the higher cost of labor for firms.

CHAPTER 3

EDUCATION SYSTEM, INEQUALITY AND PRODUCTIVITY: A QUANTITATIVE ANALYSIS

3.1 Introduction

The positive relationship between human capital accumulation and economic growth is well known. Human capital plays an important role in various endogenous growth models because of investments in the R&D sector of firms (Romer, 1990). Barro (1991), using school enrollment rates as proxy for human capital, shows that tconvergence in levels of per capita income between poor and rich countries found by Solow (1956) is only possible if poor countries have a high level of human capital. Galor and Zeira (1993) show that frictions in the credit market and non-convexity in human capital investment imply sub-optimal investment in human capital of poor households, resulting in high and persistent income inequality and lower growth. Therefore, human capital accumulation is a key factor for growth and for a more egalitarian economy.

Latin America has one of the highest and most persistent rates of income and wealth inequality in the world. Latin America and the Caribbean have a Gini Index nearly 10 points higher than Asia, 17.5 points higher than the countries in OECD, and 20.4 points higher than Eastern Europe. Moreover, despite the market reforms in most Latin American countries in middle 90s, income inequality slightly increased in this period. On the other hand, from 2000 to 2006, income inequality experienced a steady decrease, mainly for Brazil and Mexico. During this period, the Mexican and Brazilian governments adopted a set of policies to provide incentives for education. Mexico's "*Oportunidades*" program helps poor families to finance educational and health costs, and Brazil's "*Programa Nacional de Bolsa Escola*" and "*Programa Bolsa Família*" offer education subsidies and direct transfers to poor families, respectively. In both countries, these policies resulted in the expansion of basic education, which substantially reduced the share of

people with only primary and less-than-primary schooling and decreased the skill premium (Lopez-Calva and Lustig, 2010). Hence, it is important to study the quantitative implications of Latin America's current education system and policies on long run inequality and productivity. This paper uses the Brazilian experience to measure the impact of such educational policies on microeconomic decisions such as the type of education and whether attend or not to attend university as well as the impact of macroeconomic variables such as production, the Gini coefficient and the skill premium.

The next section presents the model. Section 3.3 shows the calibrated parameters for the Brazilian economy. Section 3.4 analyses different government policies related to the education system. Section 3.5 provides the concluding remarks.

3.2 The Model

The economy is inhabited by overlapping generations of individuals who live for three periods: childhood, young adulthood and adulthood[10]. In each generation, there is a mass of agents. Young adults and adults are endowed with one unit of productive time. Their productivity is proportional to their ability as workers (or human capital), h, and adults can also be skilled or unskilled, depending whether or not they attended a university during the young adulthood period.

Young adults are heterogeneous in two dimensions. Their initial value of assets, b, and their realized level of ability, which depends on: (i) an idiosyncratic term $z \in \{z_1, ..., z_5\}$, where $z_i > z_j \geq 0$ for $i > j$ and once z is realized it stays the same during the individual's lifetime, but across generations it follows a Markov process with transition matrix $\mathcal{P}(z, z') = \Pr(z_{t+1} = z' | z_t = z)$; and (ii) on the human capital investment made by parents, s_{-1}. Parents are altruistic towards their offspring. Parents can send their children to a private school, $s_{-1} = 1$, or public school $s_{-1} = 0$. There is a premium, θ, in terms of additional human capital to attend a private school, such that:

$$h = z(1 + \theta s_{-1}) \tag{1}$$

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[10] The introduction of old adults would not change the main properties of the model.

Private schools cost p_s per student and public school students do not pay any fee. Young adults can choose to not attend a university or to attend one. If they choose to not attend a university, they can work as an unskilled worker and in the next period they will also be unskilled. If they choose to acquire a university degree, then they will work only a fraction $\gamma \in (0,1)$ of their time endowment as unskilled workers during the young adulthood period. However, they will become skilled workers with probability $\pi_{Pr} \in (0,1]$ if attending a private university and with probability $\pi_{Pu} \in (0,1]$ if attending a public university. Skilled adults can also do unskilled tasks. There is no fee to attend a public university, but entrance is restricted by a fixed number of places and students are selected according to their human capital acquired during childhood, which is a proxy for an entrance exam. The public education system (schools and universities) are financed through taxes, τ, levied on labor income. There is a price $p_u > 0$ per student to attend a private university. Bellow we describe the model in detail.

3.2.1 The production sector

The consumption good is produced with a CES technology that uses skilled, S, and unskilled labor, U, as inputs, as well as capital, K. The technology exhibits constant returns to scale and is given by:

$$Y = \left[\mu(\alpha K^\rho + (1-\alpha)S^\rho)^{\frac{\eta}{\rho}} + (1-\mu)U^\eta \right]^{\frac{1}{\eta}}, \mu, \alpha \in (0,1), \text{ and } \rho, \eta \in (-\infty, 1) \tag{2}$$

The elasticity of substitution between capital and unskilled labor is equal to $\frac{1}{1-\eta}$ and the elasticity of substitution between capital and skilled labor is $\frac{1}{1-\rho}$. As in Krussell et al, 2000, we assume that there is capital skill complementarity and we assume that $\eta > \rho$.

Factor markets are competitive, and input prices equal the marginal product of each factor, such that:

$$w_s = \mu(1-\alpha)Y^{1-\eta}(\alpha K^\rho + (1-\alpha)S^\rho)^{\frac{\eta}{\rho}-1}S^{\rho-1}, \tag{3}$$

$$w_U = (1-\mu)Y^{1-\eta}U^{\eta-1}, \tag{4}$$

$$r^K = \mu\alpha Y^{1-\eta}(\alpha K^\rho + (1-\alpha)S^\rho)^{\frac{\eta}{\rho}-1}K^{\rho-1}. \tag{5}$$

Capital fully depreciates between periods.

3.2.2 The household sector

Households value consumption according to the following one-period utility function:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad \sigma > 0.$$

Households also care about the utility of their offspring. A young adult with human capital h and asset α^1 might decide to attend or not a university and acquire a high education degree. Let $V_{nd}(\alpha^1, h)$ and $V_d(\alpha^1, h)$ be the utility value of not attending and attending a university, respectively, for a young adult with ability h and asset α^1. She will choose the option which gives the highest utility, such that:

$$V(\alpha^1, h) = \max_{\{no\ degree, degree\}}\{V_{nd}(\alpha^1, h), V_d(\alpha^1, h)\} \tag{6}$$

Below, we characterize the problem of this educational choice.

3.2.2.1 The problem of a young adult not attending a university

If a young adult decides to not acquire a university degree, she will solve the following problem.

$$V_{nd}(\alpha^1, h) = \max_{\{c, a^2 \geq 0\}}\{u(c) + \beta W_U(a^{2'}, h)\}, \tag{7}$$

subject to

$$c + a^{2'} \leq (1-\tau)w_U h + (1+r)a^1, \text{ and } a^1 = b \tag{8}$$

Function $W_U(a^{2'}, h)$ describes the continuation value of an unskilled adult with asset $a^{2'}$ and ability h and r stands for the interest rate, and $\beta \in (0,1)$ is the subjective discount factor. By arbitrage, we have that $r^K = 1 + r$.

3.2.2.2 The problem of a young adult attending a university

Let $p_u > 0$ be the price to attend a private university. Young adults that attend a university can work a fraction $\gamma \in (0,1)$ of their time endowment as unskilled workers. In addition, with probability π_S they become skilled workers in the next period and with probability $1 - \pi_S$ they remain unskilled workers. Young adults who choose to attend a university solve the following problem:

$$V_d(a^1, h) = \max_{\{c, a^{2'} \geq 0\}} \{u(c) + \beta[\pi_j W_S(a^{2'}, h) + (1 - \pi_j)W_U(a^{2'}, h)]\}, \tag{9}$$

subject to

$$c + a^{2'} + p_u\left(1 - \Pi(h \geq \bar{h})\right) \leq \gamma_1(1 - \tau)w_U h + (1 + r)a^1, \text{ and } a^1 = b \tag{10}$$

$\pi_j = \pi_{Pu}$ if $h \geq \bar{h}$

$\pi_j = \pi_{Pr}$, otherwise.

$\Pi(h \geq \bar{h})$ is an indicator function that takes value one if h is larger than a threshold value $\bar{h}$ and zero otherwise. Since both universities have the same quality, young adults prefer to attend a public university since there is no direct private cost associated with a public university, but entrance is restricted. Only adults with human capital higher than $\bar{h}$ will attend public university.

Define the indicator variable $\mathbb{D}(a^1, h)$, such that $\mathbb{D}(a^1, h)$ is equal to one if the young adult attended a university and zero otherwise.

3.2.2.3 The problem of an adult

After one period, young adults become adults. There are two types of adults: unskilled and skilled, $j \in \{U, S\}$. Besides the consumption/bequest decision, adults have also to decide whether to send their offspring to a private, $s = 1$, or to a public school, $s = 0$. Private schools have higher quality than public schools. The problem of an adult is summarized by:

$$W_j(a^2, h_{-1}) = \max_{\{c, b' \geq 0, s \in \{0,1\}\}} \{u(c) + \beta E_z V(b', h')\}, \tag{11}$$

subject to

$$c + b' + p_s s \leq (1 - \tau) w_j h + (1 + r) a^2, j \in \{S, U\} \tag{12}$$

$$h' = z'(1 + \theta s), \tag{13}$$

$$z' \in \{z_1, z_h\} \text{ with } \Pr(z' = \hat{z}|z). \tag{14}$$

Indicator function $\mathbb{S}(a^2, h - 1)$ is equal to one if the adult is skilled and zero if she is unskilled.

Let Y be the wealth (asset) distribution at some period, which evolves endogenously across periods. Define $P(a^j, A) = \Pr\{a' \in A|a^j\}$ as a non-stationary transition probability function, which assigns a probability for an asset in $t + 1$ to be at A for a household that has asset a^j. The law of motion of the asset distribution is:

$$Y' = \sum_{j=1}^{2} \int P(a^j, A) Y(da^j) \tag{15}$$

3.2.3 Government

The public education (schools and universities) system is financed through a labor income tax, τ. We assume that a student in a public school (university) costs the same as a student in a private school (university). We also assume that the government runs a balanced budget in every period, such that:

$$\tau \int_z \int_a [w_s h_{-1} + w_U h_{-1}] \Upsilon(da^2) dh_{-1} +$$

$$\tau \int_z \int_a \left[w_U h(1 - \mathbb{D}(a^1, h)) + \gamma w_U h \mathbb{D}(a^1, h) \right] \Upsilon(da^1) dh =$$

$$p_s \int_z \int_a \left(1 - s(a^2, h_{-1})\right) \Upsilon(da^2) dh_{-1} + p_u \int_z \int_a \mathbb{D}(a^1, h) \Upsilon(da^1) dh \qquad (16)$$

3.3 Competitive equilibrium

In a competitive equilibrium, agents optimally solve their problems and all markets clear. The agents' optimal behavior was previously described in detail. It remains, therefore, to characterize the market equilibrium conditions. Since the consumption good is the numeraire, three market clearing conditions are required to determine wages and the interest rate in each period. The labor and capital market equilibrium equations are:

$$S = \int_z \int_a \mathbb{S}(a^2, h_{-1}) h_{-1} \Upsilon(da^2) dh_{-1}, \qquad (17)$$

$$U = \int_z \int_a h_{-1}(1 - \mathbb{S}(a^2, h_{-1})) \Upsilon(da^2) dh_{-1} + \int_z \int_a \left[h(1 - \mathbb{D}(a^1, h)) + \gamma h \mathbb{D}(a^1, h) \right] \Upsilon(da^1) dh \qquad (18)$$

$$K' = \int_z \int_a b'(a^2, h_{-1}) \Upsilon(da^2) dh_{-1} + \int_z \int_a a^{2'}(a^1, h) \Upsilon(da^1) dh \qquad (19)$$

In addition, the government budget constraint is satisfied with equality, such that equation (16) holds.

3.4 Calibration strategy and Brazilian Benchmark Case

In order to obtain quantitative results, we must calibrate the model to match key statistics of the Brazilian economy. We assume that the logarithm of idiosyncratic productivity follows an AR(1) process with persistence ρ_z and variance σ_z^2. Therefore, these two parameters define a Markov chain for labor productivity. There are 18 parameters to be calibrated. The time period of the model is 15 years. The subjective discount factor is chosen such that it matches the yearly real interest rate of 4.34%. Following Bugarin (2006), the risk aversion parameter, σ, is set to 1.43.

The production function is a CES function of capital, skilled and unskilled labor, where μ and α are the parameters for the income share and η and ρ are the parameters for the elasticity of substitution between capital (skilled labor) and unskilled labor and capital and skilled labor respectively. These 4 parameters were calibrated in Krusell et al, 2000.

The calibrated parameters γ, π_{Pu} and π_{Pr} are 0.2786, 63.34% and 36.05% respectively. The data on these parameters are from Brazil's Ministry of Education, where π_{Pu} is the ratio of college graduates over entrants in the public universities and π_{Pr} is the ratio of college graduates over entrants in the private universities. Parameter θ, the premium of attending a private school, is set to 17.48% to match the difference in the means in the Brazil's high school national exam, ENEM, of students from public and private high schools (see Binelli, Meghir and Menezes-Filho, 2009).

The remaining six parameters: $\rho_z, \sigma_z^2, \bar{h}, p_u, p_s$ and τ are chosen to match Brazil's income Gini index, the share of skilled labor, the share of young attending private universities, the share of young not attending university, total private expenditure on education as a share of GDP and total public expenditure on education as a share of GDP. Tables 3.1 and 3.2 below show the calibrated parameters and the targets.

Parameters	Calibrated Value
Intergenerational persistence of productivity: ρ_z	0.7
Variance of the productivity: σ_z^2	0.35
Threshold human capital needed to enter in a public university: $\bar{h}$	1.59[11]
Cost in real terms of private university: p_u	0.1
Cost in real terms of private school: p_s	0.3
Labor income tax rate: τ	0.035

Table 3.1: Parameters calibrated jointly to match the targets.

We used the weighted minimum squared errors procedure to calibrate the parameters jointly.[12] More weight was assigned to the income Gini Index, the percentage of skill workers and the public expenditures on education. The parameters for idiosyncratic productivity, ρ_z and σ_z^2, generate the following state vector: Z=[0.5325   0.7297   1.0000   1.3703   1.8778] and the following transition matrix:

$$
P_z = \begin{bmatrix}
0.4582 & 0.3695 & 0.1493 & 0.0218 & 0.0012 \\
0.2004 & 0.3828 & 0.3130 & 0.0934 & 0.0104 \\
0.0576 & 0.2422 & 0.4005 & 0.2422 & 0.0576 \\
0.0104 & 0.0934 & 0.3130 & 0.3828 & 0.2004 \\
0.0012 & 0.0218 & 0.1493 & 0.3695 & 0.4582
\end{bmatrix}
$$

---

[11] Given the discrete nature of the model, $\bar{h}$ can assume any value between (1.5451; 1.8778).
[12] The weights are [0.25 0.25 0.25 0.10 0.10 0.05] for income Gini index, total public expenditure on education as a share of GDP, share of workers with a degree, share of students in private university, share of students not attending university and total private expenditure on education as a share of GDP, respectively.

	Baseline Model	Brazilian Economy
Targets:		
Income Gini Index	0.55	0.55
Total public expenditure on education as a share of GDP	5.3%	5.3%
Share of workers with a degree	6.6%	7%
Share of students in private university	74.0%	75.5%
Share of students not attending university	78.1%	81%
Total private expenditure on education as a share of GDP	4.7%	2.3%

Table 3.2: Targets for baseline model. Sources: IBGE, IPEAdata, Brazilian Ministry of Education and Curi,Menezes-Filho (2010).

The benchmark model also matches very well some Brazilian statistics that were not calibrated such as the capital output ratio. In the model, the ratio equals 1.4, while in data it is equal to 1.3 (OECD Economic Outlook Database, Brazil). The skill premium computed in the benchmark model is 2.3 (in log differences) while in Brazil it is 1.9 (Gonzaga et al, 2006). A slight overestimation of the skill premium in this model is expected since we have only 2 groups of workers: skilled and unskilled. The model also overestimates the percentage of students in public schools (94% versus 83% in the data). In the model, only parents who can afford and expect their child to get a degree will send their offspring to a private school. Finally, in the baseline model, the percentage of public expenditures on public schools is 80%. In Brazil, the same share is 77% (OECD Report, 2007).

3.5 Policy Results

In order to study how different educational policies affect inequality and productivity, we will conduct policy experiments and compare them with the benchmark model. We will focus on five different policies: changes in the quality of public schools, changes in the threshold level of ability needed to enter in a public university, changes in the quality of public universities, a "same opportunity" policy and a private universities only economy.

3.5.1 Changes in the quality of public schools

In order to analyze how changes in the quality of public schools affect the baseline model results, it is necessary to change equation 1 to:

$$h = z(1 + \theta_{Pr}s_{-1} + \theta_{Pu}(1 - s_{-1}))$$ (21)

Let θ_{Pr} and θ_{Pu} be the premium in terms of human capital for agents who studied in private and public schools, respectively. In the baseline model, $\theta_{Pu} = 0$. We want to analyze how an increase in θ_{Pu} affects inequality and productivity.

Figure 3.1 below shows how changes in the quality of public schools affect production, the skilled labor market and the skill premium.
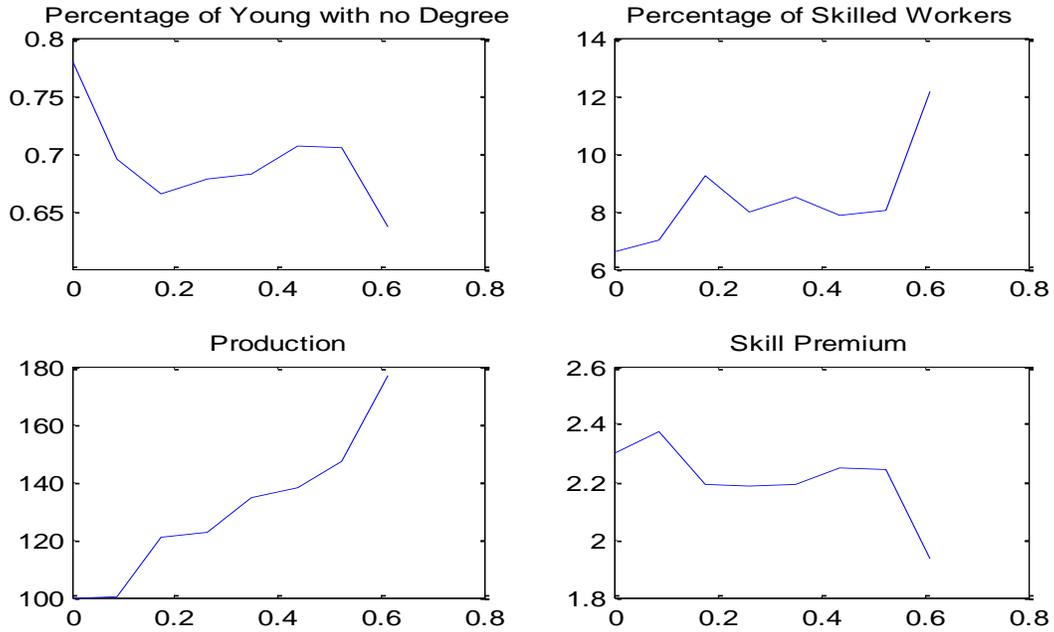
Figure 3.1: Gini coefficient, skill premium, production and percentage of agents attending university when changing the quality of public schools.

Figure 3.2 below shows how an increase in the quality of public universities affects inequality.
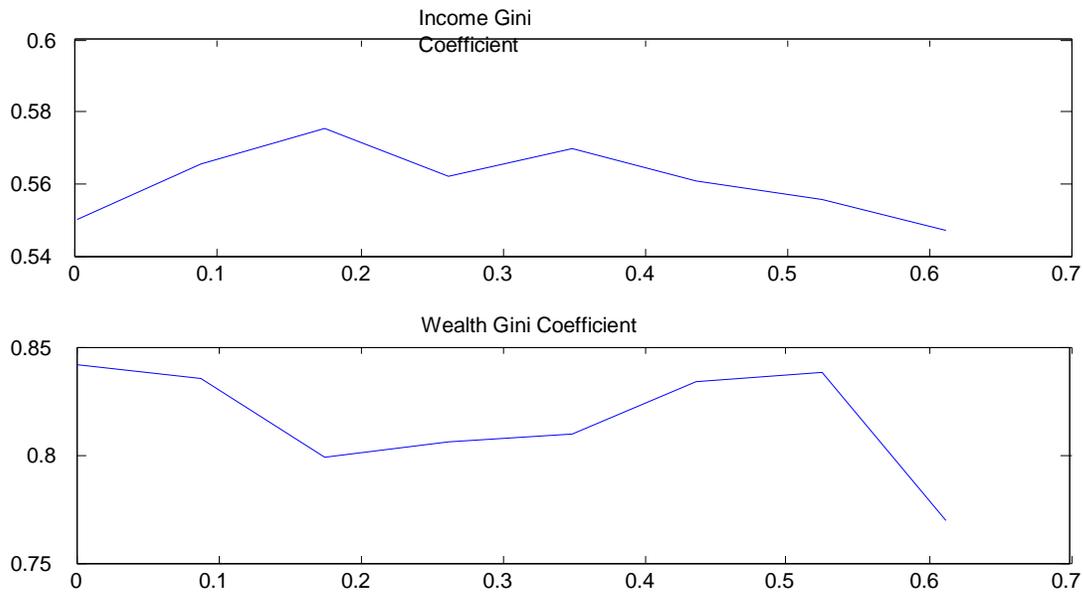

Figure 3.2: Policy 1 Income and Wealth Gine Coefficient.

In comparison with the benchmark model, an increase of the quality of public schools to the level of private schools, $\theta_{Pr} = \theta_{Pu} = 0.1748$, is responsible alone for a decrease in wealth inequality of 5 points and an increase in per capita output of more than 25%. Note that the increase in productivity and the decrease in the skill premium is higher when $\theta_{Pr} < \theta_{Pu}$, i.e., all children go to public school. This result shows that the best policy is to offer a high quality free basic education for all children and it comes directly from the fact that basic education increases productivity for both types: skilled and unskilled workers. The effect of an increase in the quality of public schools has a lower effect on the income Gini coefficient. Since public schools are provided for free, all agents, independently of income, will pay no cost for primary and secondary education.

3.5.2 Changes in the quality of public universities

In this experiment, we change the probability of being a skilled worker when attending a public university. Figure 3.3 below shows the main results:
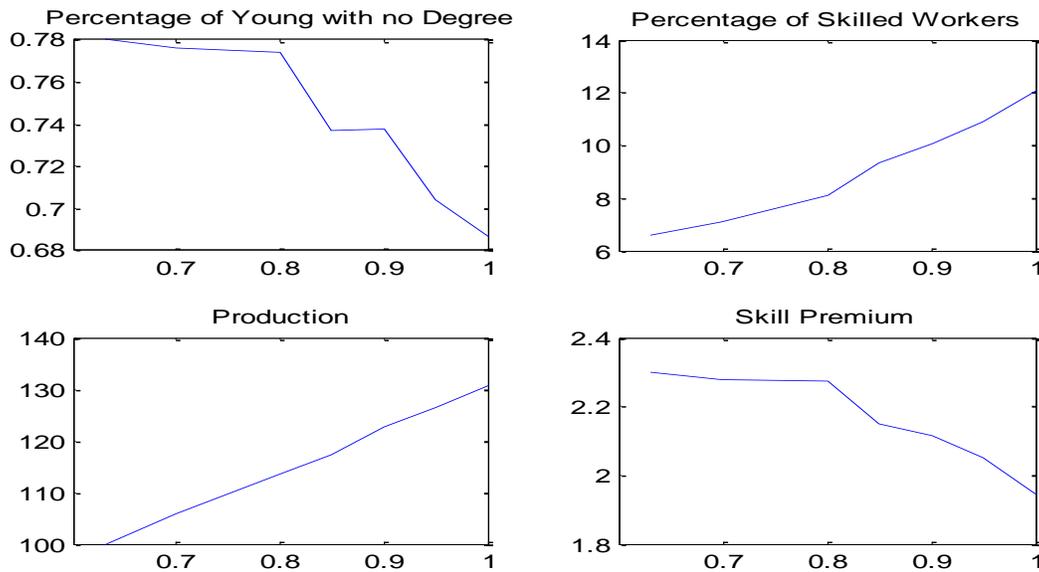

Figure 3.3: Gini coefficient, skill premium, production and percentage of agents attending university when changing the quality of public universities.
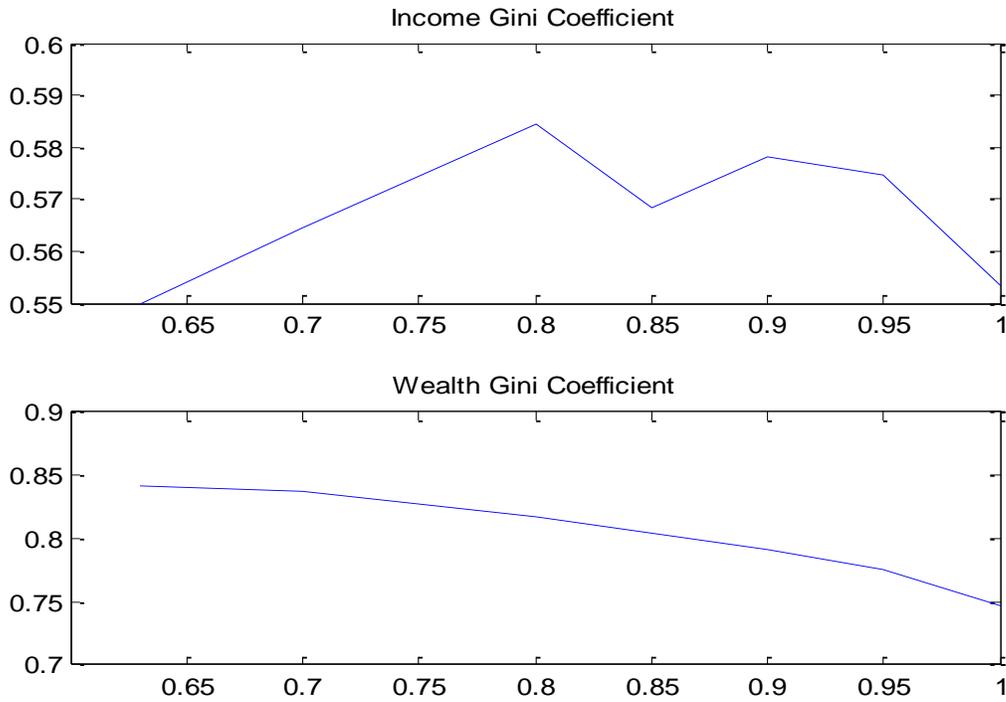
Figure 3.4: Policy 2 Income and Wealth Gine Coefficient.

Note that this policy shows a slightly decrease in the wealth Gini coefficient and it seems non-monotonic with respect to income inequality. In comparison with the baseline model, an increase in the quality of the public university system slightly increases inequality. This happens because wealthier agents have a lower opportunity cost to attend the university, moreover, they have greater chances to enter a public university because they studied in private schools. Therefore, the model presents an endogenous positive correlation between the idiosyncratic productivity (h) and wealth. So, this policy alone might increase inequality. On the other hand, an increase in the quality of public universities produces more skilled agents and therefore, it increases per capita output and decreases the skill premium.

An interesting result arises if we consider the extreme case where there is no uncertainty about being skilled if attending a public university, $\pi_{Pu} = 1$. In this case, the wealth Gini coefficient decreases 9 points with respect to the baseline model. The number of students attending public university is more than the double of the baseline model. Therefore, there is a shift in the percentage of skilled workers in the economy that implies in an increase in production and a decrease in the skill premium.

3.5.3 Changes in the threshold level of ability

A decrease in the threshold ability needed to enter the public university, $\bar{h}$, decreases the Gini Index, the skill premium and increases output per capita. The curves in figure 3 are step functions since the human capital level is discrete and given by equation 1. Therefore, a marginal change in the threshold level might not change the macroeconomic variables unless it reaches the next human capital level point in the grid. In comparison with the benchmark model, a decrease of 30% in the threshold level decreases the Gini index 4 points and increases output by approximately 10%.
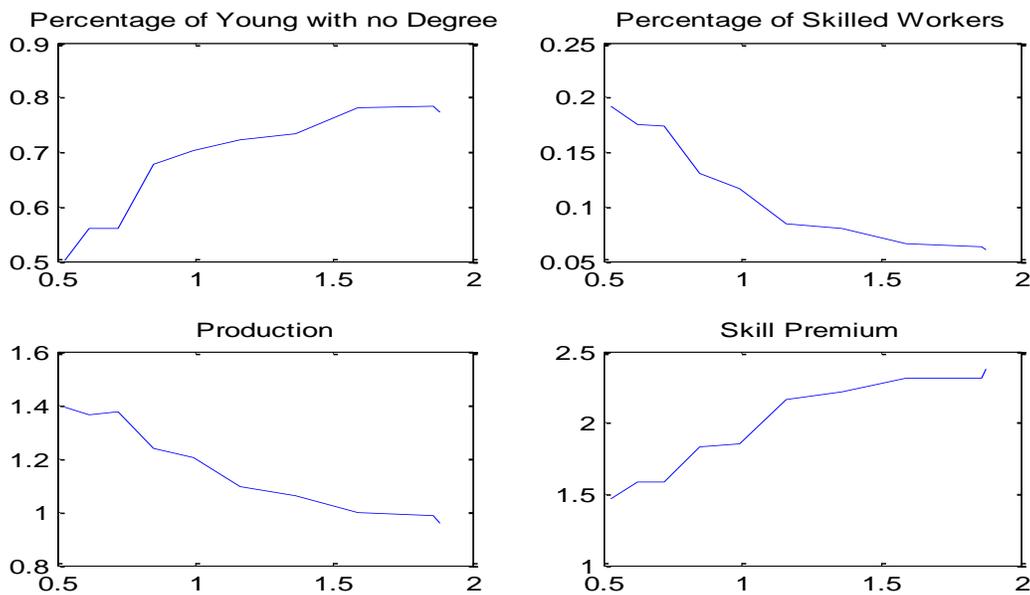

Figure 3.5: Gini coefficient, skill premium, production and percentage of agents attending university when changing the threshold level of ability.
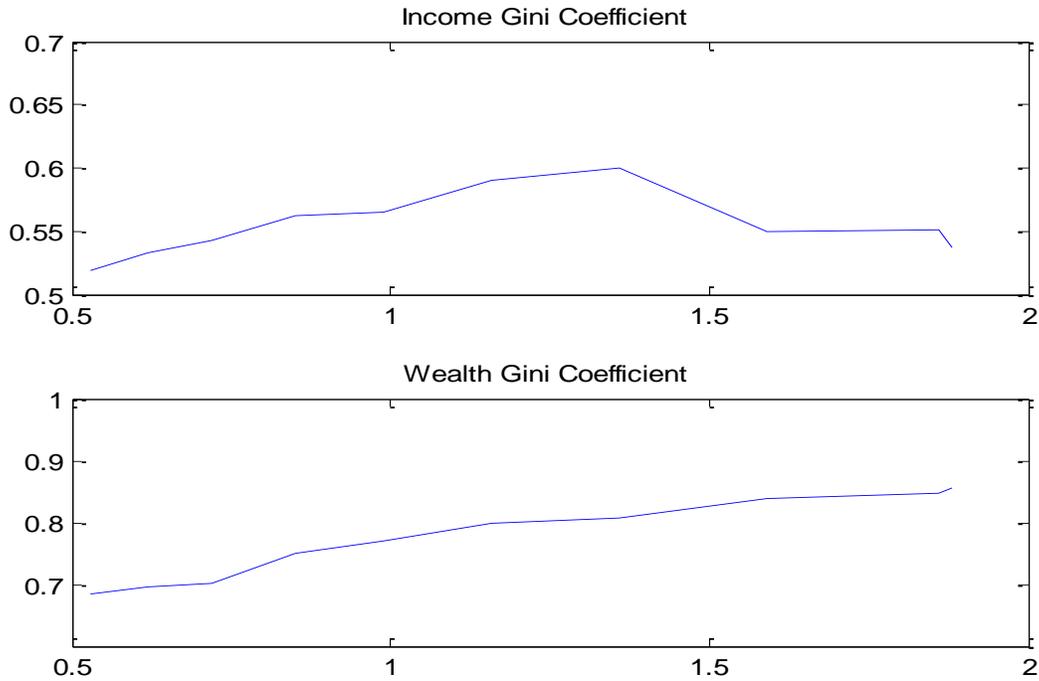
Figure 3.6: Policy 3 Income and Wealth Gine Coefficient.

It is interesting to compare the results of figure 1 with this policy. The Gini coefficient in this economy, shown by figure 3, is lower simply because by decreasing $\bar{h}$, the number of vacancies in the public universities increases and, therefore, the number of skilled agents increases, decreasing the skill premium. Note that the increase in output is less than half of the increase in output per capita in policy 1. In the model, the probability of being skilled just depends on the type of university the agent attends, public or private. If we consider that human capital affects this probability, then policy 1 is better in terms of the decrease in inequality and increase in productivity.

3.5.4 "Same opportunity" policy

Another interesting result arises when we consider the following policy, given by $\bar{a} \geq 0$. For agents poor enough, $a^1 \leq \bar{a}$, who studied in public schools, the human capital needed to enter a

public university is $h \geq \frac{\bar{h}}{(1+\theta)}$. The increase in h for poor agents is just a shift in the score for entering in the university; it does not increase the human capital level for these agents and is used as an attempt to compensate for the small chance a student has to be skilled because his family could not afford a high quality private school.

The results show that this policy has no effects in macroeconomic variables, the difference in the Gini coefficient, output per capita, percentage of skilled agents and skill premium were insignificant. Despite the fact that there is a slight increase in agents attending public universities (2%), this increase is not enough to change the skill premium and therefore, inequality. This result comes from the fact that poor young agents have a higher opportunity cost to attend university since they cannot work full time as unskilled worker while young.

3.5.5 Private universities only

In this last experiment, we compare the baseline model with an economy where there exists just private universities. The probability of being skilled is the average with respect to the baseline model. Table 3.3 shows the results:

Results	Baseline model	Just Private Universities Economy
Output per capita	100	97.42
K/Y Ratio	1.4	1.27
Income Gini Index	0.55	0.52
Wealth Gini Index	0.84	0.82
Skill Premium	2.3	2.2310
Size of Government (G/Y)	5.3%	5%
Percentage of students in public schools	94%	100%
Percentage of young not attending University	78.1	72.7%
Percentage of skilled workers	6.6%	7.3%

Table 3.3: Policy 5: Results.

Despite the fact that the percentage of skilled workers increases, output per capita slightly decreases. This happens because no child is studying in private school has a decreased the human capital level. Inequality shows a small improvement.

3.6 Concluding Remarks

In order to study how different educational policies affect productivity and inequality, we conducted an experiment calibrating the baseline economy using Brazilian data. The results show that the best policy for a decrease in inequality and for an increase in productivity is to improve the quality of primary and secondary public schools. Another important result is that for the Brazilian economy, the opportunity cost of poor young agents to attend university is too high, therefore, policies that facilitate poor young agents to enter public universities such as the introduction of quotas or shifts on score entrance exams are not very effective if not tied together with other policies such as subsidies and scholarships.

APPENDIX A

A.1. Self-Employed Managers Problem

Consider the problem of an agent with ability x that chooses to be self-employed, he maximizes capital for given θ:

$$\Pi_{se}(x) = max_{\{k\}}(1-\theta)xk^{\alpha}(\theta x)^{\beta} - (1+r)k \tag{6}$$

The necessary first order conditions are given by:

$$\frac{\partial \pi_{se}}{\partial k} = (1-\theta)\alpha xk^{(\alpha-1)}(\theta x)^{\beta} - (1+r) = 0 \tag{7}$$

Therefore:

$$k_{se}^*(x) = \left[\frac{\alpha}{1+r}(1-\theta)\theta^{\beta}x^{1+\beta}\right]^{\frac{1}{1-\alpha}} \tag{8}$$

Therefore,

$$\Pi_{se}^*(x) = x^{\frac{1+\beta}{1-\alpha}}\left(\frac{(1-\theta)\theta^{\beta}\alpha^{\alpha}}{(1+r)^{\alpha}}\right)^{\frac{1}{1-\alpha}}(1-\alpha) \tag{9}$$

Note that $k_{se}^*(x)$ and $\Pi_{se}^*(x)$ are strictly increasing and strictly convex functions in ability, x, since $\frac{dk_{se}}{dx}$; $\frac{d^2k_{se}}{dx^2} > 0$ and $\frac{d\Pi_{se}}{dx}$; $\frac{d^2\Pi_{se}}{dx^2} > 0$, for all $x > 0$.

A.2. Full Time Managers Problem

Consider the problem of an agent that chooses to be a full time manager, with $\alpha + \beta < 1$:

$$\Pi_{fm}(x) = max_{\{k;n\}}xk^{\alpha}n^{\beta} - wn - (1+r)k \tag{10}$$

The necessary first order conditions are:

$$\frac{\partial \pi_{fm}}{\partial k} = \alpha xk^{(\alpha-1)}(n)^{\beta} - (1+r) = 0 \tag{11}$$

$$\frac{\partial \pi_{fm}}{\partial n} = \beta xk^{\alpha}(n)^{(\beta-1)} - w = 0 \tag{12}$$

Therefore:

$$n^*(x) = x^{\left(\frac{1}{1-\alpha-\beta}\right)}\left(\frac{\beta}{w}\right)^{\left(\frac{1-\alpha}{1-\alpha-\beta}\right)}\left(\frac{\alpha}{1+r}\right)^{\left(\frac{\alpha}{1-\alpha-\beta}\right)} \tag{13}$$

$$k_{fm}^{*}(x) = \left[\left(\frac{\alpha}{1+r}\right)^{(1-\beta)} \left(\frac{\beta}{w}\right)^{\beta} x\right]^{\frac{1}{1-\alpha-\beta}} \tag{14}$$

and

$$\Pi_{fm}^{*}(x) = x^{\frac{1}{1-\alpha-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\alpha-\beta}} \left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{1-\alpha-\beta}} (1-\alpha-\beta) \tag{15}$$

Note that $k_{fm}^{*}(x)$ and $\Pi_{fm}^{*}(x)$ are strictly increasing and strictly convex functions in ability x, since, $\frac{dk_{fm}}{dx} > 0, \frac{d^2k_{fm}}{dx^2} > 0$ and $\frac{d\Pi_{fm}}{dx} > 0, \frac{d^2\Pi_{fm}}{dx^2} > 0$, for all $x > 0$.

APPENDIX B

Let x* be such that: $\Pi^*_{fm}(x^*) = wx^*$. Since $\Pi^*_{fm}(x)$ is strictly increasing and convex and labor income, wx, is linear in the ability level and $\Pi^*_{fm}(0) = 0$, then, there exists a unique, single cross-point: $x^* > 0$ such that for all x>x*, agent strictly prefer to be full-time manager instead of worker. Equation 10 can be used to explicitly solve for the threshold ability level x*:

$$x^* = \left[w^{(1-\alpha)} \left(\frac{1+r}{\alpha}\right)^\alpha \left(\frac{1}{\beta}\right)^\beta \left(\frac{1}{1-\alpha-\beta}\right)^{1-\alpha-\beta} \right]^{\frac{1}{\alpha+\beta}} \tag{16}$$

In the same way, we can compute x** such that $\Pi^*_{se}(x^{**}) = wx^{**}$:

$$x^{**} = \left[\left(\frac{w}{1-\alpha}\right)^{1-\alpha} \left(\frac{1+r}{\alpha}\right)^\alpha \left(\frac{1}{(1-\theta)\theta^\beta}\right) \right]^{\frac{1}{\alpha+\beta}} \tag{17}$$

Therefore, there exists a unique $x^{**} > 0$ such that for all x>x**, agent strictly prefer to be self-employed than worker.

Since $\Pi^*_{fm}(x)$ and $\Pi^*_{se}(x)$ are both strictly convex it is necessary to show that:

 (i)   ∃ a unique $\bar{x}$ s.t $\Pi^*_{se}(\bar{x})= \Pi^*_{fm}(\bar{x})$,
 (ii)   ∀ $x < \bar{x}, \Pi^*_{se}(x) > \Pi^*_{fm}(x)$,
 (iii) ∀ $x > \bar{x}, \Pi^*_{fm}(x) > \Pi^*_{se}(x)$

Let $\bar{x}$ be such that $\Pi^*_{se}(\bar{x})= \Pi^*_{fm}(\bar{x})$, using equations (9) and 10:

$$\bar{x} = \left(\frac{w}{\beta}\right)^{\frac{(1-\alpha)}{\alpha+\beta}} \left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{\alpha+\beta}} ((1-\theta)\theta^\beta)^{\frac{(1-\alpha-\beta)}{\beta(\alpha+\beta)}} \left(\frac{1-\alpha}{1-\alpha-\beta}\right)^{\frac{(1-\alpha-\beta)(1-\alpha)}{\beta(\alpha+\beta)}} \tag{18}$$

It is clear than $\bar{x} > 0$. Moreover $\Pi^*_{fm}(x) > \Pi^*_{se}(x)$ if and only if $x > \bar{x}$ and $\Pi^*_{fm}(x) < \Pi^*_{se}(x)$ if only if $x < \bar{x}$.

Therefore, note that $x^{**} > x^*$ is a sufficient condition for the no-existence of a self-employment using equations (16) and (17):

$x^{**} > x^*$ implies:

$$(1 - \theta)\theta^\beta(1 - \alpha)^{1-\alpha} < \beta^\beta(1 - \alpha - \beta)^{1-\alpha-\beta}$$

Condition abone shows assumption 1. As long as the full time entrepreneur share, $(1 - \alpha - \beta)$, is high enough, in the optimal, there is no self-employment agent in the economy.

Finally note that the demand for capital for both types of entrepreneurs is strictly increasing and convex in the ability level. Comparing equations (8) and (14):

$$k^*_{fm}(x) \geq k^*_{se}(x)$$

$$x \geq \left(\frac{1+r}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{w}{\beta}\right)^{\frac{1-\alpha}{\alpha+\beta}} \left((1-\theta)\theta^\beta\right)^{\frac{1-\alpha-\beta}{\beta(\alpha+\beta)}} \qquad (19)$$

Let $\bar{\bar{x}}$ be such that $k^*_{fm}(\bar{\bar{x}}) = k^*_{se}(\bar{\bar{x}})$. Using equation (18), one can show that $\bar{\bar{x}} < \bar{x}$, therefore, if it is optional for agent to be full-time manager in the model without frictions, his demand for capital is higher than if self-employed, i.e, any policy that changes his occupational choice result in a lower demand for capital.

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