MEASUREMENTS AND IMPLICATIONS OF THE SDSS DR7 GALAXY ANGULAR POWER SPECTRUM

BY

BRETT P. HAYES

DISSERTATION

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Doctoral Committee:

Associate Professor Robert Brunner, Chair
Professor Brian Fields
Associate Professor Leslie Looney
Associate Professor Paul Ricker
Abstract

We calculate the angular power spectrum of galaxies selected from the Sloan Digital Sky Survey (SDSS) Data Release 7 (DR7) by using a quadratic estimation method with KL-compression. The primary data sample includes over 18 million galaxies covering more than 5,700 square degrees after masking areas with bright objects, reddening greater than 0.2 magnitudes, and seeing of more than 1.5 arcseconds. We also construct a volume-limited sample of 3.2 million galaxies in the same area, consisting of galaxies with absolute r-band magnitudes $M_r < -21.2$ and photometric redshifts $z < 0.4$. We test for systematic effects by calculating the angular power spectrum on simulated data and by SDSS stripe, and we find that these measurements are minimally affected by seeing and reddening. We calculate the angular power spectrum for $\ell \leq 200$ multipoles by using 40 bands for the full area data, $\ell \leq 1000$ multipoles using 50 bands for individual stripes, and $\ell \leq 1600$ multipoles using 64 bands for a selected area near the North Galactic Pole at high resolution. We also calculate the angular power spectra for the main galaxy sample separated into 3 magnitude bins, as well as the volume-limited sample separated into 2 redshift shells and early- and late-type galaxies to examine the evolution of the angular power spectrum. We determine the theoretical linear angular power spectrum by projecting the 3D power spectrum to two dimensions for a basic comparison to our observational results for the SDSS DR7 main galaxy sample and subsamples separated by magnitude. For our high resolution and volume-limited samples, we generate nonlinear angular power spectra using CAMB nonlinear 3D matter power spectra for our projections. By minimizing the $\chi^2$ fit between these data and the theoretical angular power spectra, we measure a fit of $\Omega_m = 0.31^{+0.18}_{-0.11}$ with a linear bias of $b = 0.94 \pm 0.04$ for...
the entire SDSS DR7 main galaxy sample, $\Omega_m = 0.267 \pm 0.038$, $\Omega_b = 0.045 \pm 0.012$, and $b = 1.075 \pm 0.056$ for our high resolution sample, and $\Omega_m = 0.282 \pm 0.026$, $\Omega_b = 0.041 \pm 0.020$, and $b = 1.545 \pm 0.057$ for our full volume-limited sample. Finally, we measure the relative linear bias between early- and late-type galaxies, and find $b_e/b_l = 1.375 \pm 0.076$. 
This work is dedicated to my parents.
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Some of the results in this paper have been derived using the HEALPix (Górski et al., 2005) package.

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Chapter 1
Introduction

Our understanding of the Universe has radically changed over the last century. The first indications that the Universe contained matter and energy other than atoms (i.e. baryonic matter) and light appeared roughly 80 years ago with the discovery of dark matter from stellar orbital velocities (Oort 1932) and galaxy orbits in clusters (Zwicky 1933). The evidence for this dark matter has grown in the interim, with measurements of galaxy rotation curves (Rubin et al. 1980), galaxy collisions (Clowe et al. 2006), and gravitational lensing (Walsh et al. 1979), though other theories such as modified gravity, collectively known as MOND, (Bekenstein 2004) are also being considered. The generally accepted results imply that dark matter not only exists, but it makes up the majority of the mass in the Universe, while normal baryonic matter only accounts for roughly a sixth of the total mass (Spergel et al. 2003). However, despite evidence of dark matter from its gravitational influence, the composition of dark matter is still unknown and remains an area of active research (Ellis et al. 2005, Steffen 2009).

More recently, over the last 15 years the cosmological paradigm was again revolutionized with the detection of the accelerating expansion of the Universe from supernova surveys (Riess et al. 1998). This acceleration has been attributed to a new form of energy, called dark energy. Dark energy has an unknown composition, and exerts a negative pressure that increases the expansion of the Universe while also providing sufficient mass-energy to make the Universe spatially flat. In fact, the measurements of dark energy, primarily from the

Cosmic Microwave Background, (CMB; Spergel et al. 2007) show that it is the dominant component of the Universe today, making up almost three-quarters of the total mass-energy. Though the study of dark energy is still in its infancy, possible explanations are provided by general relativity with a cosmological constant (Einstein 1916) or quintessence (Zlatev et al. 1999).

These cosmological parameters (the densities of baryonic matter, dark matter, and dark energy) determine the composition and fate of the Universe, as well as giving us insight into the history of structure formation. During the early Universe, quantum fluctuations are believed to have grown exponentially to cosmological scales in a period of intense inflation (Guth 1981). The baryons and dark matter attract gravitationally, but the dominant energy in photons interacted with the baryons via Thomson scattering of electrons to smooth the density variations. After recombination, the photons free stream which we now observe as the CMB, and the baryons are able to collapse into dark matter halos to form galaxies. Therefore, the cosmological parameters directly affect the distribution of galaxies, not only in the early Universe, but throughout its history as later structure is expected to grow hierarchically from these initial perturbations. However, while galaxy positions are known with high precision in two dimensions, redshift information which gives the third dimension is less precise. Hence, we analyze the projected angular distribution of galaxies to investigate large scale structure in the Universe.

The angular power spectrum, \( C_\ell \), is a statistical measure that quantitatively characterizes the large scale angular distribution of matter (Peebles 1973). Therefore, calculating the angular power spectrum of galaxies is useful since the \( C_\ell \) values derived from the observations can be easily compared to theoretical predictions, and secondarily as a method of data compression, reducing clustering information of an arbitrary number of galaxy positions down to a set of \( C_\ell \) and their corresponding window functions.

Calculations of angular power spectra are well known to cosmologists for their usefulness in studying the CMB, as the CMB provides a detailed and precise measurement of the density
variations in the early Universe (e.g., Smoot et al. 1992; Netterfield et al. 2002). However, to study large scale structure in other eras, it is necessary to analyze how mass clusters by using galaxies as a tracer of the underlying dark matter distribution.

Angular power spectra of galaxies have been calculated for galaxy surveys with various depth and survey area (e.g., Huterer et al. 2001; Blake et al. 2004; Frith et al. 2005) including the SDSS (Tegmark et al. 2002, hereafter T02; Blake et al. 2007; Thomas et al. 2010). By using angular power spectra to calculate galaxy clustering, we study the Fourier modes of the galaxy distribution; this method is most sensitive to large scale effects. Recent galaxy surveys such as the APM Galaxy Survey (Maddox et al. 1990), the Two Micron All Sky Survey (Skrutskie et al. 2006), and the SDSS (Abazajian et al. 2009) have cataloged large areas of the sky, thereby providing enormous numbers of galaxies for which we can measure angular clustering. However, to date the galaxy angular power spectrum has not been calculated for the full SDSS main galaxy sample. In this thesis, we address this deficiency.

The angular power spectrum is useful for large scale clustering, and it is complemented by the two-point angular correlation function on small scales. The two-point angular correlation function (e.g., Brunner et al. 2000; Myers et al. 2007; Ross et al. 2010), which is related to the angular power spectrum by the Legendre transform (T02), is more applicable to smaller scale clustering because the calculation is done in configuration space where the distances between nearby pairs of galaxies can be calculated faster. This makes the two-point angular correlation function advantageous to use on scales where non-linear evolution is important. This regime is also where the angular power spectrum at large ℓ is more difficult to measure and model, partly due to correlations introduced between the $C_\ell$.

To calculate the angular power spectrum, we want to find the most probable parameters $C_\ell$ that could produce the data we observe. To do this, we need the likelihood function of the angular power spectrum, which is proportional to the probability of the data given the $C_\ell$. Though in theory we would like to know the entire likelihood function, calculating this $\ell_{\text{max}}$-dimensional function is difficult (Oh et al. 1999). Fortunately, since we are only interested
in the most probable $C_\ell$, we only really need to know the maximum of this function.

To determine the $C_\ell$ that maximize the likelihood function, we use the quadratic estimation method (Tegmark 1997; Bond et al. 1998, hereafter BJK98). This technique fits a quadratic function to the shape of the likelihood function for some initial angular power spectrum, finds the $C_\ell$ that maximize this quadratic, and uses these $C_\ell$ for a new quadratic fit to iteratively converge to the true maximum of the likelihood function. Once we have found the angular power spectrum of galaxies, we can use the results to infer what cosmological parameters are consistent with the measurement (e.g., Jaffe et al. 1999).

With this technique, we use our calculated angular power spectra together with theoretical angular power spectra to retrieve constraints on cosmological parameters. We generate these theoretical angular power spectra by combining the redshift distributions of our samples with 3D power spectra, integrating over redshift to project the 3D power spectra to angular (2D) power spectra. Initially, we begin by using linear 3D spectra that depend on the matter density $\Omega_m$ and the linear bias, and through comparison of the resulting angular power spectra with our estimated SDSS galaxy angular power spectra, we are able to loosely constrain these values.

The cosmological parameters of greatest interest, however, impact the angular power spectrum at multipoles that correspond to small scales, that is in the nonlinear regime. To more tightly constrain cosmological parameters, it is necessary to fit estimated angular power spectra to nonlinear 3D matter power spectra, for example, by using the Code for Anisotropies in the Microwave Background, also known as CAMB (Lewis et al. 2000). With these nonlinear 3D matter power spectra, we can create nonlinear angular power spectra which can be fit to a greater range of multipoles. With this increased fitting range, we are not only able to place tighter constraints on $\Omega_m$ and the linear bias, but we can also constrain the baryon density $\Omega_b$ and the power spectrum spectral index $n_s$.

Even though the quadratic estimation method is more efficient than direct likelihood evaluation, it is still a computationally challenging algorithm. It requires matrix multiplica-
tions and inversions, as well as storage, of non-sparse matrices with several tens of millions of elements. The computational complexity of this angular power spectrum estimator is generally what limits the extent of our calculation, not the scientific limitations of signal-to-noise or observational systematics. Hence, we have explored methods of accelerating this calculation using innovative platforms.

In this thesis, we discuss the SDSS DR7 data, our selected sample and subsamples, and our systematic tests and masking process in Chapter 2. In Chapter 3, we discuss our pixelization scheme, KL-compression, and the quadratic angular power spectrum estimation method of BJK98 in detail. In Chapter 4, we apply this estimator to the complete SDSS DR7, selected subsamples, individual SDSS stripes, volume-limited samples and their subsamples, and present the results. We construct both linear and nonlinear theoretical angular power spectra to compare with the observational results, and we extract cosmological parameters and linear bias from this computation using $\chi^2$ minimization in Chapter 5. Finally, we discuss the computational constraints involved in the quadratic estimation method and our experiments in accelerating this calculation in Chapter 6, and conclude the thesis and discuss future work to be done following this work in Chapter 7.
Chapter 2

Data

The data for these measurements were taken from the SDSS Data Release 7, the final data release of SDSS-II. The Sloan Digital Sky Survey \cite{Abazajian2009} is an imaging and spectroscopic survey using the 2.5 meter telescope at Apache Point Observatory that begun operation in 2000, and ended with the SDSS-II in 2008. The imaging observations are taken simultaneously in 5 filters (u, g, r, i, and z) as the telescope drift scans across the sky \cite{Gunn1998}. The SDSS DR7 covers 11,663 square degrees in a striped fashion.

The SDSS DR7 also provides photometric redshifts and redshift errors for each galaxy \cite{Abazajian2009}. The SDSS has measured over 900,000 galaxy spectra and uses these as a training set to find the 100 nearest neighbors of a photometrically observed galaxy in color-color space. The photometric redshift is estimated by fitting a hyperplane to these neighbors, and the error is determined by the mean deviations from the best-fit hyperplane \cite{Csabai2007}. As we require the galaxy redshift for the cosmological analysis of our results in Chapter 5, any galaxy without both a photmetric redshift and associated error is not used in our calculation. In SDSS DR7, the rms error of the estimated photometric redshifts is 0.025, while for our samples it varies from 0.038 in the brightest sample to 0.064 in the dimmest.

\footnotetext{This chapter includes material that has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.}
2.1 Sample Area

For the full SDSS DR7 main galaxy sample, we begin by selecting a large, contiguous area from DR7 by using stripes 9–37, which gives an area of 7,646 square degrees before masking. Each stripe is 2.5 degrees wide in eta (the survey latitude), and variable length in lambda (the survey longitude). Typically, however, the stripes are 100–120 degrees long. Using this large area allows us to use a band resolution of up to 4 multipoles per band when calculating the angular power spectrum for the full sample (see Section 3.2). Since this area is centered around the North Galactic Cap, we avoid the worst areas of reddening due to the Galactic disk. After masking for observational effects (e.g., reddening, seeing, bright stars; see Section 2.3), our full sample includes 18.9 million galaxies over 5,763 square degrees of the SDSS Northern Galactic Cap ellipsoid.

2.2 Large Multipole Area

We would like to determine the angular power spectrum to the highest multipoles allowed by the SDSS DR7 galaxy data, and to examine these small scales requires a high resolution pixelization. However, we are computationally limited by the $O(n_p^3)$ scaling dependence of the quadratic estimator, where $n_p$ is the number of pixels in the map (see Section 6.1). By using large shared-memory supercomputers, this limits the analysis to maps with $n_p \sim 10^4$; thus, in order to go to higher resolution and multipoles, we are forced to constrain our analysis to smaller areas than the full SDSS DR7. Performing this calculation with a smaller area necessitates using larger bands as the band width is inversely proportional to the smallest linear dimension of the area under consideration (Peebles 1980). Therefore, obtaining the angular power spectrum at high multipoles involves sacrificing band resolution and survey area, which means losing information about the angular power spectrum at all scales. However, this sacrifice is offset by being able to constrain the angular power spectrum at small scales where cosmological parameters have a greater effect, ideally this will allow
tighter constraints on those parameters.

When restricting our analysis to a smaller area, we are at least free to choose the particular area we examine. We select an area of low reddening near the North Galactic Pole, and limit our analysis to the $\sim 53.7$ square degree area corresponding to nested pixel number 162 at HEALPix resolution 8. We use $\sim 0.013$ square degree pixels at HEALPix resolution 512, which allows us to compute bands of width $25\ell$ out to $\ell \sim 1600$ (see Section 3.1).

2.3 Systematics

We tested the effects of seeing and reddening on the calculation of the angular power spectrum by varying seeing cuts from 1.0 to 3.0 arcseconds in 0.1 arcsecond intervals, and reddening cuts from 0.1 to 0.5 magnitudes in 0.05 magnitude intervals. We found that neither seeing nor reddening had a significant impact so long as a sufficient galaxy density remained to calculate the angular power spectra. This is consistent with the cross correlations between galaxy density and reddening/seeing calculated by T02 for stripe 10. Nevertheless, to minimize systematics in the SDSS galaxy sample, we have eliminated areas of seeing greater than 1.5 arcseconds and reddening worse than 0.2 magnitudes to be consistent with similar angular correlation function results (Ross et al. 2007), though others have used more stringent cuts (Wang & Brunner 2012).

To test the homogeneity and observational character of the data, we calculate the angular power spectrum separately for each stripe, using the method discussed in Section 3. If there is a significant deviation in the angular power spectrum from stripe to stripe, observational systematics might dominate over the real density variations of the combined stripe data that makes our full sample. To test for these systematics, we have calculated angular power spectra of each SDSS stripe from stripe 9 to stripe 37 after masking for seeing and reddening, with each of the $C_\ell$ including an identical range of $\ell$. The angular power spectra from each stripe are remarkably consistent with each other, as shown in the box-whisker plot in Figure
This shows that these observational systematics do not significantly alter the angular power spectra. The only notable variation between stripes is that the edge stripes 9 and 37 have much larger error bars due to these stripes having the most pixels eliminated due to the seeing and reddening cuts.

Finally, the data that we use span a wide range of Galactic latitudes, so we also consider the effect of varying stellar density on our galaxy samples. Bright stars in our Galaxy could possibly obscure background galaxies (Ross et al. 2011), or faint stars could be misclassified as galaxies by the star-galaxy classification algorithm. To examine these possibilities, we calculated the galaxy overdensity and stellar overdensity separately, applied our masks, and plotted these overdensities versus Galactic latitude in Figure 2.2. We see two exponential falloffs in the stellar overdensity, which correspond to the two edges of the SDSS dipping toward the Galactic disk. The high Galactic latitude exponential comes from the side of the SDSS in the general direction of the Galactic center and the low Galactic latitude exponential from the side near the Galactic anticenter, while the galaxy overdensity is consistent with zero at all Galactic latitudes in our sample. For the large pixel sizes we use in the following calculations, obscuration by bright stars does not have a large effect on the galaxy overdensity, and at even at the lowest magnitude we use, star-galaxy separation is accurate at the 95% confidence level (Lupton et al. 2001) so we observe no major effect on the galaxy overdensity from misclassified stars.

2.4 Subsamples

We have chosen our main sample to be from 18–21 magnitude in the extinction corrected r-band (Stoughton et al. 2002), with the faint limit chosen due to concerns about completeness in the sample past magnitude 21. Though the 95% completeness r-band magnitude limit is 22.2 (Abazajian et al. 2009), some galaxies at the fainter end of the 21-22 magnitude range are not detected or are unusable due to large errors. We therefore choose to limit our analysis
Figure 2.1: Box plot of the angular power spectra of galaxies with r-band magnitudes between 18 and 21 for the individual stripes 9 through 37. The median is in red, the 25% and 75% quartiles marked as the edge of the boxes, and the minimums and maximums marked at the end of the whiskers. This figure has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
Figure 2.2: Points in black are the pixelized stellar overdensities, as a function of Galactic latitude at HEALPix resolution 64. The exponential falloff of the Galactic disk is seen here twice, at high Galactic latitude we see the falloff of the stars toward the Galactic center and at low Galactic latitude we see the stars in the direction of the Galactic anticenter. We group the pixelized galaxy overdensities by Galactic latitude into 20 bins, which are displayed as a box-whisker plot. For each bin, the median galaxy ovedensity is plotted in red, the end of the boxes mark the 25% and 75% quartiles, and the end of the whiskers mark the minimum and maximum overdensities in that bin. This figure has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
to the brighter and more complete samples.

We have chosen subsamples of our main sample for comparison to previous results, and to test for potential systematic errors on galaxy selection. We first confirm our technique is consistent with the results from T02 up to $21^{st}$ magnitude, so we have separated stripe 10 into 3 magnitude bins from 18–19, 19–20, and 20–21. The comparison can be expected to be slightly different due to the use of the more complete DR7 data as opposed to the Early Data Release results that used galaxy probabilities (T02), in addition to the photometry calculation difference of magnitudes in SDSS data prior to DR2 (Abazajian et al. 2004). We show these results in Section 4.

We also measure the clustering attributes based on the brightness of the galaxies. The apparently brighter galaxies cluster more strongly and are generally at lower redshift, thus we expect those to have more power in the angular power spectrum. We create three new samples by separating the SDSS galaxies into 3 different r-band magnitude bins from magnitudes 18–19, 19–20, and 20–21. These magnitude ranges are sufficiently bright to minimize any systematic effects from star-galaxy separation and variable sky brightness. These samples have intrinsically different redshift distributions and luminosity functions, therefore the angular power spectra of these samples will reflect these differences, and they are also useful as a secondary systematic test.

### 2.5 Volume-Limited Sample

For the entire SDSS DR7 galaxy sample, we are magnitude-limited, which biases the sample toward detecting brighter galaxies at higher redshifts. In addition to the full SDSS DR7 galaxy sample, we construct a volume-limited sample out to redshift $z = 0.4$ to examine a complete sample that is relatively free from Malmquist bias. This volume-limited sample was created by Ross et al. (2010), selecting over 3.2 million DR7 galaxies with de-reddened r-band apparent magnitudes $m_r < 21$, r-band absolute magnitudes $M_r < -21.2$, and photometric
redshifts $z < 0.4$. This cut is more stringent than the SDSS detection limit to account for differences in k-corrections between early- and late-type galaxies.

We have additionally separated this volume-limited sample into two mutually exclusive redshift slices from $z < 0.3$ and $0.3 < z < 0.4$ to examine the possible evolution of the angular power spectrum with redshift and possible variation of the cosmological parameters obtained from fits (see Chapter 5). We have also created subsamples of the full volume-limited and redshift shell samples by separating each sample into early- and late-type galaxies based on the pztype parameter provided by the SDSS data. The pztype parameter is an estimate of the spectral type of the galaxy calculated in the photometric redshift pipeline (Abazajian et al. 2009), and we classify galaxies with a pztype value of less than 0.1 as early-type and those with pztype greater than 0.1 as late-type (Ross et al. 2010). The comparison of the galaxy angular power spectra in these volume-limited morphological samples is particularly useful in determining the relative linear bias between early- and late-type galaxies.

### 2.6 Simulated Data Set

In addition to matching the published results from T02 and verifying that our results from all stripes across the SDSS DR7 are consistent, we performed one additional test of the veracity of our quadratic angular power spectrum estimator. We generated simulated sky maps and compared the results from our quadratic estimator to the results from the HEALPix angular power spectrum estimator anafast. We first generated a linear angular power spectrum as described in Section 5.1 and used the HEALPix synfast routine to create ten pixelated sky maps at HEALPix resolution 2048. Second, we convert the pixel values in each of these ten sky maps to galaxy overdensities by using the average galaxy density of the SDSS DR7. Third, as we want to verify our estimator matches anafast results to high multipoles, we use HEALPix resolution 256 pixelization which confines us to a subsample of the full DR7 area. We choose to use the stripe 10 area as we also compare the angular power spectra of this

\footnote{See http://healpix.jpl.nasa.gov}
stripe from T02 with our results in Chapter 4. Therefore we mask, in an identical manner to our treatment of the galaxy samples, each of these simulated full sky maps to the stripe 10 boundary as described in Section 3.1.2. Finally, we combine pixels to produce a degraded map with HEALPix resolution 256. With these degraded sky maps, we calculate the angular power spectrum by using our quadratic estimator to these ten samples out to $\ell = 510$.

We also use synfast to generate the same ten maps at Healpix resolution 256, and calculate the angular power spectrum by using HEALPix angular power spectrum estimator anafast to provide a direct comparison to the results from our quadratic estimator. At resolution 256, we use the recommended $\ell = 512$ for synfast and anafast, and we performed a standard analysis with anafast of the entire pixelated sky with no regression, masking, or cuts. We show these results along with the results from our quadratic estimator in Figure 2.3. Both estimators show remarkable agreement, despite the fact that anafast is operating on a full sky map and our quadratic estimator is operating with the Stripe 10 window function. As a result, we feel our implementation of the quadratic estimator and the results we derive are robust.
Figure 2.3: The results of our quadratic angular power spectrum estimation analysis of these 10 simulated maps is plotted as a box-whisker plot with the median in red, 25% and 75% quartiles at the ends of the boxes, and the minimum and maximum results at the ends of the whiskers. The yellow band shows the minimum and maximum angular power spectrum measurements determined by the ten anafast measurements as described in the text. This figure has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
Chapter 3

Method

Any scalar field on the sky can be expanded as a linear combination of spherical harmonics

\[ s(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi). \]  

(3.1)

We model the data as the sum of a signal and an uncorrelated noise, \( x_i = s_i + n_i \), so that the correlation matrix, which is equivalent to the covariance matrix, becomes

\[ C_{ij} = \langle x_i x_j \rangle = \langle s_i s_j \rangle + \langle n_i n_j \rangle. \]  

(3.2)

Due to the cosmological principle, the Universe is believed to be statistically isotropic, and we can, using the orthogonality of spherical harmonics coefficients for isotropic fields (Baldi & Marinucci 2006), define the angular power spectrum, \( C_\ell \), as

\[ \langle a_{lm} a_{l'm'}^* \rangle \equiv C_\ell \delta_{ll'} \delta_{mm'}. \]  

(3.3)

Thus, for a statistically isotropic Universe with a Gaussian density field, the angular power spectrum represents the entire statistical content of the angular distribution of the data (Bond et al. 1998).

Angular power spectra attempt to measure the multipole moments, \( \ell \), of a two-dimensional distribution, in our case the galaxy density (Jaffe et al. 1999). However, since the

available photometric surveys only observe portions of the sky, all multipole moments cannot be individually determined (Tegmark 1996) as nearby multipole moments cause similar density variations over a small area; what is measured instead is a group of them simultaneously. Multipole moments are grouped into contiguous bands, and we make the assumption that all moments in the band are equal (e.g., Huterer et al. 2001, see Chapter 6.1). The same computation is subsequently performed on the bands as would normally be done on the individual multipole moments. This also serves to reduce the computation needed for the calculation (Borrill 1999). First, we calculate the angular power spectrum by using the smallest bands allowed, and the resulting bandpowers are averaged together into larger bands to improve the signal-to-noise and minimize errors (BJK98).

Typically, Fourier methods are used to describe the distribution of a continuous population, but the galaxy distribution is discrete. To calculate an angular power spectrum, we transform the discrete galaxy counts into a continuous galaxy density distribution. To do this, the sky is divided into “pixels” and the galaxy density in each pixel is calculated. The calculation continues in the same way as it would with a CMB temperature map (e.g., BJK98). Smaller pixels can tell us more information about the angular power spectrum, but the computation required is highly dependent on the number of pixels (Tegmark 1997).

In this chapter, we first discuss how we pixelize and mask the data, followed by our selection of bands in Section 3.2. In Section 3.3 we extensively detail how we calculate an angular power spectrum, beginning with KL-compression, and the quadratic estimation technique. In Section 3.4 we describe how these bandpowers can be combined to produce higher signal-to-noise angular power spectrum estimates, and how to calculate the window functions associated with these measurements.
3.1 Pixelization

We have chosen to use a quadratic estimation approach to calculate the maximum likelihood of the angular power spectrum using KL-compression (Bond 1995; Bunn 1995). To force the discrete galaxy observations into a continuous population, the sky is pixelated to determine the galaxy overdensity per pixel. Although necessary for this quadratic estimation technique, the pixelization of the sky causes a loss of information on scales near the pixel size, where the equivalent multipole moment is $\ell \approx \frac{180^\circ}{\theta}(T02)$. The amount of power lost at each multipole moment is quantified by the pixel window functions, and approximations to these have been provided by the HEALPix package out to roughly 75% of the pixel size. We have chosen to estimate the angular power spectrum of our samples out to the maximum multipole moment for which the pixel window functions are available, but we restrict our reported results to scales larger than the equivalent pixelization linear scale.

We pixelate the sky using equal area pixels and remove areas that are outside the survey geometry, or have high seeing or reddening values as described in Section 2.3. Any pixels with less than 75% usable area are not considered in the calculation. In the end, the galaxy overdensity is calculated:

$$x_i \equiv \frac{G_i}{\Omega_i} - 1$$  \hspace{1cm} (3.4)

where $G_i$ is the galaxy count in pixel $i$, $G$ is the average number of galaxies per square degree over the survey area, and $\Omega_i$ is the area of the pixel in square degrees. Thus the data set of possibly millions or more galaxies is reduced to a set of pixels that encodes the galaxy overdensities. The actual choice of pixelization technique, however, is important; and we have tested two different pixelization schemes, each with its own advantages.

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\[\text{The calculation of HEALPix pixel window functions are available at}\]

3.1.1 Pixelization Schemes

SDSSPix is a hierarchical, equal area pixelization scheme developed specifically for the SDSS by Max Tegmark, Yongzhong Xu, and Ryan Scranton\(^2\). It uses the natural SDSS stripe geometry to divide the sky into pixels aligned with the SDSS survey coordinates, \(\eta/\lambda\). Pixels at a particular resolution have a constant width in \(\eta\), and a variable width in \(\lambda\) to satisfy the equal area requirement. While SDSSPix is useful because of the alignment of pixels with survey boundaries which makes seeing and reddening in pixels easier to quantify, the elongation of pixels away from the survey center interfered with the convergence properties of our algorithm described below. This is because elongated pixels smooth density variations preferentially in the direction of elongation while retaining that information in the perpendicular direction. This increases the covariance between the smaller scale modes and drives increasing oscillation in high \(\ell\) bandpowers with each iteration.

HEALPix is also a hierarchical, equal area pixelization scheme (Górski et al. 2005), created for CMB experiments such as WMAP and Planck. It divides the sphere into 12 pixels at the base resolution, and higher resolutions recursively quarter these large pixels. The benefit of using HEALPix is that while pixel boundaries have no relation to our observational data, the pixels are not elongated as they are with SDSSPix. Due to the stability of the quadratic estimation method using HEALPix, we have opted to pixelize our data with HEALPix for all calculations.

3.1.2 Pixel Masks

Masking with HEALPix is more complicated than with SDSSPix since pixels may overlap the survey boundaries. For unbiased results, any pixel that overlaps a boundary must not be considered in the calculation since it may have an unphysical overdensity. Thus, many pixels on stripe edges are masked. We also eliminate pixels that are not contiguous with the primary SDSS observing footprint. A random sample of 100,000 of the pixels not used

due to the boundary are shown in Figure 3.1, we have plotted only a sample to prevent obscuration of the coordinate lines. Furthermore, we must mask pixels due to areas with poor image quality, these pixels are shown in Figure 3.2. Finally, we remove pixels where the mean seeing is more than 1.5 arcseconds, and pixels where the mean reddening is greater than 0.2, shown in Figures 3.3 and 3.4 respectively.

3.2 Selecting Bands

The first step in our approach is to select the initial fine bands. Multipole resolution is limited by $\Delta \ell \approx 180^\circ / \phi$, where $\phi$ is the analyzed area’s smallest angular dimension (Peebles 1980). For this reason, we want the broadest survey possible. Aside from being restricted to choosing bands wider than this limit, the choice of the starting, ending, initial value, and widths of each band is unrestrained, although some choices of initial values may cause non-convergence or singular matrices. We chose initial bands of equal widths, each $5\ell$ wide for the full sample and $20\ell$ wide for the individual stripes. We use initial values based on a prior angular power spectrum; however the final result is fairly insensitive to the input angular power spectrum after the iterative application of the quadratic estimation method.

While the convergence of the quadratic estimator does not significantly depend on the prior angular power spectrum, the KL-compression in Section 3.3.1 does require a reasonable prior to calculate the signal-to-noise eigenmodes. Consequently, we use prior angular power spectra generated by projecting the 3D matter power spectra obtained using CAMB with WMAP 7-year best fit cosmological parameters (Larson et al. 2011), as described in Chapter 5. We assume all $C_\ell$ within a band to be constant (Huterer et al. 2001):

$$C_\ell \equiv \frac{\ell(\ell+1)C_\ell}{2\pi} = \sum_b \chi_{b(\ell)}C_b$$

(3.5)

where $\chi_{b(\ell)} = 1$ while $\ell \in b$ and zero otherwise, and we define $C_\ell$ according to standard convention (Bond et al. 2000).
Figure 3.1: The HEALPix pixels removed for being outside the chosen SDSS boundary, for clarity we have plotted a random sample of the masked pixels. This figure has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
Figure 3.2: The HEALPix pixels removed due to poor image quality. This figure has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
Figure 3.3: HEALPix pixels removed for average seeing greater than 1.5 arcseconds. This figure has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
Figure 3.4: HEALPix pixels removed due to average reddening greater than 0.2 magnitudes. This figure has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
We start with an initial fine binning, to determine where the power is inside the larger bands that we later use. The Fisher information matrix (defined in Equation 3.11) is used to construct the bandpower window functions, and after we have performed the quadratic estimation to find the maximum likelihood, we will use these window functions to determine the correlation between bandpowers and individual multipole moments $\ell$.

### 3.3 Calculating $C_b$

Using only a knowledge of the survey geometry (or at least the region under consideration) and the assumed values for the bandpowers, we construct the covariance matrix $C$:

$$C_{ij} \equiv \langle x_i x_j \rangle = S + N$$  \hspace{1cm} (3.6)

where, $S$ is the signal matrix and $N$ is the noise matrix. The assumed bandpower values $C_b$ will only be approximate, which will make the covariance matrix approximate; but this covariance matrix will be compared to the data and iteratively corrected to converge to the true bandpower values. The signal matrix is calculated directly from the pixelated survey geometry using the assumed set of multipole values $C_\ell$. Using Equation 3.3 and the addition theorem for spherical harmonics, we can determine the mathematical form of the signal matrix to which we will compare our data (Tegmark 1997):

$$S_{ij} = \sum_{\ell} \frac{2\ell + 1}{2\ell(\ell + 1)} C_\ell P_\ell(\cos \theta_{ij}) e^{-\ell(\ell+1)\tau^2} = \sum_b C_b P_b.$$  \hspace{1cm} (3.7)

where $\theta_{ij}$ is the angle between pixels $i$ and $j$. The exponential factor is introduced to compensate for the smearing caused by a beam of width $\tau$. For pixels much larger than the beam, as is the case for a galaxy survey, this factor is negligible. The noise matrix, $N$, is
modeled as a Gaussian random process and is diagonal (Huterer et al. 2001):

\[ N_{ij} = \sigma_i^2 \delta_{ij} = \frac{1}{G} \delta_{ij}, \]  

(3.8)

where \( \sigma_i \) is the rms noise in pixel \( i \).

### 3.3.1 Karhunen-Loéve Compression

Rather than perform the full calculation on the vector of overdensities \( \mathbf{x} \), we instead choose to transform into a signal-to-noise basis. We estimate our signal-to-noise of each mode based on our prior angular power spectrum and then eliminate noisy modes in a process known as KL-compression (Vogeley & Szalay 1996; Tegmark et al. 1997). This is often useful for data compression, though this is not always the case: with the high signal in our main galaxy samples generally only tens of the several thousand modes are discarded, while in our volume-limited samples hundreds to thousands of modes are identified as low signal-to-noise. Additionally, KL-compression also provides an important sanity check of the quality of the data and the input power spectrum.

We begin by solving the generalized eigenvalue equation:

\[ \mathbf{Sb}_i = \lambda_i \mathbf{Nb}_i \]  

(3.9)

and normalizing such that \( \mathbf{b}_i^T \mathbf{Nb}_i = 1 \). We reorder the vectors \( \mathbf{b}_i \) by the signal-to-noise ratio, \( \lambda_i \), in descending order. We discard modes with insufficient signal-to-noise, and we choose to keep those with \( \lambda_i \geq 1 \). The remaining vectors \( \mathbf{b}_i \) form the columns of the matrix \( \mathbf{B}' \) that we use to transform the data vector \( \mathbf{x}' \equiv \mathbf{B}'^T \mathbf{x} \), as well as the signal, Legendre polynomial, and noise matrices \( \mathbf{S}' = \mathbf{B}'^T \mathbf{SB}' \), \( \mathbf{P}' = \mathbf{B}'^T \mathbf{PB}' \), and \( \mathbf{N}' = \mathbf{B}'^T \mathbf{NB}' \) (T02).

As we estimate the mean galaxy density from the survey itself, we constrain the data vector \( \mathbf{x} \) to have zero mean; this is known as the integral constraint (see Tegmark et al. 1998 for a detailed discussion). If we fail to account for the integral constraint we can
underestimate the power on large scales (Huterer et al. 2001), so we correct for this by adding a large number $M$ to the mean mode in the noise matrix $N$ before KL-compression. The KL-compression stage will determine that the signal-to-noise of the mean mode is low and it will be discarded with other low signal-to-noise modes.

### 3.3.2 Quadratic Estimation

From the new data vector $x'$, we perform the outer product to calculate the observed covariance matrix, $x'x'^T$, which will be compared to the constructed covariance matrix $C' = S' + N'$.

Now that we have a set of bandpowers that we want to determine, we calculate the $C_b$ that have the highest probability of creating the observed data. A complete calculation of the likelihood function, although slow, is possible (Oh et al. 1999), but a local maximum can be found by using iteration with the following estimator (BJK98):

$$\delta C_b = \frac{1}{2}(F^{-1/2})_{bb'} \text{Tr} \left[ (x'x'^T - N')(C'^{-1}P'_bC'^{-1}) \right]$$  

(3.10)

where, the Fisher information matrix $F$ is defined as:

$$F_{bb'} = \frac{1}{2} \text{Tr} \left( C'^{-1}P'_bC'^{-1}P'_b \right)$$  

(3.11)

Equation 3.11 provides the mechanism by which we can compare the covariance matrix obtained from the data $x'x'^T$ with the constructed covariance matrix $C'$. What this equation accomplishes is retrieving the $C_b$ that produce a covariance matrix $C'$ that is identical to $x'x'^T$. Note that we use $F^{-1/2}$ in Equation 3.10 as advocated by Tegmark (1998) for uncorrelated error bars and well behaved window functions. This factor is one choice among several with different properties (Padmanabhan et al. 2003), but it is important to realize that the total information content in the angular power spectrum with its associated Fisher matrix doesn’t change. This choice only affects how this information is displayed as simply points, which are the values of the $C_b$, with error bars that are the square root of only the diagonal elements.
of the inverse Fisher matrix. Therefore, the plots of angular power spectra alone do not contain the entirety of the information about the angular power spectrum; the entire Fisher matrix is needed for a full description.

By making an initial estimate of $C_b$, and iteratively applying this equation, the estimator quickly converges on a maximally probable set of bandpower values. The error in bandpower $b$, given by $\sigma_b = \sqrt{(F^{-1})_{bb}}$, is the smallest error any estimator can measure while estimating parameters from the sample itself due to the Cramer-Rao inequality (Kenney et al. 1951; Tegmark 1997).

If we assume that the primordial fluctuations that seeded the large scale structure that we see today were Gaussian (e.g., Guth 1981), the angular power spectrum contains all clustering information on linear scales. However, there has been some evidence that this might not be the case (e.g., Elsner & Wandelt 2010). Furthermore, non-linear effects from gravitational collapse become more pronounced at higher $\ell$, which also causes a departure from Gaussianity. Though the quadratic estimator we employ assumes Gaussian fluctuations, the maximum likelihood angular power spectrum values we determine are unaffected by potential non-Gaussianities in the galaxy density field. We note, however, that the presence of such non-Gaussianities would generally cause us to underestimate our error bars (T02).

### 3.4 Interpreting $C_b$

#### 3.4.1 Averaging $C_b$

After defining the bands and calculating the $C_b$, we use the Fisher Information matrix to determine the correlation between bandpowers (Knox 1998). Narrow bandpower window functions are preferred so that the error in one band measurement minimally affects other bands.

Though the Fisher matrix and $C_b$ have already been calculated for the choice of bands, we want to have a method of combining bandpowers to improve the signal-to-noise without
recalculating using the computationally demanding quadratic estimator method. For this we use the BJK98 method.

First, smaller bandpowers $b$ are averaged together into larger bandpowers $B$ (not to be confused with the KL-compression matrix $B$ defined earlier) using Equation \ref{eq:cl_avg}. We can combine any number of adjacent bandpowers to improve signal-to-noise, though combining bandpowers from sections of the angular power spectrum with significant structure will result in a loss of resolution in the areas of interest (BJK98).

$$C_B = \frac{\sum_{b \in B} \sum_{b' \in B'} C_b F_{bb'}}{\sum_{b \in B} \sum_{b' \in B'} F_{bb'}} \quad \text{(3.12)}$$

$$F_{BB'} = \sum_{b \in B} \sum_{b' \in B'} F_{bb'} \quad \text{(3.13)}$$

The averaged Fisher matrix must be calculated to determine the errors on $C_B$, which are \(\sigma_B = \sqrt{(F^{-1})_{BB}}\) (Tegmark 1997).

### 3.4.2 Calculating Window Functions

To represent the angular power spectrum visually, the data points are characterized not only by the values and errors, but also by the width and position of the bandpowers they represent. The bandpower window functions are given by (T02):

$$W = DF^{1/2} \quad \text{(3.14)}$$

where $D$ is the diagonal matrix that makes the rows of $W$ sum to unity. The midpoints of the bandpowers, $\ell_{eff}$, can also be calculated. Algorithmically, $\ell_{eff}$ is where half the power in the band comes from below and half from above that multipole (BJK98):

$$f_{Bb} = \sum_{b' \in B} F_{bb'} \quad \text{(3.15)}$$
\[ \ell_{\text{eff}} = \frac{\sum_{b \in B} \ell f_{Bb}}{\sum_{b \in B} f_{Bb}} \]  

(3.16)

We calculate the filter \( f_{Bb} \) while doing the averaging in Section 3.4.1. This filter function tells us how the power in larger bands is related to the power in the component smaller bands, and gives us information about how the power is distributed within the new larger bands (BJK98). The edges of the band, \( \ell^- \) and \( \ell^+ \), are defined to be where \( \ell f_{Bb} \) drops to \( e^{-1/2} \) of the peak power, and we plot these as horizontal error bars. The angular power spectrum at \( \ell_{\text{eff}} \) can be plotted with horizontal error bars ranging from \( \ell^- \) to \( \ell^+ \), with value \( C_B \) and vertical error bars \( \pm \sqrt{(F^{-1})_{BB}} \).
Chapter 4
Measurements of SDSS Galaxy Angular Power Spectra

In this chapter, we use the quadratic estimator of Chapter 3 to calculate the angular power spectra of the SDSS DR7 data described in Chapter 2. We begin by calculating the angular power spectrum of the SDSS DR7 main galaxy sample in an area corresponding to stripe 10, for comparison to the results of T02 in Section 4.1. This sample is split into three subsamples, separated by magnitude, corresponding to the three brightest samples in T02. After finding our results to be in general agreement with T02 after corrections to account for differences between the Early Data Release and Data Release 7, we calculate the angular power spectrum of the entire main galaxy sample in the contiguous area from stripes 9 to 37, as well as magnitude separated subsamples, in Section 4.2. Following that, in Section 4.3, we estimate the angular power spectrum of the main galaxy sample out to high $\ell$ by reducing the area of consideration in our analysis.

In addition to the entire main galaxy sample analysis, we also examine a volume-limited sample to compare the large scale distributions of galaxy samples to each other, minimizing the effects of Malmquist bias. In Section 4.4.1, we investigate the possible evolution of the angular power spectrum with redshift by comparing two volume-limited subsamples of approximately equal cosmic volume. We also separate the volume-limited sample into early- and late-type galaxies, and, in addition to clearly showing the stronger clustering of early-type galaxies, we are able to effectively determine the relative linear bias between these two morphological types in Section 4.4.2. Finally, in Section 4.4.3, we combine the redshift and

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galaxy type cuts together to produce four more subsamples that allow us to examine the possible redshift evolution of the angular power spectrum of early- and late-type galaxies.

4.1 The SDSS Angular Power Spectrum: Stripe 10

Given the complexity of the quadratic estimator and pixelization process, we want to verify that our results match those for stripe 10 previously published by T02. Combined with our systematic and simulation tests in Sections 2.3 and 2.6, this will provide a final confirmation of our approach. The results of our angular power spectrum calculation for stripe 10 for $\ell < 1000$ are shown in Figure 4.1, separated into three magnitude subsamples. These results are consistently higher than those in T02 in all samples (see Figure 4.2), but we find that our results are still in general agreement. This is due to a known magnitude calculation error in early SDSS data, where galaxy model magnitudes were miscalculated by roughly 0.2 magnitudes (Abazajian et al. 2004). When we shift our magnitude subsamples by 0.2 magnitudes to account for this difference, our results are very consistent with the previous results, typically within one standard deviation as shown in Figure 4.2. Additionally, as we are using DR7 data instead of the EDR data, galaxy counts versus galaxy probabilities, and a more robust HEALPix pixelization rather than SDSSPix, we do not expect the results to exactly coincide.

In addition, we not only need to know the final $C_\ell$, but to completely characterize the errors and the structure of each bandpower, we need to know the window functions. The variance and covariance of the $C_\ell$ are derived from the Fisher matrix. The bandpower window functions show from which $\ell$ the power in a band originates, so ideally our bandpower window functions are as narrow as possible. For illustration and comparison to T02, the bandpower window functions for the 18–19 magnitude bin of stripe 10 are shown in Figure 4.3. We see that at about $\ell \sim 750$, the window functions become wider signifying that our signal has dropped below shot noise fluctuations. Therefore we do not use bands beyond that $\ell$.
Figure 4.1: The angular power spectra of the 3 magnitude samples: 18–19, 19–20, and 20–21 for stripe 10. This figure has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
Figure 4.2: The magnitude shifted angular power spectrum in comparison with the results of T02. This figure has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
value. In the other magnitude bins, our signal does not drop below shot noise fluctuations to $\ell = 1000$ as shown in Figure 4.4. The window functions for other stripes are similar, and we have made these publically available.\footnote{All results discussed in this thesis are available at http://lcdm.astro.illinois.edu/research/aps.html}

4.2 The SDSS Angular Power Spectrum: Main Galaxy Sample

We can now calculate the angular power spectrum of the entire main galaxy sample of the SDSS DR7 for the contiguous area of the Northern Galactic Cap ellipsoid, as well as our chosen subsamples. The results of the angular power spectrum to $\ell = 200$ for the entire sample and magnitude separated subsamples are summarized in Figure 4.5 and in Table 4.1.

The brightest and on average closest galaxies in the 18–19 r-band magnitude bin are the most highly clustered at all $\ell$ as expected. Below that is the 19–20 magnitude bin, and the least clustered at all $\ell$ is the 20–21 magnitude bin. Also plotted are the linear theoretical angular power spectra discussed in Section 5.1 for $\ell < 90$.

4.3 The SDSS Angular Power Spectrum: Large Multipoles

We are computationally restricted from analyzing the full main galaxy sample at higher resolution (see Chapter 6) and are thus limited to $\ell \sim 200$ in Section 4.2. To gain more information on the shape of the angular power spectrum out to higher multipoles, we have restricted our analysis to a smaller area from which we determine the angular power spectrum at higher resolution. At HEALPix resolution 512, we are able to estimate the angular power spectrum out to $\ell = 1600$ with bands of width $\Delta \ell = 25$. We present the results from this measurement in Figure 4.6.
Figure 4.3: The window functions for each of the 50 bands, for the 18–19th magnitude bin of stripe 10, demonstrating a widening of the window functions beyond $\ell \sim 700$, where the signal drops below shot noise fluctuations. This figure has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
Figure 4.4: The window functions for each of the 50 bands, for the 20–21st magnitude bin of stripe 10, showing no corresponding widening of window functions as signal-to-noise remains strong throughout the entire range of \( \ell \). This figure has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
Figure 4.5: The angular power spectra of stripes 9–37, magnitudes 18–21 in black, 18–19 in red, 19–20 in green, and 20–21 in blue. The solid lines are the best-fit theoretical linear power spectrum for $\ell < 90$, see Chapter 5 for more details. This figure has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
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Table 4.1: The SDSS Angular Power Spectrum for our full main galaxy sample and each of the 3 magnitude subsamples. ℓ<sub>eff</sub> is the point in the band where half the power is from ℓ < ℓ<sub>eff</sub> and half the power is from ℓ > ℓ<sub>eff</sub>, not necessarily the center of the band. This table has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
Figure 4.6: The angular power spectrum out to $\ell = 1600$ of a $\sim 53.7$ square degree area of the SDSS DR7 near the North Galactic Pole, at an eighth of the linear scale of the full area results.
4.4 The SDSS Angular Power Spectrum: Volume-Limited Samples

Having measured the angular power spectrum for the entire main galaxy sample, we now look to see if we can detect evolution in the angular power spectrum with either redshift or galaxy type. For this, we use the volume-limited sample described in Section 2.5 for a fair comparison of the angular clustering properties of different galaxy samples. We first split this sample into redshift subsamples of approximately equal cosmic volume \(0 < z < 0.3\) and \(0.3 < z < 0.4\) to investigate evolution with redshift. Following that, we separate the volume-limited sample by morphological type where we certainly expect to see a marked difference in the behaviour of early- and late-type galaxies, with early-type galaxies expected to have more power at all scales due to the morphology-density relation. Finally, we combine these redshift and type cuts to probe the redshift evolution of structure based on galaxy type.

4.4.1 Redshift Shells

We have calculated the angular power spectra of our low redshift \(z < 0.3\) and high redshift \(0.3 < z < 0.4\) samples and these are shown in Figure 4.7. We can see that generally these two angular power spectra look roughly indistinguishable, though perhaps there is some variation at multipoles \(100 < \ell < 150\). To examine this closer, we have taken the ratio of these angular power spectra, which is shown in black in Figure 4.8. We see that the ratio is fairly consistent with a value of one over the entire range of \(\ell\) which implies that we cannot confidently detect any significant evolution at these scales.

Recall that although our samples are of galaxies with measured photometric redshifts \(z < 0.3\) and \(0.3 < z < 0.4\) and are therefore mutually exclusive, each of these galaxies has a photometric redshift error associated with them. We assume a Gaussian probability distribution function for each galaxy and repeatedly sample these PDFs to build up the estimated “true” redshift distribution for each sample (see Section 5.1), which is shown
Figure 4.7: The angular power spectra of the volume-limited samples with the $z < 0.3$ sample in black and the $0.3 < z < 0.4$ sample in green. We see that these two angular power spectra are in general agreement, with the exception of multipoles $100 < \ell < 150$ which are slightly higher for the high redshift sample.
Figure 4.8: The ratio of the angular power spectra of the volume-limited samples $z < 0.3$ and $0.3 < z < 0.4$. There is a slight dip for $100 < \ell < 150$ where the high redshift sample has more power, but generally the results are consistent with a ratio of one.
in Figure 5.8. There is some overlap, which implies that structures near the photometric redshift cut may be detected in both samples, therefore we expect some similarities in the features of the angular power spectra.

### 4.4.2 Galaxy Morphology Results

As early-type galaxies are believed to preferentially form in high density environments, we wish to explore evolution in the angular power spectra of galaxies split by galaxy type. In our volume-limited sample, we can compare the angular power spectra of early-type galaxies, which tend to be larger and brighter, to generally dimmer late-type galaxies without concerns of Malmquist bias selectively detecting only brighter galaxies, which, as we have shown in Section 4.2, have more power at all scales.

We also want to estimate the linear bias of early-type and late-type galaxies, and calculate the relative bias between them. The results of our galaxy morphology angular power spectra are given in Figure 4.9. Since the relative linear bias is roughly the ratio between these two angular power spectra, we can easily estimate the relative bias between early- and late-type galaxies by examining the ratio in black in Figure 4.10. We can see that the relative bias is remarkably consistent across all but the very largest scales down to $\sim 1$ degree, and the bias of early-type galaxies is roughly $30 - 40\%$ greater than the bias of late-type galaxies. After fitting in Chapter 5 we can determine the linear bias and relative biases of these samples more precisely.

#### Late-Type Large Scale Power

In Figures 4.9 and 4.10 it is clear that the power in our first bandpower for late-type galaxies is unusually high, and this is consistent across all late-type samples. The smallest scale probed by this range of $\ell$ is over 30 degrees, a great deal larger than where we expect significant structure to exist. This suggests that despite our masking process correcting for seeing, reddening, and poor observing quality, there may be a systematic that preferentially
Figure 4.9: The angular power spectra of the early- and late-type galaxies in the volume-limited samples.
Figure 4.10: The ratio of the angular power spectra of the early- to late-type galaxies in the volume-limited samples.
affects late-type galaxies on large scales. We have examined our volume-limited late-type sample in more detail, and have found that the average density of late-type galaxies drops at either end (in lambda, the survey longitude) of the SDSS observing footprint, nearer to the Galactic plane, as shown in Figure 4.11. While we already discussed in Section 2.3 and Figure 2.2 that the overall galaxy overdensity was consistent with zero at all Galactic latitudes, the late-type sample is more significantly affected and this large scale systematic effect is apparent in the first bandpower of these angular power spectra.

As this effect occurs closer to the Galactic plane, systematics that could reduce late-type galaxy counts include stellar obscuration of background galaxies, variable star-galaxy separation efficiency, and an insufficiently strict reddening mask cut. However, closer to the Galactic plane is also higher in stripe longitude lambda, so it is possible that variable sky brightness could influence the observed galaxy density. To test the possibility that stars are interfering with late-type galaxy densities, we pixelated the stars in the SDSS data in the same manner as galaxies, and masked those pixels that have an overdensity of stars greater than zero (see Figure 4.11). Due to the increased density of stars near the Galactic plane, this masks nearly all pixels at low Galactic latitudes and high lambda. However, we see in Figure 4.12 that this does begin to lower the large scale power in the first bandpower, suggesting a correlation between the increased density of stars and the lower than expected late-type galaxy density.

In order to identify the issue causing this underdensity of late-type galaxies, we need to first determine how to distinguish these various, nearly degenerate, possible causes. Though we see a correlation with stellar density in Figure 4.12, tightening the restriction on the stellar overdensity mask to overdensities of -0.1 and -0.25 provides no further reduction in the late-type large scale power. First, we extended our algorithm to calculate a cross-correlation angular power spectrum. By using this new code, we compute the cross power spectrum between late-type galaxies and stars, early-type galaxies and stars, and late-type galaxies and reddening. From these measurements, we find that only the late-type/star cross power
Figure 4.11: The average overdensity of early- and late-type galaxies plotted in red and blue respectively, plotted against the stellar overdensity in black. As we can see, the average late-type overdensity closely matches the average early-type overdensity in all but the lowest Galactic latitude bin, where the late-type overdensity deviates very slightly. This slight average underdensity, though well within one standard deviation of the survey average overdensity, is detectable with our angular power spectrum estimator and results in unexpected large-scale power in the first bandpower.
Figure 4.12: The angular power spectrum of late-type galaxies compared to the late-type sample that has masked high-stellar density pixels. Also plotted here are early-type galaxies for comparison at low $\ell$. 
spectrum shows significant correlation at small $\ell$. This evidence implies high stellar densities are associated with the slight drop in density of late-type galaxies, and suggests that either stellar obscuration or star-galaxy separation could be responsible.

With these results in mind, we next investigated how the properties of the late-type galaxy distribution changes as a function of Galactic latitude. By comparing the measured size of SDSS galaxies using the Petrosian radius containing half the Petrosian flux (Petrosian 1976), we generated binned image maps of the late- and early-type galaxy distributions, shown in Figures 4.13 and 4.14. From these maps, we see the smaller radius side of the distribution of late-type galaxies rises more at low latitude than for early-type galaxies, raising the average of low latitude late-type galaxy size. Therefore, it appears that small late-type galaxies at low latitudes are undercounted, either due to stellar obscuration or star-galaxy misclassification.

### 4.4.3 Combining Volume-Limited and Galaxy Morphology Cuts

Finally, we want to look for the possible evolution of early- and late-type galaxies between our two redshift samples. The results of our estimation of the angular power spectra of the different redshift slices for early- and late-type galaxies are given in Figures 4.15 and 4.16. We see that, as in Section 4.4.1, we see no clear evidence of evolution in these samples in different redshift ranges. Note that the high redshift late-type results are also signal-to-noise limited around $\ell \sim 120$, we therefore truncate that angular power spectrum. The ratio of angular power spectra of these samples are plotted along with the results for all types above in Figure 4.17 and are also generally consistent with a value of one suggesting no evidence of evolution.

Similarly, we use the same data to compare the early- and late-type samples in each redshift shell in Figures 4.18 and 4.19, while the ratio of these angular power spectra are provided in Figure 4.10. We find that the relative bias between early- and late-type galaxies behaves similarly and remains roughly constant at all scales regardless of the sample.
Figure 4.13: 2D histogram of the distribution of late-type galaxy radii as a function of Galactic latitude $b$. Higher galaxy counts at larger Galactic latitude reflect the greater survey area at those latitudes. Note the slight rise of the small radius side of the distribution at lower Galactic latitudes.
Figure 4.14: 2D histogram of the distribution of early-type galaxy radii as a function of Galactic latitude $b$. Higher galaxy counts at larger Galactic latitude reflect the greater survey area at those latitudes. Note the relatively flat small radius side of the distribution at lower Galactic latitudes.
Figure 4.15: The angular power spectra of the early-type galaxies in the volume-limited samples. We see a slightly greater amount of power in the range $\ell \sim 100$ to $\ell \sim 150$ for the high redshift sample, more pronounced in the early-type galaxies than in Figure 4.7.
Figure 4.16: The angular power spectra of the late-type galaxies in the volume-limited samples.
Figure 4.17: The ratio of the angular power spectra of the volume-limited samples $z < 0.3$ and $0.3 < z < 0.4$, separated by type. The late-type high redshift sample is signal-to-noise limited beyond $\ell = 120$ and we truncate the angular power spectrum for that sample. We see a slightly more pronounced dip for $100 < \ell < 150$ in the early-type galaxies compared to Figure 4.16, but we still can’t strongly conclude that there is evidence of significant evolution in redshift.
Figure 4.18: The angular power spectra of the volume-limited samples, separated by type.
Figure 4.19: The angular power spectra of the volume-limited samples, separated by type.
Chapter 5

Theory

The statistical characterizations of galaxy clustering provided by our angular power spectrum measurements are only the first step. In order to constrain cosmological parameters, we must compare these results to theoretical angular power spectra that are dependent only on cosmological parameters and the bias. Thus we can determine the most probable cosmological parameters by generating a suite of theoretical angular power spectra and finding the best-fit between the theoretical and observed angular power spectra.

5.1 Linear Power Spectrum

We begin by using a linear 3D power spectrum in this calculation, acknowledging that we must restrict our fits to large scales where nonlinear effects are negligible. However, the overall process is the same, facilitating a more detailed comparison later. To obtain theoretical $C^T_\ell$, we project the linear 3D power spectrum $P(k)$, modeled with the fitting formulae of Eisenstein & Hu (1998), down to two dimensions. With $P(k)$, we can calculate the $C^T_\ell$ we expect from a given theory (e.g., Huterer et al. 2001). From Crocce et al. (2010) we have the exact calculation for the theoretical angular power spectrum:

$$C^T_\ell = \frac{\ell(\ell + 1)}{\pi^2} \int k^2 P(k) \Phi_\ell(k)^2 \, dk \quad (5.1)$$

where:

\[ \Phi_\ell(k) = \int \phi(z)D(z)j_\ell(kr(z)) \, b \, dz \]  \hspace{1cm} (5.2)

\[ \phi(z) = \frac{1}{G} \frac{dG}{dz} \]  \hspace{1cm} (5.3)

where \( D(z) \) is the growth function \cite{Carroll1992} and \( j_\ell(kr) \) are spherical Bessel functions of the first kind, \( b \) is the bias, and \( r \) and \( g \) are the comoving distance and number density respectively. This simplifies if we use Limber’s approximation for \( \ell > 30 \) \cite{Limber1953} to approximate the calculation of the Bessel functions, which are both computationally expensive to calculate and oscillatory, which can induce unwanted oscillations in our angular power spectra:

\[ C_\ell^T \approx \frac{2\pi}{\ell(\ell + 1)} \int \phi^2(z)D^2(z)P\left(\frac{\ell + 1/2}{r(z)}\right)H(z)r^2(z)b^2 \, dz \]  \hspace{1cm} (5.4)

The theoretical power spectrum depends only on cosmological parameters through the 3D power spectrum and the bias, so we can use this dependence to infer constraints on these values. The only knowledge it requires about the sample is the redshift distribution. We calculate the redshift distribution by assuming the redshift of each galaxy is distributed as a Gaussian with mean equal to the observed photometric redshift and standard deviation equal to the error on the photometric redshift. We sample the distribution of each galaxy and weight by volume and luminosity function constraints as in \cite{Ross2010} with the luminosity function of \cite{Montero-Dorta2009}.

In Figure 5.1, we show the photometric redshift distribution of our main sample of over 18 million galaxies, separated into photometric redshift bins of width 0.001 with 0.0 \( \leq z < 1.0 \). We see that the peak of the sample is at \( z \sim 0.2 \) and falls off rapidly for \( z \geq 0.3 \). The redshift distribution is important because we must use it to project the 3D power spectra to compare with our angular power spectra. Also in Figure 5.1 we have separated the
redshift distribution into magnitude bins, which demonstrates that the photometric redshift distributions vary by magnitude, with the brighter bins being on average closer than the fainter bins. The average redshifts of these samples are $z = 0.171$ for the 18–19 magnitude bin, $z = 0.217$ for 19–20, $z = 0.261$ for 20–21, and $z = 0.243$ for the entire sample. We integrate the 3D power spectrum over these redshifts, weighted by the number of galaxies in each redshift bin, to produce an angular power spectrum using Equation \ref{eq:5.4}.

### 5.2 Fitting Full SDSS DR7 Samples

To constrain cosmological parameters, we use a $\chi^2$-fitting technique to determine the calculated theoretical linear angular power spectrum that best fits the observed bandpower measurements (Tegmark 1997). First, an average over the chosen bandpowers of the newly calculated $C_T^{\ell}$ is made so that these can be compared (Knox 1999):

$$\langle C_B^{\ell} \rangle = \sum_{B'} W_{BB'} C_{B'}^{\ell}$$ \hspace{1cm} (5.5)

with the bandpower window function $W_{BB'}$ from Equation \ref{eq:3.14}. We evaluate the following $\chi^2$ where $F$ is the Fisher matrix and $a_p$ are the cosmological parameters (Bond et al. 2000):

$$\chi^2(a_p) = \sum_{BB'} (\ln C_B - \ln C_B^{T}) C_B F_{BB'} C_{B'} (\ln C_{B'} - \ln C_{B'}^{T})$$ \hspace{1cm} (5.6)

We assume a flat cosmology and the WMAP baryon to matter ratio of $\Omega_b/\Omega_m = 0.168$ (Larson et al. 2011) to perform this $\chi^2$ minimization for $\ell < 90$. Over this range, the equivalent k is less than 0.16 $h$/Mpc at our median redshift of $\sim 0.2$; and, we therefore expect the linear $P(k)$ to be a good approximation. We note that, given the limited range of the data used with this cut, the $\ell < 90$ restriction is not likely to yield competitive constraints on $\Omega_m$, and to fit the data past $\ell = 90$ we would need to use a nonlinear power spectrum. Indeed, we find a wide range of allowed $\Omega_m$ values as shown in Table \ref{tab:5.1} which we illustrate.
Figure 5.1: The normalized photometric redshift distribution of all galaxies in stripes 9 to 37, from magnitude 18–21 in black, magnitude 18–19 in red, 19–20 in green, and 20–21 in blue. This figure has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
Table 5.1: The best fit biases and $\Omega_m$ for the four SDSS main galaxy samples.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Bias</th>
<th>$\Omega_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mag 18–21</td>
<td>0.94 ± 0.04</td>
<td>0.31$^{+0.18}_{-0.11}$</td>
</tr>
<tr>
<td>Mag 18–19</td>
<td>1.09 ± 0.05</td>
<td>0.26$^{+0.20}_{-0.15}$</td>
</tr>
<tr>
<td>Mag 19–20</td>
<td>1.03 ± 0.04</td>
<td>0.26$^{+0.17}_{-0.11}$</td>
</tr>
<tr>
<td>Mag 20–21</td>
<td>0.92 ± 0.04</td>
<td>0.33$^{+0.17}_{-0.10}$</td>
</tr>
</tbody>
</table>

by displaying the results of our $\chi^2$ minimization for the 18–21 magnitude sample in Figure 5.2. We display these best-fit models against our measurements in Figure 4.5.

5.3 Nonlinear Power Spectrum

As we see in Section 5.2, using a linear power spectrum model restrains us from using a large portion of our estimated angular power spectrum in the fitting process, especially for the small area, high multipole angular power spectrum. Consequently, though the bias is relatively well constrained, the cosmological matter and baryon densities affect the resulting angular power spectrum weakly on linear scales and are therefore not strongly constrained. Due to this restriction, we have chosen to only fit $\Omega_m$ and the bias when using a linear power spectrum model as there is too little variation to constrain the other cosmological parameters in a meaningful way. To improve the precision of our measured constraints on cosmological parameters, we also implemented a theoretical angular power spectrum calculation using nonlinear 3D power spectra obtained with the Code for Anisotropies in the Microwave Background (CAMB: Lewis et al. 2000).

Among its many functions, CAMB is capable of producing nonlinear 3D matter power spectra using HALOFIT (Smith et al. 2003). HALOFIT generates nonlinear matter power spectra using the halo model to describe galaxy correlations (Seljak 2000; Peacock & Smith 2000). The halo model suggests that the collisionless dark matter forms dark matter haloes within which baryons collapse to form some number of galaxies. Thus, the matter power spectrum is separated into two regimes: the correlation of galaxies is determined by the
Figure 5.2: The black point at $\Omega_m = 0.31$, $b = 0.94$ is the minimum of the $\chi^2$ test for the entire sample, the area in red covers the 68% confidence level. This figure has been previously published in Monthly Notices of the Royal Astronomical Society as Hayes, B., Brunner, R. and Ross, A. (2012), MNRAS, 421, 2043.
correlation between different dark matter haloes on large scales, and the correlation between galaxies within the same haloes on small scales. HALOFIT has determined empirical fitting functions for the power spectra of these two regimes by matching N-body simulations over a range of cosmological parameters. The sum of the quasi-linear large scale term and the nonlinear small-scale term produces an overall nonlinear matter power spectrum.

By running CAMB and HALOFIT with high accuracy options enabled, we produce nonlinear matter power spectra with an accuracy of $\sim 0.2\%$ (Howlett et al. 2012). However, this accuracy does assume a number of factors, including: the ionization history of the Universe, the Hubble parameter, the dark energy equation of state parameter, and the initial power spectrum. Lesser issues in these calculations include a number of other parameters such as the primordial Helium fraction and effective number of neutrino species. Where appropriate, we assumed a flat cosmology and WMAP 7-year best fit results and left other parameters at the CAMB default settings, which correspond to the current best measurements of these parameters. We generate 50 matter power spectra from $z = 0$ to $z = 1$ with $\Delta z = 0.02$ for each set of cosmological parameters that are fit, and proceed in the calculation from Equation 5.3 as before.

5.3.1 Nonlinear Fits

Now that this calculation can be extended to nonlinear scales, we can determine cosmological parameters more precisely. In addition to the bias and $\Omega_m$, we also chose to fit the baryon density, $\Omega_b$. We also investigated fitting the spectral index, $n_s$, but we found that generally the spectral index altered the angular power spectra similarly to $\Omega_m$ and thus caused a degeneracy in the parameter fits. As a result, we chose not to fit this parameter.

Now that we are no longer restricted to the linear regime, we must determine the maximum multipole to which we can fit the estimated and theoretical angular power spectra. In some cases, we are limited by the signal-to-noise of the data and can’t fit into the noise dominated region of our observed angular power spectrum results. But in most samples, the
signal-to-noise is sufficient for the full range of $\ell$ in our estimation and we are instead limited by the pixelization scale. Unfortunately, the pixelization process causes a loss of information on scales near the pixel size, but this is not a sharply defined boundary. Instead, the pixelization discards steadily more information as you approach the pixel size. The power lost due to pixelization is given by the pixel window functions, $w_\ell$, calculated and supplied by HEALPix, and this is demonstrated in Figure 5.3 for resolution 64. We see that by $\ell \sim 175$, the pixelization suppresses 50% of the power in the angular power spectrum, even though the equivalent linear scale of a pixel is $\ell \sim 200$. The pixel window functions relate the observed pixelated angular power spectrum $C^\text{pix}_\ell$ and the underlying unpixelized angular power spectrum $C^\text{unpix}_\ell$ by:

$$C^\text{pix}_\ell = w_\ell^2 C^\text{unpix}_\ell$$  \hspace{1cm} (5.7)

This has the effect of strongly suppressing the theoretical angular power spectrum at high $\ell$, but also reducing power at all scales. We have chosen to fit to a maximum multiole equivalent to twice the pixel scale for all nonlinear theory fits, where the bandpower value is reduced roughly 20% by the pixel window function but still dominated by the unpixelized angular power spectrum.

Finally, we must be aware of a known systematic in this quadratic estimation method that causes an excess of power at the small scale (i.e. high $\ell$) end of the angular power spectrum. Typically, this only affects the last few bandpowers in our estimation; and to correct for this effect, we use the quadratic estimator to calculate out to scales equivalent to $\sim 80%$ of the pixel size, and discard the extra bandpowers. However, due to covariance between the bandpowers, some of this power is inevitably transferred onto small scales, complicating the exact calculation.
Figure 5.3: The pixel window function at HEALPix resolution 64 in black, demonstrating the percentage of power lost at each multipole for any angular power spectrum calculated at this pixelization scale. In red, we integrate the pixel window function up to $\ell$ to show the cumulative power lost by that multipole. We also show the maximum multipole we use in our $\chi^2$ fits at $\ell = 100$, equivalent to twice the linear scale of the pixel.
5.3.2 Fitting Large Multipole Sample

Using the same technique as above, but with the nonlinear power spectra produced by CAMB, we have fit the results of our large multipole sample for the parameters $\Omega_m$, $\Omega_b$, and bias. We have fit out to a maximum of $\ell = 800$ or twice the pixel size to avoid signal lost by the pixelization. We find that the best fit $\Omega_m = 0.267 \pm 0.038$, $\Omega_b = 0.045 \pm 0.012$, and $b = 1.075 \pm 0.056$, and show this fit in Figure 5.4.

5.3.3 Fitting Volume-Limited Samples

We have also produced fits out to a maximum of $\ell = 100$ (twice the pixel size) of the nonlinear theoretical angular power spectra to the volume-limited samples. Due to the power associated with high stellar density in the first band of the late-type galaxy angular power spectra, discussed in Section 4.4.2, we have not included the first bandpower in these fits. To be consistent, we have excluded the first bandpower from all volume-limited fits, though the effect on the non-late-type galaxy samples was small, changing the best-fit parameters by $\sim 0.001$. These fits are generally consistent with both the above results in Sections 5.2 and 5.3.2 and WMAP 7-year results and show no strong evidence of evolution in redshift; however, the linear bias is strongly dependent on galaxy type. These best fit theoretical spectra shown in Figures 5.5, 5.6, and 5.7. Of interest to note is that the baryon acoustic oscillations (BAOs) (Eisenstein et al. 2005; Seo et al. 2012) are clearly visible in the $0.3 < z < 0.4$ sample due to its narrow redshift range, whereas the BAOs are smoothed over in the larger redshift ranges of the $z < 0.3$ and $z < 0.4$ samples. The redshift distributions for the two redshift shells are shown in Figure 5.8 demonstrating the narrower redshift distribution of the high redshift sample. The best fit values of bias and cosmological parameters for these nine samples are given in Table 5.2.

By comparing the linear biases of the early- and late-type galaxies, we can also determine the relative linear bias of these galaxy types. We find that the relative bias $b_e/b_l = 1.375 \pm 0.076$ for our entire $z < 0.4$ volume-limited sample, while $b_e/b_l = 1.421 \pm 0.083$ for $z < 0.3$. 

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Figure 5.4: The best fit theoretical angular power spectrum to the large multipole, high resolution sample out to $\ell = 800$, equivalent to twice the linear scale of the HEALPix resolution 512 pixels.
Figure 5.5: The best fit theoretical angular power spectrum to the samples split up by redshift shell with the $z < 0.3$ sample in green and the $0.3 < z < 0.4$ sample in red. Also apparent in the theoretical curves for the high redshift sample are the wiggles from baryon acoustic oscillations. The BAOs are smoothed over in the low redshift sample due to the larger redshift range.
Figure 5.6: The best fit theoretical angular power spectrum to the early-type samples split up by redshift shell with the $z < 0.3$ sample in green and the $0.3 < z < 0.4$ sample in red. Also apparent in the theoretical curves for the high redshift sample are the wiggles from baryon acoustic oscillations. The BAOs are smoothed over in the low redshift sample due to the larger redshift range.
Figure 5.7: The best fit theoretical angular power spectrum to the late-type samples split up by redshift shell with the $z < 0.3$ sample in green and the $0.3 < z < 0.4$ sample in red. Also apparent in the theoretical curves for the high redshift sample are the wiggles from baryon acoustic oscillations. The BAOs are smoothed over in the low redshift sample due to the larger redshift range.
Figure 5.8: The redshift distributions of the volume-limited samples with the \( z < 0.3 \) sample in black and \( 0.3 < z < 0.4 \). These samples cover approximately equal cosmic volumes, but the high redshift sample is a smaller redshift range so is more peaked than the low redshift sample.
Table 5.2: The best fit biases and cosmological parameters for the nine volume-limited samples.
and \( b_e/b_l = 1.200 \pm 0.071 \) for \( 0.3 < z < 0.4 \). These are fairly consistent across all samples, and show no evidence of scale dependence on these large scales.

### 5.4 Comparison with Previous Results

Though weakly constrained, our linear theory measurements of \( \Omega_m \) are consistent with other recent measurements of \( \Omega_m \) from galaxy angular power spectra such as Huterer et al. (2001); Frith et al. (2005), as well as measurements through other methods such as the 7-year WMAP results from the cosmic microwave background (Larson et al. 2011). These constraints are much improved in our fits to nonlinear theoretical matter power spectra for our large multipole and volume-limited samples. We find that our results are generally consistent with an \( \Omega_m \approx 0.27 \) with errors on the order of 0.03, which is typical of galaxy angular power spectra (Blake et al. 2007). This agrees well with the WMAP7 CMB results of \( \Omega_m = 0.267 \pm 0.026 \) as well as the combination of SDSS DR8 luminous galaxy angular power spectrum results with WMAP7 and supernova data \( \Omega_m = 0.267 \pm 0.0163 \) (Ho et al. 2012).

We find \( \Omega_b \approx 0.03 \) with errors generally about 0.02 in our volume-limited and large multipole samples, which is consistent with the \( \Omega_m = 0.0449 \pm 0.0028 \) constraints produced by WMAP7. The errors on our measurements are an order of magnitude larger than WMAP7, however, as our samples are less sensitive to this parameter, largely due to uncertainties in the photometric redshifts, and this is similar in other galaxy angular power spectra results (Thomas et al. 2010).

Measurements of the bias parameter vary with the sample under consideration. As we’ve shown in Section 5.3.3, bias is strongly type dependent, so the ratio of early- and late-type galaxies in a sample has a profound effect on the bias. Even the relative bias between early- and late-type galaxies has wide variations from a relative bias of \( 1.2 \pm 0.15 \) (Willmer et al. 1998) to \( \sim 1.75 \) (Ross et al. 2006), and is partially dependent on the cut used to separate
the different galaxy types.

With our assumptions of a flat Universe, and various properties of the initial angular power spectrum and reionization, these results imply that the mass-energy content in the Universe is not dominated by mass, but by another form of energy, believed to be dark energy. Even the mass in the Universe is not primarily normal, baryonic matter, but collisionless dark matter. The galaxies that we observe make up just a few percent of the mass-energy, but these galaxies trace the underlying dark matter distribution with a type-dependent bias describing how clustered galaxies are compared to the underlying dark matter. It is worth noting that these implications are quite consistent with the WMAP7 results, despite relying on an entirely independent measurement process and data set during a completely different cosmic epoch.
Chapter 6
Computational Investigation

6.1 Computational Requirements

The quadratic estimation method is computationally complex, due to both a large number of calculations required by matrix operations as well as large memory requirements to store these matrices. We must, therefore, consider computational feasibility when making choices about the extent of the data that we will analyze. At the scales of interest, we have found the processing time for a single processor scales as:

\[ T \approx 6 \text{ days } \left( \frac{n_b}{40} \right) \left( \frac{n_i}{3} \right) \left( \frac{n_p}{6836} \right)^3 \]  

(6.1)

and the memory requirements scale as:

\[ M \approx 60 \text{ GB } \left( \frac{n_b}{40} \right) \left( \frac{n_p}{6836} \right)^2 \]  

(6.2)

where \( n_b, n_i, \) and \( n_p \) are the number of bands, iterations, and pixels respectively. Typically, only a few iterations are necessary; we allow three iterations to achieve convergence. These are obviously highly dependent on the number of pixels, \( n_p, \) and processing time and memory requirements become prohibitive much beyond \( 10^4 \) pixels (Borrill 1999).

The calculations in this quadratic estimation method that take a significant percentage

of the total computation time are the signal matrix calculation (Equation 3.7), the KL-compression step (described in Section 3.3.1), and the large number of matrix operations on the covariance matrix (Equation 3.10). The signal matrix calculation is computationally expensive due to the recursive calculations of the Legendre polynomials that are evaluated for every pixel-pixel pair out to the maximum multipole moment under consideration. The computational demands of KL-compression and the rest of the algorithm are due to the complexity of matrix multiplication and matrix inversion, which are $O(n_p^3)$ operations.

In this thesis, we have already made two simplifying calculations that reduce the computational requirements of the quadratic method. First, the assumption that all multipole moments in a band are equal allows us to factor the bandpower value out of the signal matrix calculation in Equation 3.7. As the remaining factors in the sum only depend on the pixelated map geometry, which is unchanged after each iteration, we save a great deal of computation time by not recalculating the signal matrix during each iteration, and instead only multiplying our adjusted bandpower values by the $P_b$ matrices. As we are forced to make some assumption about the distribution of multipole moments inside each band by the limited sky coverage of galaxy surveys, the assumption that they are equal is both computationally helpful and expected to be approximately true based on large scale structure formation theory.

Another calculation that reduces the computational demand of this algorithm is the KL-compression of the data. By transforming the pixel basis to a signal-to-noise basis and discarding the low signal-to-noise modes in the data, we effectively reduce the number of “pixels” in the subsequent matrix multiplications and inversions, speeding up each of those $O(n_p^3)$ processes. Although the KL-compression step requires matrix multiplications and inversions with the number of pixels in the original data set and is a significant computation on its own, the many more matrix operations performed in the iterative process (the inversion of the covariance matrix every iteration followed by two matrix multiplications per bandpower per iteration) allows this to potentially speed up the calculation if enough low signal-to-noise
modes are removed. While we perform this KL-compression primarily to remove noisy modes, its usefulness as a method of computational acceleration must be acknowledged.

6.2 Platforms

6.2.1 Supercomputers

Naturally, the first option is to consider supercomputers when faced with computational challenges. With an $n^3_p$ scaling though, 10 times more pixels equates to a 1,000 times increase in computation time. To put this in perspective, in the same time a home computer can calculate an angular power spectrum of a data set at a particular resolution, a supercomputer would need 64 processors to calculate the same data at twice the resolution, and this is ignoring the fact that you can double the number of bands when doubling the resolution. Double the resolution again, and you would need over four thousand ($64^2 \approx 4000$) times the processing power of a desktop. Clearly, using a supercomputer will get better results, but the resolution cannot be pushed too much farther without making approximations.

When parallelizing the quadratic estimator, there are many options to consider. Though the parallelization of the signal matrix calculation by calculating each Legendre polynomial in a separate thread is relatively straightforward, there are at least two approaches to parallelizing the matrix operations. The first approach we used was to refer to Equations 3.10 and 3.11 to realize that these equations reduce to just three computationally intensive tasks per bandpower: the inversion of the covariance matrix, the multiplication of the inverted covariance matrix with the $P_b$ matrices, and the multiplication of that resultant matrix with the inverse covariance matrix.

The other multiplications are simpler because we are taking the trace of the result, allowing us to simplify those multiplications to only consider diagonal terms. Likewise, the inversion of the Fisher matrix is fast due to the comparatively smaller size of the Fisher matrix. The result is that we are able to calculate these matrix multiplications in parallel.
with each thread calculating the result for each bandpower. The most serious drawback of this method is that we are limited in the amount of parallelization that can be done by the number of bandpowers that are calculated, and further increasing the number of cores will only accelerate the signal matrix calculation and have no effect on the matrix multiplications.

Alternatively, we can parallelize the matrix operations themselves, calculating each matrix multiplication faster but doing all matrix operations sequentially. Due to the dependence of the previous method on the number of bandpowers, and the availability of libraries able to perform this parallelization automatically (namely the Intel Math Kernel Library), this has been the method of parallelization that we have chosen to use. Also, as matrix multiplications are the dominant computational hurdle in this method, it is accelerating this operation that we focus on in the next sections.

As a result, we have made use of the National Center for Supercomputing Applications’ (NCSA) 1,024 processor SGI Altix (Cobalt); its successor, the 1,536 processor SGI Altix (Ember), as well as the Pittsburgh Supercomputing Center’s (PSC) 768 core SGI Altix (Pople) and 4,096 core SGI UV 1000 (Blacklight) for these calculations.

6.2.2 Field Programmable Gate Arrays

Though supercomputers are presently the best approach to extending this method to the current technological limit, we have explored different innovative platforms that may make using this brute-force quadratic estimator method faster and less costly than using a supercomputer. After the success of using Field Programmable Gate Arrays (FPGAs) to calculate the two point angular correlation function much more efficiently than with a traditional processor (Kindratenko et al. 2007), we attempted to use FPGAs to accelerate quadratic estimation.

Naturally, the first place to start reducing the computational time involved is to attempt to optimize the operation that takes the bulk of the computation time: the matrix multiplication. We have attempted to do the matrix multiplications on a Field Programmable
Gate Array (FPGA)-based platform, which can be configured to our specific application. Our platform of choice was the SRC-6 MAP Series E processor. FPGAs excel in speeding up simple repetitive calculations; however, our results indicate that they may not be well suited to our particular problem. While the large matrix multiplication can be implemented, its performance is limited both by the off-chip memory bandwidth and limited memory read/write capability. Since SRC-6 MAP Series E processor provides eight on-board memory banks that allow only six read/writes to memory per clock cycle, only a fraction of the MAP processors peak floating point performance could be effectively utilized.

We attempted several different implementations of matrix multiplications, starting with copying the full matrices into the MAP’s on-board memory using Direct Memory Access (DMA), but considering the large matrix sizes used in scientific calculations, we also implemented matrix multiplications by transferring individual rows and columns of the product matrices into the FPGA for matrices too large to fit in on-board memory. Finally, we also tried an implementation that streamed in each matrix element as needed into the calculation.

At low resolution, we performed the angular power spectrum estimation with double precision for maps with 50, 100, 150, … pixels and fit a cubic to the computation time results, shown in Figure 6.1. Each of these methods varied in efficiency, with the calculation that had the full matrices in on-board memory understandably performing fastest. However, all these methods were slower than the CPU implementation primarily due to the restriction on the number of read/writes per clock cycle that allows only two multiplications per cycle (two reads for the product elements plus one write for the result). We used the ratio of our cubic fits to estimate the performance for larger data sets similar to scientifically interesting calculations and the results are shown in Figure 6.2. As we can see, the calculation with the full matrices in on-board memory is expected to perform best; however, even if the memory was sufficient to store matrices large enough to rival results from supercomputers, we still expect that the CPU implementation would be several times faster.

While using reconfigurable computing technologies can slightly speed up this multiplica-
Figure 6.1: The calculation time of a variety of algorithms for matrix multiplication on a Field Programmable Gate Array (FPGA). The solid lines depict our measured calculation times for small, low resolution data sets, while the dashed lines are our cubic fits to these measurements. The CPU only implementation shown in magenta is the fastest at all values of $n_p$, showing that overall the FPGA is inefficient at calculating matrix multiplications and increases the overall computation time.
Figure 6.2: The overall speedup of each of the matrix multiplication implementations on an FPGA when compared to the CPU implementation. We use the cubic fits to extrapolate the results for larger data sets similar to those currently used in scientific calculations, shown as dashed lines. We see that at for all data sets, even the most efficient implementation is expected to be an overall slowdown.
tion for single precision, we see that even with the full matrices stored in memory, at double precision we estimate an overall slowdown compared to the CPU implementation, similar to previous results (Smith et al. 2005). So while FPGAs can greatly accelerate many computations with repetitive calculations, we do not find it to be a good fit to our angular power spectrum estimator which requires heavy memory access.

6.2.3 Graphics Processing Units

We have also implemented this algorithm on Graphics Processing Units (GPU), which are part of every modern home computer. GPUs are specifically designed to parallelize simple computations across many small multiprocessors, which make them ideal for calculations such as matrix multiplication. At the time that this experiment was performed, the best available GPU was the Nvidia 8800 GTX, which was capable of single precision floating point operations using the Nvidia specific GPU programming language CUDA, and all reported calculations were performed on that device.

However, as effectively a GPU computes highly parallelizable calculations, it is subject to many constraints. The primary restriction is that batches of threads of fixed size (dependent upon the individual GPU specifications) must perform the exact same calculation simultaneously. For example, the Nvidia 8800 GTX we used is incapable of performing the multiplication of two 5x5 matrices, instead those matrices must be padded to 16x16 matrices, then multiplied. For matrix multiplication this is an easy fix; however, for other operations, it may be more difficult.

Before we implemented the KL-compression in this method, the dominant operations were the signal matrix construction, inversion of the covariance matrix, and matrix multiplications. Although we were unable to implement a parallelized matrix inversion, we calculated the highly parallelizable signal matrix and matrix multiplications on the GPU while allowing the CPU to compute the rest of the code. Using HEALPix pixelated data at resolutions 8 and 16 (respectively an eighth and a quarter of the full SDSS area resolution
results presented in this thesis), we compared the calculation of the angular power spectra using a single GPU to the same calculation done by the CPU alone. We varied the size of the blocks of threads that were calculated simultaneously and found that blocks of 64 or 128 threads performed best. These results are shown in Figure 6.3.

As can be seen, we found an extreme speedup compared to the CPU results, with the speedup even increasing for the higher resolution data due to the higher percentage of the calculation time taken by matrix multiplications. Our best speedup of the entire application was 337 times faster than the CPU only version, and this occurred for resolution 16 using a block size of 128. However, as much as we might have liked to continue these tests at higher resolution, we were limited by the memory requirements of the quadratic estimator. The Nvidia 8800 GTX has an on-board memory of 768 Mb that limited the scope of our calculations. So while this platform seems very promising in accelerating this computation, the memory is not yet sufficient to allow us to meet or exceed the calculations that can be performed using current supercomputers. However, subsequent development has gone into creating GPUs specifically for computational purposes. A recent result is the Tesla brand of General Purpose GPUs, now capable of double precision operations, which may prove useful for higher resolution calculations.
Figure 6.3: The speedups measured using the Nvidia 8800 GTX to calculate the signal matrix and matrix multiplications. Plotted in magenta and red are the speedups of the kernel (defined as the computation of the signal matrix alone) and entire application respectively measured calculating a data set pixelated at HEALPix resolution 8. In blue and cyan are the speedups of the kernel and application at HEALPix resolution 16. The x-axis shows the number of threads calculated simultaneously, which in CUDA is restricted to multiples of 2.
Chapter 7

Conclusions and Future Work

7.1 Conclusions

We have used the quadratic estimation method with KL-compression to determine the SDSS DR7 angular power spectrum, first as a means of radical compression of the angular clustering information, and second to match these observed angular power spectra with theoretical angular power spectra to extract the linear bias and cosmological parameters.

We masked for observational effects and applied this method to over 18 million SDSS DR7 galaxies, and to these data split into three magnitude subsamples out to $\ell \leq 200$. We also measured the angular power spectrum for each individual stripe out to $\ell \leq 1000$ for stripes 9–37. We have used the photometric redshift distribution of these galaxies to project the 3D power spectrum to two dimensions to obtain theoretical linear angular power spectrum, and employed a $\chi^2$ minimization to determine the best fit cosmological parameters given these observations. As the linear angular power spectrum approximation is not valid for the entire range of our estimated angular power spectrum, these parameter constraints have a large allowed range of values.

We have also estimated the angular power spectra for a small area of the SDSS DR7 main galaxy sample out to $\ell \leq 1600$ as well as nine volume-limited samples out to $\ell \leq 200$, separated by redshift, galaxy type, and the combination of redshift and type. Due to the

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weak constraints provided using a theoretical linear 3D power spectra, we have also generated nonlinear 3D matter power spectra using CAMB and used these to fit results from our volume-limited and large multipole angular power spectrum results.

We found that the linear bias of our magnitude separated SDSS main galaxy samples was $b = 1.09 \pm 0.05$ in the 18–19 magnitude range, $b = 1.03 \pm 0.04$ for 19–20, and $b = 0.92 \pm 0.04$ for 20–21, with an overall bias of $b = 0.94 \pm 0.04$ for our combined 18–21 magnitude sample. We have also calculated the cosmological density of matter as $\Omega_m = 0.31^{+0.18}_{-0.11}$ from our entire SDSS main galaxy sample using a linear power spectrum fit. In our large multipole sample, we found $b = 1.075 \pm 0.056$, $\Omega_m = 0.267 \pm 0.038$, and $\Omega_b = 0.045 \pm 0.012$. The results of our nine volume-limited samples are presented in Table 5.2.

7.2 Future Work

In the future, the quadratic angular power spectrum estimation code that we’ve developed can be applied to surveys other than the SDSS, such as the upcoming Dark Energy Survey (DES). In September 2012, the DES is scheduled to begin taking data for a 5-year survey, but even before that data is available, the simulations of the DES data can be analyzed with our angular power spectrum estimation code similar to what was done in T02 with the SDSS Early Data Release. To support this and similar efforts, we have made our parallelized estimation code publicly available.

We also can see the evidence of baryon acoustic oscillations in the angular power spectrum for narrow redshift slices, as in the theoretical angular power spectra in Figures 5.5, 5.6, and 5.7. Though the variations in the angular power spectra from BAOs is generally smaller than our error bars for these samples, detection of the BAOs in SDSS galaxy angular power spectra in the $30 < \ell < 300$ range is an area of current research (Seo et al. 2012). Indeed, the Baryon Oscillation Spectroscopic Survey (BOSS: Eisenstein et al. 2011) of SDSS-III aims to explore the BAO signal through examining correlations of Luminous Red Galaxies (LRGs)

\footnote{Codes are available at \url{http://lcdm.astro.illinois.edu/code/apscode.html}}
A closely related measurement that can be done is the cross-power spectrum estimation. The method discussed in Chapter 3 has referred only to auto-correlation, that is correlating galaxy overdensity with itself; but Equation 3.6 can be trivially extended to calculate the cross-correlation, for example between quasars and LRGs, by taking the outer product of two different data sets rather than of the same data set. This will not only provide interesting science, such as constraints on local primordial non-Gaussianity (Slosar et al. 2008), but can also be a test of systematics by calculating the cross-correlation of two samples that should be uncorrelated. T02 used cross-power spectrum estimation between galaxy overdensity and seeing and reddening to show that these systematics are very weakly correlated in comparison to the galaxy-galaxy auto-correlation, and thus do not significantly affect the SDSS galaxy angular power spectrum.

The most unfortunate drawback of this method of angular power spectrum estimation is the limitation imposed by computational constraints. The galaxy data we currently have is sufficient to calculate the angular power spectrum to much higher spatial resolution as we have shown in Section 4.3 but performing this calculation for the entire survey is currently not feasible using this method because of the inordinately large number of pixels. Other estimation methods exist, such as the anafast estimator included in the HEALPix package, but this estimator is not well suited to surveys with limited sky coverage, such as partial sky galaxy surveys. It is very useful to explore other methods of overcoming this computational limitation to explore the angular power spectrum to the limit allowed by the data.

A method proposed by Doré et al. (2001) involves computing angular power spectra for smaller submaps of a larger data set, and recombining these high-$$\ell$$ measurements from the small maps with low-$$\ell$$ measurements from a coarsely pixelized large map. This hierarchical decomposition technique can be applied to data sets with a traditionally intractable number of pixels, though the smaller submaps necessitate a broadening in multipole resolution. This drawback is minor compared to the advantage gained by surpassing the pixel limit, and this
technique can possibly be used to combine our large multipole sample results with our full SDSS DR7 main galaxy sample results to improve the resulting angular power spectrum.

Furthermore, the quick pace of technological advancement in computer hardware means that work that we have done in Chapter 6 may already be outdated. The production of the Tesla line of Nvidia General Purpose GPUs specifically designed for computing applications means that perhaps the technological limitations that prevented us from calculating scientifically interesting sized data sets may already be obsolete. Tesla GPGPUs now power the Tianhe-1A supercomputer in Tianjin, China which was the world’s most powerful supercomputer until July 2011. Given that the only limitation that we faced with our GPU calculation was the available memory, performing angular power spectrum estimation on this platform should soon be revisited.
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