DYNAMICS OF VOLTAGE COLLAPSE

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FOREWORD

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1. INTRODUCTION

1.1. Motivation

Voltage collapse is characterized by a widespread reduction in voltage levels throughout the system. While voltage collapse is often described by a sequence of events, it has not as yet been described by a set of dynamic equations. Voltage stability is normally presented as a steady state sensitivity phenomenon. In this research, the subjects of voltage collapse and/or voltage stability are studied from a dynamic model of a multimachine system.

System voltage levels are normally strongly linked to reactive power reserves. The reactive power controls are largely automated and scattered through the system. Further, these controls depend upon the local voltage levels for their operation. While reactive power reserves may be the critical factor in voltage control, most of the voltage collapse cases reported have been related to heavy real power tie line flows or loads. Many interchange schedules are limited by voltage sensitivity to real power. That is, the limiting factor is not the voltage level itself, rather the sensitivity of voltage levels to real power transfer. The sharp voltage sensitivity to real power load is not understood in the context of dynamic stability. The main question to be answered in this research is: Can voltage collapse/stability be observed as an unstable mode in a dynamic model?

1.2. Literary Summary

Voltage collapse which is characterized by widespread voltage reduction throughout the system has occurred for two situations: When there has been a reactive power deficit and a reactive power surplus. The two cases illustrating a reactive power deficit are the North East American disturbance of November
1965 and the French collapse of December 1978 [7]. The New York disturbance of July 1977 was due to a reactive power surplus [6], [7], [8].

The above mentioned cases of power system collapses have stimulated considerable research into the cause of such occurrences. Walter R. Lachs, a native Austrian, has made significant contribution in this area through a series of papers giving a detailed account right from initiation of such events to describing various corrective procedures [1] - [6].

For the basis of a general description of the events leading to system voltage collapse, an EHV transmission system shall be considered. All elements of the power system generators, transformers, transmission lines, capacitors, load, rotating machines, etc., are involved in the process.

Breakdown of a system occurs when coordination between the scattered automatic controls is disrupted. In order to operate effectively this would require maintaining a continuing balance between the supply and demand of both real and reactive power. The initiating event is the loss of a highly loaded EHV line between a generator and a major load center or a sudden load spike in the system's hourly profile. In either case, this places a substantially increased reactive burden on the system by the latter loading on adjacent EHV lines.

Immediately following a disturbance there is a stabilizing effect caused by:

a) Over excitation of rotating units which gives an additional supply of reactive power to the system.

b) Load magnitudes are reduced following a sharp reduction in voltage.

Although the generator MW loading would be reduced, the MVar output would increase to meet the extra transmission reactive losses. To avoid thermal damage to the rotor, over excitation must be restricted, which is ensured by auto-
matic control devices. With an abrupt reduction in excitation the system is destabilized considerably.

Following the initial stabilizing effect the TCUL transformers operate, in order to restore distribution voltages to predisturbance levels. Each tap changing operation increased the load and the EH voltage level is reduced by interrelated factors.

Each extra increment of load tends to increase the MW and MVar losses on an EHV line and hence, produces a greater voltage drop at the receiving end. The reactive losses increase substantially as the voltage level is reduced.

The operation of the distribution transformer tap changes while increasing the secondary distribution voltage tends to reduce the subtransmission and similarly the subtransmission transformers tap changing would reduce the high subtransmission voltage levels. This effectively increases the total reactive loading by

a) reducing the charging of subtransmission cables and capacitors,

b) increasing series losses in subtransmission circuits and transformers. Each load transformer tap changing operation directly increases the loads which in turn are to be supplied by the generators. The extra reactive loading through the generator transformers would reduce the EH voltage levels.

System voltage stability can be maintained while there are sufficient "rotating reactive power reserves." Timing of transformer tap changing should be graded such that the higher the voltage the faster the tap changing. With the suggested tap change time there is a benefit of fewer tapping operations and less tap change maintenance at the lower voltage areas where the greatest number of transformers are situated.
Protective measures have 3 basic objectives:

(i) To raise EHV transmission voltage levels.
(ii) To restrict system load.
(iii) To improve system reactive power reserves.

Two relatively new measures that have been proposed are:

(i) Strategic load shedding which utilizes the change of voltages to select the correct level of load shedding at each major load substation.
(ii) Rearranging generator MW outputs to modify transmission line loading so as to significantly reduce the series reactive power losses.

The above description treats voltage collapse as a quasi-steady-state phenomenon. In this research, we attempt to observe voltage collapse as a prelude to or consequence of an unstable equilibrium condition.

The equilibrium condition of a power system is changing constantly during the daily load cycle. The stability of a given equilibrium is normally evaluated through an analysis of the linearized dynamic equations describing the system. Several papers have discussed this process. The linearized set of equations for the entire system is derived from the dynamic model of each of the machines including its exciter, as well as the nonlinear loads, all of which are interconnected through the transmission network.

The linearization is done along the same lines as the references [9], [11], [16] and [17].

This analysis is complicated mainly due to the size and complexity of various systems and in addition, machines interact with each other across the interconnection network. The construction of such a model starts with the linearization of a set of differential and algebraic equations describing the
system around an operating point, which is then formulated in terms of the state space variables. There are different choices when the dynamics of the machine are studied, e.g.,

1) Classical swing model.
2) Flux decay model (no damper winding).
3) Two axis model (neglecting stator transients).

In the paper [9] the flux decay model of the machine is considered where in the effect of damping is taken into account by an equivalent damping factor in the torque equation. The author has taken into account various nonlinear loads including dynamic loads like induction motors.

The work by Nealon [16] differs from the present thesis work in the sense he has considered only constant impedance type of loads whereas this thesis deals with constant power loads in addition to constant impedance loads. This results in a considerable difference in the behaviour of the system's eigenvalues. With the constant impedances absorbed in the \([Y]\) matrix, the physical modes of the generators are augmented by their respective transient reactances so that current injection at all except the internal nodes is zero. This greatly simplifies the problem of expressing the machine terminal currents in terms of the static variables. Although transient saliency is neglected, this report does not deal with the internal node model; instead, the formulation is done using a general system model which can be used with any type of nonlinear loads. In contrast to [9], only a part of the \([Y]\) matrix is inverted which reduces the computation considerably.
2. MULTIMACHINE DYNAMIC MODELING

2.1. Introduction

A basic model of a synchronous machine consists of three stator phase windings and two rotor windings. The windings on the rotor include a field winding whose axis coincides with the direct or d-axis and a damper winding along the quadrature or q-axis. Being an electromechanical system, the set of electrical and mechanical equations are coupled through the interdependence of flux linkages on the rotor position $\theta$ and the torque $T_q$ on the flux linkages. The differential equations describing the synchronous machine have time varying coefficients as can be seen due to the sinusoidally time varying inductances. Therefore, it seems only logical to introduce a symmetrical transformation which takes into account the relative motion between the stator and rotor flux linkages. This is done using Park's transformation, which converts the three stator phase windings into two equivalent fictitious windings: One along the d-axis and the other along the q-axis. We have thus transformed the stator variables onto a reference frame rotating with the rotor giving use to mutual inductances that are independent of rotor position and time. Since the axes of the stator are now orthogonal, there is no mutual coupling between them. The overall system consisting of synchronous machines, excitation system and network constraints finally give rise to a complete state space model, the procedure of which will be explained in detail in later chapters.

2.2. Machine and Excitation System Models

In the model used for this analysis, the transient effects of the synchronous machine will be dominated by the rotor, namely the field and damper windings along the d and q axes, respectively. Another assumption that is made is that the stator flux linkages along the d and q axes are fast variables represented by their quasi-steady-state algebraic relationships. After a lengthy
manipulation of the basic differential equations, we arrive at the following set of two algebraic and four differential equations [11]

\[ V_d = -I_d R_s + I_q x_q' + E_d' \]  

\[ (2.1) \]

\[ V_q = -I_q R_s - I_d x_d' + E_q' \]  

\[ (2.2) \]

\[ T_{do} E_q' = -E_q' - (x_d' - x_d) I_d + E_{fd} \]  

\[ (2.3) \]

\[ T_{qo} E_d' = -E_d' + (x_q' - x_q) I_q \]  

\[ (2.4) \]

\[ \dot{s} = \omega - \omega_s \]  

\[ (2.5) \]

\[ \dot{\omega} = P_m - P_e - D(\omega - \omega_s) \]  

\[ (2.6) \]

All exciters are assumed to have IEEE type DC1 model [Appendix 1]. The equations describing the excitation system are

\[ \dot{E}_{fd} = -K_E/T_E E_{fd} + 1/T_E V_R \]  

\[ (2.7) \]

\[ \dot{V}_R = -1/T_A V_R - K_A/T_A V_F + K_A T_A (V_t - V_{ref}) \]  

\[ (2.8) \]

\[ \dot{V}_F = -K_E K_F T_E T_F E_{fd} + K_F T_E T_F V_R - 1/T_F V_F \]  

\[ (2.9) \]

where:

\[ V_d + jV_q = \text{machine terminal voltage in p.u. in machine reference frame} \]

\[ I_d + jI_q = \text{machine terminal current in p.u. (generator notation) in machine reference frame} \]
$E_{d}'$ = voltage proportional to the damper winding flux linkage

$E_{q}'$ = voltage proportional to the field winding flux linkage

$X_d$ = unsaturated d-axis synchronous reactance

$x_q$ = unsaturated q-axis synchronous reactance

$x_d'$ = unsaturated d-axis transient reactance

$x_q'$ = unsaturated q-axis transient reactance

$R_s$ = stator winding resistance in p.u.

$T_{q0}$ = q-axis transient open circuit time constant in sec

$\omega$ = actual rotor speed in rad/sec

$\omega_s$ = synchronous rotor speed (377 rad/sec)

$E_{fd}$ = field excitation voltage in p.u.

$H$ = inertia constant of the machine in MJ·sec/MVA

$P_m$ = mechanical power

$P_e$ = electrical power output of the machine in p.u.

$\delta$ = relative load angle in radians

$D$ = machine load damping coefficient in p.u.

Define a new variable [Appendix 1]

$$R_f = -V_F + \frac{K_F}{T_T} E_{fd}$$
so that

\[ R_f = -V_F + \frac{K_F}{T_F} E_{fd} \]

Substituting for \( V_F \) and \( E_{fd} \) from (2.7) and (2.9) the set of equations for the excitation system modifies to,

\[ \dot{E}_{fd_i} = -\frac{K_A}{T_A} E_{fd_i} + \frac{1}{T_E} V_{R_i} \]  \hspace{1cm} (2.10)

\[ \dot{V}_{R_i} = \frac{K_A}{T_A} E_{fd_i} - \frac{1}{T_A} V_{R_i} + \frac{K_A}{T_A} R_{fi} + \frac{K_A}{T_A} V_{r} \]  \hspace{1cm} (2.11)

\[ \dot{R}_{fi} = \frac{K_F}{T_F} E_{fd_i} - \frac{1}{T_{Fi}} R_{fi} \]  \hspace{1cm} (2.12)

Two more assumptions that are made are:

1) Transient saliency is neglected \( (x_d' = x_q') \).

2) Stator resistance \( R_s \) is considered negligible.

2.3. **Load and Network Models**

In this section the following static load models are considered.

(i) Constant power loads.

(ii) Constant current loads.

(iii) Constant impedance loads.

Denoting the complex injected power by \( \vec{S}_{in} \) and the injected current by \( \vec{I}_{in} \), the load can be written as a function of voltage magnitude as

\[ \vec{S}_{in} = \vec{V} \vec{I}_{in}^* = P_k |\vec{V}|^k + jQ_k |\vec{V}|^k \]  \hspace{1cm} (2.13)
where \( k \) can take up any real value and \( P_k \) and \( Q_k \) are constants.

**Case - 1**

\[ k = 0, \quad S_{in} = P_0 + jQ_0 \] which is a constant power type of load.

**Case - 2**

\[ k = 1; \quad S_{in} = P_1 |\mathbf{V}| + jQ_1 |\mathbf{V}| = \mathbf{V} T_{in}^* \]

\[
S_{in} = (P_1 + jQ_1) |\mathbf{V}| = |\mathbf{V}| |T_{in}| \angle \theta_v - \theta_i
\]

\[
= \sqrt{P_1^2 + Q_1^2} |\mathbf{V}| \angle \phi
\]

(2.14)

where \( \mathbf{V} = |\mathbf{V}| \angle \theta_v \), \( T_{in} = |T_{in}| \angle \theta_i \) and \( P_1 + jQ_1 = |P_1 + jQ_1| \angle \phi \).

From the above expressions, \( |T_{in}| = (P_1^2 + Q_1^2)^{1/2} \), \( \theta_v - \theta_i = \phi \). This gives rise to a constant current, constant power factor load.

**Case - 3**

\[ k = 2; \quad S_{in} = P_2 |\mathbf{V}|^2 + jQ_2 |\mathbf{V}|^2 = \mathbf{V} T_{in}^* \]

\[ |\mathbf{V}|^2 \] can be written as \( \mathbf{V}\mathbf{V}^* \). Hence

\[
S_{in} = \mathbf{V}[P_2 \mathbf{V}^* + jQ_2 \mathbf{V}^*]^2,
\]

\[
= \mathbf{V}[(P_2 - jQ_2) \mathbf{V}]^*
\]

(2.15)

Thus, \( T_{in} = (P_2 - jQ_2) \mathbf{V} \). This gives rise to a constant impedance load with the impedance \( Z = 1/(P_2 - jQ_2) \).

A passive transmission network consisting of transmission lines, transformers, shunt reactors, etc., can be represented by the mathematical model

\[
\mathbf{T} = [\mathbf{V}] \mathbf{V}
\]

(2.16)
The elements of this matrix can be written from the conventional nodal method of circuit analysis.

\[ Y_{ii} = \text{sum of admittances connected to the } i^{th} \text{ bus.} \]

\[ Y_{ij} = \text{negative of the admittance connected between the } i^{th} \text{ and } j^{th} \text{ bus.} \]

Equation (2.16) can be rewritten in summation notation as

\[ I_i = \sum_{j=1}^{n} Y_{ij} V_j \]

The orientation of \( I_i \) and \( V_i \) are given as

![Figure 2.1. Voltage, current orientation.](image)

2.4. Steady State Operating Point

This section gives a brief outline on axis transformation from the synchronously rotating reference to the individual rotor reference frame. Using Park's transformation
\[ F_{0DQ} = T_a F_{abc} \bigg|_{\theta_a = \omega_s t} \]  
(2.18)

and

\[ F_{odq} = T_a F_{abc} \bigg|_{\theta_a = \theta} \]  
(2.19)

where the capital subscript letters indicate a transformation into the synchronously rotating reference frame and the small subscript letters denote the rotor reference frame. \( F \) can represent any of the following quantities - current, voltage or flux linkage. From (2.18) and (2.19)

\[ F_{0DQ} = T_a \begin{bmatrix} \omega_s t \\ \theta_a \end{bmatrix}^{-1} F_{odq} \]  
(2.20)

where

\[
T_a \bigg|_{\theta_a = \omega_s t}^{-1} \begin{bmatrix} \omega_s t \\ \theta_a \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_s t - \theta) & \sin(\omega_s t - \theta) \\ 0 & -\sin(\omega_s t - \theta) & \cos(\omega_s t - \theta) \end{bmatrix}
\]  
(2.21)

Substitute \( \delta = \theta - \omega_s t + \pi/2 \) to get

\[
\begin{bmatrix} F_d \\ F_q \end{bmatrix} = \begin{bmatrix} \sin \delta & \cos \delta \\ -\cos \delta & \sin \delta \end{bmatrix} \begin{bmatrix} F_d \\ F_q \end{bmatrix} \Rightarrow F_{DQ} = T^{-1} F_{dq}
\]  
(2.22)

and

\[
\begin{bmatrix} F_d \\ F_q \end{bmatrix} = \begin{bmatrix} \sin \delta & -\cos \delta \\ \cos \delta & \sin \delta \end{bmatrix} \begin{bmatrix} F_d \\ F_q \end{bmatrix} \Rightarrow F_{dq} = T F_{DQ}
\]  
(2.23)
The interpretation of this transformation is that the q-axis of the machine reference frame leads the D-axis of the network reference frame by an angle $\delta$, with the q, Q axes leading the d, D axes, respectively. Diagrammatically it is given by

Figure 2.2. Figure showing the orientation of the direct and quadrature axes of the machine and network.

At the operating point all quantities are denoted with a superscript '0'. For $v_{ti}^0 = V_{Di}^0 + jV_{Qi}^0$ and $i_{i}^0 = I_{Di}^0 + jI_{Qi}^0$ given from a load flow solution; other equilibrium values are

$$E_i^0 = (V_{Di}^0 + jV_{Qi}^0) + jx_{qi}(I_{Di}^0 + jI_{Qi}^0)$$

and

$$\delta_i = \tan^{-1} \left( \frac{\text{Im} E_i^0}{\text{Re} E_i^0} \right)$$

$$\begin{bmatrix} v_{di}^0 \\ v_{qi}^0 \end{bmatrix} = T \begin{bmatrix} V_{Di}^0 \\ V_{Qi}^0 \end{bmatrix}; \quad \begin{bmatrix} I_{di}^0 \\ I_{qi}^0 \end{bmatrix} = T \begin{bmatrix} I_{Di}^0 \\ I_{Qi}^0 \end{bmatrix}$$
\[ E_{di}^0 = V_{di}^0 - I_{qi} \times q_i \]  
\[ E_{qi}^0 = V_{qi}^0 + I_{di} \times q_i \]  
(2.27)  
(2.28)

From (2.3)

\[ E_{fdi}^0 = E_{qi}^0 + (x_{di} - x_{d1}) I_{di}^0 \]  
(2.29)

From (2.7) - (2.9), on setting all derivatives to zero,

\[ V_{Ri}^0 = (K_E + S_E) E_{fdi}^0 \]  
(2.30)

\[ V_{Fi}^0 = K_F/T_E V_{Ri} - \frac{(K_E + S_E) K_F}{T_E} E_{fdi}^0 \]  
(2.31)
3. LINEARIZED ANALYSIS

3.1. Introduction

The machine, excitation system, load and network were represented in the last chapter by a set of equations

\[
\dot{x} = F(x, y) \quad (3.1)
\]

\[
g(x, y) = 0
\]

where \( x = [E_d, E_q, \delta, \omega, E_{fd}, V_F]^T \) is the vector of state variables. \( y \) is the vector of interface variables relating the interconnection of each set of dynamic equations. The objective of this research is to study the stability properties of the interconnected multimachine system. In order to do this, linearization of the dynamic equations is necessary. Considering deviations from the equilibrium point as

\[
\Delta x = x - x^0
\]

\[
\Delta y = y - y^0
\]

the linearized state equations have the form

\[
\dot{\Delta x} = M_1 \Delta x + M_2 \Delta y \quad (3.2)
\]

\[
0 = M_3 \Delta x + M_4 \Delta y
\]

which can be finally written as

\[
\dot{\Delta x} = A \Delta x
\]

The following sections show how to obtain the matrix \( A \) in a systematic manner.
3.2. **Linearized State Equations**

The linearized form of Equations (2.1) - (2.12) in matrix notation is

\[
\begin{bmatrix}
\Delta E'_{di} \\
\Delta E'_{qi} \\
\Delta \delta_i \\
\Delta \omega_i
\end{bmatrix} =
\begin{bmatrix}
-1/T_{qoi} & 0 & 0 & 0 \\
0 & -1/T_{dqi} & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & -D_i/M_i \\
\end{bmatrix}
\begin{bmatrix}
\Delta E'_{di} \\
\Delta E'_{qi} \\
\Delta \delta_i \\
\Delta \omega_i
\end{bmatrix}
+
\begin{bmatrix}
0 & x_{qi} - x_{qi}'/T_{qoi} & 0 \\
0 & x_{di} - x_{di}'/T_{dqi} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/M_i \\
\end{bmatrix}
\begin{bmatrix}
\Delta I_{di} \\
\Delta I_{qi} \\
\Delta P_{fe_i} \\
\end{bmatrix}
\]

(3.4)

For the excitation system,

\[
\begin{bmatrix}
\Delta E'_{fdi} \\
\Delta V_{R_i} \\
\Delta V_{P_i}
\end{bmatrix} =
\begin{bmatrix}
-\frac{K_E}{T_{E_i}} & 1/T_{E_i} & 0 \\
-\frac{K_A F_i}{T_{A_i} T_{E_i}} & -1/T_{A_i} & K_A/T_{A_i} \\
K_F/T_{F_i} & 0 & -1/T_{F_i}
\end{bmatrix}
\begin{bmatrix}
\Delta E'_{fdi} \\
\Delta V_{R_i} \\
\Delta V_{P_i}
\end{bmatrix}
+
\begin{bmatrix}
0 & 0 & 0 \\
0 & K_A/T_{A_i} & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta I_{di} \\
\Delta I_{qi} \\
\Delta P_{fe_i}
\end{bmatrix}
\]

(3.5)

where the state variables are in vector form. The following interface variables
must be eliminated to obtain the closed form A matrix: $\Delta I_d$, $\Delta I_q$, $\Delta V_t$, $\Delta P_e$.

The change in voltage $\Delta V_{t_i}$ and power $\Delta P_{e_i}$ can be expressed as functions of state and interface variables as

$$\Delta V_{t_i} = \frac{V_{D_i}^0}{V_{t_i}^0} \Delta V_{D_i} + \frac{V_{Q_i}^0}{V_{t_i}^0} \Delta V_{Q_i} \quad (3.6a)$$

$$\Delta P_{e_i} = \begin{bmatrix} I_{d_i}^0 & I_{q_i}^0 \end{bmatrix} \begin{bmatrix} \Delta E_d' \\ \Delta E_q \end{bmatrix} + \begin{bmatrix} E_{d_i} & E_{q_i} \end{bmatrix} \begin{bmatrix} \Delta I_{d_i} \\ \Delta I_{q_i} \end{bmatrix} \quad (3.6b)$$

Thus the objective of this section is to eliminate all $\Delta I_{d_i}$, $\Delta I_{q_i}$, $\Delta V_{D_i}$, $\Delta V_{Q_i}$ from the above equations. These interface variables must satisfy the load and network constraints. The nonlinear interface equations were given in Chapter 2.

Going back to the transformation matrices

$$\begin{bmatrix} F_{d_i} \\ F_{q_i} \end{bmatrix} = \begin{bmatrix} \sin \delta_i & -\cos \delta_i \\ \cos \delta_i & \sin \delta_i \end{bmatrix} \begin{bmatrix} F_{D_i} \\ F_{Q_i} \end{bmatrix} \quad (2.23)$$

On linearizing about the equilibrium point,

$$\begin{bmatrix} \Delta F_{d_i} \\ \Delta F_{q_i} \end{bmatrix} = \begin{bmatrix} \sin \delta_i^0 & -\cos \delta_i^0 \\ \cos \delta_i^0 & \sin \delta_i^0 \end{bmatrix} \begin{bmatrix} \Delta F_{D_i} \\ \Delta F_{Q_i} \end{bmatrix} + \begin{bmatrix} F_{d_i}^0 \\ F_{q_i}^0 \end{bmatrix} \Delta \delta_i \quad (3.7)$$

For the reverse transformation,

$$\begin{bmatrix} F_{D_i} \\ F_{Q_i} \end{bmatrix} = \begin{bmatrix} \sin \delta_i & \cos \delta_i \\ -\cos \delta_i & \sin \delta_i \end{bmatrix} \begin{bmatrix} F_{d_i} \\ F_{q_i} \end{bmatrix} \quad (3.8)$$

Once again, on linearizing about the operating point,
\[
\begin{bmatrix}
\Delta F_{D_i} \\
\Delta F_{Q_i}
\end{bmatrix} = \begin{bmatrix}
\sin \delta_i^0 & \cos \delta_i^0 \\
-cos \delta_i^0 & \sin \delta_i^0
\end{bmatrix} \begin{bmatrix}
\Delta F_{D_i} \\
\Delta F_{Q_i}
\end{bmatrix} + \begin{bmatrix}
-F_{Q_i}^0 \\
-F_{D_i}^0
\end{bmatrix} \Delta \delta_i \\
\]

(3.9)

Starting with the stator algebraic equations

\[
\begin{bmatrix}
V_{D_i} \\
V_{Q_i}
\end{bmatrix} = \begin{bmatrix}
E_{D_i} \\
E_{Q_i}
\end{bmatrix} - Z_i \begin{bmatrix}
I_{D_i} \\
I_{Q_i}
\end{bmatrix} \\
\]

(3.10)

where

\[
Z_i = \begin{bmatrix}
0 & -x_{di} \\
-x_{di} & 0
\end{bmatrix}
\]

Applying the transformation \( T \) (2.23)

\[
T \begin{bmatrix}
V_{D_i} \\
V_{Q_i}
\end{bmatrix} = T \begin{bmatrix}
E'_{D_i} \\
E'_{Q_i}
\end{bmatrix} - Z_i T \begin{bmatrix}
I_{D_i} \\
I_{Q_i}
\end{bmatrix} \\
\]

(3.11)

Premultiplying by \( T^{-1} \) throughout

\[
\begin{bmatrix}
V_{D_i} \\
V_{Q_i}
\end{bmatrix} = \begin{bmatrix}
E'_{D_i} \\
E'_{Q_i}
\end{bmatrix} - T^{-1} Z_i T \begin{bmatrix}
I_{D_i} \\
I_{Q_i}
\end{bmatrix} \\
\]

(3.12)

Now,

\[
T^{-1} Z_i T = \begin{bmatrix}
\sin \delta_i & -\cos \delta_i \\
\cos \delta_i & \sin \delta_i
\end{bmatrix} \begin{bmatrix}
0 & -x_{di}' \\
x_{di} & 0
\end{bmatrix} \begin{bmatrix}
\sin \delta_i & \cos \delta_i \\
-\cos \delta_i & \sin \delta_i
\end{bmatrix} = \begin{bmatrix}
0 & -x_{di}' \\
x_{di} & 0
\end{bmatrix} = Z_i
\]
On linearizing about the operating point,

\[
\begin{pmatrix}
\Delta V_{Di} \\
\Delta V_{Q_i}
\end{pmatrix}
= \begin{pmatrix}
\Delta E_{Di} \\
\Delta E_{Q_i}
\end{pmatrix}
- Z_i \begin{pmatrix}
\Delta I_{Di} \\
\Delta I_{Q_i}
\end{pmatrix}
\]  

(3.13)

In full vector form

\[ [\Delta V_N] = [\Delta E_N] - [M][\Delta I_N] \]  

(3.14)

From now on it should be noted that the subscript 'N' represents quantities in the network reference frame and the subscript 'm' represents the quantities in the machine reference frame. M is a \(2m \times 2m\) matrix given by

\[
\begin{bmatrix}
Z_1 \\
& Z_2 \\
& & \ddots \\
& & & Z_m
\end{bmatrix}
\]

\[ m = \text{number of machines} \]

The linearized form of the equations for \(V_t\) and \(P_e\) have been given by (3.5) and (3.6). In full vector form the equations get modified to

\[ [\Delta V_{tN}] = [G][\Delta V_N] \]  

(3.15)

\(G\) is an \(m \times 2m\) matrix given by
\[
[\Delta p_e] = [E][\Delta I_m] + [F][\Delta E_m]
\]

(3.16)

\(E\) is an \(m \times 2m\) matrix given by

\[
\begin{bmatrix}
E'_{d_1} & E'_{d_1} \\
E'_{q_1} & E'_{q_1} \\
E'_{d_2} & E'_{q_2} \\
& \ddots \\
E'_{d_m} & E'_{q_m}
\end{bmatrix}
\]

\(F\) is an \(m \times 2m\) matrix whose elements are

\[
\begin{bmatrix}
I'_{d_1} & I'_{q_1} \\
I'_{d_2} & I'_{q_2} \\
& \ddots \\
I'_{d_m} & I'_{q_m}
\end{bmatrix}
\]
From axis transformation relations

\[
\begin{bmatrix}
\Delta I_{di} \\
\Delta I_{qi}
\end{bmatrix} = T \bigg|_{\delta = \delta^0} \begin{bmatrix}
\Delta I_{Di} \\
\Delta I_{Qi}
\end{bmatrix} + \begin{bmatrix}
0 \\
I_{qi}
\end{bmatrix} \begin{bmatrix}
\Delta \delta_i
\end{bmatrix}
\]

where the superscript '0' indicates that it is evaluated at the operating point.

In full vector form,

\[
[\Delta I_m] = [S][\Delta I_N] + [R][\Delta \delta]
\]  \hspace{1cm} (3.17)

where \( S \) is a \( 2m \times 2m \) matrix given by

\[
\begin{bmatrix}
\sin \delta_1 & -\cos \delta_1 \\
\cos \delta_1 & \sin \delta_1
\end{bmatrix}
\]

\[
\begin{bmatrix}
\sin \delta_2 & -\cos \delta_2 \\
\cos \delta_2 & \sin \delta_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
\sin \delta_m & -\cos \delta_m \\
\cos \delta_m & \sin \delta_m
\end{bmatrix}
\]

\( R \) is a \( 2m \times m \) matrix given by
By a similar process

\[
[A\Delta E_N] = [N][A\Delta E_m] + [P][\Delta \delta]
\]  

(3.18)

N is a $2m \times 2m$ matrix given by

\[
\begin{pmatrix}
\sin \delta_1^0 & \cos \delta_1^0 \\
-cos \delta_1^0 & \sin \delta_1^0 \\
\end{pmatrix}
\begin{pmatrix}
\sin \delta_2^0 & \cos \delta_2^0 \\
-cos \delta_2^0 & \sin \delta_2^0 \\
\end{pmatrix}
\begin{pmatrix}
\sin \delta_m^0 & \cos \delta_m^0 \\
-cos \delta_m^0 & \sin \delta_m^0 \\
\end{pmatrix}
\]

P is a $2m \times m$ matrix given by
Finally, we have arrived at the following set of equations for the interfacing of $m$ machines between their individual reference frames and the network reference frame:

\[
\begin{align*}
[\Delta V_N] &= [\Delta E_N'] - [M][\Delta I_N] \\
[\Delta E_N'] &= [N][\Delta E_m'] + [P][\Delta \delta] \\
[\Delta I_m] &= [S][\Delta I_N] + [R][\Delta \delta] \\
[\Delta P_e] &= [E][\Delta I_m] + [F][\Delta E_m'] \\
[\Delta V_t] &= [G][\Delta V_N]
\end{align*}
\] (3.19) (3.20) (3.21) (3.22) (3.23)

Clearly $\Delta I_m$, $\Delta P_e$ and $\Delta V_t$ can be found in terms of the network reference currents $\Delta I_N$ and the state variables $\Delta E_m'$ and $\Delta \delta$. The remaining task is to express these network referenced currents $\Delta I_N$ in terms of the state variables. This requires information about the interconnecting network and loads.
For the complex power injected at each load bus

\[ P_i + jQ_i = (V_{D_i} + jV_{Q_i})(I_{D_i} - jI_{Q_i}) \]  \hspace{1cm} (3.24)

modeled with constants \( P_{ik}, Q_{ik} \) and \( k \) as

\[ P_i = P_{ik}|V_i|^k \]  \hspace{1cm} (3.25)

\[ Q_i = Q_{ik}|V_i|^k \]  \hspace{1cm} (3.26)

where

\[ |V_i| = (V_{D_i}^2 + V_{Q_i}^2)^{1/2} \]

the nonlinear load constraints are,

\[ V_{D_i}I_{D_i} + V_{Q_i}I_{Q_i} = P_{ik}(V_{D_i}^2 + V_{Q_i}^2)^{k/2} \]  \hspace{1cm} (3.27)

\[ -V_{D_i}I_{Q_i} + V_{Q_i}I_{D_i} = Q_{ik}(V_{D_i}^2 + V_{Q_i}^2)^{k/2} \]  \hspace{1cm} (3.28)

Linearizing these load constraints gives the load bus terminal conditions,

\[
\begin{bmatrix}
I_{D_i}^0 & I_{Q_i}^0 & \Delta V_{D_i} \\
-I_{Q_i}^0 & I_{D_i}^0 & \Delta V_{Q_i}
\end{bmatrix}
+ \begin{bmatrix}
V_{D_i}^0 & V_{Q_i}^0 \\
V_{Q_i}^0 & -V_{D_i}^0
\end{bmatrix}
\begin{bmatrix}
\Delta I_{D_i} \\
\Delta I_{Q_i}
\end{bmatrix}

= \begin{bmatrix}
kP_{ik}|V_i|^{k-2} & V_{D_i}^0 \\
kQ_{ik}|V_i|^{k-2} & V_{Q_i}^0
\end{bmatrix}
\begin{bmatrix}
\Delta V_{D_i} \\
\Delta V_{Q_i}
\end{bmatrix}
\]

\hspace{1cm} (3.28a)

which simplifies to

\[
\begin{bmatrix}
\Delta I_{D_i} \\
\Delta I_{Q_i}
\end{bmatrix} = [D_i]
\begin{bmatrix}
\Delta V_{D_i} \\
\Delta V_{Q_i}
\end{bmatrix}
\]

\hspace{1cm} (3.29)
where $D_i$ is a $2 \times 2$ matrix whose elements are given by

$$\begin{align*}
D_i(1,1) &= -v_{D_i}^0 I_{Q_i}^0 - H_k k v_{D_i}^0 |v_i|^k - 2 + Q_k v_{D_i}^0 |v_i|^k - 2 + v_{Q_i}^0 I_{Q_i}^0, \\
D_i(1,2) &= P_k k v_{D_i}^0 y_{Q_i}^0 v_{Q_i}^0 |v_i|^k - 2 - v_{D_i}^0 I_{D_i}^0 - v_{Q_i}^0 I_{Q_i}^0 + Q_k k v_{D_i}^0 |v_i|^k - 2, \\
D_i(2,1) &= -v_{Q_i}^0 I_{Q_i}^0 + P_k k v_{D_i}^0 v_{Q_i}^0 |v_i|^k - 2 - v_{D_i}^0 I_{D_i}^0 - v_{Q_i}^0 I_{Q_i}^0 - Q_k k v_{D_i}^0 |v_i|^k - 2, \\
D_i(2,2) &= -v_{Q_i}^0 I_{Q_i}^0 + v_{D_i}^0 I_{D_i}^0 + P_k k v_{Q_i}^0 v_{Q_i}^0 |v_i|^k - 2 - Q_k k v_{Q_i}^0 |v_i|^k - 2.
\end{align*}$$

Taking into account all the loads $i = 1, \ldots, L$

$$\begin{align*}
[\Delta I_N]_{\text{Load}} &= [Y_{\text{Load}}] [\Delta V_N]_{\text{Load}} \\
[\Delta I_N]_{\text{Load}} &= [\Delta I_{D_i} \Delta I_{Q_i} \ldots \Delta I_{D_L} \Delta I_{Q_L}]^T \\
[\Delta V_N]_{\text{Load}} &= [\Delta V_{D_i} \Delta V_{Q_i} \ldots \Delta V_{D_L} \Delta V_{Q_L}]^T
\end{align*}$$

$Y_{\text{Load}}$ is a $2L \times 2L$ matrix formed by the $(2 \times 2)$ $D_i$ blocks, $i = 1, 2, \ldots, L$

\[
Y_{\text{Load}} = \begin{bmatrix}
D_1 & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & D_L
\end{bmatrix}
\]

The linear form of the interconnected network is,
where $Y_{mm}(2m \times 2m)$, $Y_{ml}(2m \times 2l)$, $Y_{lm}(2l \times 2m)$ and $Y_{ll}(2l \times 2l)$ are submatrices constituting the entire system $Y_{BUS}$ matrix.

From (3.34) and (3.35)

$$
[Y_{load} - Y_{ll}] [\Delta V_{N_{load}}] = [Y_{lm}] [\Delta V_{N_{mach}}] \tag{3.36}
$$

which gives

$$
[\Delta V_{N_{load}}] = [Y_{load} - Y_{ll}]^{-1} [Y_{lm}] [\Delta V_{N_{mach}}] \tag{3.37}
$$

The interconnection constraint is then,

$$
[\Delta I_{N_{mach}}] = [Y_{mm} + Y_{ml} (Y_{load} - Y_{ll})^{-1} Y_{lm}] [\Delta V_{N_{mach}}] \tag{3.38}

=> [\Delta I_{N_{mach}}] = [Y_A] [\Delta V_{N_{mach}}] \tag{3.39}
$$

where

$$
[Y_A] = [Y_{mm} + Y_{ml} (Y_{load} - Y_{ll})^{-1} Y_{lm}]
$$

which is like the familiar reduced admittance matrix.

From now on we delete the subscripts 'mach' and 'load' and $[\Delta I_N]$, $[\Delta V_N]$ represent the machine currents and terminal voltage in the network reference frame. To complete the interface, substitute for $[\Delta I_N]$ in (3.19) to get

$$
[I + MY_A] [\Delta V_N] = [\Delta E_N^r] \tag{3.40}
$$
which gives

\[
[\Delta V_N] = [Q][\Delta E_N']
\]  

(3.41)

where

\[
[Q] = [I + (M)(Y_A)]^{-1}
\]

substitute for \([\Delta E_N']\) from (3.20) to get

\[
[\Delta V_N] = [Q][N][\Delta E_m'] + [P][\Delta \delta]
\]

\[
= [x_1][\Delta E_m'] + [z_1][\Delta \delta]
\]

(3.42)

where

\[
[x_1] = [Q][N]\text{ and } [z_1] = [Q][P]
\]

substitute for \([\Delta V_N]\) from (3.42) in (3.39)

\[
[\Delta I_N] = [Y_A][x_1][\Delta E_m'] + [Y_A][z_1][\Delta \delta]
\]

\[
= [x_2][\Delta E_m'] + [z_2][\Delta \delta]
\]

(3.43)

(3.44)

where

\[
[x_2] = [Y_A][x_1]\text{ and } [z_2] = [Y_A][z_1]
\]

substitute for \([\Delta I_N]\) from (3.44) into (3.21)

\[
[\Delta I_m] = [S][x_2][\Delta E_m'] + [z_2][\Delta \delta] + [R][\Delta \delta]
\]

\[
[\Delta I_m] = [s_3][\Delta E_m'] + [z_3][\Delta \delta]
\]

(3.45)

(3.46)
where

\[ [x_3] = [S][x_2], [z_3] = [R] + [S][z_2] \]

Substitute for \([\Delta I_m]\) from (3.46) into (3.22)

\[ \Delta P_e = [E][x_3][\Delta E_m] + [z_3][\Delta \delta] + [F][\Delta \delta] \]

\[ = [x_4][\Delta E_m] + [z_4][\Delta \delta] \]  

(3.47)

(3.48)

where \([x_4] = [E][x_3]\) and \([z_4] = [E][z_3] + [F]\). Finally substitute for \([\Delta V_N]\) in (3.42) into (3.23)

\[ \Delta V_t = [G][x_1][\Delta E_m] + [z_1][\Delta \delta] \]

\[ = [x_5][\Delta E_m] + [z_5][\Delta \delta] \]  

(3.49)

(3.50)

where \([x_5] = [G][x_1]\) and \([z_5] = [G][z_1]\). Finally, all the interface variables have been expressed in terms of the machine state variables. The three relevant equations are rewritten

\[ \Delta I_m = [x_3][\Delta E_m] + [z_3][\Delta \delta] \]  

(3.51)

\[ \Delta P_e = [x_4][\Delta E_m] + [z_4][\Delta \delta] \]  

(3.52)

\[ \Delta V_t = [x_5][\Delta E_m] + [z_5][\Delta \delta] \]  

(3.53)

3.3. Formulation of the 'A' Matrix

Consider the linearized equations in matrix notation (3.4). These equations were written down for a single machine. Consider the first two equations
In full vector form, taking into account all the 'm' machines,

\[
[\Delta E_m'] = [B_1][\Delta E_m'] + [B_2][\Delta I_m] + [B_3][\Delta E_{fd}]
\]

where 'B_1' and 'B_2' are 2m x 2m matrices given by,

\[
B_1 = \begin{bmatrix}
-1/T_{q_{oi}}' & 0 \\
0 & -1/T_{d_{oi}}'
\end{bmatrix}
\]

\[
B_2 = \begin{bmatrix}
0 & -1/T_{q_{oi}}' \\
1/T_{d_{oi}} & 0
\end{bmatrix}
\]
$$B_2 = \begin{pmatrix}
0 & (x_{q_i}' - x_{q_i})/T_{q_{oi}} \\
(x_{d_i}' - x_{d_i})/T_{d_{oi}} & 0 \\
. & . & . & . \\
. & . & . & . \\
(x_{d_m}' - x_{d_m})/T_{d_{om}} & 0 & (x_{q_m}' - x_{q_m})/T_{q_{om}} \\
x_{d_m}' & x_{d_m}' & . & . & . & . & . & . \\
\end{pmatrix}$$

'B_3' is a $2m \times 2m$ matrix given by

$$B_3 = \begin{pmatrix}
0 & 1/T_{d_{01}} \\
1/T_{d_{01}} & 0 \\
. & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . \\
1/T_{d_{02}} & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . \\
0 & 1/T_{d_{om}} \\
\end{pmatrix}$$
Substituting for \([\Delta I_m]\) from (3.51) into (3.54)

\[
[\Delta E_m] = [B_1][\Delta E_m] + [B_2][\Delta I_m] + [B_3][\Delta \delta]
\]

\[
= [A_1][\Delta E_m] + [A_2][\Delta \delta] + [A_3][\Delta E_f]
\]  \(3.55\)

where \(A_1(2m \times 2m), A_2(2m \times m), A_3(2m \times m)\) are given by

\[
A_1 = [B_1] + [B_2][x_3]
\]

\[
A_2 = [B_2][z_3]
\]

\[
A_3 = [B_3]
\]

Taking the third equation from (3.4)

\[
[\Delta \delta_i] = [\Delta \omega_i]
\]

In full vectorform, i.e., taking all the 'm' machines into account,

\[
[\Delta \delta] = [\Delta \omega]
\]  \(3.56\)

Taking the last equation of (3.4)

\[
[\Delta \omega_i] = [-D_i/M_i][\Delta \omega_i] + [1/M_i][\Delta P_{ei}]
\]  \(3.57\)

Using matrix notation for all m machines and substituting for \([\Delta P_e]\) from (3.52)

\[
[\Delta \omega] = [A_4][\Delta \omega] + [B_4][\Delta I_m] + [B_5][\Delta \delta]
\]

\[
= [A_4][\Delta \omega] + [A_5][\Delta E_m] + [A_6][\Delta \delta]
\]  \(3.59\)
\( [A_4] \) is an \( m \times m \) matrix given by

\[
\begin{bmatrix}
-D_1/M_1 \\
\vdots \\
-D_m/M_m
\end{bmatrix}
\]

\( [B_4] \) is also an \( m \times m \) matrix given by

\[
\begin{bmatrix}
1/M_1 \\
\vdots \\
1/M_m
\end{bmatrix}
\]

\( [A_5] = [B_4][x_4] \)

and

\( [A_6] = [B_4][z_4] \)

The first equation of (3.5) in full vector form is written as

\[
[\Delta E_{fd}] = [A_7][\Delta E_{fd}] + [A_8][\Delta V_R] + [0][R_p] \tag{3.60}
\]

\( [A_7] \) and \( [A_8] \) are \( m \times m \) matrices given by
The second equation of (3.5) in full vector form is written as

\[
[A_7] = \begin{bmatrix}
-\frac{K_{E1}}{T_{E1}} & \cdot & \cdot & \cdot \\
\cdot & -\frac{K_{E2}}{T_{E2}} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -\frac{K_{Em}}{T_{Em}}
\end{bmatrix}
\]

\[
[A_8] = \begin{bmatrix}
\frac{1}{T_{E1}} & \cdot & \cdot & \cdot \\
\cdot & \frac{1}{T_{E2}} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \frac{1}{T_{Em}}
\end{bmatrix}
\]

The second equation of (3.5) in full vector form is written as

\[
[\Delta \dot{V}_R] = [A_9][\Delta E_{fd}] + [A_{10}][\Delta V_R] + [A_{11}][\Delta R_F] + [B_3][\Delta \nu_t]
\] (3.61)

Substituting for \([\Delta V_t]\) from (3.53)

\[
[\Delta \dot{V}_R] = [A_9][\Delta E_{fd}] + [A_{10}][\nu_R] + [A_{11}][\Delta R_F] + [A_{12}][\Delta E_m] + [A_{13}][\Delta \delta]
\] (3.62)

\(A_9, A_{10}, A_{11}\) are \((m \times m)\) matrices given by

\[
[A_9] = \begin{bmatrix}
-\frac{K_{A1}K_{F1}}{T_{a1}T_{f1}} & \cdot & \cdot & \cdot \\
\cdot & -\frac{K_{A2}K_{F2}}{T_{a2}T_{f2}} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -\frac{K_{Am}K_{Fm}}{T_{Am}T_{Fm}}
\end{bmatrix}
\]
where $A_{14}$ and $A_{15}$ are $m \times m$ matrices given by:

$$
A_{12} = [B_5][x_5] \quad \text{and} \quad A_{13} = [B_5][z_5]. \quad \text{The last equation of (3.5) in full vector form is written as}

$$
[\Delta R_F] = [A_{14}][\Delta E_{fd}] + [0][\Delta V_R] + [A_{15}][\Delta R_F]
$$

(3.63)
The 'A' matrix is now formed by a proper arrangement of the matrices $A_1 - A_{15}$ as shown in Figure 3.1.
SYSTEM 'A' MATRIX

Figure 3.1. The system 'A' matrix.
4. SYSTEM STUDY AND RESULTS

4.1. Introduction

In this chapter systematic study is done on the test system under different loading conditions. The one line diagram of the test system is shown in Figure 4.1, [14].

The system is characterized by 9 buses (whose transmission and distribution voltage levels are 230 kV), 9 transmission lines and 3 generating units. Each of the machines is represented by a set of seven equations, four of which describe the machine dynamics and the remaining three describe the dynamic model of its exciter. Machines (2) and (3) are rated at 18 kV, 163 mW and 13.8 kV, 85 mW, respectively. All network and machine data were per unitized on a 100 MVA base. All machine data, transmission line parameters and base case system load levels are summarized in Tables 4.1 - 4.3B. The system eigenvalues at the operating point are shown in Table 4.4.

This chapter deals with the actual process of loading the system under study in steps at each load bus and observing the terminal voltages of each machine as well as the eigenvalues of the system 'A' matrix. The whole process is repeated for two different types of loads, namely

1) Constant Power loads and
2) Constant impedance loads.

Curves showing the relationship between the load (P) and the terminal voltage (V_t) are plotted under each of the loading conditions Figures 4.2 - 4.4. Further, the migration of the critical eigenvalues as a function of system load is shown in Figures 4.5-4.6. The eigenvalues under constant power and constant impedance loading conditions for various loads at each of the load buses are shown in Tables 4.5-4.10. The plot of the critical eigenvalue as a function of load is
Figure 4.1. Three-machine, nine-bus test system.
### TABLE 4.1.

TRANSMISSION LINE DATA ON A 100 MVA BASE

<table>
<thead>
<tr>
<th>From Bus Number</th>
<th>To Bus Number</th>
<th>Series Resistance (V)</th>
<th>Series Reactance (x)</th>
<th>Shunt Susceptance (B/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>0</td>
<td>0.0576</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>0.017</td>
<td>0.092</td>
<td>0.079</td>
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<tr>
<td>6</td>
<td>9</td>
<td>0.039</td>
<td>0.170</td>
<td>0.179</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>0</td>
<td>0.0586</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>0.0119</td>
<td>0.1008</td>
<td>0.1045</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>0.0085</td>
<td>0.072</td>
<td>0.0745</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>0</td>
<td>0.0625</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>0.032</td>
<td>0.161</td>
<td>0.153</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0.01</td>
<td>0.085</td>
<td>0.088</td>
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</table>

### TABLE 4.2.

BUS DATA FOR THE 3-MACHINE 9-BUS STUDY SYSTEM

<table>
<thead>
<tr>
<th>Bus Number</th>
<th>Bus Type</th>
<th>Load Level</th>
<th>Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Swing</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>2</td>
<td>PV</td>
<td>1.025</td>
<td>1.025</td>
</tr>
<tr>
<td>3</td>
<td>PV</td>
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<td>1.025</td>
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<tr>
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<td>---</td>
<td>---</td>
</tr>
<tr>
<td>5</td>
<td>PQ</td>
<td>1.25+j0.5</td>
<td>1.25+j0.5</td>
</tr>
<tr>
<td>6</td>
<td>PQ</td>
<td>0.9+j0.3</td>
<td>0.9+j0.3</td>
</tr>
<tr>
<td>7</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>8</td>
<td>PQ</td>
<td>1.0+j0.35</td>
<td>1.0+j0.35</td>
</tr>
<tr>
<td>9</td>
<td>---</td>
<td>---</td>
<td>---</td>
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TABLE 4.3A.
INDIVIDUAL MACHINE PARAMETERS FOR THE 3-MACHINE, 9-BUS SYSTEM UNDER STUDY

SYNCHRONOUS MACHINE DATA ON A 100 MVA BASE

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Machine 1</th>
<th>Machine 2</th>
<th>Machine 3</th>
</tr>
</thead>
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<tr>
<td>H</td>
<td>23.64</td>
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<td>3.01</td>
</tr>
<tr>
<td>x_d</td>
<td>0.146</td>
<td>0.8958</td>
<td>1.3125</td>
</tr>
<tr>
<td>x_d'</td>
<td>0.0608</td>
<td>0.1198</td>
<td>0.1813</td>
</tr>
<tr>
<td>x_q</td>
<td>0.0969</td>
<td>0.8645</td>
<td>1.2578</td>
</tr>
<tr>
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<td>0.1198</td>
<td>0.1813</td>
</tr>
<tr>
<td>T_d0</td>
<td>8.96</td>
<td>6.0</td>
<td>5.89</td>
</tr>
<tr>
<td>T_q0</td>
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<td>0.6</td>
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TABLE 4.3B.
EXCITATION SYSTEM PARAMETERS FOR THE STUDY SYSTEM

EXCITATION SYSTEM DATA ON A 100 MVA BASE

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<thead>
<tr>
<th>Quantity</th>
<th>Exciter 1</th>
<th>Exciter 2</th>
<th>Exciter 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>K_A</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>T_A</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>K_E</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>T_E</td>
<td>0.314</td>
<td>0.314</td>
<td>0.314</td>
</tr>
<tr>
<td>K_F</td>
<td>0.063</td>
<td>0.063</td>
<td>0.063</td>
</tr>
<tr>
<td>T_F</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Figure 4.2. Plot of voltage versus load (P) under constant power loading conditions at Bus-5.
Figure 4.3. Plot of voltage versus load (P) under constant power loading conditions at Bus-6.
Figure 4.4. Plot of voltage versus load (P) under constant power loading conditions at Bus-8.
### TABLE 4.4.

**EIGENVALUES AT THE OPERATING POINT**

<table>
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<th>Row</th>
<th>Real Part</th>
<th>Imag. Part</th>
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<td>2</td>
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<td>12.99618</td>
</tr>
<tr>
<td>3</td>
<td>-1.03695</td>
<td>-12.99618</td>
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</tr>
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<td>-8.37601</td>
</tr>
<tr>
<td>6</td>
<td>-3.65119</td>
<td>7.21603</td>
</tr>
<tr>
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<td>-3.65119</td>
<td>-7.21603</td>
</tr>
<tr>
<td>8</td>
<td>-3.73618</td>
<td>7.35416</td>
</tr>
<tr>
<td>9</td>
<td>-3.73618</td>
<td>-7.35416</td>
</tr>
<tr>
<td>10</td>
<td>-3.75055</td>
<td>7.42551</td>
</tr>
<tr>
<td>11</td>
<td>-3.75055</td>
<td>-7.42551</td>
</tr>
<tr>
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<td>-3.69876</td>
<td>0.00000</td>
</tr>
<tr>
<td>14</td>
<td>-0.20208</td>
<td>1.50406</td>
</tr>
<tr>
<td>15</td>
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</tr>
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</tr>
<tr>
<td>17</td>
<td>-0.17784</td>
<td>-0.92227</td>
</tr>
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<td>0.64142</td>
</tr>
<tr>
<td>19</td>
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<td>-0.64142</td>
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</tr>
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<td>$P = 2.2$</td>
</tr>
<tr>
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<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-1.05±j12.99</td>
<td>-1.087±j12.96</td>
</tr>
<tr>
<td>$\omega$</td>
<td>-0.33±j8.405</td>
<td>-0.35±j8.38</td>
</tr>
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<td>0.000</td>
</tr>
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<td>$e_{fd}$</td>
<td>-5.11±j7.766</td>
<td>-5.12±j7.79</td>
</tr>
<tr>
<td>$v_R$</td>
<td>-5.17±j7.94</td>
<td>-5.17±j7.91</td>
</tr>
<tr>
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<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$e_q$</td>
<td>-0.63±j1.256</td>
<td>-0.64±j1.106</td>
</tr>
<tr>
<td></td>
<td>-0.42±j0.763</td>
<td>-0.41±j0.63</td>
</tr>
<tr>
<td>$R_f$</td>
<td>-0.41±j0.5194</td>
<td>-0.1127</td>
</tr>
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</tr>
<tr>
<td>-------</td>
<td>---------</td>
<td>---------</td>
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</tr>
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<td>0.000</td>
</tr>
<tr>
<td>$E_{fd}$</td>
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<td>-0.1807</td>
</tr>
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<td>$E_{fd}$</td>
<td>-2.8571</td>
<td>-2.8571</td>
</tr>
<tr>
<td>$E_{eq}$</td>
<td>-0.42±j0.765</td>
<td>-0.42±j0.765</td>
</tr>
<tr>
<td>$R_f$</td>
<td>-0.41±j0.534</td>
<td>-0.41±j0.534</td>
</tr>
<tr>
<td>$V_R$</td>
<td>2.8571</td>
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</table>
### TABLE 4.7
LOADING AT BUS-8
(CONST. P)

<table>
<thead>
<tr>
<th>$P$</th>
<th>$E'_{d}$</th>
<th>$E'_{f}$</th>
<th>$\delta$</th>
<th>$\omega$</th>
<th>$V_{f}$</th>
<th>$E'_{q}$</th>
<th>$R_{f}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>-6.2724</td>
<td>-3.7148</td>
<td>-1.07+j12.98</td>
<td>-0.34+j8.43</td>
<td>0.000</td>
<td>-0.42+j1.243</td>
<td>-0.40+j1.243</td>
</tr>
<tr>
<td></td>
<td>-6.1839</td>
<td>-3.6701</td>
<td>-1.1+j12.81</td>
<td>-0.38+j8.34</td>
<td>0.000</td>
<td>-0.409+j1.016</td>
<td>-0.402+j1.013</td>
</tr>
<tr>
<td>2.0</td>
<td>-6.3393</td>
<td>-3.6407</td>
<td>-1.03+j12.44</td>
<td>-0.38+j8.18</td>
<td>0.000</td>
<td>-0.402+j1.065</td>
<td>-0.386+j1.065</td>
</tr>
<tr>
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<td>-6.5676</td>
<td>-3.5110</td>
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<td>-0.29+j7.26</td>
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<tr>
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<td>-0.25+j6.30</td>
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</table>

47
<table>
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<td>-6.3342</td>
<td>-6.4786</td>
<td>-6.6861</td>
</tr>
<tr>
<td>(\delta)</td>
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<td>-1.062±j12.986</td>
<td>-1.079±j12.966</td>
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<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
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<td>(\gamma)</td>
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<td>-2.8571</td>
<td>-2.8571</td>
<td>-2.8571</td>
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<td>----------</td>
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<td>----------</td>
</tr>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$E_{q}'$</td>
<td>-0.467+j1.085</td>
<td>-0.45+j1.971</td>
<td>-0.436+j1.929</td>
<td>-0.439+j1.926</td>
<td>-0.439+j1.926</td>
<td>-0.439+j1.9179</td>
<td>-0.44+j1.907</td>
</tr>
<tr>
<td></td>
<td>-0.425+j1.7509</td>
<td>-0.432+j6.31</td>
<td>-0.2215</td>
<td>-0.2198</td>
<td>-0.2184</td>
<td>-0.2167</td>
<td>-0.2143</td>
</tr>
<tr>
<td></td>
<td>-0.414+j5.309</td>
<td>-0.1502</td>
<td>-0.1287</td>
<td>-0.1256</td>
<td>-0.1206</td>
<td>-0.1119</td>
<td>-0.0839</td>
</tr>
<tr>
<td></td>
<td>2.8571</td>
<td>-2.8571</td>
<td>-2.8571</td>
<td>-2.8571</td>
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</tr>
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<td></td>
<td>$P = 1.5$</td>
<td>$P = 2.0$</td>
<td>$P = 2.2$</td>
<td>$P = 2.4$</td>
<td>$P = 2.5$</td>
<td>$P = 2.6$</td>
<td>$P = 2.7$</td>
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<td>-----------</td>
</tr>
<tr>
<td>$E_{\theta}^d$</td>
<td>-5.14±j7.808</td>
<td>-5.15±j7.85</td>
<td>-5.15±j7.84</td>
<td>-5.15±j7.84</td>
<td>-5.15±j7.84</td>
<td>-5.15±j7.84</td>
<td>-5.15±j7.85</td>
</tr>
<tr>
<td>$E_{\theta}^q$</td>
<td>-0.46±j1.087</td>
<td>-0.436±j1.945</td>
<td>-0.439±j1.9276</td>
<td>-0.439±j1.9276</td>
<td>-0.439±j1.9276</td>
<td>-0.439±j1.9276</td>
<td>-0.44±j1.904</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-1.04±j1.301</td>
<td>-1.06±j1.252</td>
<td>-0.987±j1.125</td>
<td>-0.866±j1.1196</td>
<td>-0.765±j1.11478</td>
<td>-0.64±j1.1081</td>
<td>-0.64±j1.1081</td>
</tr>
<tr>
<td>$\omega$</td>
<td>-0.25±j8.38</td>
<td>-0.255±j8.244</td>
<td>-0.254±j8.2017</td>
<td>-0.241±j8.017</td>
<td>-0.235±j7.84</td>
<td>-0.24±j7.43</td>
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<tr>
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<td>0.000</td>
<td>0.000</td>
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</tr>
<tr>
<td>$\omega$</td>
<td>-0.1979</td>
<td>-0.185</td>
<td>-0.1771</td>
<td>-0.1641</td>
<td>-0.152</td>
<td>-0.1422</td>
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</tr>
<tr>
<td>$\omega$</td>
<td>-0.1979</td>
<td>-0.185</td>
<td>-0.1771</td>
<td>-0.1641</td>
<td>-0.152</td>
<td>-0.1422</td>
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<tr>
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<td>-0.185</td>
<td>-0.1771</td>
<td>-0.1641</td>
<td>-0.152</td>
<td>-0.1422</td>
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<tr>
<td>$\omega$</td>
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<td>-0.185</td>
<td>-0.1771</td>
<td>-0.1641</td>
<td>-0.152</td>
<td>-0.1422</td>
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<td>-0.1771</td>
<td>-0.1641</td>
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<td>-0.1422</td>
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<td>-0.152</td>
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<tr>
<td>$\omega$</td>
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<td>-0.185</td>
<td>-0.1771</td>
<td>-0.1641</td>
<td>-0.152</td>
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<tr>
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<td>-0.1641</td>
<td>-0.152</td>
<td>-0.1422</td>
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</tr>
<tr>
<td>$\omega$</td>
<td>-0.1979</td>
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<td>-0.1771</td>
<td>-0.1641</td>
<td>-0.152</td>
<td>-0.1422</td>
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<tr>
<td>$\omega$</td>
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<td>-0.1641</td>
<td>-0.152</td>
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<tr>
<td>$\omega$</td>
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<td>-0.1641</td>
<td>-0.152</td>
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<tr>
<td>$\omega$</td>
<td>-0.1979</td>
<td>-0.185</td>
<td>-0.1771</td>
<td>-0.1641</td>
<td>-0.152</td>
<td>-0.1422</td>
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<tr>
<td>$\omega$</td>
<td>-0.1979</td>
<td>-0.185</td>
<td>-0.1771</td>
<td>-0.1641</td>
<td>-0.152</td>
<td>-0.1422</td>
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<tr>
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<td>-0.1641</td>
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<tr>
<td>$\omega$</td>
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<tr>
<td>$\omega$</td>
<td>-0.1979</td>
<td>-0.185</td>
<td>-0.1771</td>
<td>-0.1641</td>
<td>-0.152</td>
<td>-0.1422</td>
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</tr>
<tr>
<td>$\omega$</td>
<td>-0.1979</td>
<td>-0.185</td>
<td>-0.1771</td>
<td>-0.1641</td>
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</tr>
<tr>
<td>$\omega$</td>
<td>-0.1979</td>
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<td>-0.1771</td>
<td>-0.1641</td>
<td>-0.152</td>
<td>-0.1422</td>
<td>-0.1422</td>
</tr>
</tbody>
</table>
shown in Figure 4.5 and 4.6 for the case of constant power and constant impedance loads, respectively.

4.2. Eigenvalue Analysis

In this section, an effort is made to establish a correlation between the eigenvalues and their respective modes, namely \( E_d', E_q', \delta, \omega, E_{fd}, V_R, R_F \). The time constant \( T_{qo} \) above governs the eigenvalue corresponding to the variable \( E_d' \) and the real eigenvalues of Table 4.4, rows (1), (12), and (13) determine the modes corresponding to \( E_d' \) of the three machines. In order to determine the eigenvalues corresponding to '\( \delta \)' and '\( \omega \)' of each machine, the following procedure was followed.

Equations (2.5) and (2.6) were written for each machine. They were then linearized to get it into the form

\[
\begin{bmatrix}
\Delta \delta_i \\
\Delta \omega_i 
\end{bmatrix} = [A_1] \begin{bmatrix}
\Delta \delta_i \\
\Delta \omega_i 
\end{bmatrix}
\]  

(4.1)

The eigenvalues of \( A_1 \) were computed and it was found that since \( A_1 \) is singular, there is a zero eigenvalue which corresponds to the center of angle (\( \delta_{COA} \)) of the system. All the machines have a damping factor of about 0.2 and the real eigenvalue of -0.1989 corresponding to the center of speed (\( \omega_{COA} \)) mode of the system. Further, the matrix \([A_1]\) had two pairs of eigenvalues of the form -0.99 \( \pm j13.001 \) and -0.21 \( \pm j8.35 \), and hence, one can conclude that the values in rows (2) - (5) correspond to the variable \( \delta \) and \( \omega \) of the machines. Referring to Appendix 1 it can be seen that \( E_{fd}, V_R \) are the fast variables while \( E_q', R_F \) are slow. Out of the remaining 6 pairs the eigenvalues in rows (6) - (11) correspond to the \( E_{fd} \) and \( V_R \) variables of the three machines while rows (14) - (19) correspond to the \( E_q', R_F \) variables of the three machines.
Figure 4.5. Plot of critical eigenvalue versus load (constant $P$).

Figure 4.6. Plot of critical eigenvalue versus load (constant $z$).
From Tables (4.5) - (4.10) it is seen that as the load is varied at each of the load buses under constant power loads, the modes associated with the direct axis flux linkage \( E'_q \) tends to become unstable. The limit on the excitation of each of the generators is simulated as follows: There is an upper and lower limit for the reactive power output for each machine. At each iteration of the load flow the value of 'Q' for each machine is checked and when any of them hits the prescribed limit the load flow program is modified by converting the PV bus which has hit the limit into a PQ bus. Further, as soon as any of the generators reach their respective reactive power supply limit the output of the amplifier of that exciter is made constant by setting the time constant \( T_A \) to a very large value.
5. CONCLUSIONS

The purpose of this thesis was twofold:

(i) To develop a systematic method of constructing the 'A' matrix including nonlinear load representation.

(ii) Using this model to correlate dynamic stability with the voltage collapse phenomena.

In the literature [9], [15], the 'A' matrix has been constructed; however, the proposed method differs from [9] in two respects.

(a) In our model an extra damper winding along the q-axis has been added.

(b) The method of eliminating the load model is done in two steps, thus making it computationally more advantageous.

Although the induction motor load has not been considered, it would constitute a minor extension of the present work.

As pointed out in the introduction, the voltage collapse has been treated essentially as a static phenomenon so far. In the recent reference [6] it is a qualitative way of explaining voltage collapse in terms of dynamics of the system. The key aspect of the dynamical instability is recognized as the progressive mismatch in reactive power reserves. The approach taken in our work is to use the linearized model to find the eigenvalues as the system gets loaded. To simplify the concept, the ULTC transformers and switching of the lines have not been modeled. The effects of progressive system loading on both the voltage profile and the system eigenvalues are examined. Since eigenvalues are identified with a particular part of the generating unit, it has been possible to trace the movement of the eigenvalues as a function of system
loading. It is concluded that when voltage instability occurs, the eigenvalues associated with the flux decay part of the generators move into the right half plane.

This happens in the vicinity of the knee of the P versus V curve. It is also observed that the reactive power losses increase at a much faster rate than real power losses. This is in agreement with the reference [19] describing the French power failure.

Further research is required to incorporate ULTC transformers and induction motor loads. To counteract the voltage instability phenomenon it is necessary to incorporate switching which will change the elements of the 'A' matrix and also incorporate a scheme for load shedding.

With this insight it should be possible to develop effective procedures for automatic system protection which will allow considerable enhancement of system security.
APPENDIX 1.

FAST AND SLOW VARIABLE ANALYSIS OF THE EXCITATION SYSTEM

The IEEE Type-1 exciter model neglecting saturation is represented by the following three equations [12], [13]

\[ T_E \dot{E}_{fd} = -K_E E_{fd} + V_R \]  \hspace{1cm} (A.1)

\[ T_A \dot{V}_R = -V_R - K_A V_F \]  \hspace{1cm} (A.2)

\[ T_F \dot{V}_F = -V_F + \frac{K_F}{T_E} V_R - \frac{K_F K_E}{T_E} E_{fd} \]  \hspace{1cm} (A.3)

The field winding dynamics is given by

\[ T_{do} \dot{E}_q = -E_q + E_{fd} \]  \hspace{1cm} (A.4)

Usually, \( T_{do} \) is big and hence \( E_q \) is a slow variable. Since \( K_A \) is very big and \( T_A \) is small \( V_R \) is a fast variable. Let

\[ K_A = 1/\varepsilon^2 \]

The fast \( V_R \) input to \( V_F \) and \( E_{fd} \) make them have fast components also. Equations (A.1)-(A.4) are modified to

\[ \dot{E}_{fd} = -\frac{K_E}{T_E} E_{fd} + \frac{1}{T_E} V_R \]  \hspace{1cm} (A.5)

\[ \varepsilon^2 \dot{V}_R = -\frac{\varepsilon^2}{T_A} V_R - \frac{1}{T_A} V_F \]  \hspace{1cm} (A.6)

\[ \dot{V}_F = -\frac{1}{T_F} V_F + \frac{K_F}{T_E T_F} V_R - \frac{K_E K_F}{T_E T_F} E_{fd} \]  \hspace{1cm} (A.7)

\[ \dot{E}_q = -\frac{1}{T_{do}} E_q + \frac{1}{T_{do}} E_{fd} \]  \hspace{1cm} (A.8)
Define \( V_R' = \varepsilon V_R \) so that

\[
\dot{E}_{fd}' = \frac{-K_E}{T_E} E_{fd}' + \frac{1}{T_E \varepsilon} V_R'
\]  
(A.9)

\[
\varepsilon V_R' = \frac{-\varepsilon}{T_A} V_R' - \frac{1}{T_A} V_F
\]  
(A.10)

\[
\dot{V}_F' = \frac{-1}{T_F} V_F' + \frac{K_F}{T_E T_F} \varepsilon V_R' - \frac{K_E K_F}{T_E T_F} E_{fd}'
\]  
(A.11)

\[
\dot{E}_q' = \frac{-1}{T} E_q' + \frac{1}{T} E_{fd}'
\]  
(A.12)

Multiplying (A.9) and (A.11) by \( \varepsilon \) throughout

\[
\dot{E}_q' = \frac{-1}{T} E_q' + \frac{1}{T} E_{fd}'
\]  
(A.13)

\[
\varepsilon V_F' = \frac{-\varepsilon}{T_F} V_F' + \frac{K_F}{T_E T_F} \varepsilon V_R' - \frac{K_E K_F}{T_E T_F} E_{fd}'
\]  
(A.14)

\[
\varepsilon E_{fd}' = \frac{-K_E}{T_E} \varepsilon E_{fd}' + 1/T_E V_R'
\]  
(A.15)

\[
\varepsilon V_R' = \frac{-\varepsilon}{T_A} V_R' - \frac{1}{T_A} V_F
\]  
(A.16)

The above system appears as if it has one slow and three fast variables, but the 3 × 3 determinant of the fast subsystem is '0'. Since (A.14) and (A.15) have \( \varepsilon \) everywhere except \( V_R' \), \( V_R' \) is eliminated from one to get a slow variable. Multiplying (A.15) by \( \frac{K_F}{T_F} \) and subtracting (A.14) \( V_R' \) is eliminated from one of them.

Define \( R_f = \frac{K_F}{T_F} E_{fd}' - V_F \)
The set of equations get modified to

\[ \dot{E}'_q = -\frac{1}{T_{do}} E'_q + \frac{1}{T_{do}} E_{fd} \]  
\[ \dot{R}'_F = \frac{-K_E K_F}{T_{EF} T_{TF}} E_{fd} + \frac{K_F}{T_{EF} T_{TF}} \varepsilon V'_R + \frac{1}{T_F} V_F \frac{-K_F}{T_{EF} T_{TF}} \varepsilon V'_R + \frac{K_E K_F}{T_{EF} T_{TF}} E_{fd} \]  
\[ \dot{E}_{fd} = \frac{-K_E \varepsilon}{T_E} E_{fd} + \frac{1}{T_E} V'_R \]  
\[ \dot{V}'_R = \frac{-\varepsilon}{T_A} V'_R + \frac{1}{T_A} V'_F - \frac{K_F}{T_{TF} T_A} E_{fd} \]

(A.17)  
(A.18)  
(A.19)  
(A.20)

The fast subsystem has a nonzero determinant and hence the change of variables introduced is all right.

In the slow \( t \) time scale

\[ 0 = \frac{1}{T_E} V'_R + 0 E_{fd} \]  
\[ 0 = \frac{1}{T_A} R_F \frac{-K_F}{T_{TF} T_A} E_{fd} \]  

(A.21)  
(A.22)

which gives

\[ V'_R = 0, E_{fd} = \frac{T_F}{K_F} R_F \]  

(A.23)

and

\[ \dot{E}'_q = -\frac{1}{T_{do}} E'_q + \frac{1}{T_{do}} E_{fd} \]  
\[ \dot{R}'_F = \frac{-1}{T_F} R_F + \frac{K_F}{T_F^2} E_{fd} \]  

(A.24)  
(A.25)
The reduced order slow model is

\[
\begin{align*}
\dot{R}_F &= 0 \\
\ddot{E}_q' &= -\frac{1}{T_{do}} E'_q + \frac{T_F}{K_F T_{do}} R_F 
\end{align*}
\] (A.26)

In the fast time scale \( \tau = t/\varepsilon \)

\[
\begin{align*}
\frac{dE_{fd}}{d\tau} &= \frac{1}{T_E} V_R' \\
\frac{dV_R'}{d\tau} &= \frac{1}{T_A} R_F - \frac{K_F}{T_A T_F} E_{fd}
\end{align*}
\] (A.27) (A.28)

which gives

\[
\lambda_\tau = \pm j \sqrt{\frac{K_F}{T_A T_E T_F}}
\]

\[
\lambda_\tau = \pm j \sqrt{\frac{K_A K_F}{T_A T_E T_F}}
\]

So, though \( V_F \) is not a pure slow variable \( R_F \) is a slow variable. With this change of variable, the equations describing the excitation system are

\[
\begin{align*}
T_E \dot{E}_{fd} &= -K_E E_{fd} + V_R \\
T_A \dot{V}_R &= -V_R + K_A R_F - \frac{K_A K_F}{T_F} E_{fd} \\
T_F \dot{R}_F &= -R_F + \frac{K_F}{T_F} E_{fd}
\end{align*}
\] (A.29) (A.30) (A.31)
APPENDIX 2.

PROGRAM LISTINGS

PROGRAM TEST(INMC, DATAG, OTAG, RESULT, DAMP, TAPE7=INMC,
CTAPE9=OTAG, TAPE8=RESULT, TAPE2=DAMP, TAPE5=DATAG)

REAL A(21, 21), IN(6, 6), F(6, 6), VD(9), VQ(9), VMO(3), VMQ(3),
AY(9), AI(40, 3), AIMG(3), X(3, 6), ED(3), EQ(3), EFD(3),
REAL Y(3, 12), XD(3), XQ(3), XDT(3), XGT(3), AKE(3), AKA(3), SE(3),
REAL Y3(12, 12), YV(12, 12), D(12, 12), YA(12, 12), YAM(12, 12), YMI(12, 12), TDQ(3),
REAL ALM, YSM(12, 6), YVM(12, 6), YA(12, 6), YA(12, 6), QA(12, 6), CA(3, 6),
REAL LQO(3, 3), YAI(6, 6), CA(3, 6), C(3, 3), D(3, 3), A(3, 3), C(3, 3),
REAL LQRC(3, 3), LQ(3, 3), CL(3, 3), CR(3, 3), E(3, 3), G(3, 3), A(3, 3), C(3, 3),
REAL LQDC(3, 3), LQDC(3, 3), CR(3, 3), E(3, 3), G(3, 3), A(3, 3), C(3, 3),
REAL G1(6, 6), G2(3, 3), AI(6, 6), A2(6, 3), A3(3, 6), AA(3, 3), R(9),
REAL WR(21), WI(21), Wi(21), Z(W, 21, 21), Y(21, 21), DMP(3),
INTEGER IV(21), KP(9), KU(9), MQ(9),
COMPLEX ET(9), VT(9), S(9), VTC(9), AI(9), YBUS(9, 9), ZKL
REAL THE(9), VMAG(9), PO(9), QU(9), T1(3, 3)

PI = 4 * (ATN(1))

READ(*, 500) MACH, LOAD

500 FORMAT(1X, 12, 12)

DO 502 I = 1, MACH

502 FORMAT(1X, 504) XQ(I), XD(I), XDT(I), XGT(I), TDQ(I), TDO(I), TDO(I), AH(I)

504 FORMAT(1X, 21, 21)

DO 501 I = 1, MACH

501 FORMAT(1X, 7, 7) AKE(I), SE(I), AKE(I), AKF(I), TE(I), TF(I), TA(I)

505 FORMAT(1X, 7, 7) TF(I), TA(I)

C READ INPUT DATA TO NRLF

650 FORMAT(18A10)

READ(5, 650) TITLE

651 FORMAT(18A10)

DO 652 I = 1, NN

652 FORMAT(1X, 5, 5) IDUM, DUM1, DUM2, PINJ(I), WINJ(I)

653 FORMAT(I4, 4E10.4)

CONTINUE

654 FORMAT(1X, 5, 5) IDUM, DUM1, DUM2, DUM3, DUM4, DUM5, DUM6

CONTINUE

655 FORMAT(1X, 4E10.4)

IF(NPV.EQ.0) GO TO 650

CONTINUE

656 FORMAT(3, 3E10.4)

CONTINUE

660 CONTINUE

DO 660 I = 1, NN

660 FORMAT(3E10.4)

YBUS(I, J) = (0, 0, 0, 0)

DO 620 I = 1, NNR

READ(5, 661) IDUM, K, L, ZKL, BKL

661 FORMAT(1X, 3E10.4)

BKL = BKL / 2

YBUS(K, K) = YBUS(K, K) + (1, 0, 0, 0) + CMPLX(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)

CONTINUE

C READ LOAD FLOW SOLUTION

9.793 I = 1, NN

READ(6, 710) S(I), VMAG(I), THE(I), MQ(I)

710 FORMAT(1X, 4E16.9, I1)

IF(MQ(I).EQ.0) TA(I) = 1, 0, 0, 12

720 CONTINUE

CONTINUE

720 CONTINUE

READ(7, 622) KP(I), KG(I)

622 FORMAT(1X, 4I2, 1X, I2)

PO(I) = PINJ(I) / VMAG(I) ** KP(I)

QO(I) = QNF(I) / VMAG(I) ** KP(I)

CONTINUE

CONTINUE

673 CONTINUE

N1 = 2, NMAK

674 LN = 2, NN

N2 = 1, 1

L2 = 1, LOAD

N4 = 0, NMAK

N8 = 1, 1

NCA = NMAK + 1
NE=NA+NMACH+1
N2=2*N1+1
N7=NE+NMACH-1
N8=NE+7+1
N9=NB+NMACH-1
NA=NA+9+1
NF=NA+NMACH-1
DO 70 I=1,N1
    DO 70 J=1,N1
    C1(I,J)=0.
    C3(I,J)=0.
    D1(I,J)=0.
    D3(I,J)=0.
    T1(I,J)=0.
    A1(I,J)=0.
    CONTINUE
    DO 71 I=1,NMACH
    DO 71 J=1,N1
        B1(I,J)=0.
        B3(I,J)=0.
        E1(I,J)=0.
        E3(I,J)=0.
        A3(I,J)=0.
    CONTINUE
    DO 72 I=1,N1
    DO 72 J=1,NMACH
        C2(I,J)=0.
        C4(I,J)=0.
        A2(I,J)=0.
        A4(I,J)=0.
    CONTINUE
    DO 73 I=1,NMACH
    DO 73 J=1,NMACH
        D2(I,J)=0.
        E2(I,J)=0.
        A4(I,J)=0.
        C4(I,J)=0.
    CONTINUE
    DO 403 I=1,NN
        VT(I)=COMPLEX(VTMAG(I)*COS(THE(I)), VTMAG(I)*SIN(THE(I))
    CONTINUE
    DO 405 I=NC,NN
        AI(I)=COMPLEX(PINJ(I),=QINJ(I))/CONJG(VT(I))
    CONTINUE
    DO 402 I=1,NMACH
        AT(I)=CONJG(S(I))/CONJG(VT(I))
    ET(I)=VT(I)*AI(I)*COMPLEX(0.,0.,XU(I))
    P(I)=ATAN(ATMAG(ET(I)))/REAL(ET(I))
    CONTINUE
    DO 401 I=1,NN
        AO(I)=REAL(AI(I))
        AO(I)=AIMAG(AI(I))
    CONTINUE
    DO 39 I=1,N1
        DO 39 J=1,N1
            R(I,J)=SIN(R(K))
            T(I,J)=COS(R(K))
            T(K,J)=T(I,J)
            K=K+1
        CONTINUE
    DO 30 I=1,N1
        DO 30 J=1,N1
            T(I,J)=T(K,J)
        CONTINUE
    DO 531 I=1,NN
        VT(I)=REAL(VT(I))
        VT(I)=AIMAG(VT(I))
        CONTINUE
DO 3 I=1,NMACH
  VMQ(I)=VMQ(I)*SIN(R(I))-VDO(I)*COS(R(I))
  VMQ(I)=VMQ(I)*COS(R(I))+VDO(I)*SIN(R(I))
3 CONTINUE
DO 4 I=1,NMACH
  AIOM(I)=AIOM(I)*SIN(R(I))+AIQ(I)*COS(R(I))
  AIOM(I)=AIOM(I)*SIN(R(I))+AIQ(I)*COS(R(I))
4 CONTINUE
DO 6 I=1,NMACH
  ENQ(I)=ENQ(I)*AIOM(I)*XDT(I)
  EMD(I)=EMD(I)*AIOM(I)*XQT(I)
  EMD(I)=EMD(I)*AIOM(I)*(XQ(I)-XQT(I))*AIMD(I)
6 CONTINUE
DO 5 J=1,N1
  E(I,J)=0
  F(I,J)=0
  G(I,J)=0
5 CONTINUE
DO 31 I=1,NMACH
  J=2*I-1
  K=J+1
  E(I,J)=AIMQ(I)
  E(I,K)=AIMQ(I)
  F(I,J)=EMQ(I)
  F(I,K)=EMQ(I)
  G(I,J)=COS(THE(I))
  G(I,K)=SIN(THE(I))
31 CONTINUE
DO 19 I=1,NMACH
  R(I,J)=F(I,J)
19 CONTINUE
DO 8 I=1,NMACH
  EN(I)=EN(I)*SIN(R(I))*EAMQ(I)*COS(R(I))
  EN(I)=EN(I)*COS(R(I))*EAMQ(I)*SIN(K(I))
8 CONTINUE
DO 9 I=1,N1
  DO 8 J=1,NMACH
9 CONTINUE
K=1
DO 32 I=1,N2,2
  J=I+1
  P(I,K)=EAM(K)
  P(I,J)=E(I,J)
K=K+1
32 CONTINUE
DO 14 I=1,N1
  DO 14 J=1,N1
Q(I,J)=0
14 CONTINUE
K=1
DO 33 I=1,N2,2
  J=I+1
  Q(I,J)=XDT(K)
  Q(J,I)=W(I,J)
K=K+1
33 CONTINUE
DO 16 J=1,N1
  DO 16 I=1,NMACH
16 CONTINUE
J=1
DO 34 I=1,N2,2
  K=I+1
  F(I,J)=AIMQ(I)
  F(K,J)=AIMQ(J)
J=J+1
34 CONTINUE
34 CONTINUE
DO 37 I=1,N1
DO 37 J=1,N1
GI(I,J)=0.
37 CONTINUE
K=1
DO 38 I=1,N2,2
J=I+1
GI(I,J)=((U(K)-XO(K))/TD(K))
GI(J,I)=((V(K)-YD(K))/TD(K))
K=K+1
38 CONTINUE
DO 17 I=1,NMACH
DO 17 J=1,NMACH
GI(I,J)=0.
17 CONTINUE
G2(I,I)=2.*PI*6*.6/(2.*AH(I))
36 CONTINUE
DO 47 I=1,N3
DO 47 J=1,N3
AI(I,J)=0.
47 CONTINUE
K=1
DO 39 I=1,N2,2
J=I+1
AI(I,J)=1./TD(I)
AI(J,I)=1./TD(J)
39 CONTINUE
A(2,13)=1./TD(1)
A(2,15)=1./TD(2)
A(4,15)=1./TD(3)
DO 49 I=4,N5
J=I+NMACH
AI(I,J)=1.
49 CONTINUE
DO 50 I=1,L2
DO 50 J=1,L2
DI(I,J)=0
Y(I,J)=0
YAM(I,J)=0.
50 CONTINUE
DO 54 I=1,LOAD
K=1+NMACH
I=I+1
DI(I,1)=VD(K)*AID(K)+PO(K)*KP(K)*VD(K)*2*VTMG(AK)**(KP(K)-2)+
1*K(K)*KD(K)*KD(K)+VD(K)*VTMG(AK)**(KV(K)-2)+
1*KV(K)*KD(K)*KD(K)+VD(K)*VTMG(AK)**(KV(K)-2).
D(I,1)=VD(K)*AVD(K)+PO(K)*KP(K)*VD(K)*AVD(K)*VTMG(AK)**(KP(K)-2)+
1*KP(K)*KD(K)*KD(K)+VD(K)*VTMG(AK)**(KV(K)-2).
D(I,J)=VD(K)*AVD(K)+PO(K)*KP(K)*VD(K)*AVD(K)*VTMG(AK)**(KP(K)-2)+
1*KP(K)*KD(K)*KD(K)+VD(K)*VTMG(AK)**(KV(K)-2).
D(J,1)=VD(K)*AVD(K)+PO(K)*KP(K)*VD(K)*AVD(K)*VTMG(AK)**(KP(K)-2)+
1*KP(K)*KD(K)*KD(K)+VD(K)*VTMG(AK)**(KV(K)-2).
D(J,1)=VD(K)*AVD(K)+PO(K)*KP(K)*VD(K)*AVD(K)*VTMG(AK)**(KV(K)-2)+
1*KP(K)*KD(K)*KD(K)+VD(K)*VTMG(AK)**(KV(K)-2).
D(J,J)=VD(K)*AVD(K)+PO(K)*KP(K)*VD(K)*AVD(K)*VTMG(AK)**(KP(K)-2)+
1*KP(K)*KD(K)*KD(K)+VD(K)*VTMG(AK)**(KV(K)-2).
54 CONTINUE
DO 79 I=1,LN
DO 79 J=1,LN
Y(I,J)=3.
79 CONTINUE
DO 75 I=1,N1
Y(I,J)=0.
75 CONTINUE
Y(AI(I,J))=0.
75 YAI(I,J)=0.
YAI(I,J)=0.
Y1M(I,J)=0.
75 CONTINUE
DO 76 I = 1, N1
76 CONTINUE
DO 77 J = 1, L2
Y3(I, J) = 0
77 CONTINUE
DO 78 I = 1, L2
78 CONTINUE
DO 510 I = 1, NN
DO 510 J = 1, NN
J1 = 2*I - 1
J2 = 2*J - 1
Y(I, J) = REAL(YBUS(I, J))
K = J + 1
Y(I, K) = AIMAG(YBUS(I, J))
L = I + 1
Y(L, J) = Y(T1, K)
Y(L, K) = Y(I1, J1)
510 CONTINUE
DO 511 I = 1, N1
DO 511 J = 1, L2
Y2(I, J) = Y(I, K)
511 CONTINUE
DO 512 I = 1, L2
DO 512 J = 1, N1
K = I + 1
Y(K, J) = Y(K, K1)
512 CONTINUE
DO 513 I = 1, L2
DO 513 J = 1, N1
513 CONTINUE
CALL SCH0(Y4M, Y4MI, L2)
CALL PROD(Y4MI, Y4M, L2, L2, N1)
CALL PROD(Y2, Y3M, Y3M, L2, N1, N1)
515 CONTINUE
CALL SCH0(YA, YA1, N1, N1, N1)
CALL PROD(YA, YA1, N1, N1, N1, N1)
CALL PROD(YA, YAI, N1, N1, NMACH)
CALL PROD(YA, C2, C2, N1, N1, NMACH)
516 CONTINUE
CALL SCH0(YA, YAI, N1, N1, NMACH)
CALL PROD(YA, YAI, N1, N1, NMACH)
CALL PROD(YA, C2, C2, N1, N1, NMACH)
517 CONTINUE
CALL SCH0(YA, YAI, N1, N1, NMACH)
CALL PROD(YA, YAI, N1, N1, NMACH)
CALL PROD(YA, C2, C2, N1, N1, NMACH)
DO 517 I = 1, N1
DO 517 J = 1, NMACH
C4(I, J) = C4(I, J) + F1(I, J)
517 CONTINUE
```fortran
FORMAT(///)
CALL PROD(B, C3, C5, NMA1C, N1, N1)
CALL PROD(B, C4, D6, NMA1C, N1, N1)
DO 518 I=1, NMA1C
DO 518 J=1, N1
C5(I,J)=C5(I,J)+E(I,J)
CONTINUE
CALL PROD(G, C1, C7, NMA1C, N1, N1)
   I=NMA1C
   CALL PROD(G, C2, C8, NMA1C, N1, N1)
   CALL PROD(G, C3, A1, N1, N1, N1)
   CALL PROD(G, C4, A2, N1, N1, N1)
   CALL PROD(G, D6, A4, NMA1C, NMA1C, N1)
DO 530 I=1, N1
DO 530 J=1, N1
A(I,J)=A(I,J)+A1(I,J)
CONTINUE
DO 530 I=1, N1
DO 519 J=1, NMA1C
K=J+N1
A(I,K)=A2(I,J)
CONTINUE
DO 521 I=1, NMA1C
DO 521 J=1, N1
K=I+N1
A(K,J)=A3(I,J)
CONTINUE
DO 522 I=1, NMA1C
DO 522 J=1, NMA1C
K=I+N1
A(K,K)=A4(I,J)
CONTINUE
DO 50 I=1, NMA1C
   T1(I,I)=A*K*A(I)/TA(I)
CONTINUE
CALL PROD(T1, C7, C9, NMA1C, NMA1C, N1)
CALL PROD(T1, C9, C9, NMA1C, NMA1C, NMA1C)
DO 525 I=1, NMA1C
DO 525 J=1, N1
K=I+N1
A(I,J)=CA(I,J)
CONTINUE
DO 526 I=1, NMA1C
DO 526 J=1, NMA1C
K=I+N1
A(K,K)=CB(I,J)
CONTINUE
DO 527 I=N6, N7
J=I+NMA1C
K=I+NMA1C
A(I,I)=A*K*A(K)+SE(K))/TE(K)
A(I,J)=1./TE(K)
CONTINUE
DO 528 I=N8, N9
K=I+NMA1C
J=I+NMA1C
A(I,I)=1./TA(K)
A(I,J)=A*K*A(K)/TA(K)
A(I,J)=A*K*A(K)/TA(K)*TF(K)
CONTINUE
DO 529 I=NA, NB
K=I+NMA1C
J=I+NMA1C
A(I,J)=1./TF(K)
A(I,J)=(AKF(K))/(TF(K)*2)
```
529 CONTINUE
WRITE(6,101)
101 FORMAT(///, "*********V1**********V2**********V3**********V4**********V5**********
*********V7**********V8**********V9**********EFD1**********EFD2**********EFD3**********", //)
WRITE(6,722) (VTMAG(I),I=1,9), (EFD(I),I=1,3)
722 FORMAT(1X,12(F8.4,1X))
WRITE(6,407)
407 FORMAT(///)
N=N3
NMM=N3
MAT2=0
CALL KG(NM,N,A,W,R,WI,MATZ,Z,IV1,IV,FV1,FV,IERR)
WRITE(6,90) (NR(I),WI(I),I=1,N3)
90 FORMAT(2X,F20.10,F4X,F20.10)
STOP
END
SUBROUTINE PROD(A,B,C,K,L,M)
REAL A(K,L),B(L,M),C(K,M),SUM
DO 10 I=1,K
DO 10 J=1,L
SUM=0.
SUM=SUM+A(I,N)*B(N,J)
10 CONTINUE
C(I,J)=SUM
RETURN
END
SUBROUTINE SCHC(X,O,N)
REAL X(N,N),O(N,N)
DO 10 L=1,N
DO 20 I=1,N
DO 20 J=1,N
IF(I .EQ. J .OR. J .EQ. L) GO TO 20
SUM=SUM+B(I,J)*X(L,J)/X(L,L)
20 CONTINUE
O(L,L)=1./SUM
DO 30 I=1,N
IF(I==L) GOTO 30
30 O(L,I)=X(L,I)*U(L,L)
O(I,L)=X(L,I)*U(L,L)
30 CONTINUE
DO 70 I=1,N
DO 70 J=1,N
70 X(I,J)=O(I,J)
70 CONTINUE
DO 50 I=1,N
DO 50 J=1,N
50 O(I,J)=O(I,J)
RETURN
END
REFERENCES
