Technical Status Report No. 14

on

STUDIES IN TRANSISTOR-RC
NETWORK SYNTHESIS

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31 May 1960

Circuit Theory Group
Electrical Engineering Research Laboratory
Engineering Experiment Station
University of Illinois
Urbana, Illinois
A. PROJECT STATUS

1. **Staff**
   
   Dr. M. E. van Valkenburg, Professor, Project Director; Dr. J. B. Cruz, Assistant Professor; Dr. S. L. Hakimi, Assistant Professor (one-third time on Project); Don L. Epley, Instructor (half time); James R. Young, Instructor (two-thirds time); D. A. Calaham, Instructor; S. S. Yau, Research Assistant (half time).

2. **Reports and Papers**
   
   Technical Note No. 11, "Synthesis of Double-Terminated Active Networks," King Sun Fu, 4 March 1960.
   
   
   Two papers from the Project were presented at the Polytechnic Institute of Brooklyn Symposium on Active Networks and Feedback Systems which was held 19-21 April 1960. These are: J. B. Cruz, Jr. and M. E. Van Valkenburg, "The Synthesis of Models for Time-Varying Linear Systems," and R. E. Thomas, "The Use of the Active Lattice to Optimize Transfer Function Sensitivities."
   
   Two letters to the editor based on Technical Note No. 12 have been accepted for publication in the IRE Transactions on Circuit Theory.

3. **Studies in Progress**
   
   (a) **Switching Theory**
   
   Technical Note No. 13 containing the results on the synthesis of transmission contact networks, the minimization of switching functions, and the design of relay circuits with error protection was finished during the past quarter. Two journal articles based on this report are now under preparation.
   
   An attempt to extend the sensitivity method of designing reliable contact circuits to the multi-terminal case was made with little progress to date. A preliminary investigation of the possibility of applying the sensitivity concept to sequential circuit problems indicates a possibility of worthwhile results. However, the problems involved are quite different from those encountered in previous work. (Epley)

   (b) **RLC Network Synthesis**
   
   The problem of synthesizing networks from transfer function specification by means of the cascade of a single RC and a single RL network (Figure 1) has
been reduced to the following problem: Given an arbitrary polynomial $N(s)$. What are the conditions necessary that it be formed as the numerator of $Z(s)$ for the network of Figure 2?

![Figure 1](RC RL)

![Figure 2](RC RL)

Figure 1  
**Figure 2**

A necessary condition on $N(s)$ was reported previously. It is now possible to state necessary and sufficient conditions on $N(s)$ for it to be so formed. They are:

1. If $N(s)$ has only pairs of complex zeros, then

   
   
   \[
   \sum_{l=1}^{m} \arg s_1 \leq \frac{\pi}{2};
   \]

   (1)

2. If $N(s)$ has real and complex zeros, then

   a) 
   
   \[
   N(s) \bigg|_{s=\sigma > 0} > 0
   \]

   (2)

   b) 
   
   \[
   \sum_{l=1}^{m} \arg s_1 < \frac{\pi}{2}
   \]

   (3)

where $\arg s_1$ is measured from the negative real axis to the upper half plane zeros of $N(s)$ (Figure 3).

On the basis of these results, sufficient conditions for the realization of any $z_{12}(s)$, $y_{12}(s)$, or $g_{12}(s)$ (within a gain constant) in the form of Figure 1 have been found. In the realization procedure, a minimum number of inductors is always used.

It has been recognized that the RL network may be realized by an inverter having the transmission matrix
The model of Figure 4, which is similar to the Linvill RC-NIC model, has been studied.

\[
[t] = \begin{bmatrix} 0 & a \\ a & 0 \end{bmatrix}, \quad a \text{ real}
\]

The poles of transfer function realizable by the RC-Inverter model are in general restricted by Equations 1, 2, and 3. These are recognized as very restrictive in comparison with the practically unrestricted pole location of the RC-NIC model. However, the minimum pole sensitivity to changes in the active element has been found for each model. It has been shown that the minimum sensitivity for the RC-Inverter model is always smaller than the minimum sensitivity for the RC-NIC model. The improvement obtainable may be as much as a factor of two.

Two main problems will be attacked in the next quarter:

1. The gain of networks realized by the RC-Inverter model is limited by the \( a \) of the transmission matrix. For a given \( a \), what is the largest gain obtainable by this model?

2. Given a transfer function to be realized, it is now known that there may be regions along the negative real axis which may not contain poles or zeros of \( z_{11} \) and \( z_{22} \) of the network of Figure 4. How may these regions be found in advance so that the freedom allowed in the synthesis procedure may be predicted? (Calahan)

(c) Studies of Sensitivity

The study of "second-order" sensitivity has been continued. It has been found that by using generalized root-locus techniques we can analyze the
sensitivity of a system with multiple parameter variations, and also possibly optimize the synthesis of a system from a prescribed system function. This work will be continued.

A new technique for analyzing the sensitivities of all immittance functions corresponding to n parameter variations at the same time is being developed. The procedure is to diagonalize the parameter matrix of a network by means of canonical transformations. Since the diagonal elements of the diagonalized matrix, which correspond to the complex frequencies of the network, are directly affected by the element values, and since the immittance matrix, which provides all informations about the immittances of the network, can be easily obtained from the diagonalized matrix, the sensitivities of all immittance functions are easily evaluated. During the coming quarter, simplifications of the transformations and a definite procedure of finding canonical transformation matrix of the parameter matrix of a network will be studied. (Yau)

A new technique for calculating a measure of sensitivity of a system function resulting from a simultaneous and large variation of system parameters has been developed. By this technique, one can set bounds on the behavior of system function caused by the tolerances of the components. A statistical measure of sensitivity is also developed. Using a pseudo Monte Carlo technique, it is possible to use a digital computer to compute this statistical measure of sensitivity. This technique will also enable one to assign a probability of occurrence to a derivation of the system function. These problems are being further studied. A paper on this subject jointly with Dr. J. B. Cruz, Jr. has been submitted for presentation at WESCON in Los Angeles. (Hakimi)

(d) Grounded RC Two-Ports

A technical report is now being written, to be published during the next quarter, on the research reported in progress February 1, 1960. Difficulty has been encountered in the synthesis of ladders with zeros of transmission at both $s = 0$ and $\infty$, but this difficulty does not appear to be insurmountable. At the present time, it is not feasible to synthesize ladders with complicated distributions of transmission zeros at frequencies other than zero and infinity, although the application of methods available as results of this
research are useful in many cases. Continued efforts are being made to increase the transmission gain realized by the proposed parallel ladder structure. (Young)

4. **Trips and Other Events**

Professors Hakimi and Van Valkenburg attended the IRE Convention in New York on 21-24 March 1960. Professor Van Valkenburg was chairman of a technical session titled "Circuit Theory: Current Contributions." This trip was made at no cost to the Project.

Professors Cruz and Van Valkenburg attended the Polytechnic Institute of Brooklyn Symposium on Active Networks and Feedback Systems held in New York on 19-21 April 1960. A joint paper was presented at this Symposium.

Dr. Don E. Epley has accepted the position of Assistant Professor at Stanford University beginning in September. During this summer months he will be employed at the Bell Telephone Laboratories, Murray Hill, New Jersey.

Submitted by:

M. E. Van Valkenburg
Project Director

31 May 1960